

Scheduling Real-Time Information-Update Flows for the Optimal Confidence in Estimation

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Abstract— This paper considers a wireless network where multiple flows are delivering status updates about their respective information sources. An end-user aims to make accurate real-time estimations about the status of each information source using its received packets. As the accuracy of estimation is most impacted by events in the recent past, we propose to measure the *Confidence-in-Estimation* by the number of timely deliveries in a window of the recent past, and say that a flow suffers from a *Loss-of-Confidence* (LoC) if this number is insufficient for the end user to make a reliable estimation with small confidence intervals.

We then study the problem of minimizing the system-wide LoC in wireless networks where each flow has a different requirement and link quality. We show that the problem of minimizing the system-wide LoC requires the control of the temporal variance of timely deliveries for each flow. This feature makes our problem significantly different from other optimization problems that only involve the average of control variables. Surprisingly, we show that there exists a simple online scheduling algorithm that is near-optimal. Simulation results show that our proposed algorithm is significantly better than other state-of-the-art policies. The practical value of this work is further evaluated by a case study of the real-time estimation problem of linear Gaussian processes, where we show that, under the optimal estimate algorithm, our scheduling policy results in better estimate accuracy, both in terms of the average mean square error and 95-percentile of mean square error, than other policies, including one that aims to optimize Age-of-Information, another performance metric for the application of real-time estimation.

Index Terms—Wireless Networks, Packet Scheduling, Optimization, Information-update Systems, Real-time Estimation

I. INTRODUCTION

Many emerging applications in the information-update system, such as industrial Internet-of-Things (IIoT) and Virtual-Reality (VR), require the real-time delivery of information. From an end user's perspective, the performance of such applications is determined by their ability to accurately estimate the real-time status of their respective information sources, such as the temperature of a machine in industrial IIoT or the location of a monster in a VR game. Further, most estimation techniques have two following features: First,

Part of this work has been presented at 17th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, WIOPT 2019, Avignon, France, 3–7 June, 2019 [1]. This work significantly expands [1] by including more theoretical analysis and simulation studies, as well as a case study on a real-time estimation problem.

information collected in the recent past is much more useful than that in the distant past for making an accurate or reliable estimate. Second, most estimation algorithms, even simple ones like linear extrapolation, require multiple data points in the recent past. However, most existing network performance metrics, ranging from traditional Quality-of-Service (QoS) metrics such as throughput, delay, and jitter, to emerging ones like Timely-Throughput and Age-of-Information (AoI), fail to directly capture those features of the users' estimation. Therefore, network algorithms aiming at optimizing these network performance metrics may result in poor performance for these emerging applications.

To address the need for such applications, we borrow the idea of a “confidence interval” from statistics, and introduce a new metric called Confidence-in-Estimation (CiE). In statistics, a small confidence interval means that the ground truth falls in a small range around the estimate with a pre-specified probability, and therefore implies that the estimate is accurate. We then aim to define CiE to reflect whether the network performance of wireless information-update systems leads to small confidence intervals for the end-user.

Our proposal for the CiE is directly based on two above features of general estimation algorithms: We consider that each information source, such as sensors generating readings or VR servers generating video frames, generates information updates periodically. Stale information is dropped in favor of the transmission of new information. The CiE of an information flow only depends on the number of packets that are delivered on time in a window of the recent past. If the number of timely deliveries in this window of the recent past is below a user-specified threshold, then the resulting estimate will have a wide confidence interval, and we therefore say that this flow suffers from a Loss-of-Confidence (LoC). Our goal is to minimize the system-wide LoC in a wireless network with multiple flows, each with different threshold and channel reliability.

Using Brownian approximation and martingale theory, we show that the problem of minimizing the system-wide LoC is equivalent to an optimization problem that involves two sets of constraints: One set of constraints are related to the average of timely deliveries of each flow, and another set of constraints are related to the temporal variance of timely

deliveries. The existence of constraints about the variance of timely deliveries makes this problem significantly different from other Network-Utility-Maximization (NUM) problems that only involve constraints about the average of variables, and hence cannot be solved by most existing techniques for NUM problems.

