Surrogate Vibration Modeling Approach for Design Optimization of Electric Machines

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***Abstract*— This article presents a surrogate modeling approach to predict the vibrational behavior during electric machine design. The methodology used in identifying and implementing the surrogate model allows a time-efficient integration of the surrogate model in multiphysics geometric optimization of electric machines, especially in scenarios of drop-in replacement. Based on the state-of-the-art review, the use of analytical or numerical techniques alone considering motor frame and support are not adequately accurate, time efficient, and flexible. The proposed method uses the 3-D finite-element (FE) structural simulations for identifying characteristic response profiles in the surrogate model and does not use structural FE simulations in the optimization loop. Experimental tests are performed on a 6/10 switched reluctance machine to validate the 3-D FE model used to identify the surrogate model. The rational fraction polynomial (RFP) method is used to fit the vibration impulse responses for a number of machines geometries defined by three stator dimensions. The impulse response of any geometry can be determined using linear interpolation of the RFP coefficients. Using the radial forces determined from the FE electromagnetic simulation, the deformation is determined using superposition.**

***Index Terms*— Design optimization, electric machines, finite-element analysis (FEA), transfer functions, vibration measurement.**

I. Introduction

IN ADDITION to the growing challenges of meeting the power density and cost requirements for electric machines, the vibroacoustic requirement has become an important issue and gained a lot of interest in recent years. In electric machine design, electromagnetically induced forces remain the main source of vibrations and acoustic noise. Several researchers have focused on the modeling and analysis of vibration and acoustic noise issues in electric machines. Most models that have investigated vibration and noise due to electromagnetic sources have shared a similar approach, where electromagnetic analysis was used to calculate the forces applied to the stator teeth surfaces, which is discussed in detail in Section III. Following this, modal analysis is performed to determine the natural frequencies and the modal shapes of the stator structures. Resulting vibration levels of the motor can be calculated by applying the electromagnetic forces to the tooth surface, which can also be extended to evaluate the acoustic noise by determining the sound pressure level at a given distance of the motor.

The choice of modeling approach of the electric machine vibroacoustic behavior is influenced by the machine structure and type—radial or axial flux machines, prediction accuracy, and computational resources. In addition, considering the machine frame, housing, gearbox, and supports is important in the choice of the modeling approach due to its influence on the machine vibration levels. The 2-D analytical and 2-D and 3-D finite-element (FE)-based numerical models were employed to predict the vibration of electric machines.

This article proposes a time-efficient and high-fidelity vibration surrogate model that maintains the low computational cost of the analytical models and preserves the accuracy of the 3-D numerical models. The novelty presented in this approach is that the structural changes in the stator dimensions are parameterized in the model, which enables adoption into the geometric optimization. This feature is particularly useful in the design of drop-in replacement motors to adhere to a very specific set of volumetric and size constraints. In this process, computationally intense 3-D finite-element analysis (FEA) simulations are only used to identify the vibration surrogate model and a minimal number of simulations are used for the identification process. In order to achieve computational efficiency, the number of identification simulations needs to be significantly lower than the number of evaluated designs in the optimization. The surrogate model is presented for the vibration model, and the electromagnetic FEA is still performed and not replaced by the surrogate model. In this article, a state-of-the-art review is presented in Section II. Section III explains the proposed modeling approach. The experimental validation of the FE model used for the identification of the model is presented in Section IV. Finally, the feasibility and the performance of the feasibility of the proposed model are discussed in Section V.

II. State-of-the-Art Review

A. Overview of Vibration Models

Most approaches in the literature that analyze the structural and vibroacoustic behavior of electric machines use 3-D

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numerical structural models accompanied by either 2-D or 3-D electromagnetic FEA models [1], [2]. In the analytical modeling approaches for vibration, the stator and housing have typically been simplified as a ring or thick cylindrical shell to determine the surface deformation [3], [4]. In [5], an analytical vibration model is developed to enable fast calculation, but the effects of end caps on the structural intrinsic property have been neglected, which may induce significant errors [6]. However, structural and acoustic behaviors, such as natural frequencies of the motor, depend highly on its 3-D shape and boundary conditions. In addition, axial flux motors cannot be described with a 2-D model.

Another approach using empirical structural models was presented in [7] using experimental data. These models were considered in [8]–[10] to analyze and improve the vibration characteristics of the machine by optimizing the electric excitation. These methods are based on identifying a transfer function to describe the relationship between the measured acceleration and the FEA-determined electromagnetic forces.

Other researchers have tried to improve the computational efficiency of the models by using the electromagnetic and structural FEA simulations only for model identification. Field reconstruction method (FRM) and FEA-identified impulse responses were proposed to predict the electromagnetic vibrations in electrical machines [11] and the acoustic noise and torque pulsation in permanent magnet synchronous motor (PMSM) with static eccentricity and partial demagnetization [12]. A similar methodology was introduced in [13], where the electromagnetic and transient structural FEA can be bypassed using modal expansion and identifying impulse responses. Similarly, the concept of vibration synthesis has been presented, where the structural vibration responses are determined for a generic set of force excitation using FEA [14]. A numerically identified modal matrix was utilized to perform an analytical calculation of electric machine vibration in [15] toward developing computationally efficient structural models.

