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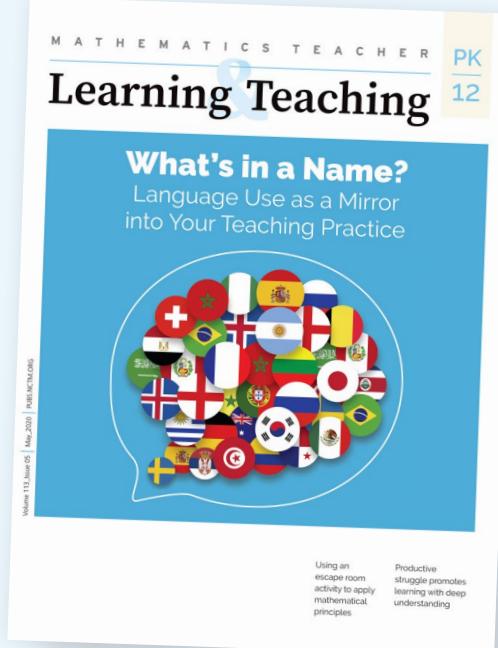
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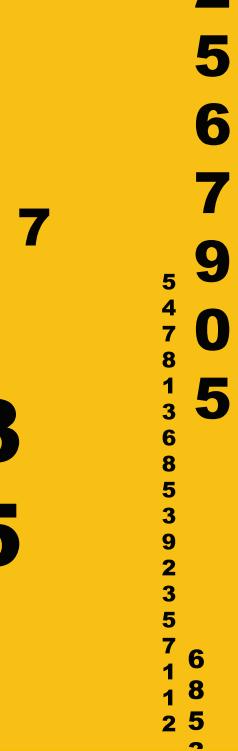
When Students Use Technology Tools, What Are You Noticing?

Lara K. Dick, Allison W. McCulloch, and Jennifer N. Lovett

Eliciting and using student thinking (NCTM 2014) is a foundational pedagogical skill that informs the decisions teachers make during instruction. It informs the questions we pose, the focus of small-group and whole-class discussions, and even the small adjustments we make during a well-planned lesson. To use student thinking to strengthen the teaching of mathematics, a teacher needs to learn how to professionally notice students' mathematical thinking (Thomas et al. 2014/2015; Thomas et al. 2015). In *Principles to Actions: Ensuring Mathematical Success for All*, this is described as "planning for ways to elicit information, interpreting what the evidence means with respect to student learning, and then deciding how to respond on the basis of students' understanding" (NCTM 2014, p. 53).

Previous articles in the NCTM journals have highlighted the importance of professional noticing; what it looks like in elementary and secondary classrooms; its connection to other pedagogical skills (e.g., orchestrating whole-class discussions); and how Jacobs, Lamb, and Philipp's (2010) framework

for professional noticing of students' thinking can support teachers as they work on developing this practice (Amador, Glassmeyer, and Brakoniecki 2020; Thomas et al. 2014/2015; Thomas et al. 2015). What is missing from these examples is how noticing student thinking is different when students are working in technology-mediated learning environments. Technology-mediated learning environments are those in which students use mathematical action technologies to interact with digital objects in mathematically defined ways (Dick and Hollebrands 2011). Mathematical action technologies or tools (e.g., virtual manipulatives, graphing calculators, and dynamic geometry programs) offer ways for students to communicate their mathematical ideas through their interactions with the technology and thus afford new avenues for teachers to elicit evidence of students' mathematical thinking. In what follows, we will explore what it means to professionally notice students' thinking in a technology-mediated learning environment.



A framework to guide teacher noticing when students are working in technology-mediated learning environments.

WHAT MAKES TECHNOLOGY-MEDIATED LEARNING ENVIRONMENTS DIFFERENT—ENGAGEMENT!

When students are working with a technology tool, the ways they interact with the tool can offer insight into their mathematical thinking and learning. Noticing student thinking in such a context requires paying attention not only to what students say and write but also to the ways they engage with the technology.

Imagine you are in a classroom in which students are working on a “mystery transformations” task. Students have been studying transformations

(e.g., translations, rotations, and reflections) and have been challenged to determine the transformation that was applied to map a preimage to an image in a dynamic geometry environment. You launched the task and are now monitoring your students as they work independently on the task. As you are walking up to these students, you hear them say, “Yes! . . . Oh, no. . . . Let’s try that again,” so you pause and watch them work (see video 1). What do you notice?