We propose a simple online scheduling algorithm for this problem. We analytically prove that the timely deliveries under our scheduling algorithm satisfy both the constraints on the average and those on the variance in the optimization problem. We also analytically prove that our algorithm is near-optimal for the optimization problem in the sense that its performance can be made arbitrarily close to a theoretical bound.

In the evaluation, we compare our algorithm against two other state-of-the-art policies, one of them is provably optimal in terms of timely-throughput, and the other achieves an approximation bound in terms of Age-of-Information. Simulation results show that our policy achieves much smaller LoC than these two policies.

Finally, to verify the practicality of our assumptions about the confidence of information-update flows, we conduct a case study on the real-time estimation problem of linear Gaussian processes. We consider the scenario where there are multiple sensors generating noisy measurements of their monitored processes, and an estimator makes real-time estimation of all processes based on its received information. We run simulations for this estimator and evaluate its resulting mean square error. The result shows that our policy receives the best performance when compared to the other two policies, in terms of the average mean square estimate error and the 95-percentile of mean square estimate error. This result also demonstrates that the concept of confidence does capture the performance of general information-update remote-estimation problems, and provides more insights than AoI-based models in the reliability of estimate in more general cases.

The rest of this paper is as following order: Section II introduces our model for confidence in real-time wireless networks. Section III shows that the problem of minimizing LoC is equivalent to an optimization problem. Section IV introduces our online scheduling algorithm. Section V analyzes the performance of our scheduling algorithm and shows that it is near-optimal for the optimization problem. Section VI presents our simulation results. Section VII conducts a case study on the real-time estimation problem of linear Gaussian processes. Section VIII reviews some related work. Finally, Section IX concludes this paper.

II. SYSTEM MODEL

We extend the model in [2], which focuses on the short-term performance for wireless networks with homogeneous links, to address the confidence in information flows estimation in real-time wireless network where different wireless links can have different channel qualities.

We consider a real-time wireless network that serves $\mathcal{N}$ clients. Time is slotted, and the duration of one time slot is the amount of time needed by a whole transmission, including

all overheads such as the transmission of poll packet or ACK. Hence, the AP can transmit to at most one client at each time slot, and it has the instantaneous feedback information on whether the transmission is successful. We consider that wireless transmissions are subject to effects of shadowing, multi-path, fading, interference, etc., and different clients experience different channel qualities as they are located at different positions. Hence, we assume that each transmission for client i is successful with probability p_i .

We consider that each client is associated with a real-time information-update flow, and use flow i to indicate the flow associated with client i . Specifically, we assume that each real-time flow generates one packet periodically every τ slots, that is at time slots $1, \tau+1, 2\tau+1, \dots$. Each packet has a stringent delay bound of τ slots, and is removed from the system if it cannot be delivered before its delay bound. In other words, each packet in a real-time flow is only valid for transmission until the next packet arrives. We thereby say that τ time slots form an *interval*. Packets arrive at the system at the beginning of each interval, and have deadlines at the end of the interval.

We note that this model for real-time flows applies to many emerging wireless applications. For example, consider multi-user virtual reality (VR) or augmented reality (AR), where an AP streams VR/AR contents to multiple VR/AR headsets. All headsets play VR/AR contents at the same frame rates, and therefore they generate traffic at the same frequency. Further, as the AP should always transmit the newest VR/AR content to a headset, packets that fail to be delivered on time should be removed and replaced by newer packets. Likewise, one can also consider industrial Internet of Things (IoT), where an AP polls measurements from multiple sensors monitoring different locations. Sensors have the same sampling frequency and therefore generate traffic at the same frequency. Also, stale measurements should be dropped when a new measurement is generated.

An important feature of real-time application such as VR/AR and industrial IoT is that each flow can typically tolerate a small amount of sporadic packet losses, but is very sensitive to a burst of packet losses. For example, in industrial IoT, a controller can use various estimation techniques to estimate the value of a lost sensor reading. However, the accuracy of the estimate significantly degrades if there is a burst packet losses. Further, it is obvious that the accuracy of the estimate only depends on the deliveries of recent sensor readings, and readings in the distant past have negligible effect on the estimation accuracy. We thereby say that the *Confidence-in-Estimation (CiE)* of one information flow relies on if its delivered packets enable the controller to make an accurate estimation.