B. Vibration Models in Design Optimization

Geometric optimization ideally requires a time-efficient multiphysics model with a realistic computational requirement, which can be related to the large number of evaluated designs in the process. In addition, the accuracy of the model is necessary to ensure that the actual performance of the evaluated designs is close to the performance predicted by the model. In [16] and [17], an analytical multiphysics model was developed for a variable speed induction machine. Here, the mechanical behavior of the stator stack was predicted using a 2-D equivalent ring model, whose natural frequencies were also computed analytically. The static deflection of the stator can then be computed analytically for each sinusoidal wave. Lumped models were implemented in [18], where a particle swarm optimization (PSO) was performed on a multiphysics model for a PMSM. Electromagnetic noise optimization was also considered a wound-field synchronous machine (WFSM) [19], where the authors developed high-fidelity 3-D FE models for the optimization to ensure sufficient accuracy. As expected, the computational cost of such models is high. Other research efforts were focused on optimizing the vibration behavior of the machine without developing structural models. In [20], noise, vibration, and harshness (NVH) optimization was performed for a synchronous reluctance machine (SynRM), where the torque ripple was the target performance. This was based on the fact that the torque ripple is the major factor that affects NVH characteristics. Cao *et al.* [21] used the 2-D FE electromagnetic model within the optimization, which focused on the minimization of electromagnetic torque ripple. Similarly, the structural model was not included in the optimization presented in [22], which was only focused on minimizing the radial forces. The structural analysis was only performed for the optimized design.

These studies showed that in terms of computational time, analytical techniques have a clear advantage over numerical techniques. However, one could conclude that the use of analytical or numerical techniques only does not lead to a model with optimal combination of accuracy, time efficiency, and flexibility.

As mentioned earlier, some approaches were aimed to reduce the computational burden of 3-D FE structural simulations. However, these approaches were developed for an electric machine with fixed stator geometry and dimensions. In the case of geometric optimization, where the dimensions of the stator are varied, the structural model should be identified for each design candidate, which is based on the fact that the vibration characteristics are dependent on the interaction of electromagnetic excitation quantities and structural quantities defined by the geometry and material. Since a large number of designs are considered in the optimization process, using the 3-D FE models in the optimization loop would increase the computation time significantly.

This article presents a structural model for electric machines that use a surrogate approach. The model identification process will allow maintaining the accuracy of the 3-D FE models with lower computation time, which will enable time-efficient geometric optimization. The vibration model is coupled with a 2-D FE electromagnetic model to determine the forces applied to the stator.

III. Proposed Surrogate Modeling Approach

A. Overview

Conventional approaches for predicting vibration in electric machines consists of three stages, as shown in Fig. 1. First, the electrical excitation of the machine is determined from a circuit or a control model of the machine. Second, the electromagnetic analysis is performed to compute the radial and tangential components of the magnetic flux density, from which the tangential and radial components of the electromagnetic force densities acting on the stator teeth using the Maxwell stress tensor are computed as in the following:

$$f_t = \frac{1}{\mu_o} B_n B_t \tag{1}$$

$$f_n = \frac{1}{2\mu_o}\left(B_n^2 - B_t^2\right) \tag{2}$$

where B_t and B_n are the tangential and normal components of the magnetic flux density, respectively, and μ_o is the magnetic permeability of air.

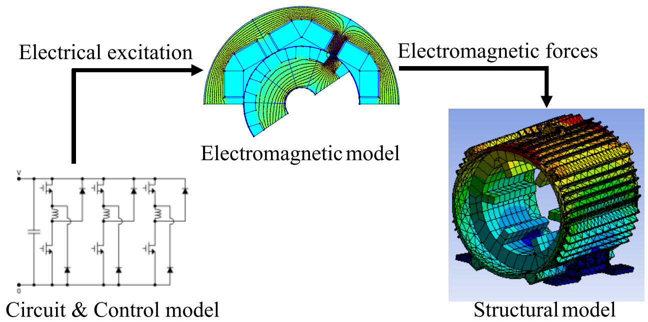

Fig. 1. Multiphysics models interconnection for vibration prediction.

Finally, the structural FEA is performed with the computed electromagnetic forces as inputs. Mode superposition method is used to perform the structural analysis, which requires modal analysis to determine natural frequencies and the corresponding mode shapes of the stator structure. Toward this goal, free vibration equation of a linear undamped system is considered [23]

$$\big([K] - \omega_i^2[M]\big)\{\emptyset_i\} = \{0\} \tag{3}$$

where ω_i and $\{\emptyset\}_i$ are the ith natural angular frequency and mode shape of the structure, respectively, and $[K]$ and $[M]$ are the stiffness and mass matrices, respectively.

The mode superposition method solves the harmonic equation in modal coordinates for a force acting on a viscous damped spring–mass system

$$[M]\{a\} + [C]\{v\} + [K]\{x\} = \{F\} \tag{4}$$

where $[C]$ is the damping matrix, $\{a\}, \{v\}$, and $\{x\}$ are the acceleration, velocity, and displacement vectors, respectively, and $\{F\}$ is the applied force vector.