What did you pay attention to as you watched the video? Did you focus on the correctness of the students' mathematical ideas? Did you focus on what was said? Did you focus on how the technology tool was used? Did you pay attention to the affordances and limitations of the technology tool or even the questions posed to the students? Although there is a lot to possibly pay attention to, our focus in this article is on the importance of paying attention to not only what students say and do but also the ways in which they engage with technology tools to inform teacher interpretations of their understandings and, ultimately, pedagogical decisions. This important skill of attending to and interpreting students' mathematical thinking followed by deciding how to respond is referred to as professional noticing of students' mathematical thinking (Jacobs, Lamb, and Philipp 2010).

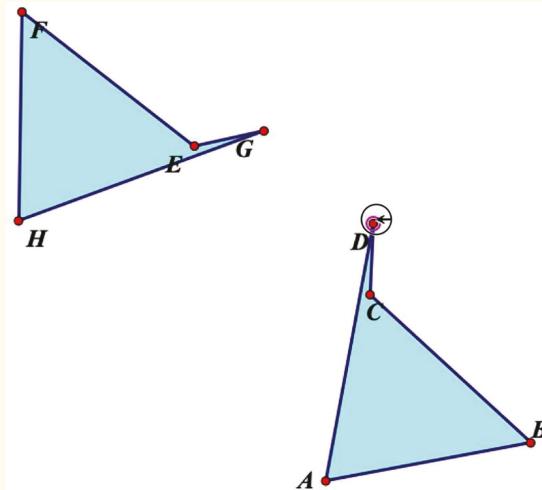
A FRAMEWORK FOR NOTICING STUDENT THINKING IN A TECHNOLOGY-MEDIATED LEARNING ENVIRONMENT

Thomas and colleagues (2015) showed how using a framework for professional noticing of students' thinking can support teachers in designing responsive instruction. Professional noticing includes three components: (1) attending to students' mathematics, (2) interpreting students' mathematical understanding, and (3) deciding how to respond instructionally (Jacobs, Lamb, and Philipp 2010). Given the power of using frameworks and the complexity of teaching in technology-mediated learning environments, we expand on Jacobs, Lamb, and Philipp's (2010)

framework to include noticing of student engagement with technology (see figure 1). The arrows in the figure indicate that all components of noticing are, by their nature, interrelated (Jacobs, Lamb, and Philipp 2010). However, we separate both attention to and interpretation of students' spoken and written mathematical thinking from attention to and interpretation of students' engagement with the technology to highlight the importance of coordinating the two.

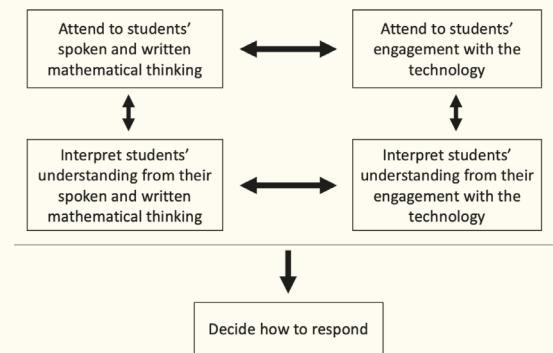
Notice the *decide how to respond* component is separated from the other components. This is to balance the importance of focusing on both spoken and written mathematical thinking and technology engagement prior to making instructional decisions; if a teacher focuses on one more than the other, then he or she may not be fully informed when making an instructional decision.

Video 1 Noticing How Students Engage with a Technology Tool



Watch the full video online.

Fig. 1



We include noticing of student engagement in our framework for noticing student thinking in a technology-mediated learning environment (*Noticing Students' Mathematical Thinking in Technology-Mediated Learning Environment [NITE] framework*).

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In addition, when deciding how to respond to a student working in a technology-mediated learning environment, one must consider how to position the technology (or not) in one's response to support the student in moving their mathematical thinking forward. For this reason, deciding how to respond does not necessarily include students' engagement with the technology. Like Jacobs, Lamb, and Philipp (2010, p. 197), we emphasize "the ability to effectively integrate these three component skills is a necessary, but not sufficient, condition for responding on the basis of children's understandings." Hence, integration of the three noticing components while coordinating attending to and interpreting is the goal of this complex teaching practice.

Think back to the students working with the dynamic geometry technology trying to figure out the mystery transformation. The students expressed excitement, followed by disappointment, and then continued to explore. Pausing to watch the students (attending to), you see them first drag one polygon on top of the other and try to make small adjustments, seemingly trying to make it match perfectly. Then students drag one vertex, and the two polygons no longer match. The students move the polygon back to its original position and start to drag the different vertices. Eventually, students match up pairs of vertices. When the pairs are on top of each other, they all lie in a line. The students then separate the matched pairs of vertices, select one pair, and drag them so they stay together.

