Received: 14 August 2020 | Revised: 11 June 2021 | Accepted: 30 June 2021

DOI: 10.1111/nrm.12324

Natural Resource Modeling WILEY

Risk averse choices of managed beach widths under environmental uncertainty

Di Jin[1] | **Porter Hoagland**[1] | **Andrew D. Ashton**[2]

[1]Marine Policy Center, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

[2]Department of Geology and Geophysics, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

Correspondence
Di Jin, Marine Policy Center, Woods Hole Oceanographic Institution, Woods Hole, MA 20543, USA.
Email: djin@whoi.edu

Funding information
National Science Foundation, Grant/Award Number: ARG 1518503; J. Seward Johnson Fund; National Oceanic and Atmospheric Administration, Grant/Award Number: NA18OAR4170104

Abstract

Applying a theoretical geo-economic approach, we examined key factors affecting decisions about the choice of beach width when eroded coastal beaches are being nourished (i.e., when fill is placed to widen a beach). Within this geo-economic framework, optimal beach width is positively related to its values for hazard protection and recreation and negatively related to nourishment costs and the discount rate. Using a dynamic modeling framework, we investigated the time paths of beach width and nourishment that maximized net present value under an accelerating sea level. We then analyzed how environmental uncertainty about expected future beach width, arising from natural shoreline dynamics, intermittent large storms, or sea-level rise, leads to economic choices favoring narrower beaches. Risk aversion can affect a coastal property owner's choice of beach width in contradictory ways: the expected benefits of hazard protection must be balanced against the expected costs of repeated nourishment actions.

Recommendations for Resource Managers

- Because of environmental uncertainty, coastal protection projects are risky investments. It is important to consider the effects of uncertainty on cost–benefit assessments of these projects.

- Without uncertainty, optimal beach width is positively related to its values for hazard protection and recreation and negatively related to nourishment costs and the discount rate.
- Uncertainty about future shoreline positions has a negative impact on the choice of an optimal beach width.
- Risk aversion can affect a coastal property owner's choice of beach width in contradictory ways: a wider beach to protect their home, and a narrower beach if the effectiveness of nourishment is uncertain.

KEYWORDS
beach nourishment, beach width, coastal protection, risk management, shoreline change

1 | INTRODUCTION

Facing imminent shoreline change, including flooding and erosion due to storms and sea-level rise (SLR), coastal managers and property owners must adapt. A key decision concerns the choice over the extent of protection. For beachfront communities, this choice may involve determining a target beach nourishment volume that would result in a beach wide enough to provide both protection and recreational amenities during a foreseeable period.

In the context of a developed coast, a beach's width is defined as the distance between the seaward limit of developed property and some reference shoreline. Assuming that the seaward extent of developed property limit remains fixed, the shoreline location determines the beach width. Sandy shorelines move naturally over time, varying seasonally in response to storms and nourishments and over longer timescales due to sediment budget imbalances and SLR (Vitousek et al., 2017), but the size of these fluctuations can be uncertain.

As the shoreline moves seasonally between winter and summer and in response to storms, beach width varies around what many models consider to be some mean long-term "equilibrium" position (Splinter et al., 2014; Yates et al., 2009). This seasonal pattern in beach width oscillation typically results from the cross-shore sediment exchange between the supratidal and lower-intertidal part of the beach (Quartel et al., 2008). Using shoreline data from the US East Coast, Zhang et al. (2002) showed that, even after large storms, beaches recovered to positions consistent with their long-term (100+ year) trend, concluding that the gradual recession of beaches along the US East Coast is controlled mainly by SLR and variations of sediment supply.

For different coastal landforms at different locations, shoreline position may be affected by many different factors, including river sand supply to beaches downcoast (A. D. Ashton & Giosan, 2011; Komar, 1973), seacliff erosion contributions to natural sediment supply (Runyan & Griggs, 2003), and sand supply from subtidal sand bank reservoirs to the upper beach flat following major storms (Anthony et al., 2006). The patterns of shoreline changes are influenced also by the grain sizes of beach sand (Frihy & Lotfy, 1997), alongshore dispersion assisted by seasonal cross-shore movements (Hicks & Inman, 1987), the shape of the shoreline (Pelnard-Considéré, 1956; Rosati et al., 2002), wave characteristics (Ashton et al., 2001), and the wind regime (Anthony et al., 2006).

Investigating severe storms of the US Atlantic and Gulf coasts, Morton (2002) found that the primary factors affecting morphological responses include storm characteristics, geographic position relative to a storm path, the timing of storm events, the duration of wave exposures, wind stresses, the degree of flow confinement, the geologic framework, sediment textures, the vegetative cover, and the type and density of coastal development. Large-scale monitoring of beach changes suggests that storm response can be heterogeneous alongshore, with significant rapid accretion of storm-eroded beaches (List et al., 2006). Long-term and large-scale shoreline change tends to accumulate over years-to-decades, with the alongshore signal showing properties of Brownian noise (Lazarus et al., 2011).

Over the past few decades, coastal monitoring programs have been used to facilitate management of coastal stability problems on sandy coastlines, including video-derived shorelines (Kroon et al., 2007; Lippmann & Holman, 1989) that allow the measurement of daily (or more frequent) shoreline erosion and accretion, as well as responses to storms, seasonal cycles, and anthropogenic interventions, such as beach nourishment. Analysis of these shoreline data over decades and longer suggests that shoreline variability occurs over a broad spectral scale, such that the signal of annual and even interannual storm events may be indistinguishable in the shoreline signal (Lazarus et al., 2019). Many models have been developed showing that the prediction of shoreline change remains elusive, and even “successful” model predictions typically require considerable training data and exact knowledge of future driving conditions (Montaño et al., 2020). In summary, the variability and uncertainty of the shoreline position (beach width) arise from a multitude of factors acting across a wide range of temporal scales.

For coastal communities, preferred beach width varies greatly depending on usage patterns and personal preferences (Davey et al., 2014). Coastal property values (residences) are affected by beach width (Landry & Hindsley, 2011; Pompe & Rinehart, 1994, 1999). Beach width also is an important criterion when considering the public's enjoyment of the beach as a recreational resource. Wider beaches may not always be better, but within reason the public tends to prefer wider beaches (King, 2006). According to Parsons et al. (2000), beaches can be too narrow or too wide for recreation. These authors proposed that the ideal beach width for recreational visitors in the US Mid-Atlantic region ranged from 23 to 61 m as measured from the dune toe to the seaward extent of the berm. For Welsh beaches in the United Kingdom, Morgan (1999) investigated preferred beach widths, finding that they should fall between 46 and 183 m at low tide and 18–46 m at high tide. In determining recreational amenity values for beaches in California, King (2006) suggested that the preferred beach widths ranged from 30 to 76 m.

Using simple geo-economic models and optimal control theory, key factors affecting optimal beach width are examined here. Typically, geological models are used to investigate shoreline changes under different natural conditions and anthropogenic interventions (Ashton et al., 2001; Komar, 1973; Lorenzo-Trueba & Ashton, 2014). In contrast, geo-economic models extend geological analyses through the incorporation of assessments of economic welfare, often with the objective of maximizing the present value of net economic benefits through the choice of socially optimal shoreline management strategies.

There is a growing literature focused on geo-economic analyses of coastal property values, beach nourishments, and shoreline changes, particularly in terms of SLR response strategies.