For a linear system, the displacement $\{x\}$ can be expressed as a linear combination of mode shapes $\{\emptyset_i\}$

$$\{x\} = \sum_{i=1}^{n} y_i\{\emptyset_i\} \tag{5}$$

where y_i are the modal coordinates for (5).

Accordingly, the impulse response of displacement at a single point of measurement can be determined for a unitary concentrated force applied to one stator tooth. In a linear system, the total displacement is the sum of the impulse responses caused by each force. Furthermore, the magnitude of the impulse response is proportional to the magnitude of force. Therefore, if the unitary impulse responses for each stator tooth are identified, the total deformation at the measurement point can be reconstructed for the impulse response and radial force spectra using the superposition principle, shown in Fig. 2, which implies additivity and scalability.

In the case of geometric optimization, radial forces and impulse responses spectra vary when the stator and rotor dimensions are changed in the optimization loop. Since FE electromagnetic analysis is already performed for every design, radial forces can be calculated for any machine geometry. For structural impulse response, the proposed vibration surrogate model is utilized to estimate the impulse responses.

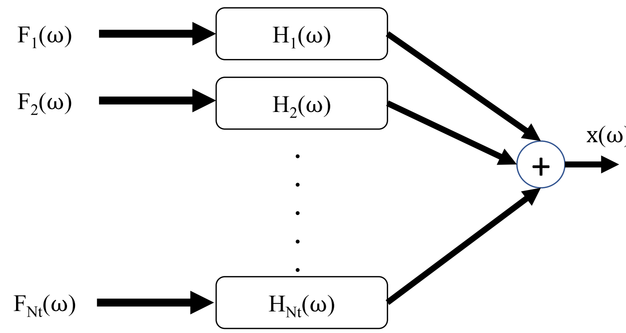

Fig. 2. Vibration reconstruction from individual impulse responses.

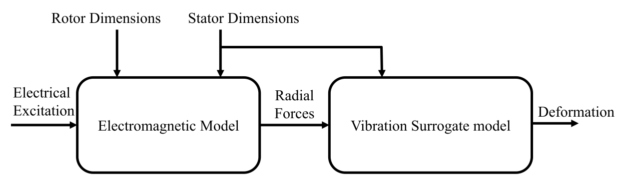

Fig. 3. Determining the deformation using the surrogate model for given machine dimensions.

In order to identify the model, impulse responses should be expressed as a function of stator dimensions. This requires the use of a suitable mathematical representation to fit the impulse responses acquired using the 3-D structural FEA for a number of geometries prior to the optimization. Afterward, the coefficients within the chosen frequency response function (FRF) representation can be interpolated for any geometry within the range of identified dimensions.

The proposed model is identified for a machine with a fixed envelope volume (fixed outer diameter and axial length), which is a reasonable assumption at the design stage, especially when the design process is started with a motor sizing procedure.

The effect of the rotor structure on the vibration levels of the machine is considered in the electromagnetic model, where the radial forces are determined. However, since the vibration measurements are performed on the stator structure, the rotor dimensions are not considered in the surrogate model. Fig. 3 shows how the machine design parameters and dimensions are considered for determining deformation using the surrogate model.

B. Mathematical Representation of the Impulse Response

Ideally, a simple and accurate representation for the impulse FRF magnitude and phase angle is desired, which enables estimation of the variations in the vibration characteristics as geometry changes. Coefficients of the mathematical form should reflect the change in the vibration characteristic as the geometry changes. Furthermore, since the main purpose of introducing the surrogate model is to reduce the computational burden, a systematic and time-efficient approach is needed for the model identification prior to the optimization.

Several methods have been considered in the literature to fit the FRF, mainly to extract the modal parameters such as the natural frequencies and damping ratios from experimental results. The Gaukroger–Skingle–Heron method is based on fitting the FRF of system with N degrees of freedom using the least-square fit, as shown in the following:

$$H(\omega) = a_0 + \left(\sum_{r=1}^{N} \frac{A_r + i\omega B_r}{\omega_r^2 - \omega^2 + i2\zeta_r\omega_r\omega}\right)e^{i\varphi} \tag{6}$$

where a_0 is a complex constant and φ is the angle representing the out-of-range modes.

This method can provide good estimates; however, the process can be slower when compared to other methods.

For slightly damped structures, the Ewins–Gleeson method can be used for fitting the FRF, as shown in the following:

$$H(\omega) = \sum_{r=1}^{N} \frac{A_r}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2}. \tag{7}$$

Due to the assumption of slightly damped system, the numerator coefficient is a real quantity. This representation works well with slightly damped structures; however, it is sensitive to the selection of the FRF points [24].