A lot happened in 80 seconds! What can we glean (interpret) from attending to the students' engagement with the polygons? The way the students tested the fit of the two polygons when one was moved (translated) on top of the other indicates an understanding of translation being a rigid motion. If the transformation was a translation, it should have maintained the coverage as vertices were dragged. When the students started to drag a pair of vertices to see if they stayed on top of each other, evidence exists that the students may have been trying to determine whether a line of reflection exists. The students' engagement with the technology, in this instance through dragging, provided a lot of information about their thinking—much more than what students actually said. With this information, you can now make an instructional decision about how to respond in a meaningful way.

In this example, the students did not speak very much, making noticing their tool engagement easier because there was little to coordinate. If students are speaking or writing while engaging with a technology tool, noticing becomes even more complex, thus making the Noticing

Students' Mathematical Thinking in Technology-Mediated Learning Environment (NITE) framework a helpful tool for guiding analysis of students' thinking.

Next, we examine what professional noticing of student thinking in technology-mediated learning environments may look like across the grade bands using examples from elementary, middle, and high school. Specifically, we present three examples. In the first, the applet Number Pieces (<https://www.mathlearningcenter.org/apps/number-pieces>) is used to introduce the addition of two three-digit numbers (Common Core State Standards for Mathematics [CCSSM] Content Standard 2.NBT.7 [NGA Center and CCSSO 2010]). In the second, a Desmos activity Make it Balance (<https://teacher.desmos.com/activitybuilder/custom/59de912a3f06a210c73513fa>) is used to introduce the idea of mean as a balance point (CCSSM 6.SP.A.3). In the third, a dynamic graphing technology (i.e., Desmos graphing calculator within a desmos activity) is used to introduce the identification of vertical asymptotes (CCSSM F-IF.C.7.D).

Although three examples are provided, the reader should focus on the example(s) that are of interest based on grade level. It is not necessary to read all three.

- Go to the elementary school example below.
- Go to the middle school example on p. 277.
- Go to the high school example on p. 279.

VIRTUAL MANIPULATIVE TECHNOLOGIES: NOTICING STUDENTS' THINKING ABOUT PLACE VALUE

Virtual manipulatives (e.g., virtual fraction tiles, number lines, geoboards, or algebra tiles) are used throughout K-12 mathematics teaching. The use of virtual versus physical manipulatives has been explored with a focus on the affordances of each modality (e.g., Clements and McMillen 1996; Moyer, Bolyard, and Spikell 2002). For this example, we consider a virtual manipulative simulating physical base-ten blocks. Base-ten blocks have been shown to increase first and second graders' computation performance and increase their understanding of the base-ten system (Fuson and Briars 1990).

Here, we share an example of a second-grade student, Irene, using the free virtual base-ten block app Number Pieces from The Math Learning Center (see figure 2). In school, Irene had added two two-digit numbers, but this was her first experience adding two