Examples of these models are summarized in Table 1, and many are summarized by recent reviews by Landry (2011) and Gopalakrishnan et al. (2016). All of these studies considered the benefits of beach width for its protection of adjacent physical structures or the maintenance of recreational uses, with repeated nourishment episodes representing management responses to erosion. Although Landry (2011) discussed optimal beach width in a dynamic context, in most of the studies in Table 1, nourishment was typically modeled as a process to restore a chosen maximum cross-shore width (e.g., Lazarus et al., 2011; Smith et al., 2009).

Our analysis is distinguished from prior work by its central focus on the choice of beach width. In particular, we investigate how uncertainty about future shoreline position can influence choices about the nourishment of beaches. More specifically, the customary geo-economic framework is extended by investigating the impacts of future uncertainty on beach width. We maintain the general style of existing geo-economic models, however, and our analysis is theoretical, ignoring many real-world complexities.

As noted above, the long-term trajectory of future shoreline position (and associated beach width) is significantly unpredictable, a factor not taken into account in previous geo-economic studies (Table 1). Although some components of this uncertainty are cyclical, operating on a seasonal or storm-driven basis around slowly changing a mean state (Splinter et al., 2014; Yates et al., 2009), on longer timescales, the lack of understanding of large-scale, long-term coastal behavior and future climate trajectories represents uncertainty at the annual-to-decadal scales of many of these models and on the implementation of management activities themselves. Further, the behavior of human engineering interventions remains poorly measured, leading to higher levels of uncertainty. Investment in coastal protection remains a risky investment. Facing SLR, coastal communities are investing in shoreline protection and climate adaptation. In many cases, the uncertainties about future payoffs of these investments are ignored in cost–benefit analyses, which may lead to incorrect conclusions. In fact, when risk and uncertainty are present, the investment rule should be modified, and a greater revenue stream will be required to justify the same level of investment.

2 | OPTIMAL BEACH WIDTH: A DETERMINISTIC MODEL

To better envision how a managed coastal system responds to a changing environment (in this case an accelerating sea level), a private property owner's (or a single coastal community's) strategy to control erosion by nourishing a fronting beach is modeled.[1] The optimal nourishment strategy for the owner of a beach-front property can be formulated as a simple deterministic optimal control problem (Jin et al., 2013). The owner seeks to maximize the net benefits of nourishment, subject to a certain understanding of proximate shoreline dynamics:

$$\max \int_0^\infty e^{-\delta t}[B(y(t)) - C(s(t))]dt, \tag{1}$$

$$\text{s.t.} \quad \frac{dy}{dt} = -\gamma(t) + s(t), \tag{2}$$

where B and C are the benefits and costs, respectively, of beach nourishment; $y(t)$ is the annual-average beach width in front of the property at time t (the property is located at $y = 0$); s is the rate of beach nourishment; γ is the long-term rate of erosion (shoreline retreat); and δ is the discount rate. Equation (2) is the state equation (equation of motion) describing the rate of

TABLE 1 Geo-economic models of shoreline change

Studies	Model	Economic benefits of beach width	Economic costs	Erosion	Storm surge	Cross- and along-shore	Control variable	Stochastic effects
McNamara and Werner (2008a, 2008b)	Coupled simulation model	Tourism, hotel revenue	Beach nourishment cost, hotel construction, and operating costs	Yes	Yes	Both	Beach nourishment, hotel construction	Yes
Slott et al. (2008)	Coupled simulation model	Home value	Beach nourishment cost	Yes	No	Both	Beach nourishment	No
Smith et al. (2009)	Dynamic optimization model	Home value	Beach nourishment cost	Yes	No	Cross-shore	Beach nourishment interval	No
Gopalakrishnan et al. (2011)	Econometric and simulation models	Home value	Beach nourishment cost	Yes	No	Cross-shore	Beach nourishment interval	No
Landry (2011)	Dynamic optimization model	Home value	Beach nourishment cost	Yes	No	Cross-shore	Beach nourishment	No
Lazarus et al. (2011)	Simulation models	Home value	Beach nourishment cost	Yes	No	Both	Beach nourishment interval	No
McNamara et al. (2011)	Simulation model	Home value	Beach nourishment cost	Yes	No	Both	Beach nourishment	No
McNamara and Keeler (2013)	Simulation model	Home value	Beach nourishment cost	Yes	Yes	Both	Beach nourishment	Yes

(Continues)

TABLE 1 (Continued)

Studies	Model	Economic benefits of beach width	Economic costs	Erosion	Storm surge	Cross- and along-shore	Control variable	Stochastic effects
Jin et al. (2013)	Dynamic optimization model	Home value	Beach nourishment cost	Yes	No	Both	Beach nourishment	No
McNamara et al. (2015)	Dynamic optimization model	Home value	Beach nourishment cost	Yes	Yes	Cross-shore	Beach nourishment interval	Yes

change in the state variable, shoreline position (y), which is affected by erosion (γ) and nourishment (s), where s is the control variable. The conceptual underpinning of this model follows that of Landry (2011). Note that, in practice, nourishment is episodic, and the above continuous formulation may be justified by a coastal protection financing mechanism in which a nourishment property tax rate on nearshore homes contributes to funding periodic nourishment.

The benefits of beach nourishment today will be realized in future years. The objective function (1) is the net present value of all future benefits. As in a financial analysis, the discount rate (δ) is a critical component of the discounted cash flow calculation, and it determines how much a series of future cash flows is worth as a single lump sum value today. The discount rate is affected by both time preference and future economic growth. In cost–benefit analysis, a higher discount rate implies that the decision-maker has taken on a more near-term perspective. In contrast, a lower discount rate would suggest that the decision-maker has adopted a more long-term perspective. The discount rates for public and private analyses often are different. Typically, in evaluating social and environmental programs, a lower discount rate is used, placing a greater value on the welfare of future generations.[2]

Typically, the value attributable to beach width (i.e., the benefit affected by nourishment, B; Slott et al., 2008) can be specified as

$$B(y(t)) = \alpha y(t)^{\beta}, \tag{3}$$

where B is the property's rental value at t, and $\alpha > 0$ and $0 < \beta < 1$ are coefficients. Thus,

$$\frac{\partial B}{\partial y} > 0 \quad \frac{\partial^2 B}{\partial y^2} < 0 \quad \frac{\partial B}{\partial \alpha} > 0 \quad \frac{\partial B}{\partial \beta} > 0. \tag{4}$$

Of the two benefit coefficients, α reflects the implicit price of a property that reflects its structural characteristics, the local neighborhood, and environmental amenities (Jin et al., 2015). β captures the unique effects of beach width on B, including both recreational benefits and the reduced damages from coastal hazards. Without uncertainty about shoreline change, benefits grow with an increasing β as the property owner would prefer to maintain additional beach width for coastal protection (Figure 1).

The costs of beach nourishment can be specified as

$$C(s(t)) = c_f + cs(t), \tag{5}$$

where c_f is a fixed cost, and c is a coefficient for the variable cost. The variable cost of a project is a function of the amount of sand required to build the beach, and the fixed cost of a project includes assessment, permitting, and the hiring of dredges and spreading equipment.