The rational fraction polynomial (RFP) method is widely used in the estimation of the modal parameters, and it can provide accurate estimates for the damping ratios [25]. The flexibility provided by the RFP method can allow a systematic identification of FRF, which can be implemented directly in MATLAB using transfer function identification and partial fraction decomposition functions. Therefore, the RFP representation is used in this article. The mathematical form is as the following:

$$H(\omega) = \frac{x(\omega)}{F(\omega)} = \sum_{r=1}^{N}\left(\frac{A_r}{i\omega - \lambda_r} + \frac{A_r^*}{i\omega - \lambda_r^*}\right) \tag{8}$$

$$\lambda_r = \sigma_r + i\omega_r. \tag{9}$$

C. Identification Procedure

In this work, the model considers three main stator dimensions, stator yoke thickness (Y_T), tooth width (T_W), and tooth depth (T_D), as shown in Fig. 4. The chosen three dimensions are considered the main dimensions to describe the geometry of the SRM stator. The 3-D FE structural analysis is performed in ANSYS for a number of geometries defined by different values of the three dimensions. For each geometry (combination of the three dimensions), the impulse response at the measurement point is determined by applying a unit force to a single stator tooth, as shown in Fig. 5, which shows the set of 3-D FE simulations for model identification.

The range of each dimension should be chosen to cover all possible dimensions dictated by the geometric template and the optimization algorithm. The number of steps in each dimension can be adjusted based on the sensitivity analysis of vibration characteristics to the variation in dimensions. In this case, the dimensions range and steps are summarized in Table I.

The resulting FRFs are fitted using the RFP method described in Section III-B, and the modal parameters are extracted, as shown in Fig. 6. This step is repeated for all stator teeth. The number of RFP terms N can be set to adapt to the frequency range of the FRF, which is usually chosen based on the speed range of the machine and the number of vibration modes of interest.

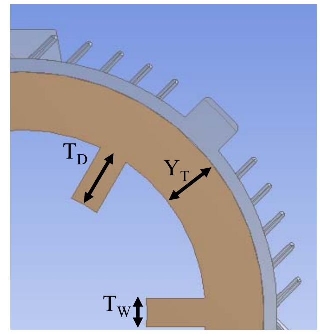

Fig. 4. Stator dimensions considered in the surrogate model.

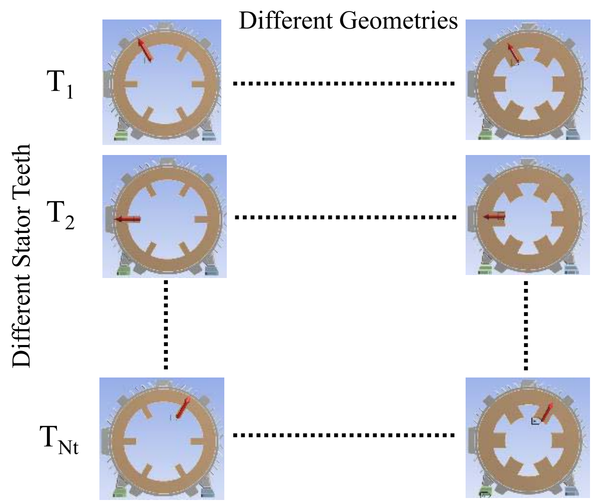

Fig. 5. 3-D FE simulation set for model identification.

TABLE I
CONSIDERED STATOR DIMENSIONS

Stator Dimensions	Range (mm)	Step Size (mm)
Y_T	10-30	5
T_W	10-30	5
T_D	10-20	5

D. Model Implementation

When the identified model is implemented in the optimization loop, the unitary impulse responses of any geometry within the identified range can be determined using linear interpolation of the coefficients in (8), as shown in Fig. 7. If more parameters are considered in the approach, a multivariate interpolation method, such as inverse distance weighting or kriging, can be used. Extrapolation of values outside the identified range cannot guarantee the accuracy of response estimation.

The deformation response can then be reconstructed from the radial force spectrum determined using the FE electromagnetic analysis and identified impulse responses from the surrogate model, as shown in Figs. 8 and 9.

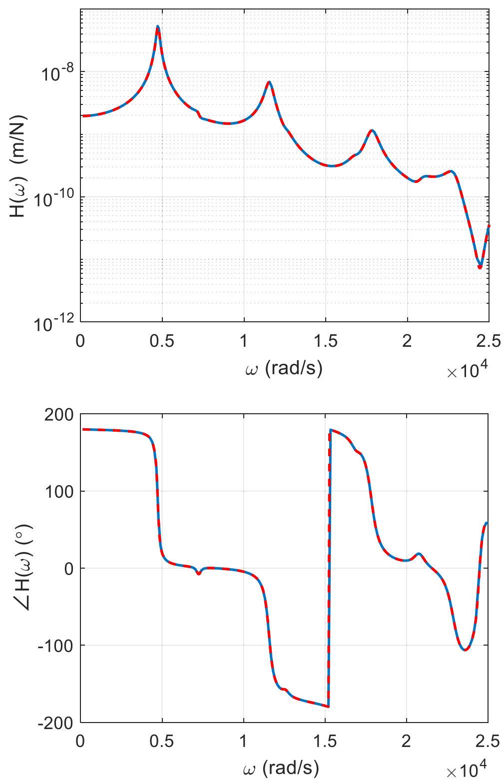

Fig. 6. Unitary FRF fitting using RFP. Top: magnitude. Bottom: phase.