The goal of this paper is to define and optimize the *CiE* for each information flow. To capture the aforementioned feature of real-time applications, we assume that the confidence of a real-time flow estimated status at a given point of time only depends on the packet deliveries in the *window* of past T intervals. Specifically, let $X_i(t)$ be the total number of timely-deliveries for flow i in the first t intervals. We then have

$X_i(t) - X_i(t-1) = 1$ if a packet is delivered to client i in interval t , and $X_i(t) - X_i(t-1) = 0$ if not. The number of timely-deliveries in the window of the last T intervals can then be represented as $X_i(t) - X_i(t-T)$, and we assume that the CiE of flow i at the end of interval t only depends on the value of $X_i(t) - X_i(t-T)$.

We assume that, to make an accurate estimate, each client i requires that there are at least $q_i T$ packets being delivered in the past T intervals, i.e., $X_i(t) - X_i(t-T) \geq q_i T$. The value of q_i depends on the context of the information flow. For example, a sensor monitoring a high-frequency signal requires a larger q_i than one that is monitoring a low-frequency signal.

Due to the unreliable nature of wireless transmissions, it is obvious that it is not possible to satisfy the requirements of all clients at all time. When the AP fails to deliver $q_i T$ packets for a client i , then the estimation of current state of client i becomes less accurate, and therefore we say that flow i *loses confidence*.

We now formally define the measure of *Loss-of-Confidence* (LoC). Suppose $X_i(t) - X_i(t-T) < q_i T$ for some i and t . Recall that every transmission for client i is successful with probability p_i . Hence, the AP would have needed to, on average, schedule $\frac{q_i T - (X_i(t) - X_i(t-T))}{p_i}$ more transmissions for client i to make $X_i(t) - X_i(t-T) = q_i T$. We therefore define the *unbiased shortage* of client i at the end of interval t as $\theta_i(t) := \max\{\frac{q_i T - (X_i(t) - X_i(t-T))}{p_i}, 0\}$. At the end of each interval t , each client i suffers from a LoC of $C(\theta_i(t))$ based on its unbiased shortage, where $C(\cdot)$ is a strictly increasing, strictly convex, and differentiable function with $C(0) = 0$ and $C'(0) = 0$.

This paper aims to evaluate the long-time average total LoC of all clients in the system, which is written as
$$\lim_{T \rightarrow \infty} \frac{\sum_{t=T+1}^{T+T} \sum_{i=1}^N C(\theta_i(t))}{T} = \lim_{T \rightarrow \infty} \frac{\sum_{t=T+1}^{T+T} \sum_{i=1}^N C(\frac{q_i T - X_i(t) - X_i(t-T)}{p_i})}{T}.$$
 It also aims to propose an online scheduling policy¹ that minimizes the total LoC.

III. THE FORMULATION OF THE OPTIMIZATION PROBLEM

In this section, we derive some fundamental properties about the minimization of total LoC. We then formulate an optimization problem.

Recall that $X_i(t)$ is the total number of timely-deliveries for client i in the first t intervals. Obviously, $\{X_i(1), X_i(2), \dots\}$ is a sequence of random variables whose distribution is determined by the employed packet scheduling policy. For simplicity, we only focus on ergodic scheduling policies in this paper. Thus, the random variable $\{X_i(t) - X_i(t-T)\}$ can be modeled by a positive recurrent Markov chain. By the law of large numbers, we can define $\bar{X}_i := \lim_{t \rightarrow \infty} \frac{X_i(t)}{t}$. Further, following the central limit theorem of Markov chains [3], $\hat{X}_i := \lim_{T \rightarrow \infty} \frac{X_i(T) - T\bar{X}_i}{\sqrt{T}}$ is a Gaussian random variable with mean 0 and some finite variance, which we denote by σ_i^2 , with $\sigma_i \geq 0$. Hence, we can approximate $X_i(t) - X_i(t-T)$

as a Gaussian random variable with mean $T\bar{X}_i$ and variance $T\sigma_i^2$ when T is reasonably large. Let $\Phi(x)$ represents the cumulative distribution function of a random variable under standard normal distribution, then, under this approximation, we have that the CDF of $(X_i(t) - X_i(t-T)) - T\bar{X}_i$ is $\Phi(\frac{x}{\sqrt{\sigma_i^2 T}})$.