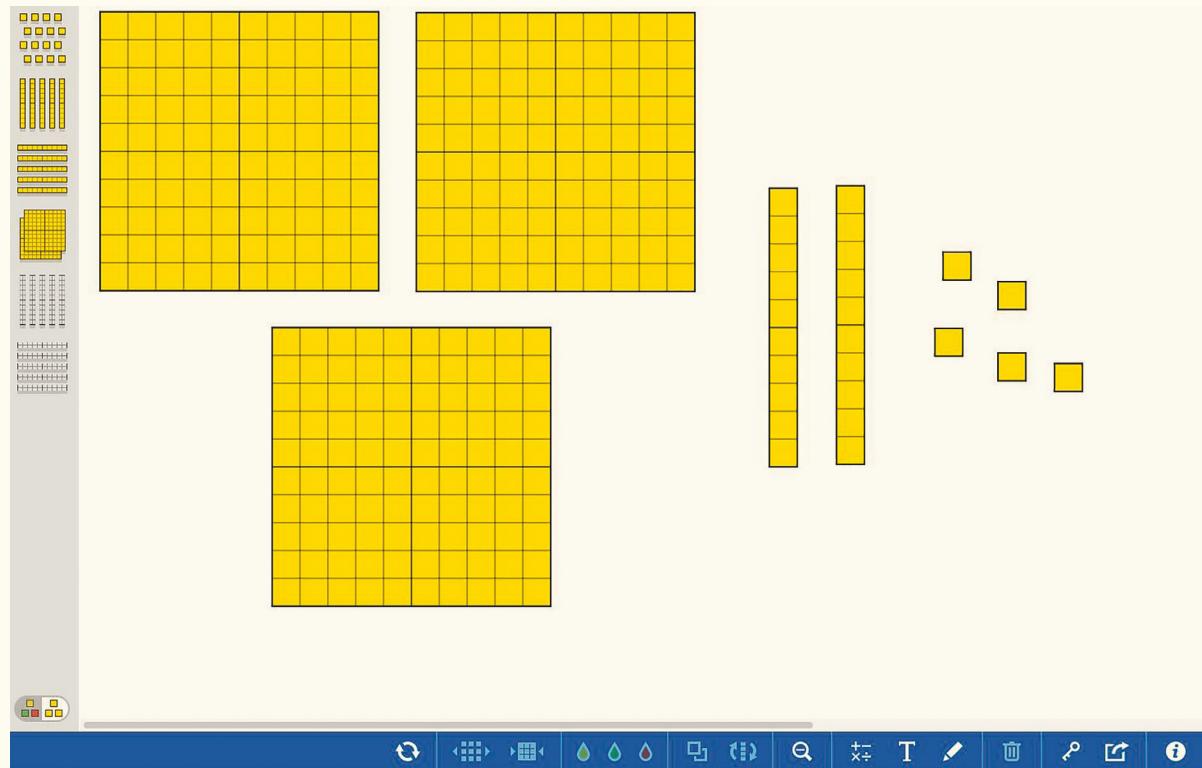
three-digit numbers. This was also Irene's first experience using a mouse though she had previously used the Number Pieces app on her school-issued iPad®. In video 2, Irene solves the problem $367 + 254$. With the NITE framework in mind, watch the video and imagine you are monitoring Irene as she is working in your classroom. Consider jotting down what you hear her saying and how she engages with the virtual manipulative.

 [Watch Video 2: Irene Solves a Problem Using the Number Pieces Applet.](#)

As Irene works, she provides a running commentary of her spoken thoughts, revealing both her understanding of the base-ten structure and her use of the technology. Using the framework to guide our noticing, in the attend component of noticing in a technology-mediated learning environment, we should go beyond what we hear Irene say or see her write and also include a consideration of her engagement with the technology. Doing this carefully may result in the following:

- Irene is initially unsure of working with two three-digit numbers. She claims, "That might be a little bit too hard for me."
- She begins by pulling out the total number of hundreds she will need, followed by the number of tens, and finally ones.
- The applet initially puts tens on top of one of her hundreds, so she moves the hundred away.
- She separates the ones to keep herself from visually mistaking the lined-up ones for a ten.
- Irene adds the hundreds first and initially counts the group of 8 tens as a hundred. She quickly corrects herself.
- Irene indicates that "we have 10 blocks with us, so that will make a hundred." She uses the applet's combine feature to make the hundred.
- Irene then wonders aloud if there are enough ones to make another ten. She counts and determines she can.
- Irene's use of the mouse causes her to glitch the "join together" feature when she tries to make the ten, so she thinks she must carefully line them up,

Fig. 2



The Number Pieces applet shows 325.

but she does not have room. The interviewer intervenes and tells her to try again.

- Irene makes a ten, moves it beside the other tens, and then looks at the screen and determines the answer to the addition problem is 621.

Through the careful coordination of Irene's spoken words and her engagement with the applet, we can interpret the following about her current understanding of addition of two three-digit numbers:

- The applet helped Irene focus on one place-value at a time and revealed her understanding of the base-ten system.
- Irene understands that 10 tens make a hundred and 10 ones make a ten.
- Irene's strategy is to combine the larger pieces first, which parallels a partial-sums method of addition.

Now that we have a grasp of Irene's approach to adding two three-digit numbers and the way the technology allowed her to show her understanding of the base-ten system, the teacher can make an informed decision about how to respond. A possible response could be to ask Irene questions about her strategy to encourage reflection and justification (Huinker and Bill 2017, p. 102). A question like, "I saw you combine tens to make a hundred and then combine ones to make a ten. Why did you combine them in that order?" A specific question focused on her strategy would allow her a chance to articulate her understanding of the base-ten structure. Another question may be to ask if she could solve the problem in a different way. If she is unsure, one option may be to have her solve the same problem again but to ask her to combine in a different order—ones first and then tens. This may help her realize she can be flexible in choosing whether to first combine the hundreds or the tens and would eventually assist her when she is introduced to the standard algorithm for addition.