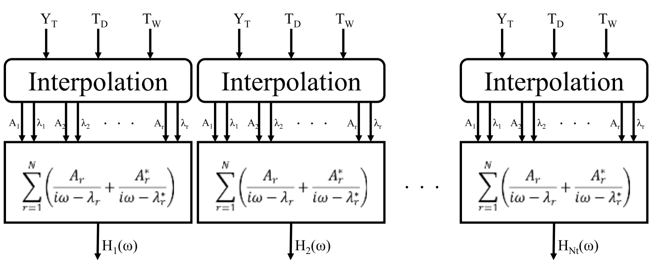

Fig. 7. Prediction of unitary impulse responses for different stator dimensions.

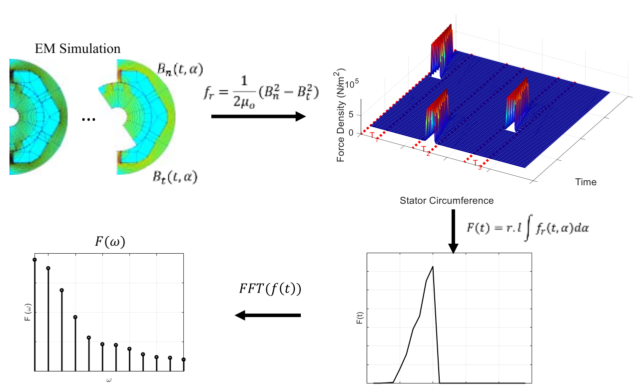

Fig. 8. Determining the radial forces using the electromagnetic model.

IV. Experimental Validation of FE Model

Since the proposed surrogate model is identified using the 3-D FE structural model developed in ANSYS, experimental validation is performed on the ANSYS model to confirm that the model settings, similar to contact regions, supports, and mesh size, are set properly. This will ensure that the identified modal quantities agree with the actual measurements.

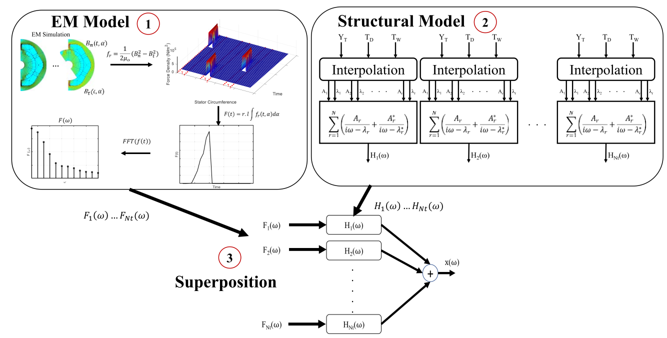

Fig. 9. Procedure of vibration prediction using the electromagnetic model, surrogate model, and superposition method.

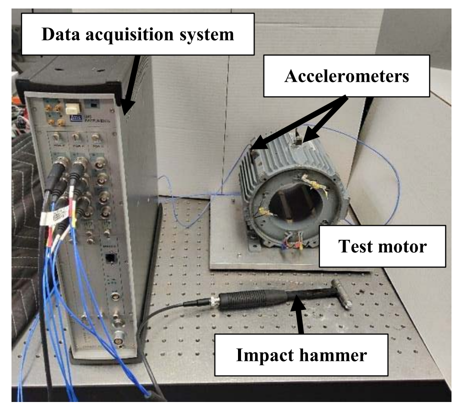

Fig. 10. Experimental setup for FE model validation.

TABLE II
MOTOR SPECIFICATIONS

Name	Value
Outer diameter	165 *mm*
Stack length	89 *mm*
Rated power	746 *W*
Rated speed	1800 *rpm*
DC bus voltage	680 *V*
Stator core material	M19 Steel

Fig. 10 shows the experimental setup for validation. The target machine is a three-phase switched reluctance motor with a higher number of rotor poles, 6/10 pole configuration [26], [27], where the stator core is encased in a cast-iron frame. The machine dimensions and parameters are shown in Table II. The experimental validation was carried with a hammer impact test with LMS TestLab for postprocessing. Two accelerometers (PCB Piezotronics 353B15) were roved across 21 locations over the frame and the FRFs were acquired.

As shown in Table III and Fig. 11, a good match was obtained between the results of the experimental setup and the FEA model in ANSYS, and the results include the natural

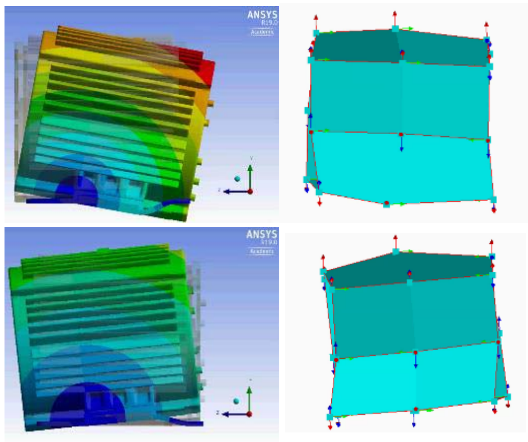

Fig. 11. Experimental validation of mode shapes obtained from FEA.

TABLE III
COMPARISON OF NATURAL FREQUENCIES DETERMINED USING FEA AND THE EXPERIMENTAL SETUP

	FEA	Experimental
1	142 *Hz*	132 *Hz*
2	1984 *Hz*	1968 *Hz*
3	2784 *Hz*	2724 *Hz*
4	2909 *Hz*	2927 *Hz*

frequencies and mode shapes of the considered machine with the housing.