Solving for an optimal rate of nourishment (Jin et al., 2013) requires that:

$$s = \begin{cases} 0 & \text{if } c > \lambda, \\ s^* & \text{if } c = \lambda, \\ s_{\max} & \text{if } c < \lambda, \end{cases} \tag{6}$$

where λ is the shadow value of shoreline position, reflecting the benefits associated with a wider beach. The shadow value of variable y is the adjoint or costate variable for y in the optimal control problem (1) and (2) (i.e., the Lagrange multiplier for constraint on y in the constrained optimization problem), and it shows the incremental change in the value of the objective function (1) from an incremental change in y (Conrad & Clark, 1987). Thus, condition (6) suggests that the property owner should nourish the beach at the maximum feasible level

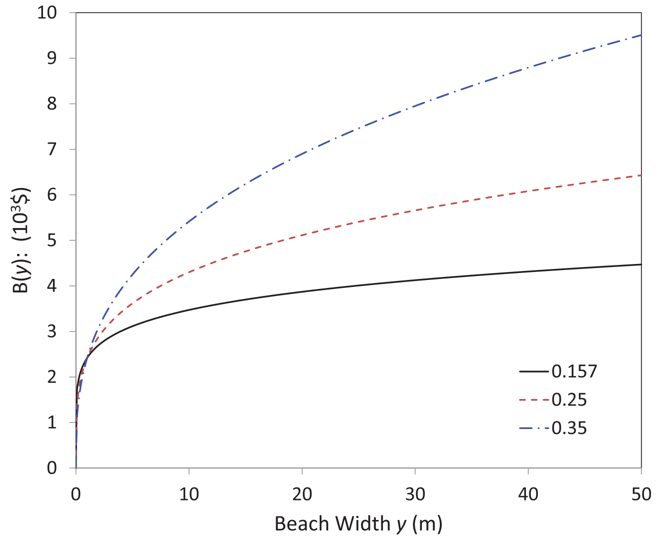

FIGURE 1 Benefit function (B) under different benefit coefficients (β)

when the benefit (i.e., the shadow value) exceeds the cost (c), and do nothing when the cost exceeds the associated benefit. The singular solution represents a long-run optimal level of nourishment (s^*) when the cost (c) equals the benefit (λ). Under the singular solution, the optimal shoreline position can be calculated as

$$y^* = \left(\frac{\delta c}{\alpha\beta}\right)^{\frac{1}{\beta-1}}. \tag{7}$$

Since $\beta < 1$, we have

$$\frac{\partial y^*}{\partial c} < 0 \quad \frac{\partial y^*}{\partial \delta} < 0 \quad \frac{\partial y^*}{\partial \alpha} > 0 \quad \frac{\partial y^*}{\partial \beta} > 0. \tag{8}$$

The optimal shoreline position (y^*) is negatively related to direct and indirect nourishment costs and the discount rate (δ), and positively related to the benefits of beach width (α) and (β). Note that as $c \to \infty$, $y^* \to 0$, and it becomes too costly to nourish the beach.

From (2) and (7), we see that the singular solution implies:

$$\frac{dy}{dt} = 0 \Rightarrow s^* = \gamma \tag{9}$$

so that the beach should be maintained at a width where the nourishment rate is equal to the rate of erosion. Under deterministic conditions, the optimal nourishment strategy is driven by observed shoreline change.

The basic idea of the deterministic model can be demonstrated using numerical simulations and dynamic programming (DP) methods. Rather than modeling a constant rate of SLR, as has been assumed in previous studies, here we assume an accelerating SLR over the future century. To implement an accelerating SLR, we assume a simple Bruun-type response of the shoreline whereby erosion rate scales linearly with the SLR rate (Bruun, 1962). Baseline input parameters for the DP model are summarized in Table 2. We conducted simulations over a planning horizon of 100 years using the baseline parameters. In addition, a sensitivity analysis was

TABLE 2 Baseline input parameters

Parameters	Unit	Value
Geology		
Sea level rise (SLR)	m/year	0.003
Beach slope		0.01
Erosion	m/year	0.3
Rate of increase in SLR	rate/year	0.02
Initial beach width	m	60
Benefit		
Beachfront length for each lot	m	50
Property value	10^6\$/properties	1.5
Benefit coefficient (α)	\$/m/year	2418
Benefit coefficient (β)		0.157
Cost		
Fixed cost of beach nourishment (c_f)	\$/m/year	10
Variable cost of beach nourishment (c_0)	\$/m^3	20
Variable cost per unit length (c)	\$/m	200
Discount rate (δ)	rate/year	0.07

Source: Jin et al. (2013) and Hoagland et al. (2012).

performed with respect to the rate of SLR, unit nourishment cost, and benefit coefficient. In all simulations the terminal condition of the DP problem is specified as objective = 0 (i.e., the discounted shadow value of the resource (beach width) being zero at year = 100). Figures 2 through 4 and Table 3 show the results of these simulations. Note that curves in these plots are not smooth because DP is a discrete implementation of the continuous time-optimal control model in (1) and (2).

Increased erosion leads to increased nourishment and therefore less net benefits for higher annual increasing rate rates of SLR (Figure 2). According to Equation (7), the optimal beach width (y*) is fixed (50 m in the baseline case) and is not a function of increased erosion resulting from accelerating SLR, following the relationship in (9). The annual rate of nourishment (m/year) increases as SLR increases (upper-right panel in Figure 2). Because of these costs, the net benefits of nourishment (net present value [NPV] in Table 3) are reduced. Note that because of the terminal condition, nourishment stops before the end of the planning horizon (100 years), leading to a decline in beach width in final years (upper-left panel in Figure 2). Accordingly, close to the end of the planning horizon, there is a steep increase in the present value of net-benefits arising from the terminal condition (lower-right panel in Figure 2).

For higher costs of nourishment, the optimal beach width will decrease from 50 m to 22 m (upper-left panel in Figure 3 and Table 3), and nourishment events will be both smaller in scale and shifted to later years (upper-right panel in Figure 3). Figure 4 depicts the effects of changing the benefit coefficient β. As in the case of changing nourishment cost, the optimal beach width is affected by the beach width benefit coefficient (Figure 4). For higher β (=0.174),