The long-term average total LoC can now be re-written as below:

$$\begin{aligned} & \lim_{T \rightarrow \infty} \frac{\sum_{t=T+1}^{T+T} \sum_{i=1}^N C(\frac{q_i T - X_i(t) - X_i(t-T)}{p_i})}{T} \\ &= \lim_{T \rightarrow \infty} \sum_{i=1}^N E[C(\frac{q_i T}{p_i} - \frac{X_i(T) - X_i(T-T)}{p_i})] \\ &\approx \lim_{T \rightarrow \infty} \sum_{i=1}^N E[C(\frac{q_i T}{p_i} - \frac{\sqrt{T}\hat{X}_i + T\bar{X}_i}{p_i})] \\ &= \sum_{i=1}^N \int_z C(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}) d\Phi(z). \end{aligned} \quad (1)$$

The approximation step is from the above definitions of $\bar{X}_i$ and $\hat{X}_i$. The last step is the expectation formula under the law of the unconscious statistician.

Let $[\bar{X}_i]$ and $[\sigma_i]$ be the vectors consisting of $\bar{X}_i$ and σ_i for all $1 \leq i \leq N$ respectively. Then, Eq. (1) can be viewed as has two sets of control variables: $[\bar{X}_i]$ and $[\sigma_i]$, since their values are determined by the employed policy. Below, we derive the corresponding constraints of these two sets of variables.

We first derive the constraints on $[\bar{X}_i]$. Previous work [4] has shown that, under any work-conserving policy², we have, for all t ,

$$E[\sum_{i=1}^N \frac{X_i(t) - X_i(t-1)}{p_i}] = \tau - I_{\{1,2,\dots,N\}}, \quad (2)$$

and

$$E[\sum_{i \in S} \frac{X_i(t) - X_i(t-1)}{p_i}] \leq \tau - I_S, \quad (3)$$

for any subset $S \subseteq \{1, 2, \dots, N\}$, where I_S is called the *idle time* and has been shown to be a well-defined constant under all work-conserving policies. Therefore, we have

$$\sum_{i=1}^N \frac{\bar{X}_i}{p_i} = \tau - I_{\{1,2,\dots,N\}}, \quad (4)$$

and

$$\sum_{i \in S} \frac{\bar{X}_i}{p_i} \leq \tau - I_S, \forall S \subseteq \{1, 2, \dots, N\}. \quad (5)$$

We further assume that, similar to the total resource pooling condition, the constraint $\sum_{i \in S} \frac{\bar{X}_i}{p_i} \leq \tau - I_S$ is not tight and can be ignored when S is not $\{1, 2, \dots, N\}$.

¹An online scheduling policy is a policy that determines which packet to transmit in each slot based on all system parameters and the entire history.

²A scheduling policy is called work-conserving if it always schedules a transmission when there is at least one packet available for transmission.

Now, we derive the constraint of $[\sigma_i]$. By (2), the sequence of random variables $\{\sum_{i=1}^N \frac{X_i(t)}{p_i} - t(\tau - I_{\{1,2,\dots,N\}}) | t = 1, 2, \dots\}$ is a martingale. By the martingale central limit theorem [5], $\hat{X}_{TOT} := \lim_{T \rightarrow \infty} \frac{\sum_{i=1}^N \frac{X_i(T)}{p_i} - T(\tau - I_{\{1,2,\dots,N\}})}{\sqrt{T}} = \lim_{T \rightarrow \infty} \frac{\sum_{i=1}^N \frac{X_i(T)}{p_i} - T(\sum_{i=1}^N \frac{\bar{X}_i}{p_i})}{\sqrt{T}}$ is a Gaussian random variable with mean 0, and its variance is

$$\sigma_{TOT}^2 := \lim_{T \rightarrow \infty} \frac{1}{T} \left[\sum_{t=1}^T \left(\sum_{i=1}^N \frac{X_i(t) - X_i(t-1)}{p_i} \right)^2 - (\tau - I_{\{1,2,\dots,N\}})^2 \right], \quad (6)$$

whose value depends on the employed scheduling policy.