V. Proposed Model Validation and Feasibility

A. Model Validation Results

The developed model accuracy was validated by comparing the frequency response predicted by the proposed surrogate model with the actual frequency responses for random geometries and forces magnitudes applied to all stator teeth. The actual frequency responses for different geometries are obtained using ANSYS. This comparison resembles the implementation of the model in a geometric optimization, where different geometries are evaluated, and different magnitudes of forces are computed from the coupled electromagnetic model. As shown in Fig. 12, frequency response estimated by the surrogate model for random geometries with different forces applied to stator teeth, shown in Table IV, provides good approximation compared with the actual values. The computation time needed for the surrogate model is ~1 s, where in the case of 3-D FEA model, ~8 min were needed to finish the structural simulation.

B. Computational Feasibility Analysis

The main purpose of introducing the surrogate model is to mitigate the computational burden of structural FE analysis in the geometric optimization loop with minimal loss of accuracy. In the implementation of the proposed model, the highest amount of computation time is taken up by the identification process. Once the surrogate model is identified, the run time

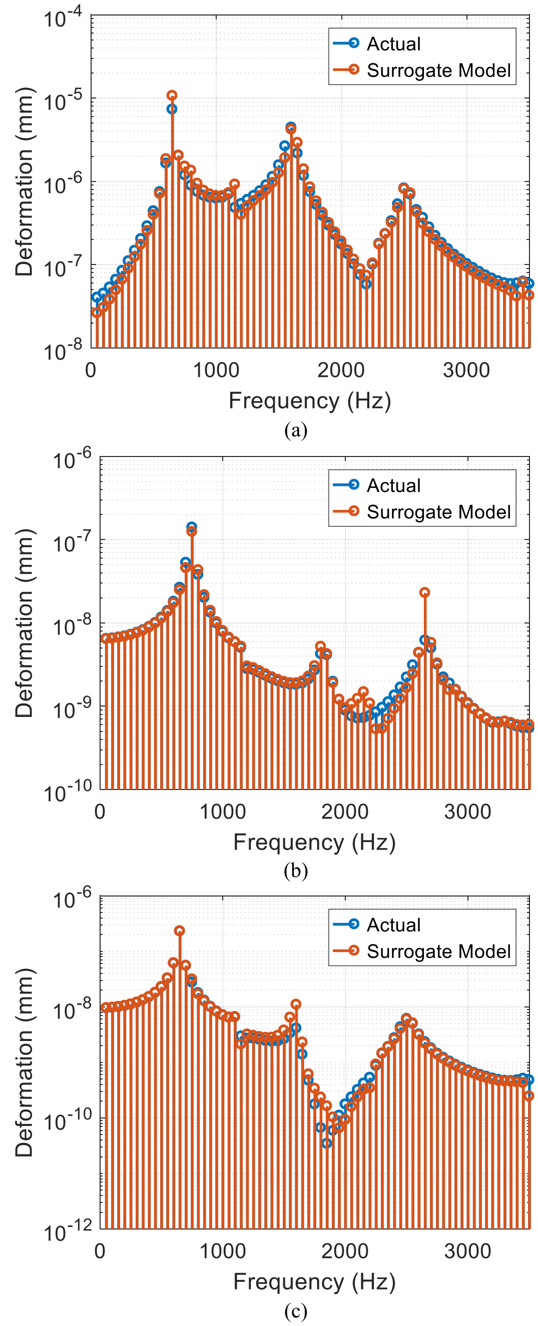

Fig. 12. Surrogate model validation results for three arbitrarily chosen combinations of geometry and forces. (a) Case 1. (b) Case 2. (c) Case 3.

of the surrogate model is very low and can be neglected. As explained earlier, the total number of simulations required to identify the model (N_{Ti}) depends on the number of stator teeth (N_T) and the number of simulations at each tooth (N_i), as shown in the following equation:

$$N_{Ti} = N_T N_i. \tag{10}$$

N_i depends on the number of design parameters (p) and the number of steps in each design parameters (s_k). For the proposed model, N_i can be expressed as follows:

$$N_i = \prod_{k=1}^{p} s_k. \tag{11}$$

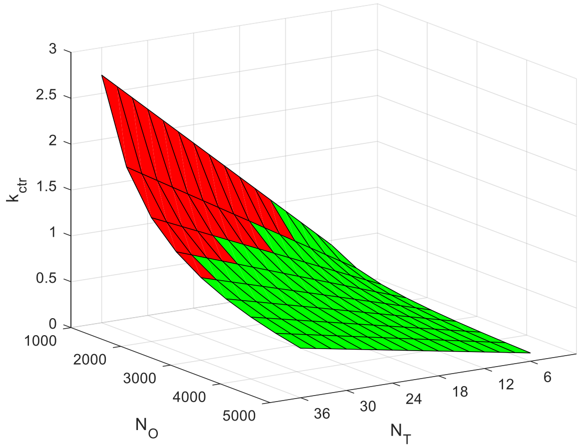

Fig. 13. Computation time reduction factor (k_{ctr}) for different numbers of stator teeth and designs considered in the geometric optimizations.