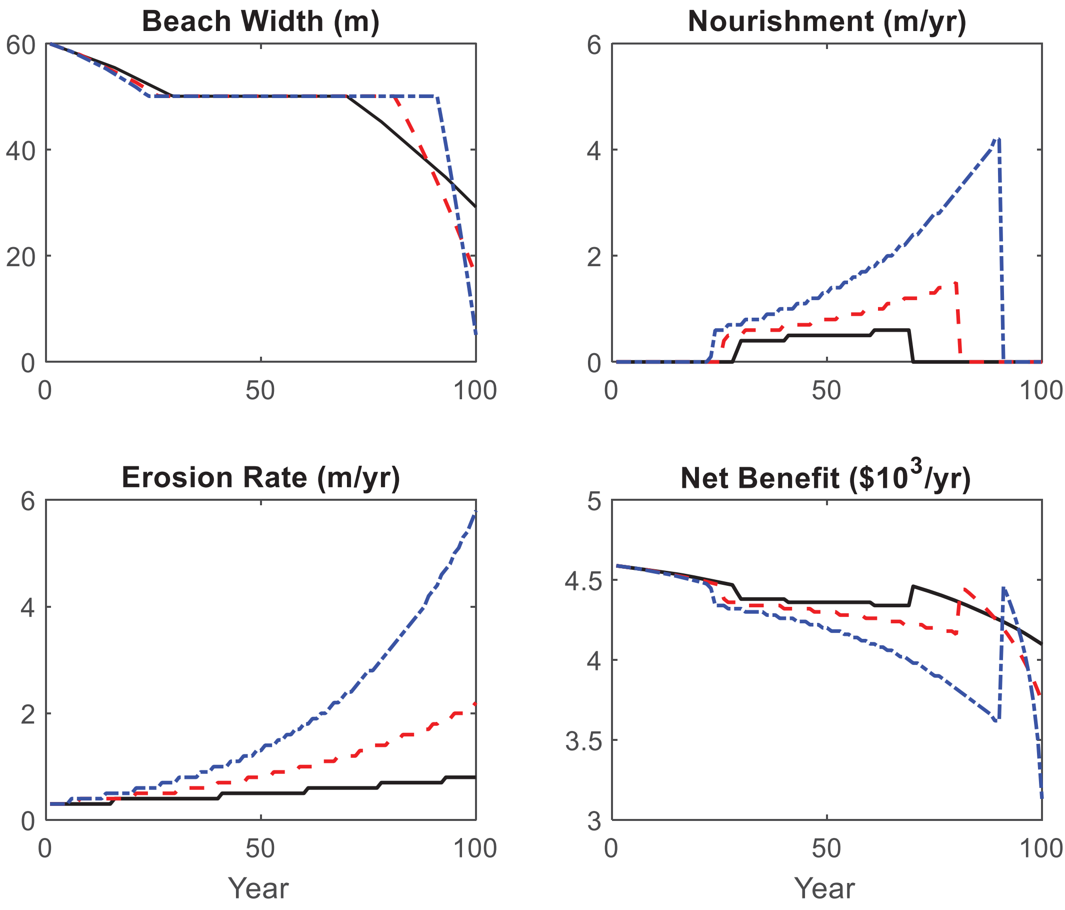

FIGURE 2 Beach width under different annual rates of increase in SLR rate: 1% (black solid line), 2% (red dashed line), and 3% (blue dash-dot line), initial beach width (y_0) = 60 m. SLR, sea-level rise

y^* becomes 62 m, which is wider than the starting beach width (60 m) (upper-left panel in Figure 4). Both nourishment efforts (costs) and net benefits are sensitive to changes in β (upper- and lower-right panels in Figure 4 and Table 3). For all cases, the scenario that maximizes NPV includes periods of time when the community nourishes its beaches to the optimal values, followed by the terminal condition reflecting the discounted shadow value of beach width being zero at the end of planning period, a result of the acceleration of sea-level-driven erosion and the discounting of far-future benefits.

The interactions among discount rate, nourishment cost, and shoreline position (Equation 7) are illustrated in Figure 5. Optimal beach widths are smaller for larger discount rates. Similarly, an increase in nourishment costs leads to narrower optimal beach widths.

3 | OPTIMAL BEACH WIDTH: A STOCHASTIC MODEL

In the DP example above, although the beach width changes over time due both to natural causes and to management, the natural component of beach width change (erosion) was assumed to be exactly known, an unlikely scenario for any beach. Here, we add a component of

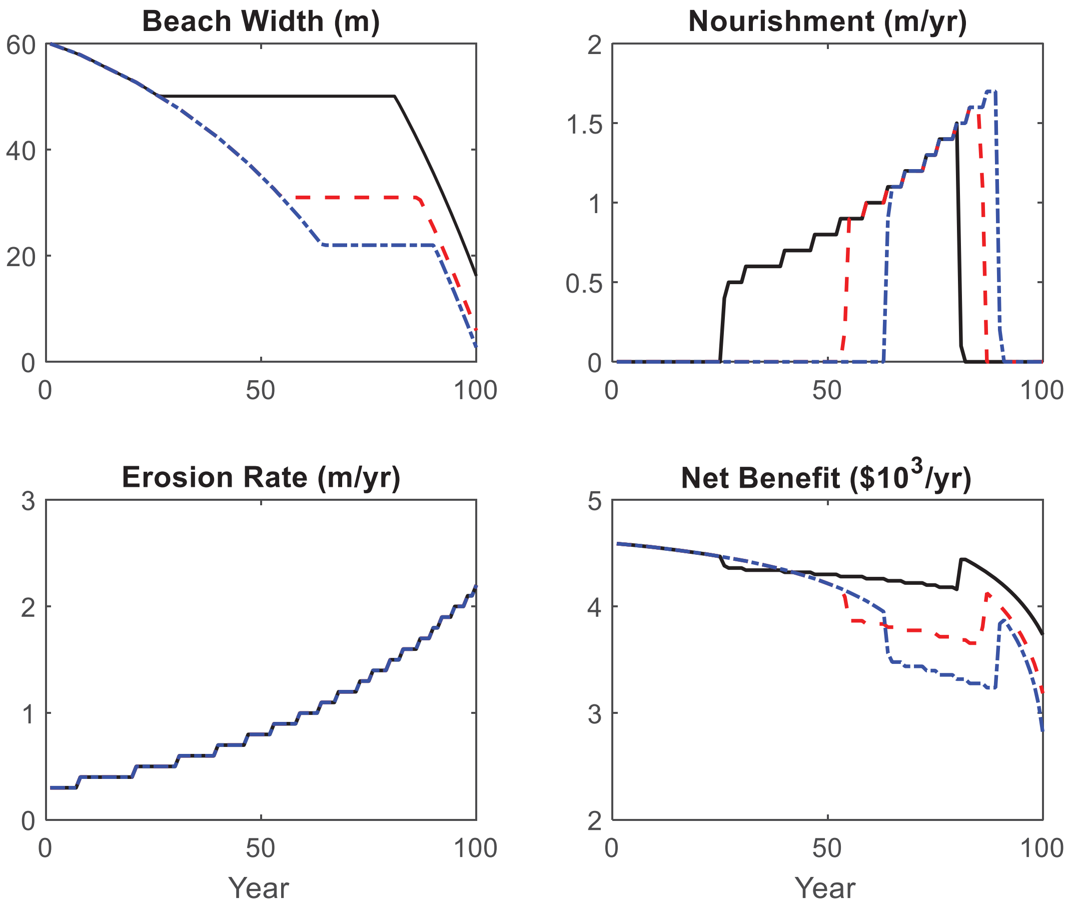

FIGURE 3 Beach width under different unit nourishment costs: \$20/m^3 (black solid line), \$30/m^3 (red dashed line), and \$40/m^3 (blue dash-dot line), initial beach width (y_0) = 60 m

uncertainty to the formulation, with the goal of understanding how uncertainties in the future shoreline position affect the optimal beach width. The uncertainties about future erosion and the consequent effectiveness of beach nourishment cause the shoreline position to become uncertain, requiring modifications to Equation (2):

$$dy = (s - \gamma)dt + \sigma y dz, \tag{10}$$

where σ is the variance parameter of shoreline position (y), and z is a Wiener process. Equation (10) implies that the current shoreline position is known with certainty, but the instantaneous change in the position is, in part, random. Note that y represents the long-term (annual-averaged) shoreline position in the study, and we ignore seasonal or other short-term fluctuations in y to avoid unnecessary complexity. Our selection of a Wiener process is motivated by prior application to other problems in natural resource economics (Pindyck, 1980); for the shoreline example, previous studies also suggested that many long-term shoreline signals exhibit characteristics of a Brownian (or Wiener) process (Lazarus et al., 2011, 2019). Essentially, the above specification reflects a long-term cumulative effect of multiple natural, social, and policy drivers. As explained by McNamara et al. (2015), rising sea levels and increased storminess threaten to accelerate coastal erosion, while growing demand for coastal real estate

