MIPRO 2020, September 28 - October 02, 2020, Opatija, Croatia

Prediction of the behavior of a pneumatic soft robot based on Koopman operator theory

E. Kamenar [1, 2], N. Črnjarić-Žic [1], D. Haggerty [3], S. Zelenika [1, 2], E. W. Hawkes [3] and I. Mezić [2, 3]

[1] University of Rijeka, Faculty of Engineering, Vukovarska 58, 51000 Rijeka, Croatia
[2] University of Rijeka, Centre for Micro- and Nanosciences and Technologies, R. Matejčić 2, 51000 Rijeka, Croatia
[3] UC Santa Barbara, Department of Mechanical Engineering, Santa Barbara, CA 93105, USA

ekamenar@riteh.hr

Abstract **– Thanks to their flexibility, soft robotic devices offer critical advantages over rigid robots, allowing adaptation to uncertainties in the environment. As such, soft robots enable various intriguing applications, including human-safe interaction devices, soft active rehabilitation devices, and soft grippers for pick-and-place tasks in industrial environments. In most cases, soft robots use pneumatic actuation to inflate the channels in a compliant material to obtain the movement of the structure. However, due to their flexibility and nonlinear behavior, as well as the compressibility of air, controlled movements of the soft robotic structure are difficult to attain. Obtaining physically-based mathematical models, which would enable the development of suitable control approaches for soft robots, constitutes thus a critical challenge in the field. The aim of this work is, therefore, to predict the movement of a pneumatic soft robot by using a data-driven approach based on the Koopman operator framework. The Koopman operator allows simplifying a nonlinear system by "lifting" its dynamics into a higher dimensional space, where its behavior can be accurately approximated by a linear model, thus allowing a significant reduction of the complexity of the design of the resulting controllers.**

Keywords – soft robots, Koopman operator, nonlinear lifting

I. INTRODUCTION

Owing to their compliant behavior, when compared to traditional rigid (or, as sometimes referred to in literature, hard [1]) robots, soft robots offer important advantages. They are made of highly compliant (soft) materials, which allow them to adapt their shape to environmental constraints and obstacles, making possible their efficient use in cluttered surroundings [2, 3]. Although such devices are currently still mostly employed by researchers in experimental systems, the number of examples of their practical use is increasing. Soft robots are, hence, used in various interesting applications, including human-safe interaction devices and soft grippers for pick-and-place tasks in industrial environments [2–4]. Furthermore, soft robotic devices represent a promising technology in medical applications, where they can be used as actuating (exoskeletal) components in active rehabilitation devices for human motion assistance [5], particularly for stroke patients [2, 4, 6].

Pneumatic actuation with air used as the working fluid, is often employed to inflate the hollow compliant structure, which enables the motion of the end-effector of the soft robot [3]. The high compliance, which represents one of the main advantages of these devices, at the same time makes their dynamical behavior highly nonlinear. In fact, the inherent nonlinear behavior, high dimensionality, as well as the compressibility of air, makes pneumatic soft robots very complex to model. On the other hand, mathematical modelling is necessary to synthetize an appropriate control system that enables the controlled movements of this class of devices. The modeling of soft robotic systems is furthermore an important subject because it shortens the time used for their design and deployment.

The Koopman operator, in turn, allows a linearized representation of nonlinear dynamical systems, exhibiting excellent performances. This mathematical tool is, therefore, increasingly suggested for modelling and control in e.g. biology, power grids, fluid dynamics, traffic or DC motors controls. What is more, the Koopman operator was recently successfully applied to model and control a factual complex positioning system, aimed at nanometric precision positioning, characterized by multiple sources of intricate mechanical nonlinearities induced by friction [7-10].

The aim of this work is, therefore, to provide an advanced tool for modelling the behavior of a pneumatically-driven soft robotic device, available at the premises of the University of California Santa Barbara (UCSB), by employing a data-driven machine learning approach based on Koopman operator theory [7-10]. The developed model creates the preconditions to synthetize a motion controller allowing to attain a precise tracking control of the end-effector of the used soft robotic device.

The remainder of this paper is organized as follows: in Section II a brief introduction to the Koopman operator theory, with application to nonlinear dynamical systems, is provided, followed in Section III by the description of the devised experimental set-up. The modelling procedure based on the experimentally acquired motion data and the thorough analyses of the obtained results are described in Section IV. In Section V, the main conclusions are finally drawn, and an outlook to future work is provided.