Recall that $\hat{X}_i := \lim_{T \rightarrow \infty} \frac{X_i(T) - T\bar{X}_i}{\sqrt{T}}$ is a Gaussian random variable with variance σ_i^2 . Hence, we have $\hat{X}_{TOT} = \sum_{i=1}^N \frac{\hat{X}_i}{p_i}$, and the variance of $\frac{\hat{X}_i}{p_i}$ is $(\frac{\sigma_i}{p_i})^2$. By Cauchy-Schwarz Inequality, we have:

$$\begin{aligned} \left(\sum_{i=1}^N \frac{\sigma_i}{p_i} \right)^2 &= \left(\sum_{i=1}^N \sqrt{\text{Var}\left(\frac{\hat{X}_i}{p_i}\right)} \right)^2 \\ &= \sum_{i=1}^N \text{Var}\left(\frac{\hat{X}_i}{p_i}\right) \\ &\quad + 2 \sum_{l=1}^N \sum_{m=l+1}^N \sqrt{\text{Var}\left(\frac{\hat{X}_l}{p_l}\right) \text{Var}\left(\frac{\hat{X}_m}{p_m}\right)} \\ &\geq \sum_{i=1}^N \text{Var}\left(\frac{\hat{X}_i}{p_i}\right) + 2 \sum_{l=1}^N \sum_{m=l+1}^N \text{Cov}\left(\frac{\hat{X}_l}{p_l}, \frac{\hat{X}_m}{p_m}\right) \\ &= \text{Var}\left(\sum_{i=1}^N \frac{\hat{X}_i}{p_i}\right) = \sigma_{TOT}^2, \end{aligned} \quad (7)$$

where $\text{Var}(X)$ denotes the variance of X and $\text{Cov}(X, Y)$ denotes the covariance.

Although the value of σ_{TOT} may be different for different scheduling policies, we first consider the special case of minimizing the total LoC when σ_{TOT} is given and fixed. By (1), (4), and (7), the optimization problem can be written as:

$$\text{Min } L = \sum_{i=1}^N \int_z C\left(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}\right) d\Phi(z) \quad (8)$$

$$\text{s.t. } \sum_{i=1}^N \frac{\bar{X}_i}{p_i} = \tau - I_{\{1,2,\dots,N\}} \quad (9)$$

$$\sum_{i=1}^N \frac{\sigma_i}{p_i} \geq \sigma_{TOT}. \quad (10)$$

Theorem 1. Let $[\bar{X}_i^*]$ and $[\sigma_i^*]$ be the optimal solution to (8) – (10). Then $\bar{X}_i^* = (\frac{\tau - I_{\{1,2,\dots,N\}}}{N} - \sum_{j=1}^N \frac{q_j}{N p_j} + \frac{q_i}{p_i}) p_i$, and $\sigma_i^* = \frac{\sigma_{TOT}}{N} p_i$, for all $1 \leq i \leq N$.

Proof. Since $C(\cdot)$ is a convex function, we have:

$$\begin{aligned} L &= \sum_{i=1}^N \int_z C\left(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}\right) d\Phi(z) \\ &\geq N \int_z C\left(\frac{1}{N} \sum_{i=1}^N \left(\sqrt{\frac{\sigma_i^2 T}{p_i^2}} z - \frac{(\bar{X}_i - q_i)T}{p_i}\right)\right) d\Phi(z) \\ &\geq N \int_z C\left(\frac{1}{N} (\sigma_{TOT} \sqrt{T} z - \sum_{i=1}^N \frac{(\bar{X}_i - q_i)T}{p_i})\right) d\Phi(z), \end{aligned}$$