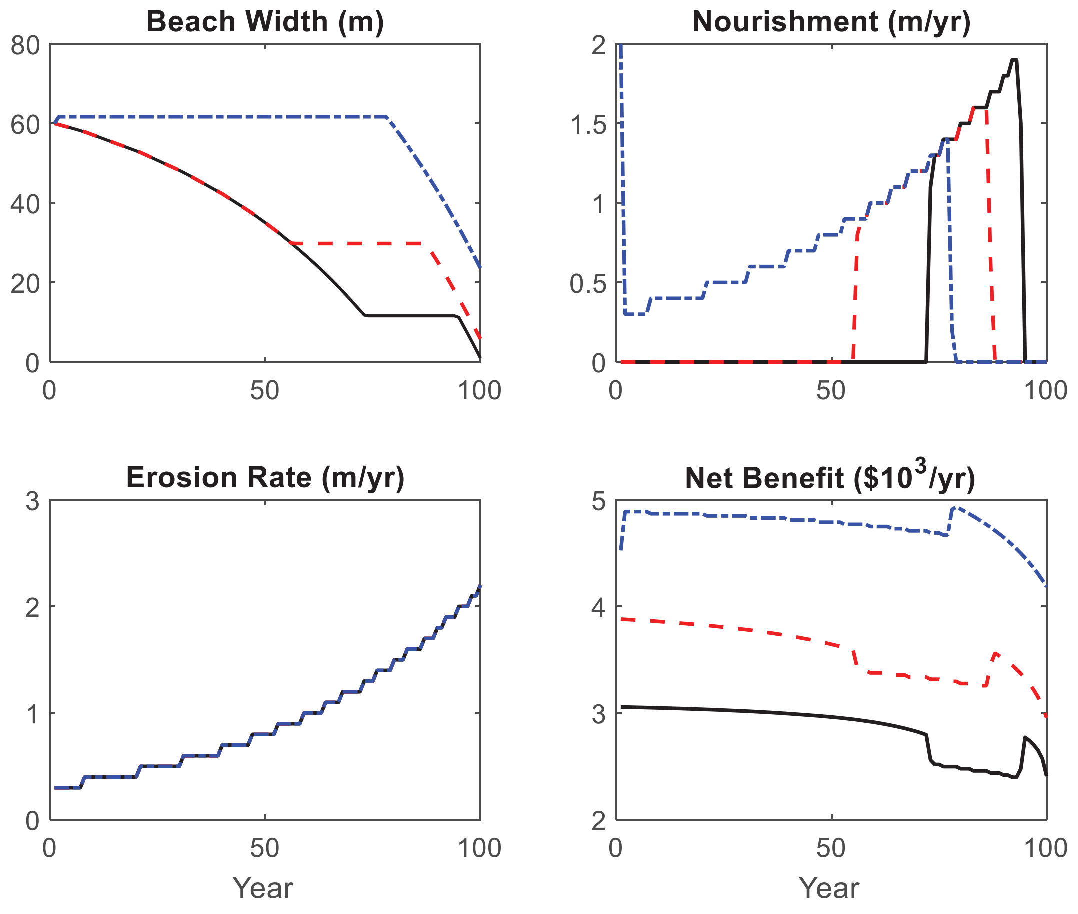

FIGURE 4 Beach width under different benefit coefficients (β): 0.058 (black solid line), 0.116 (red dashed line), and 0.174 (blue dash-dot line), initial beach width $(y_0) = 60$ m

encourages more spending to hold back the sea, in spite of the shrinking federal budget for beach nourishment and other policy uncertainties. According to Lazarus et al. (2011), increases in the rate of SLR can destabilize economically optimal nourishment practices into a regime characterized by the emergence of chaotic shoreline evolution. As coastal managers in charge of the nourishment process typically work with imperfect information, and their local beach width changes as a function of neighbors' nourishment actions (due to the lateral redistribution of sand between neighboring towns by alongshore sediment transport), they react to that information by recalculating their optimal nourishment interval. The management decisions add uncertainly to an already difficult to predict coastal morphodynamical system.

The property owner's value function is

$$V(y) = \max_s E_t \int_t^{\infty} [B(y) - C(s)] e^{-\delta(\tau - t)} d\tau. \tag{11}$$

The owner's problem is to maximize V in Equation (11), subject to shoreline dynamics (Equation 10). As in the deterministic model, s is the control variable, and y is the state variable.

The Bellman equation for this constrained optimization is

TABLE 3 Total costs and net benefits of beach nourishment

Parameters	Beach width (m)	Total nourishment cost (10^3\$)	NPV (10^3\$)
Annual rate of increase in SLR			
1%	50	339	69,094
2%	50	514	68,865
3%	50	791	68,554
Unit nourishment cost			
\$20/m^3	50	514	68,865
\$30/m^3	31	270	68,787
\$40/m^3	22	243	68,758
Benefit coefficients (β)			
0.058	12	179	46,334
0.116	30	225	58,447
0.174	62	1,759	73,927

Abbreviations: NPV, net present value; SLR, sea-level rise.

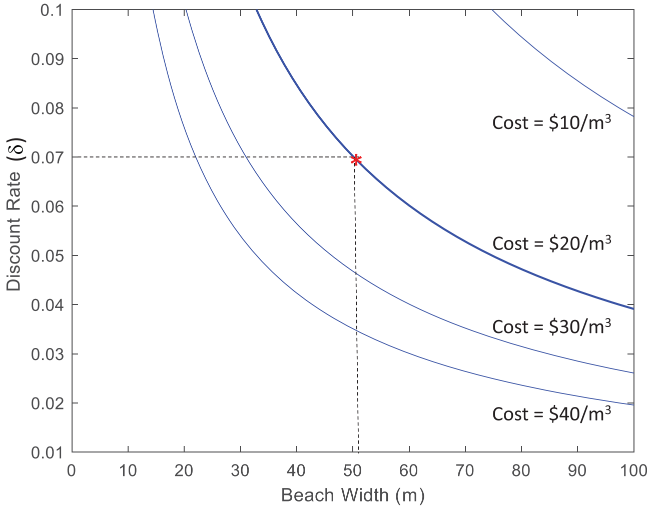

FIGURE 5 Effects of discount rate (δ) on optimal beach width (y^*) under different unit nourishment costs (\$/m3). Red asterisk denotes the baseline case of the deterministic model (unit nourishment cost = \$20/m^3, $\delta = 0.07$, and $y^* = 50$ m)

$$\delta V(y) = \max_s \{B(y) - C(s) + \frac{1}{dt} E[dV(y)]\}. \quad (12)$$

This equation is also known as the fundamental equation of optimality (Bellman, 1957), and relevant discussions can be found in Dixit and Pindyck (1994) and Kamien and Schwartz (1981). We use Ito's Lemma (Ito, 1944) to expand dV and obtain:[3]

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial y}dy + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}(dy)^2. \tag{13}$$

Substituting Equation (10) for dy into Equation (13) and noting that V is not an explicit function of t ($\partial V/\partial t = 0$),[4] we have

$$dV = \left[(s - \gamma)\frac{\partial V}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2}\right]dt + \sigma y \frac{\partial V}{\partial y}dz. \tag{14}$$

Next, substituting Equation (14) into (12) and noting that $E(dz) = 0$, the optimization problem (Equation 12) becomes

$$\delta V = \max_s \left[\alpha y^\beta - C(s) + (s - \gamma)\frac{\partial V}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2}\right]. \tag{15}$$

Maximizing the right-hand side of Equation (15) with respect to nourishment, s, gives rise to the first-order conditions:

$$\frac{\partial V}{\partial y} = \frac{dC(s*)}{ds}. \tag{16}$$

Equation (16) is the optimality condition for the rate of beach nourishment (s). The marginal cost of nourishment is equal to its marginal benefit; that is, the increase in value function (V) with respect to an increase in beach width (y), resulting from the nourishment (s).