Authorized licensed use limited to: Univ of Calif Santa Barbara. Downloaded on August 01,2021 at 19:17:56 UTC from IEEE Xplore. Restrictions apply.

II. Koopman Operator Theory in Systems' Modelling and Control

The Koopman operator theory allows a linearization of highly-nonlinear dynamical systems, exhibiting, in terms of accuracy, performances superior to other linear predictors, such as those based on local linearization or the so-called Carleman linearization [7]. The respective method for calculating linear predictors is relatively simple and, contrary to physically-based models grounded on knowledge of material properties and first-order principles, it is data-driven, i.e., it depends on experimental input-output data only, while it provides a global and smooth view of systems' behavior even for longer prediction intervals [7, 10, 11]. The Koopman operator represents, thus, a machine learning data-driven method that can be used to obtain a state-space representation of the model of the studied device [11, 12]. Its numerical approximations allow "lifting" the nonlinear dynamics of the considered factual device (i.e., of its state-space model) into a higher dimensional space of so-called observables (i.e., simple scalar functions), where the behavior of the considered system can be predicted by a simpler yet accurate linear (though higher dimensional [12]) model [7, 10].

What is more, the nonlinearities pertaining to many physical systems would often imply the need to use nonlinear control approaches that are associated with large computational times, as well as stability and robustness issues, especially in higher-dimensional problems [8]. Koopman operator theory, recently introduced to the control community as well [7, 13], allows, in turn, a significant simplification of the model-based design of the controller and reduces the computational complexity in real-time applications [8]. In other words, the obtained Koopman-based linear predictors can be used to design controllers that can cope with the nonlinear dynamics of the controlled nonlinear system, although they rely on linear control design approaches such as the Model Predictive Controller (MPC), the Linear Quadratic Regulator (LQR), the H-infinity method (H_∞) or similar [7, 10].

In the herein considered case, a suitable modelling of soft robot is particularly important, as it allows developing feedforward predictors that should, as precisely as possible, predict the behavior of the physical system. Feedforward controllers are complemented in this case with a feedback term that compensates for the eventual minor model uncertainties and external disturbances only [11]. If, in turn, the control system would rely mostly on the feedback term, the advantages induced by the compliance of the soft robot would be highly reduced, as it is well-known that the feedback term tends to increase the overall stiffness of the controlled system [14].

Taking into account the stated advantages, Koopman operator theory is applied to an innovative soft robotic device described in the following section.

III. Experimental Set-up

The experiments conducted in the framework of this study are performed on a soft robotic experimental device developed at UCSB and depicted in Fig. 1 [15]. The device consists of a flexible air-tight envelope ("skin") and two "muscles", inflated each via an independent pressure input unit (designated in Fig 1 with "A"). A PhaseSpace Inc. Impulse X2E Motion Capture System (indicated with "B"), consisting of six cameras and eight active LED trackers attached to the characteristic points at the surface of the soft robot (see label "C" in Fig 1), is used to collect motion data. The used motion tracking device enables a sampling frequency of up to ~1 kHz and a sub-millimeter motion resolution [16]. Given its high sampling rate, the same motion capture system will likely be used in the second phase of the work as the displacement feedback for the closed-loop position control of the robot's end-effector.

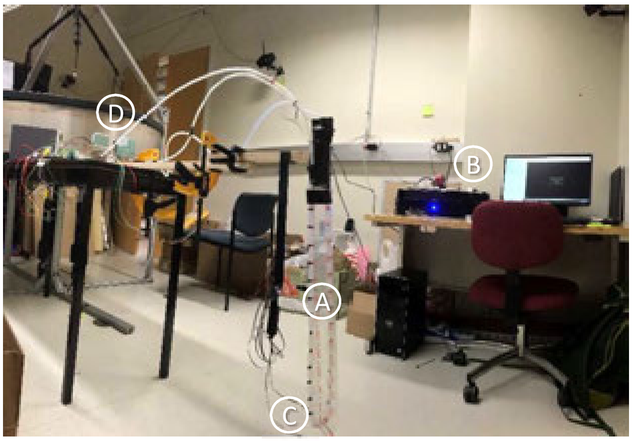

Figure 1. Experimental set-up at UCSB: soft robot (A), motion capture device (B), LED tracker (C) and electromagnetic pressure valves (D)