Because this was Irene's first exposure to adding two three-digit numbers, choosing another problem for her to solve is a possible next step for her. She could use the applet again to solve another problem to help solidify her use of combining 10 tens to make a hundred, or we may ask her to solve another three-digit addition problem without the applet to determine if she can apply her understanding in the absence of a pictorial representation. Any of these choices would both provide more information about Irene's current understandings

and support her in making connections between her strategy and her understanding of the base-ten system.

- Go to the middle school example below.
- Go to the high school example on p. 279.
- Go to the "Affordances of the NITE Framework" section on p. 281.

DYNAMIC DATA TECHNOLOGIES: NOTICING STUDENTS' THINKING ABOUT MEAN

When students are first exposed to conceptual underpinnings of measures of center, they can use technology to assist them in investigating the effects of changing data points. For example, students may explore how data representations are affected by changes in data (Roy, Hodges, and Graul 2016) or how data affect the location of the mean or median (Watson et al. 2008). For this example, we consider a dynamic data technology that develops an understanding of the mean as a balance point. Although not always taught conceptually, understanding of the mean as a balance point emphasizes that the mean is between the minimum and maximum of the data set, that it does not have to belong to the data set, and that for any data set, the sum of the signed distances from the mean is zero (O'Dell 2012).

We share an example of a sixth grader, Carly, engaging with the Make it Balance Desmos activity. In school, Carly had been introduced to the concepts of mean, median, and mode as measures of center and had already been taught the procedure for finding the mean, namely, adding the values of the data points and dividing by the total number of data points. She had not been exposed to the mean from a conceptual point of view. In video 3, Carly is working on page 4 of the activity (see figure 3). She is asked to place the triangle such that the three bears will be balanced. Note: She is not told that what she is doing is finding the mean until page 8. With the framework in mind, watch the video and imagine you are monitoring Carly as she is working in your classroom. Consider jotting down what you hear her saying and how she engages with the dynamic applet as she explores the data.

 Watch Video 3: Carly Engages with the Make It Balance Desmos Activity.

Paying attention to what Carly says makes it apparent that she used a trial-and-error method to find the correct placement. However, using the NITE framework

to guide our noticing, in the attend component of noticing in a technology-mediated learning environment, we should go beyond what we hear her say or what she types and also consider her engagement with the technology. Doing this carefully may result in the following:

- Carly reads the prompt and slides the triangle to the 4 and then back to the 0, saying she needs to think for a second.
- She then considers the location of the bears and says one bear is at the 1, and two bears are at the 8 and 9 (she initially says one bear is at the 2 but corrects herself).
- Carly says the triangle has to be closer to the bear at the 1 to “make it equal,” so she slides it to the 3.
- Carly then focuses on the distances between the bears. She states, “From [the] 1 to 9, there are eight ones, and from [the] 1 to 8, there are seven ones.”
- Carly slides the triangle to the 5 and states, “That looks about halfway between them,” but after considering it, she decides she does not want it halfway because “the two bears would be heavier.”
- Carly slides the triangle to the 4 and checks to see if it is balanced by pressing “check my work.”

- Upon getting feedback that the scale is not balanced, Carly looks at the screen and ponders.
- She decides the triangle needs to be closer to the two bears, so she slides it to the 6 and once again checks the balance by pressing “check my work.”

Through the careful coordination of these statistically salient features of her work, we can interpret the following about her current understanding of mean as balance:

- Carly understands that the balance point is not just halfway between the two groups of bears. In other words, it is not the middle.
- Carly understands that to determine the balance point, she needs to consider the relationship of the distance between the various bears and their associated “heaviness.”
- Carly understands that the position of the balance point should be closer to the “heavier” set of bears.
- No evidence exists yet that Carly understands exactly how to use the “heaviness” of the bears to determine the balance point.

Fig. 3

STUDENT SCREEN PREVIEW 

4 of 14  

Three Bears

Here's a beam and three bears.

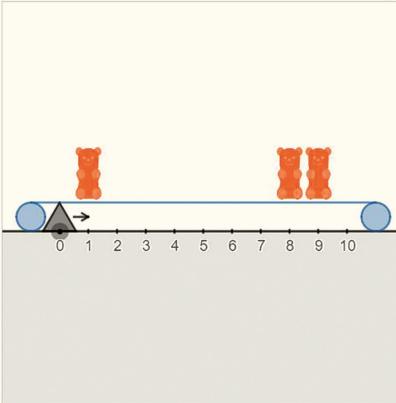
Place the triangle to make them balance.

When you finish, press "Check My Work."