Letting $s*$ be the solution to Equation (16), we can rewrite Equation (15) as

$$\delta V = \alpha y^\beta - C(s*) + (s* - \gamma)\frac{\partial V}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^2 V}{\partial y^2}. \tag{17}$$

Differentiating Equation (17) with respect to y yields:

$$\delta \frac{\partial V}{\partial y} = \alpha\beta y^{\beta-1} - \frac{dC}{ds}\frac{\partial s*}{\partial y} + (s* - \gamma)\frac{\partial^2 V}{\partial y^2} + \frac{\partial s*}{\partial y}\frac{\partial V}{\partial y} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^3 V}{\partial y^3} + \sigma^2 y \frac{\partial^2 V}{\partial y^2}. \tag{18}$$

Note that:

$$\frac{1}{dt}Ed\left(\frac{\partial V}{\partial y}\right) = (s* - \gamma)\frac{\partial^2 V}{\partial y^2} + \frac{1}{2}\sigma^2 y^2 \frac{\partial^3 V}{\partial y^3}. \tag{19}$$

Substituting Equations (5), (16), and (19) into (18) yields the optimality condition for shoreline position y:

$$\delta c = \alpha\beta y^{\beta-1} + \sigma^2 \frac{\partial^2 V}{\partial y^2} y. \tag{20}$$

Comparing Equations (7) and (20), we see that the second term on the RHS of (20) captures the effect of uncertainty about future shoreline position on the choice of beach width. Because $\partial^2 V/\partial y^2 < 0$, as shown in Figure 6, the optimal beach width with uncertainty (y**) is less than that in the deterministic case (y*).

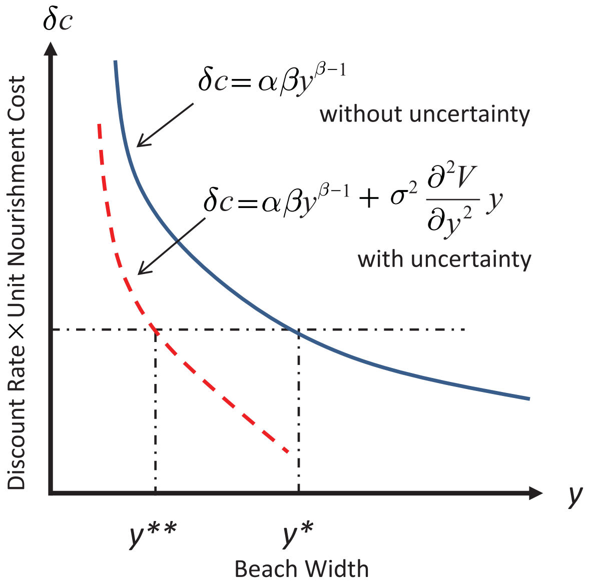

FIGURE 6 Effect of uncertainty (σ) on optimal beach width (y)

Differentiating $C(s(t))$ with respect to $s(t)$ from Equation (5) and using the optimality condition in Equation (16), we can rewrite Equation (20) as

$$\frac{\alpha\beta}{c}y^{\beta-1} = \delta + \sigma^2 yA(y), \tag{21}$$

where $A(y) = -\frac{\partial^2 V}{\partial y^2}/\frac{\partial V}{\partial y}$ is an index of absolute risk aversion (Pindyck, 1984). Results of the deterministic model (Equation 7) indicate that the optimal beach width (y^*) is inversely related to the discount rate (δ). Equation (21) suggests that, with uncertainty about future beach width, the negative effects of the discount rate (δ) are augmented by a risk premium ($\sigma^2 yA(y)$) that the property owner would pay to eliminate the uncertainty about future shoreline positions. The above results that optimal width will be narrower when the effectiveness of nourishment is uncertain are consistent with McNamara et al. (2015), which shows that the introduction of stochastic storms reduces the frequency of nourishment.[5]

Figure 7 depicts the effects of uncertainty (σ) on the optimal beach width (y^{**}) under two different levels of risk aversion. Note that for a risk-neutral property owner, $A = 0$. Thus, $A = 10^{-6}$ represents a situation where the property owner is nearly risk neutral, and $A = 10^{-3}$ represents a situation where the owner is highly risk averse. With increasing uncertainty (σ), the optimal beach width narrows. For example, in the upper panel ($\beta = 0.157$) and for $A = 10^{-3}$, the optimal beach width decreases from close to 50 m to below 10 m as the uncertainty parameter (σ) increases from 0.1 to 5. In the discussion of Equation (4) and Figure 1, however, we have explained that increasing risk aversion, willingness to pay for coastal protection, is associated with a larger benefit coefficient β, which leads to a wider beach (Equation 8). The lower panel of Figure 7 shows that when $\beta = 0.174$ and the uncertainty parameter is low, the optimal beach width rises to over 60 m. Thus, variability in shoreline position (σ) and the benefits of beach width (β) have opposite effects on the optimal beach width.

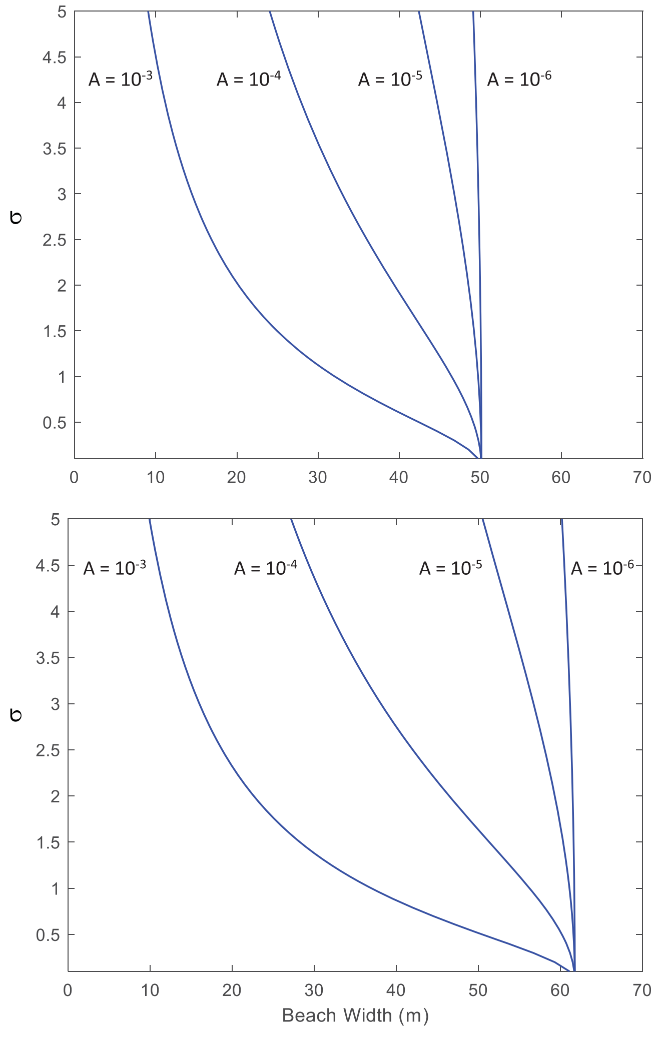

FIGURE 7 Effects of uncertainty parameter (σ) on optimal beach width (y^{**}) under different levels of risk aversion and two different benefit coefficients (β): 0.157 (upper panel) and 0.174 (lower panel)

4 | CONCLUSIONS

With climate change and SLR, developing effective and economically efficient coastal management plans has become increasingly important to coastal communities around the world. We have examined key economic factors affecting optimal decisions for beach nourishment and optimal managed beach widths using simple geo-economic models and optimal control theory. The basic framework is also applicable to other coastal protection approaches. Results of our deterministic model analysis suggest that the optimal beach width is positively related to its values for recreation and hazard protection (β) and the overall property value (α), and negatively related to variable nourishment costs (c) and the discount rate (δ). We analyzed the time paths of beach width and nourishment that maximized net present value under an accelerating sea level using a dynamic programming model.