The herein considered open-loop experiments are conducted by exciting the system via two randomized (white-noise) inputs in the form of PWM voltage signals applied to the proportional electromagnetic valves (indicated in Fig. 1 with "D") that generate the respective pressure output on each of the two "muscles", while the "skin" is inflated to a constant pressure. The same controller utilized for commanding the control signals to the muscles, and recording the real-time pressure in each of them, is also used to trigger the motion capture system and inherit its sampling frequency, thus synchronizing the input-output data. The acquired data consists, then, of time-related sets of *x-y* (planar) coordinates relating to the motion of each of the LED trackers. This information is used next to obtain the corresponding data-driven model of the soft robot by employing Koopman operator theory.

IV. Modelling of the Response of the Soft Robot

As pointed out, the infinite-dimensional Koopman operator is an emerging tool for the advanced modeling of highly-nonlinear dynamical systems. Identifying a nonlinear model by using experimental data implies, in turn, solving non-linear optimization problems based on an extensive knowledge of system's behavior [11, 17].

A. *Application of Koopman Operator Theory to the Soft Robotic Device*

It is supposed here that the discrete-time representation of the used soft robotic dynamical system in the time step t_{i+1} can be defined in the state space as [7, 11]:

$$x_{i+1} = f(x_i, u_i), \qquad (1)$$

Authorized licensed use limited to: Univ of Calif Santa Barbara. Downloaded on August 01,2021 at 19:17:56 UTC from IEEE Xplore. Restrictions apply.

where $x_i = x(t_i) \in X \subseteq \mathbb{R}^n$ is the state of the system, while $u_i \in \mathbb{R}^m$ is the control input in the discrete time step t_i. It is assumed next that, by applying to the state space the vector of lifting functions $\Psi(x) = (\psi_1(x), \dots, \psi_N(x))^T = z \in M \subseteq \mathbb{R}^N$, where $\psi_i: X \to \mathbb{R}$, $i = 1, \dots, N$ are the chosen observable functions, the evolution in the lifting space M has the form:

$$z_{i+1} = \mathbf{A} z_i + \mathbf{B} u_i \qquad x_{i+1} = \mathbf{C} z_{i+1} \tag{2}$$

where $\mathbf{A}$ is system's matrix, $\mathbf{B}$ is the control matrix, $\mathbf{C}$ is the projection operator from the lifting space to the state space, while $z_0 = \Psi(x_0)$.

The aim of the work is, therefore, to predict the motion behavior of the soft robotic device based on the standardized state-space model defined by (2).

The Koopman operator $\mathcal{K}$ associated to the original system is the infinite-dimensional linear operator acting on the generalized observables $\phi: X \to \mathbb{R}$ as $\mathcal{K}\phi(x) = \phi(f(x,u))$. Since the aim here is to obtain the time-domain prediction of the behavior of the studied dynamical system, the Extended Dynamic Mode Decomposition (EDMD) algorithm, which relies mainly on least square regression, is adopted next to construct the finite-dimensional approximations of the Koopman operator [17, 18]. By applying EDMD on the observable functions $\{\psi_1(x), \dots, \psi_N(x)\}$, the finite-dimensional approximation of the compression of the Koopman operator on the linear subspace, spanned by these functions, is hence obtained [17, 18]. Furthermore, with the assumption that in the lifted space the dynamical system evolves according to (2), in the considered problem the representation of this finite dimensional approximation will be of the form [**A**, **B**], and can be obtained as the solution of the minimization problem [7]:

$$\min_{\mathbf{A},\mathbf{B}} \sum_{i=0}^{K-1} \|\Psi(x_{i+1}) - \mathbf{A}\Psi(x_i) - \mathbf{B}u_i\|_2^2 . \tag{3}$$

In the data-driven framework, by denoting $\mathbf{X}_{lift} = [\Psi(x_0), \dots, \Psi(x_{K-1})]$, $\mathbf{Y}_{lift} = [\Psi(x_1), \dots, \Psi(x_K)]$ and $\mathbf{U} = [\mathrm{u}_0, \dots, \mathrm{u}_{K-1}]$, the matrices **A** and **B** can be obtained by finding an analytical solution to (3) by using [7]:

$$[\mathbf{A}, \mathbf{B}] = \mathbf{Y}_{lift}\left[\mathbf{X}_{lift}, \mathbf{U}\right]^{\dagger} \tag{4}$$

where † denotes the Moore-Penrose pseudoinverse of a function. It has to be noted here that matrix **U**, which contains the input control signals, remains non-lifted.