TABLE IV
DIMENSIONS AND MAGNITUDES OF FORCES USED IN MODEL VALIDATION

Stator Dimensions			
	Y_T (mm)	T_D (mm)	T_W (mm)
Case 1	14.6	19.7	22.1
Case 2	18.9	17.1	23.1
Case 3	24.8	19.0	16.2

Radial Forces (N)						
	F_1	F_2	F_3	F_4	F_5	F_6
Case 1	200∠0°	250∠30°	50 ∠45°	300∠60°	350 ∠0°	100∠30°
Case 2	1 ∠45°	2 ∠30°	1 ∠30°	1 ∠120°	1 ∠0°	2 ∠30°
Case 3	10∠30°	15 ∠0°	10 ∠30°	12 ∠45°	15 ∠30°	10∠30°

Based on this information, the model is considered computationally feasible if the number of simulations used to identify the model is fewer than the number of designs evaluated in the geometric optimization N_o.

To evaluate the computational feasibility of the surrogate model, a figure of merit (k_{ctr}) defined in (12) is used to describe the reduction in the computation time

$$k_{\text{ctr}} = \frac{N_T N_i}{N_o}. \tag{12}$$

Accordingly, the proposed model is considered to offer a reduction in computation time, if k_{ctr} is less than 1. Fig. 13 shows the computational feasibility described by k_{ctr} for different values of N_T and N_o. N_i was set at the same value 75, used in the identification process of the target machine. The portions of the surface shown in green and red represent $k_{\text{ctr}} < 1$ and $k_{\text{ctr}} \geq 1$, respectively.

Furthermore, for the considered machine 6/10 SRM, the surrogate model offered a reduction in computation time for a wide range of N_o. For example, if 5000 designs are considered in the optimization, the computation time can be reduced by almost 90%. This allowed inclusion of the structural model within a multiphysics geometric optimization, without being restricted by the number of designs evaluated in the optimization process.

VI. CONCLUSION

This work presents a surrogate approach for predicting vibration frequency responses of electric machines with different stator geometries. This approach can be used to implement a vibration model within a geometric optimization loop, where a large number of designs need to be evaluated.

The presented model considers typical constraints imposed by the motor housing and fixed structure support, where the identification method is based on expressing the FRF of a few geometries in RFP form.

The model used to identify the surrogate model was validated with ANSYS for a 6/10 SRM switched reluctance machine. It was demonstrated that the model can provide good accuracy and significant reduction in computational time. This method can serve as an effective approach toward developing drop-in replacements for motors in the existing applications.

REFERENCES

[1] F. Chai, Y. Li, Y. Pei, and Y. Yu, "Analysis of radial vibration caused by magnetic force and torque pulsation in interior permanent magnet synchronous motors considering air-gap deformations," *IEEE Trans. Ind. Electron.*, vol. 66, no. 9, pp. 6703–6714, Sep. 2019, doi: 10.1109/TIE.2018.2880707.

[2] L. Gao, H. Zheng, L. Zeng, and R. Pei, "Evaluation method of noise and vibration used in permanent magnet synchronous motor in electric vehicle," in *Proc. IEEE Transp. Electrific. Conf. Expo. (ITEC)*, Detroit, MI, USA, Jun. 2019, pp. 1–4, doi: 10.1109/ITEC.2019.8790612.

[3] J. F. Gieras, C. Wang, and J. C. Lao, *Noise of Polyphase Electric Motors*. Boca Raton, FL, USA: Taylor & Francis, 2006.

[4] M. Boesing and R. W. De Doncker, "Exploring a vibration synthesis process for the acoustic characterization of electric drives," *IEEE Trans. Ind. Appl.*, vol. 48, no. 1, pp. 70–78, Jan./Feb. 2012.

[5] R. Islam and I. Husain, "Analytical model for predicting noise and vibration in permanent-magnet synchronous motors," *IEEE Trans. Ind. Appl.*, vol. 46, no. 6, pp. 2346–2354, Nov./Dec. 2010.

[6] D. Torregrossa, F. Peyraut, B. Fahimi, J. M'Boua, and A. Miraoui, "Multiphysics finite-element modeling for vibration and acoustic analysis of permanent magnet synchronous machine," *IEEE Trans. Energy Convers.*, vol. 26, no. 2, pp. 490–500, Jun. 2011.

[7] B. Kaku, I. Miyashita, and S. Sone, "A novel prediction method of acoustic magnetic noise based on induction motor's NHCC function," *IEEE Trans. Ind. Electron.*, vol. 46, no. 2, pp. 398–406, Apr. 1999.

[8] K.-H. Ha, Y.-K. Kim, G.-H. Lee, and J.-P. Hong, "Vibration reduction of switched reluctance motor by experimental transfer function and response surface methodology," *IEEE Trans. Magn.*, vol. 40, no. 2, pp. 577–580, Mar. 2004.

[9] Z. Tang, P. Pillay, and A. M. Omekanda, "Vibration prediction in switched reluctance motors with transfer function identification from shaker and force hammer tests," *IEEE Trans. Ind. Appl.*, vol. 39, no. 4, pp. 978–985, Jul./Aug. 2003.