Our stochastic analysis shows that the uncertainty about future shoreline positions (σ) has a negative impact on the choice of an optimal beach width. The risk perception of a homeowner affects their choice of beach width in two opposing ways: a wider beach to protect their home and a narrower beach if the effectiveness of nourishment is uncertain. The expected benefits of hazard protection must be balanced against the expected costs of repeated nourishment actions. An improved characterization of the uncertainties in future shoreline position, through scientific research on shoreline dynamics, will help coastal managers to implement cost-effective coastal protection systems. With projected SLR, coastal communities are investing in shoreline protection and climate adaptation often without considering the uncertainties about future payoffs of these investments, leading to erroneous cost–benefit analyses. Results of the study call for a recognition of the important effects of uncertainty on cost–benefit assessments of investments in coastal protection. Essentially, the investment rule is to invest if the value of the project is at least as large as the investment cost plus a risk premium.

To become a useful decision tool for practical uses in coastal management, the theoretical framework developed in the study must be expanded to include multiple geological and environmental variables to capture some key effects on shoreline changes (e.g., energetic storms and seasons). In particular, the proliferation of shoreline datasets can help constrain the uncertainty in shoreline position, although these uncertainties may increase with climate change. In addition, a wide range of benefits and costs associated with ecosystem services and economic activities should be incorporated into the models to assist public or private decision making.

ACKNOWLEDGMENTS

We thank Eli Lazarus and an anonymous reviewer for their constructive comments. Support for this study was provided by NSF Grant No. ARG 1518503, WHOI Sea Grant (NOAA Award Number: NA18OAR4170104), and the J. Seward Johnson Fund in Support of the Marine Policy Center.

AUTHOR CONTRIBUTIONS

Di Jin: conceptualization (equal); data curation (equal); formal analysis (equal); funding acquisition (equal); methodology (equal); software (equal); writing original draft (equal). **Porter Hoagland**: conceptualization (equal); funding acquisition (equal); investigation (equal); methodology (equal); writing review & editing (equal). **Andrew D. Ashton**: conceptualization (equal); funding acquisition (equal); methodology (equal); project administration (equal); writing review & editing (equal).

ENDNOTES

[1]This general idea is applicable also to the management of a public beach by converting the private property owner problem to a social planner's problem. In the social planner's model, nourishment decisions are made at the beach town or community level with net benefits reflecting capitalized value of beach width to beach-front homes.

[2]In economic project analysis, the rate at which future benefits and costs are discounted relative to current values often determines whether a project passes a benefit-cost test. This is especially true of projects with long time horizons, such as those dealing with long-term environmental changes (Arrow et al., 2013).

[3]Ito's Lemma is used to differentiate or integrate a function of a generalized Brownian motion process. See, for example, Dixit and Pindyck (1994) and Ross (1996).

[4]This is an autonomous infinite-horizon problem (Judd, 1998).

[5]In the model of McNamara et al. (2015), hurricanes are Poisson distributed, and if a storm occurs at some time before the next nourishment period, then the benefits from nourishment accrue only to the moment the storm occurs.

ORCID

Di Jin https://orcid.org/0000-0002-6403-7983

REFERENCES

Anthony, E. J., Vanhee, S., & Ruz, M.-H. (2006). Short-term beach–dune sand budgets on the North Sea Coast of France: Sand supply from shoreface to dunes, and the role of wind and fetch. *Geomorphology*, *81*(3), 316–329.

Arrow, K., Cropper, M., Gollier, C., Groom, B., Heal, G., Newell, R., Nordhaus, W., Pindyck, R., Pizer, W., Portney, P., Sterner, T., Tol, R. S. J., & Weitzman, M. (2013). Determining benefits and costs for future generations. *Science*, *341*, 349–350.

Ashton, A., Murray, A. B., & Arnoult, O. (2001). Formation of coastline features by large-scale instabilities induced by high-angle waves. *Nature*, *414*, 296–300.

Ashton, A. D., & Giosan, L. (2011). Wave-angle control of delta evolution. *Geophysical Research Letters*, *38*(13). https://doi.org/10.1029/2011GL047630

Bellman, R. (1957). *Dynamic programming*. Princeton University Press.

Bruun, P. (1962). Sea-level rise as a cause of shore erosion. *Journal of the Waterways and Harbors Division*, *88*(1), 117–130.

Conrad, J. M., & Clark, C. W. (1987). *Natural resource economics: Notes and problems*. Cambridge University Press.

Davey, E., Coghlan, I., Carley, J., & Lofthouse, J. (2014). *Beach width: The ideal, expectations and observations*. Water Research Laboratory (WRL), School of Civil and Environmental Engineering, UNSW Australia, Manly Vale NSW.

Dixit, A. K., & Pindyck, R. S. (1994). *Investment under uncertainty*. Princeton University Press.

Frihy, O. E., & Lotfy, M. F. (1997). Shoreline changes and beach-sand sorting along the Northern Sinai Coast of Egypt. *Geo-Marine Letters*, *17*(2), 140–146.

Gopalakrishnan, S., Landry, C. E., Smith, M. D., & Whitehead, J. C. (2016). Economics of coastal erosion and adaptation to sea level rise. *Annual Review of Resource Economics*, *8*, 13.1–13.21.

Gopalakrishnan, S., Smith, M. D., Slott, J. M., & Murray, A. B. (2011). The value of disappearing beaches: A hedonic pricing model with endogenous beach width. *Journal of Environmental Economics and Management*, *61*, 297–310. https://doi.org/10.1016/j.jeem.2010.09.003

Hicks, D. M., & Inman, D. L. (1987). Sand dispersion from an ephemeral river delta on the Central California Coast. *Marine Geology*, *77*(3), 305–318.

Hoagland, P., Jin, D., & Kite-Powell, H. L. (2012). The costs of beach replenishment along the US Atlantic coast. *Journal of Coastal Research*, *28*(1A), 199–204.

Ito, K. (1944). Stochastic integral. *Proceedings of the Imperial Academy (Tokyo)*, *20*, 519–524.

Jin, D., Ashton, A. D., & Hoagland, P. (2013). Optimal responses to shoreline changes: An integrated economic and geological model with application to curved coasts. *Natural Resource Modeling*, *26*(4), 572–604.

Jin, D., Hoagland, P., Au, D. K., & Qiu, J. (2015). Shoreline change, seawalls, and coastal property values. *Ocean and Coastal Management*, *114*, 185–193.

Judd, K. L. (1998). *Numerical methods in economics*. MIT Press.

Kamien, M. I., & Schwartz, N. L. (1981). *Dynamic optimization: The calculus of variations and optimal control in economics and management*. North-Holland.

King, P. G. (2006, July). *The economics of regional sediment management in Ventura and Santa Barbara counties: A pilot study*. Interim Report to the Coastal Sediment Management Workgroup. Prepared at the Economics Department of San Francisco State University.

Komar, P. D. (1973). Computer models of delta growth due to sediment input from rivers and longshore transport. *GSA Bulletin*, *84*(7), 2217–2226.

Kroon, A., Davidson, M. A., Aarninkhof, S. G. J., Archetti, R., Armaroli, C., Gonzalez, M., Medri, S., Osorio, A., Aagaard, T., Holman, R. A., & Spanhoff, R. (2007). Application of remote sensing video systems to coastline management problems. *Coastal Engineering*, *54*(6), 493–505.