The projection operator relating to the data vector **X** is determined next as [7]:

$$\mathbf{C} = \mathbf{X}\,\mathbf{X}_{lift}^{\dagger}. \tag{5}$$

Based on the obtained model, the p step horizon prediction of the behavior of the system $\hat{z}_{j+p}$, from an initial state x_j can be computed recursively, for $i = 1, \dots, p$, where the initial state of recursion is given by $\hat{z}_j = \Psi(x_j)$, as [7, 11]:

$$\hat{z}_{j+i} = \mathbf{A}\hat{z}_{j+i-1} + \mathbf{B}u_{j+i-1} \qquad \hat{x}_{j+i} = \mathbf{C}\,\hat{z}_{j+i}. \tag{6}$$

B. Results and Discussion

In the considered case the open loop planar motion coordinates x and y of the end-effector of the used soft robot, as measured at discrete time points for determined input pressures (i.e., control inputs u_i) at each of the two muscles, represent the attained data points x_i.

The sampling time in the numerical experiments is, in turn, set to 1 ms. Two sets of lifting functions are then considered in the EDMD algorithm. The first set consist of first-order monomials (r=1), where the lifting space coincides with the original state-space. On the other hand, the second batch of lifting functions is enlarged with second-order monomials (r=2). Taking into account the number of data points, the dimensions of matrices $\mathbf{X}_{lift}$ and $\mathbf{Y}_{lift}$ are $N \times 160\,000$.

The preliminary experimental motion data of a specific LED tracker on the end effector of the studied UCSB soft robotic device, overlaid by the respective learned responses for different prediction horizons, as obtained by applying the above Koopman operator-based approach, are thus compared in Figures 2 – 4. The depicted responses are given for an approximate 1 s time-frame, randomly chosen in the experimental data set. From the presented results, it can be deduced that the modeled responses follow very well the experimental data for a 10 ms prediction step. For larger prediction steps, the delay between the modeled and the experimental responses increases. However, it can be noticed that even for a 30 ms prediction time, the modeled response follows still the

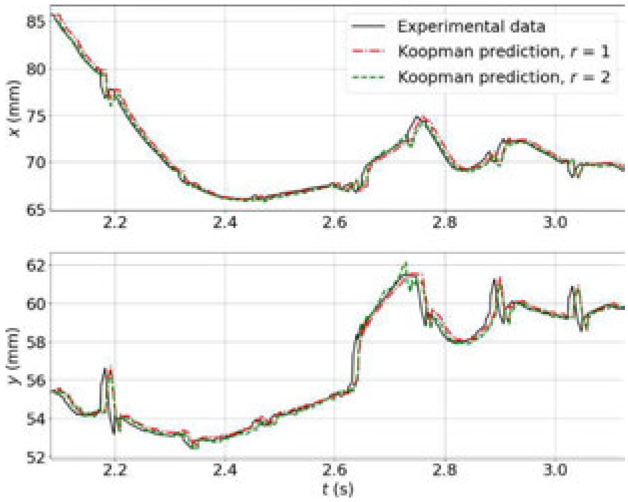

Figure 2. Comparison of the measured and predicted planar motion of the end effector of the soft robot for 10 time steps equaling a 10 ms prediction interval

Authorized licensed use limited to: Univ of Calif Santa Barbara. Downloaded on August 01,2021 at 19:17:56 UTC from IEEE Xplore. Restrictions apply.

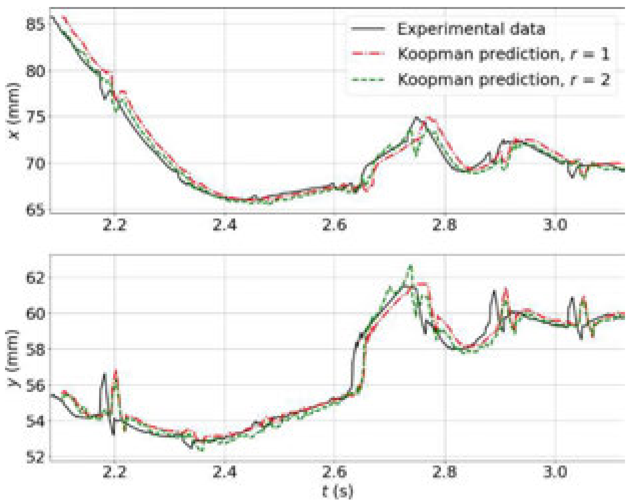

Figure 3. Comparison of the measured and predicted planar motion of the end effector of the soft robot for 20 time steps equaling a 20 ms prediction interval