Check My Work

Teacher Moves

This is a great place to check student progress. Use the "Summary" view in the teacher dashboard to identify students who may need individual support. 



Although Carly is not aware that what she is doing is finding the mean, she is asked to place the triangle such that the three bears will be balanced on page 4 of the Make It Balance Desmos activity.

Given the information about how Carly thinks about the balance (which will soon be identified as the mean) and the ways the technology applet may have helped her develop her understanding, the teacher can make a fully informed decision about how to respond at this moment. For example, noting Carly's reliance on the distance between the bears, you may ask her about the distance between the bears and the balance point. Another possible response could be to ask her to use the sliding triangle to show you what she meant by heaviness and how the weight affected the location of the triangle. The first response does not require the use of the technology tool, whereas the second does. However, either response would move Carly away from trial and error and push her toward making a connection between the location of the balance and the sum of the distances on either side.

- Go to the elementary school example on page 275.
- Go to the high school example below.
- Go to the “Affordances of the NITE Framework” section on page 281.

DYNAMIC GRAPHING TECHNOLOGIES: NOTICING STUDENTS' THINKING ABOUT VERTICAL ASYMPTOTES

A common use of technology in middle and high school is to use sliders to investigate the structure of transformations of functions and their graphs. For example, students may explore the structure of linear functions in slope intercept form, $y = mx + b$, by creating sliders for m and b and then investigating the ways in which the graph changes as a result of dragging the sliders to change the value of m or b (e.g., Walker and Edwards 2017) or the classic problem of explaining the effect of b on the graph of the quadratic function written in standard form (e.g., Edwards and Özgün-Koca 2009). Such explorations support students in making sense of key characteristics of the graphs of function families and can (and should) be done with all function families (see CCSSM.HSF.BF.B.3).

Here, we share an example of Integrated Math 3 students working on a task in which they are being introduced to the idea of a vertical asymptote. In this task, students use sliders to explore the parameters of rational functions and their effect on the number of vertical asymptotes a function has and their location. The task is created in Desmos Activity Builder and includes pages with the Desmos graphing calculator in the activity. Thirteen pages are in the full activity (you

can find the activity at <https://teacher.desmos.com/activitybuilder/custom/5d82a5a6288f7374687fe429>), and students explore more complex rational functions as they move through the activity (see figure 4). In addition, the dotted line depicting the location of the vertical asymptote(s) is eventually removed from the graph. At the end of the activity, students are asked to respond to the following questions:

- On the basis of what you have learned, how could you explain to a friend how to determine the number of vertical asymptotes a rational function might have?
- How would you explain how to find the location of those vertical asymptotes given the rational function?

Imagine you are monitoring students as they are working on this task in your classroom. You are about to walk up to a pair of students, Eden and McKenzie, working on page 5 of the activity. Here, they are using sliders to explore the parameters k , a , and b in the function $f(x) = k/(ax + b)$ and their effect on the location (and existence) of vertical asymptotes. With the NITE framework in mind, watch video 4. Consider jotting down what you hear the students saying, what they type, and how they engage with the technology.

Paying attention to what the students say makes it apparent that they have determined a way to find the location of a vertical asymptote for rational functions of the form $f(x) = k/(ax + b)$. However, using the framework to guide our noticing in the attend component of noticing in a technology-mediated learning environment, we should go beyond what we hear them say or see them write and also consider their engagement with the technology. Doing this carefully may result in the following:

- Students recognize that only b and a affect the location of the vertical asymptote.
- Students identify $x = -b/a$ (or $-x = b/a$) as the location for a vertical asymptote.
- Students change each of the sliders and watch and discuss how they do (or do not) affect the location of the vertical asymptote.
- Students set $k = 0$ so that the function is no longer rational.
- Students change b and then move a and notice both have something to do with the asymptote (e.g., “I think whatever b is, [that] is your vertical

asymptote, but it has something to do with a too.”)