Landry, C. E. (2011). Coastal erosion as a natural resource management problem: An economic perspective. *Coastal Management*, *39*, 259–281.

Landry, C. E., & Hindsley, P. (2011). Valuing beach quality with hedonic property models. *Land Economics*, *87*, 92–108.

Lazarus, E., Ashton, A., Murray, A. B., Tebbens, S., & Burroughs, S. (2011). Cumulative versus transient shoreline change: Dependencies on temporal and spatial scale. *Journal of Geophysical Research. Earth Surface*, 116. https://doi.org/10.1029/2010JF001835

Lazarus, E. D., Harley, M. D., Blenkinsopp, C. E., & Turner, I. L. (2019). Environmental signal shredding on sandy coastlines. *Earth Surface Dynamics*, *7*(1), 77–86. https://doi.org/10.5194/esurf-7-77-2019

Lazarus, E. D., McNamara, D. E., Smith, M. D., Gopalakrishnan, S., & Murray, A. B. (2011). Emergent behavior in a coupled economic and coastline model for beach nourishment, *Nonlinear Processes in Geophysics*, *18*, 989–999.

Lippmann, T. C., & Holman, R. A. (1989). Quantification of sand bar morphology: A video technique based on wave dissipation. *Journal of Geophysical Research*, *94*, 995–1011.

List, J. H., Farris, A. S., & Sullivan, C. (2006). Reversing storm hotspots on sandy beaches: Spatial and temporal characteristics. *Marine Geology*, *226*, 261–279. https://doi.org/10.1016/j.margeo.2005.10.003

Lorenzo-Trueba, J., & Ashton, A. D. (2014). Rollover, drowning, and discontinuous retreat: Distinct modes of barrier response to sea-level rise arising from a simple morphodynamic model. *Journal of Geophysical Research: Earth Surface*, *119*, 779–801.

McNamara, D. E., Gopalakrishnan, S., Smith, M. D., & Murray, A. B. (2015). Climate adaptation and policy-induced inflation of coastal property value. *PLOS One*, *10*(3), e0121278. https://doi.org/10.1371/journal.pone.0121278

McNamara, D. E., & Keeler, A. (2013). A coupled physical and economic model of the response of coastal real estate to climate risk. *Nature Climate Change*, *3*, 559–562.

McNamara, D. E., Murray, A. B., & Smith, M. D. (2011). Coastal sustainability depends on how economic and coastline responses to climate change affect each other. *Geophysical Research Letters*, *38*(L07401), 2011–2.

McNamara, D. E., & Werner, B. T. (2008a). Coupled barrier island–resort model: 1. Emergent instabilities induced by strong human-landscape interactions. *Journal of Geophysical Research*, *113*, F01016. https://doi.org/10.1029/2007JF000840

McNamara, D. E., & Werner, B. T. (2008b). Coupled barrier island–resort model: 2. Tests and predictions along Ocean City and Assateague Island National Seashore, Maryland. *Journal of Geophysical Research*, *113*, F01017. https://doi.org/10.1029/2007JF000841

Montaño, J., Coco, G., Antolínez, J. A. A., Beuzen, T., Bryan, K. R., Cagigal, L., Castelle, B., Davidson, M. A., Goldstein, E. B., Ibaceta, R., Idier, D., Ludka, B. C., Masoud-Ansari, S., Méndez, F. J., Murray, A. B., Plant, N. G., Ratliff, K. M., Robinet, A., Rueda, A., … Vos, K. (2020). Blind testing of shoreline evolution models. *Scientific Reports*, *10*, 2137. https://doi.org/10.1038/s41598-020-59018-y

Morgan, R. (1999). Preferences and priorities of recreational beach users in Wales, UK. *Journal of Coastal Research*, *15*(3), 653–667.

Morton, R. A. (2002). Factors controlling storm impacts on coastal barriers and beaches: A preliminary basis for near real-time forecasting. *Journal of Coastal Research*, *18*(3), 486–501.

Parsons, G. R., Massey, D. M., & Tomasi, T. (2000). Familiar and favorite sites in a random utility model of beach recreation. *Marine Resource Economics*, *14*(4), 299–315.

Pelnard-Consideré, R. (1956). Essai de theorie de l'evolution des formes de rivage en plages de sable et de galets. *4th Journees de l'Hydraulique, Les Energies de la Mer, III*, *1*, 289–298.

Pindyck, R. S. (1980). Uncertainty and exhaustible resource markets. *Journal of Political Economy*, *88*(6), 1203–1225.

Pindyck, R. S. (1984). Uncertainty in the theory of renewable resource markets. *Review of Economic Studies*, *51*, 289–303.

Pompe, J. J., & Rinehart, J. R. (1994). Estimating the effect of wider beaches on coastal housing prices. *Ocean & Coastal Management*, *22*(2), 141–152.

Pompe, J. J., & Rinehart, J. R. (1999). Establishing fees for beach protection: Paying for a public good. *Coastal Management*, *27*(1), 57–67.

Quartel, S., Kroon, A., & Ruessink, B. G. (2008). Seasonal accretion and erosion patterns of a microtidal sandy beach. *Marine Geology*, *250*(1), 19–33.

Rosati, J. D., Walton, T. L., & Bodge, K. (2002). Chapter III-2: Longshore sediment transport. In D. B. King (Ed.), *Coastal engineering manual, Part III, coastal sediment processes*. U.S. Army Corps of Engineers. III-2-i - III-2-113.

Ross, S. M. (1996). *Stochastic processes* (2nd ed.). Wiley.

Runyan, K., & Griggs, G. B. (2003). The effects of armoring seacliffs on the natural sand supply to the beaches of California. *Journal of Coastal Research*, *19*(2), 336–347.

Slott, J. M., Smith, M. D., & Murray, A. B. (2008). Synergies between adjacent beach-nourishing communities in a morpho-economic coupled coastline model. *Coastal Management*, *36*(4), 374–391.

Smith, M. D., Slott, J. M., McNamara, D., & Murray, A. B. (2009). Beach nourishment as a dynamic capital accumulation problem. *Journal of Environmental Economics and Management*, *58*(1), 58–71.

Splinter, K. D., Turner, I. L., Davidson, M. A., Barnard, P., Castelle, B., & Oltman-Shay, J. (2014). A generalized equilibrium model for predicting daily to interannual shoreline response. *Journal of Geophysical Research: Earth Surface*, *119*, 1936–1958. https://doi.org/10.1002/2014JF003106

Vitousek, S., Barnard, P. L., Limber, P., Erikson, L., & Cole, B. (2017). A model integrating longshore and cross-shore processes for predicting long-term shoreline response to climate change. *Journal of Geophysical Research: Earth Surface*, *122*, 782–806. https://doi.org/10.1002/2016JF004065

Yates, M. L., Guza, R. T., & O'Reilly, W. C. (2009). Equilibrium shoreline response: Observations and modeling. *Journal of Geophysical Research*, *114*(C9), C09014. https://doi.org/10.1029/2009jc005359

Zhang, K., Douglas, B., & Leatherman, S. (2002). Do storms cause long-term beach erosion along the U.S. East Barrier Coast? *The Journal of Geology*, *110*(4), 493–502.

How to cite this article: Jin, D., Hoagland, P., & Ashton, A. D. (2021). Risk averse choices of managed beach widths under environmental uncertainty. *Natural Resource Modeling*, e12324. https://doi.org/10.1111/nrm.12324