experimental data with good accuracy. A further mathematical analysis of the goodness of fit of the modeled vs. experimental data is performed next, based on the order of monomials and the length of the prediction horizon, using as the relevant indicator the Mean Absolute Error (MAE) normalized over the mean value of the measured data points (the normalized MAE indicator is hence designated as NMAE):

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N}|x_i - \hat{x}_i|$$
$$\mathrm{NMAE} = \left(\frac{\mathrm{MAE}}{\bar{x}}\right). \tag{7}$$

The calculated NMAE values for the considered experimental set-up are hence reported in Table 1. It can be seen that the deviation between the modeled and the experimental response slightly decreases when higher order monomials are used as the lifting functions while,

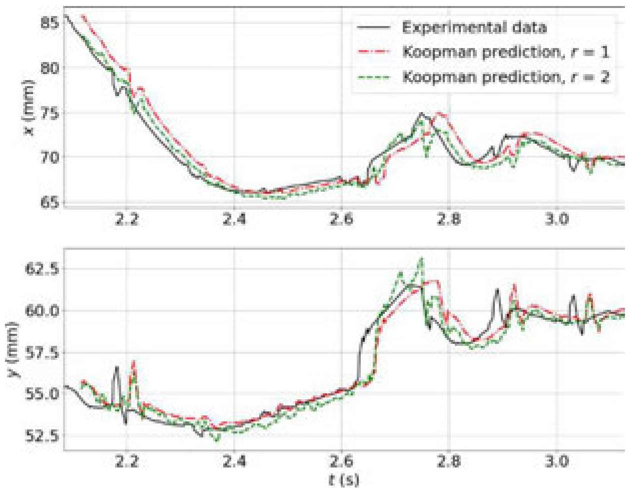

Figure 4. Comparison of the measured and predicted planar motion of the end effector of the soft robot for 30 time steps equaling a 30 ms prediction interval

TABLE I.
CALCULATED NMAE (%) FOR DIFFERENT PREDICTION HORIZONS AND USED LIFTING FUNCTIONS

τ (ms)	r=1		r=2	
	error x (%)	error y (%)	error x (%)	error y (%)
10	1.9	1.55	1.7	1.5
20	3.55	2.84	3.22	2.75
30	5.2	4	4.7	3.94
40	6.86	5.24	6.16	5.16
50	8.43	6.42	7.63	6.38

evidently, the value of the NMAE indicator increases with longer prediction step.

The obtained results represent, therefore, a good basis for the development of controllers that would enable the precise tracking of the end-effector of soft robots.

V. CONCLUSIONS AND OUTLOOK

The experimental identification of the behavior, and the respective Koopman operator-based modelling of an innovative soft robotic experimental device, is performed in this work. The acquisition of experimental motion data is based in this frame on the excitation of the system by random input signals. Based on the attained experimental data, a state-space model of the system is built by employing a finite-dimensional Extended Dynamic Mode Decomposition Koopman operator approximation. The comparison of the experimental responses with those obtained from simulations on the obtained Koopman-based model, for different prediction steps, allows establishing that the modelled responses follow accurately the experimental data.

Based on the proposed modelling approach, in the next phases of the work further experimental tests will be conducted to acquire larger set of experimental data with the goal of extending the accurate prediction window. Furthermore, advanced control typologies, e.g. based on MPC, LQR, H_∞ or similar linear control design methods, will be synthetized with the aim of attaining the precise tracking control of soft robotic devices.

ACKNOWLEDGMENTS

The work performed by E. Kamenar and S. Zelenika is made possible by using the equipment funded via the ERDF project RC.2.2.06-0001 "Development of Research Infrastructure at the University of Rijeka Campus - RISK", and supported by the University of Rijeka grant uniri-tehnic-18-32 "Advanced mechatronics devices for smart technological solutions".