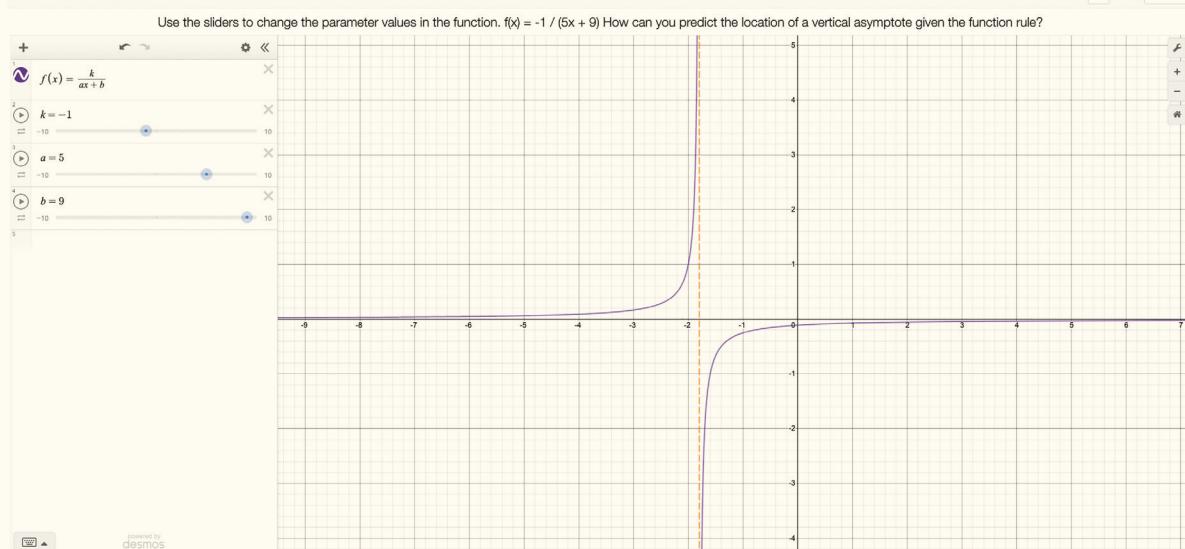
- Students test the conjecture with multiple values of a and b on the sliders.
- Students recognize that rather than (b/a) , it is the opposite $(-b/a)$ and explain it by saying it is one of those “weird flippy things that graphs do.”

- Once students conjecture that the vertical asymptote is located at $x = -b/a$, they test the conjecture with additional values of a and b on the sliders.

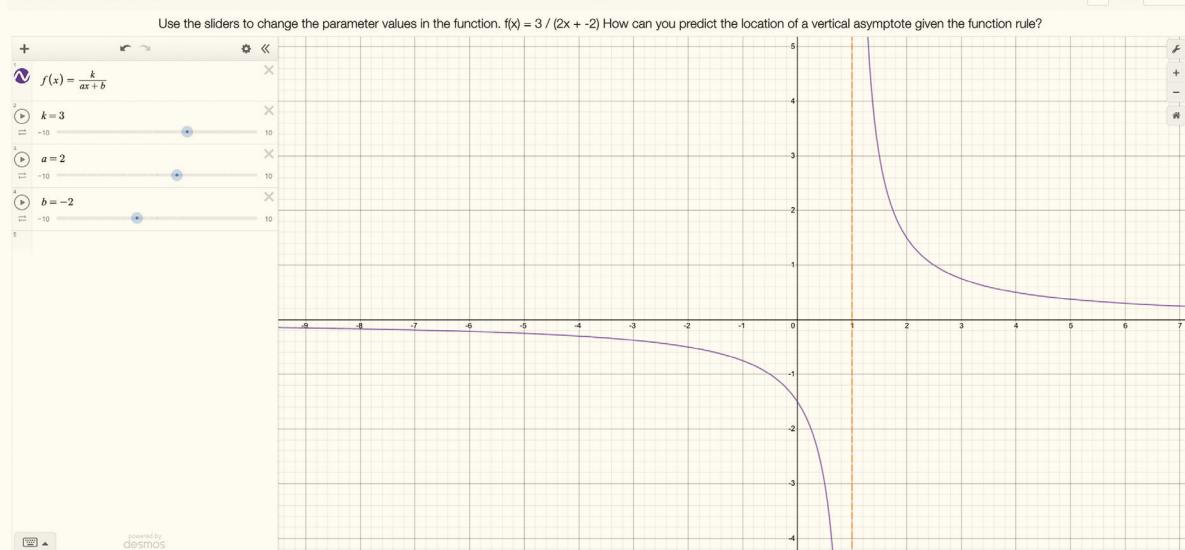
Through the careful coordination of these mathematically salient features of their work, we can

Fig. 4

(a) STUDENT SCREEN PREVIEW



(b) STUDENT SCREEN PREVIEW



Students explore more complex rational functions as they move through page 5 of the introduction to vertical asymptotes Desmos activity.

interpret the following about their current understandings of vertical asymptotes:

- The students understand that the location of a vertical asymptote for a rational function of the form $f(x) = k/(ax + b)$ can be determined by $x = -b/a$ ($-x = b/a$) (i.e., they have a procedure for locating the vertical asymptote); this is evidenced by their statement of the rule and their confidence after testing it with multiple values of a and b .
- The students have not yet connected their rule to the structure of the function equation (i.e., they have not connected a and b to the denominator of the rational function and the fact that it cannot be 0). It appears this may be because of the way the technology allowed them to see the asymptote separate from the graph of the rational function. (In fact, the function is not rational when students are determining and testing their rule!)
- The students have not yet connected their rule to setting the denominator of the function equal to 0 to solve to explain why the vertical asymptote is located at $x = -b/a$ rather than $x = b/a$. This is evident in the way the technology allowed students to test and make sense of the “weird flippy thing” they used the sliders to test and make sure the pattern held true, but there is no evidence they connect this to setting the denominator equal to 0.

Now, armed with information about students' understanding of asymptotes and ways in which the

technology may have played into the development of those understandings, the teacher can make a much more informed decision about how to respond. For example, noticing the students' rule for finding the location of the asymptote was determined when the function they were examining was not rational, you may start by drawing their attention to the function structure (i.e., $f(x) = k/(ax + b)$) and asking them to predict (and then verify using the dynamic graphing tool) what the graph of the function will look like for varying values of k , a , and b in addition to the location of the asymptote. Another possible response would be to ask the students to explain why it makes sense that the asymptote is located at $x = -b/a$ given the structure of the function. The first response requires the use of the technology tool, and the second one does not. However, either response would push the students toward connecting the existence and location of an asymptote to the structure of the function equation.

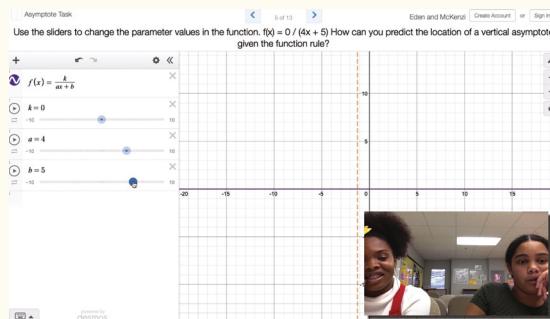
- Go to the elementary school example on page 275.
- Go to the middle school example on page 277.
- Go to the “Affordances of the NITE Framework” section below.

AFFORDANCES OF THE NITE FRAMEWORK

If we were to look just at the students' final responses in each of the earlier examples, we would be missing important insights into their understanding. For example, we would know that Irene was able to find the sum of two three-digit numbers successfully, but we would not know that she did so by combining the hundreds first. For Carly, we would know she made the bears balance, but we would not know she had focused on only the distance between the two groups of bears. Similarly, for Eden and McKenzie, we would know they had determined how to find the location of a vertical asymptote, but we would not know that they did so without paying any attention to the structure of a rational function. However, the use of the NITE framework to guide our noticing ensured we did coordinate what the students were doing with the technology tool with what they said and recorded.

The NITE framework pushes us to not only consider the technology in our noticing but also carefully coordinate students' engagement with the technology to inform our interpretation of their current understanding and our pedagogical decisions. For example, when Eden and McKenzie change the parameter k to

Video 4 Using Sliders to Explore Vertical Asymptotes



Watch the full video online.

0, the function is no longer rational. It appears Eden and McKenzie did this so that they could focus on the asymptote only, without the graph of the function being “in the way.” However, this is really important to notice because it provides insight into the connections Eden and McKenzie are making between the asymptote and the function (i.e., they are not making any connection to the function in the excerpt we see). Similarly, noticing the way in which Irene moves the pieces on her screen to combine and compare them provides important insight into her understanding of base-ten numerals and the strategies she is developing for adding them. Because this was her first time working with three-digit numbers, paying attention to how she actually moves the pieces is important information to influence our pedagogical decisions.

Through the lens of the NITE framework, we were able to make sense of the students’ current understandings and how they were connected to the technological representations students created. In doing so, we

are now better equipped to make pedagogical decisions that support the students’ learning.

PEDAGOGICAL DECISIONS TO SUPPORT STUDENT LEARNING

Coordinating students’ mathematical thinking in a technology-mediated learning environment is not trivial. However, we have found that providing a framework to support one’s learning of this important pedagogical skill to be very helpful (Dick et al. 2020).

Through the lens of the NITE framework, teachers are guided to coordinate what students say and write and the ways they engage with the technology to make sense of their current understandings, including how they connect to the technological representations that students create. As a result, teachers are better equipped to make pedagogical decisions that support the students’ learning—whether those decisions include using the technology tool or not. [_____](#)

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