[10] A. Saito, M. Kuroishi, and H. Nakai, "Empirical vibration synthesis method for electric machines by transfer functions and electromagnetic analysis," *IEEE Trans. Energy Convers.*, vol. 31, no. 4, pp. 1601–1609, Dec. 2016.

[11] D. Torregrossa, B. Fahimi, F. Peyraut, and A. Miraoui, "Fast computation of electromagnetic vibrations in electrical machines via field reconstruction method and knowledge of mechanical impulse response," *IEEE Trans. Ind. Electron.*, vol. 59, no. 2, pp. 839–847, Feb. 2012.

[12] J. Krotsch and B. Piepenbreier, "Radial forces in external rotor permanent magnet synchronous motors with non-overlapping windings," *IEEE Trans. Ind. Electron.*, vol. 59, no. 5, pp. 2267–2276, May 2012.

[13] C. Lin and B. Fahimi, "Prediction of acoustic noise in switched reluctance motor drives," *IEEE Trans. Energy Convers.*, vol. 29, no. 1, pp. 250–258, Mar. 2014.

[14] M. Boesing, T. Schoenen, K. A. Kasper, and R. W. De Doncker, "Vibration synthesis for electrical machines based on force response superposition," *IEEE Trans. Magn.*, vol. 46, no. 8, pp. 2986–2989, Aug. 2010.

[15] D. Braunisch, B. Ponick, and G. Bramerdorfer, “Combined analytical–numerical noise calculation of electrical machines considering nonsinusoidal mode shapes,” *IEEE Trans. Magn.*, vol. 49, no. 4, pp. 1407–1415, Apr. 2013.

[16] J. Le Besnerais, V. Lanfranchi, M. Hecquet, and P. Brochet, “Multiobjective optimization of induction machines including mixed variables and noise minimization,” *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1102–1105, Jun. 2008.

[17] J. Le Besnerais *et al.*, “Multiphysics modeling: Electro-vibro-acoustics and heat transfer of induction machines,” in *Proc. 18th Int. Conf. Electr. Mach.*, Vilamoura, Portugal, Sep. 2008, pp. 1–6.

[18] N. Bracikowski, M. Hecquet, P. Brochet, and S. V. Shirinskii, “Multiphysics modeling of a permanent magnet synchronous machine by using lumped models,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 6, pp. 2426–2437, Jun. 2012.

[19] A. Frias *et al.*, “Rotor and stator shape optimization of a synchronous machine to reduce iron losses and acoustic noise,” in *Proc. IEEE Vehicle Power Propuls. Conf.*, Seoul, South Korea, Oct. 2012, pp. 98–103.

[20] G.-J. Park, J.-S. Kim, B. Son, and S.-Y. Jung, “Optimal design of PMa-synRM for an electric propulsion system considering wide operation range and demagnetization,” *IEEE Trans. Appl. Supercond.*, vol. 28, no. 3, Apr. 2018, Art. no. 5205804.

[21] G.-Z. Cao, J.-L. Fang, S.-D. Huang, J.-A. Duan, and J. F. Pan, “Optimization design of the planar switched reluctance motor on electromagnetic force ripple minimization,” *IEEE Trans. Magn.*, vol. 50, no. 11, Nov. 2014, Art. no. 8203404.

[22] Q. Wang, J. Li, R. Qu, and Y. Lu, “Design and optimization of a permanent magnet synchronous machine for low vibration and noise applications,” in *Proc. 21st Int. Conf. Electr. Mach. Syst. (ICEMS)*, Jeju-do, South Korea, Oct. 2018, pp. 280–284.

[23] K. J. Bathe, *Finite Element Procedures*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1996.

[24] O. Omar, N. Tounsi, E.-G. Ng, and M. A. Elbestawi, “An optimized rational fraction polynomial approach for modal parameters estimation from FRF measurements,” *J. Mech. Sci. Technol.*, vol. 24, no. 3, pp. 831–842, Mar. 2010.

[25] B. J. Schwarz and M. H. Richardson, “Experimental modal analysis,” *CSI Rel. Week*, vol. 35, no. 1, pp. 1–12, 1999.

[26] P. C. Desai, M. Krishnamurthy, N. Schofield, and A. Emadi, “Novel switched reluctance machine configuration with higher number of rotor poles than stator poles: Concept to implementation,” *IEEE Trans. Ind. Electron.*, vol. 57, no. 2, pp. 649–659, Feb. 2010.

[27] B. Bilgin, A. Emadi, and M. Krishnamurthy, “Design considerations for switched reluctance machines with a higher number of rotor poles,” *IEEE Trans. Ind. Electron.*, vol. 59, no. 10, pp. 3745–3756, Oct. 2012.

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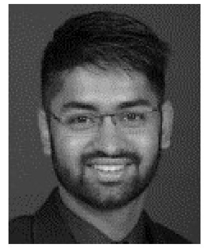

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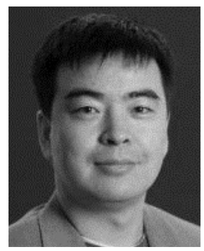

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