REFERENCES

[1] R. V. Martinez, C. R. Fish, X. Chen, and G. M. Whitesides, "Elastomeric origami: Programmable paper-elastomer composites as pneumatic actuators", *Adv Funct Mater*, vol. 22(7), pp. 1376-84, 2012.

[2] C. Majidi, "Soft robotics: A perspective - Current trends and prospects for the future", *Soft Robot*, vol. 1(1), pp. 5-11, 2014.

Authorized licensed use limited to: Univ of Calif Santa Barbara. Downloaded on August 01,2021 at 19:17:56 UTC from IEEE Xplore. Restrictions apply.

[3] D. Rus, and M.T. Tolley, “Design, fabrication and control of soft robots”, *Nature*, vol. 521(7553), pp. 467-75, 2015.

[4] P. Polygerinos, S. Lyne, Z. Wang, L. F. Nicolini, B. Mosadegh, G. M. Whitesides, and C. J. Walsh, “Towards a soft pneumatic glove for hand rehabilitation”, in *2013 IEEE/RSJ Int. Conf. Intel. Robots & Sys.*, pp. 1512-17, November 2013.

[5] S. Li, D. M. Vogt, D. Rus, and R. J. Wood, “Fluid-driven origami-inspired artificial muscles”, *P Natl Acad Sci USA*, vol. 114(50), pp. 13132-7, 2017.

[6] C. Majidi, “Soft-Matter Engineering for Soft Robotics”, *Adv Mater Technol*, vol. 4(2), pp. 1800477, 2019.

[7] M. Korda, and I. Mezić, “Linear predictors for nonlinear dynamical systems: Koopman operator meets model predictive control”, *Automatica*, vol. 93, pp. 149-60, 2018.

[8] E. Kaiser, J. N. Kutz, and S. L. Brunton, “Data-driven approximations of dynamical systems operators for control”, *arXiv preprint arXiv:1902.10239*, 2019.

[9] I. Abraham, G. De La Torre, and T. D. Murphey, “Model-based control using Koopman operators”, in *Robotics: Science and Systems XIII*, Cambridge, USA, January 2017.

[10] S. Zelenika, E. Kamenar, M. Korda, and I. Mezic, “Application of Koopman-Based Control in Ultrahigh-Precision Positioning,” in *The Koopman Operator in Systems and Control: Concepts, Methodologies and Applications*, vol. 484. A. Mauroy, I. Mezic, Y. Susuki, Eds., Springer Nature, 2020 (in press).

[11] D. Bruder, C. D. Remy, and R. Vasudevan, “Nonlinear system identification of soft robot dynamics using Koopman operator theory”, *arXiv preprint arXiv:1810.06637*, 2018.

[12] D. Bruder, B. Gillespie, C. D. Remy, and R. Vasudevan, “Modeling and control of soft robots using the Koopman operator and model predictive control”, *arXiv preprint arXiv:1902.02827*, 2019.

[13] H. Arbabi, M. Korda, and I. Mezic, “A data-driven Koopman model predictive control framework for nonlinear flows”, *arXiv preprint arXiv:1804.05291*, 2018.

[14] C. Della Santina, M. Bianchi, G. Grioli, F. Angelini, M. Catalano, M. Garabini, and A. Bicchi, “Controlling soft robots: Balancing feedback and feedforward elements”, *IEEE Robot Autom Mag*, vol. 24(3), pp. 75-83, 2017.

[15] J. D. Greer, T. K. Morimoto, A. M. Okamura, and E. W. Hawkes, “Series pneumatic artificial muscles (sPAMs) and application to a soft continuum robot”, in *2017 IEEE International Conference on Robotics and Automation*, pp. 5503-10. May 2017.

[16] “The Impulse X2E Motion Capture System”. Internet: http://phasespace.com/x2e-motion-capture/, 2019.

[17] C. Folkestad, D. Pastor, I. Mezic, R. Mohr, M. Fonoberova, and J. Burdick, “Extended dynamic mode decomposition with learned Koopman eigenfunctions for prediction and control”, *arXiv preprint arXiv:1911.08751*, 2019.

[18] N. Črnjarić-Žic, S. Maćešić and I. Mezić, “Koopman operator spectrum for random dynamical system”, arXiv preprint arXiv:1711.03146, 2019.

Authorized licensed use limited to: Univ of Calif Santa Barbara. Downloaded on August 01,2021 at 19:17:56 UTC from IEEE Xplore. Restrictions apply.