with equality occurs when $\frac{\bar{X}_i^*}{p_i} - \frac{q_i}{p_i} = \frac{\bar{X}_j^*}{p_j} - \frac{q_j}{p_j}$ and $\frac{\sigma_i^*}{p_i} = \frac{\sigma_j^*}{p_j}$ for any $i, j \in \{1, 2, \dots, N\}$. By (9) and (10), we have $\bar{X}_i^* = (\frac{\tau - I_{\{1,2,\dots,N\}}}{N} - \sum_{j=1}^N \frac{q_j}{N p_j} + \frac{q_i}{p_i}) p_i$ and $\sigma_i^* = \frac{\sigma_{TOT}}{N} p_i$. $\square$

Theorem 1 establishes the optimal $\{\bar{X}_i\}$ and $\{\sigma_i\}$ that minimizes the total LoC when σ_{TOT} is given and fixed. Obviously, smaller σ_{TOT} leads to smaller total LoC. Therefore, we seek to solve the optimization problem below, which aims to minimizing σ_{TOT} while satisfying the results of Theorem 1:

$$\text{Min } \sigma_{TOT}^2 := \lim_{T \rightarrow \infty} \frac{1}{T} \left[\sum_{t=1}^T \left(\sum_{i=1}^N \frac{X_i(t) - X_i(t-1)}{p_i} \right)^2 - (\tau - I_{\{1,2,\dots,N\}})^2 \right] \quad (11)$$

$$\text{s.t. } \bar{X}_i = \bar{X}_i^*, \forall 1 \leq i \leq N \quad (12)$$

$$\sigma_i = \frac{\sigma_{TOT}}{N} p_i, \quad \forall 1 \leq i \leq N, \quad (13)$$

where $\bar{X}_i^* := (\frac{\tau - I_{\{1,2,\dots,N\}}}{N} - \sum_{j=1}^N \frac{q_j}{N p_j} + \frac{q_i}{p_i}) p_i$.

We note that the problem (11) – (13) involves both a constraint on the average of $X_i(t)$ (12) and a constraint on the variance of $X_i(t)$ (13) for each i . Most existing studies on network utility maximization (NUM) problem only addresses constraints on the average of decision variables, and therefore cannot be applied to solve (11) – (13). In fact, no stationary randomized policies can optimally solve (11) – (13). In the following sections, we will establish the surprising result that there exists a simple online scheduling policy that is near-optimal for the problem (11) – (13).

IV. AN ONLINE SCHEDULING POLICY

In this section, we propose a simple online scheduling policy for the problem (11) – (13). We first provide a brief outline of the construction of our algorithm. First, we remove the constraint on variance (13) and focus on the following optimization problem:

$$\text{Min } \lim_{T \rightarrow \infty} \frac{1}{T} \left[\sum_{t=1}^T \left(\sum_{i=1}^N \frac{X_i(t) - X_i(t-1)}{p_i} \right)^2 - (\tau - I_{\{1,2,\dots,N\}})^2 \right] \quad (14)$$

$$\text{s.t. } \bar{X}_i = \bar{X}_i^*, \forall 1 \leq i \leq N. \quad (15)$$

Obviously, this optimization problem is a lower bound to the original problem (11) – (13). It is also a standard NUM problem that only involves a constraint on the average of $X_i(t)$

for each i . We can therefore derive a near-optimal online scheduling algorithm using the Drift-Plus-Penalty approach [6]. We further demonstrate the surprising result that, due to the specific choice of our Lyapunov function, our algorithm also satisfies the constraint on variance (13). Therefore, our algorithm is near-optimal to the original problem (11) – (13).

We now introduce some notations that are necessary for the design and analysis of our algorithm. Let $d_i(t) := \frac{\bar{X}_i^* t}{p_i} - \frac{X_i(t)}{p_i}$ be the *deficit* of client i in interval t . Obviously, we have $\bar{X}_i := \lim_{t \rightarrow \infty} \frac{X_i(t)}{t} = \bar{X}_i^*$ if and only if $\lim_{t \rightarrow \infty} \frac{d_i(t)}{t} = 0$. We also define $\Delta d_i(t) := d_i(t+1) - d_i(t) = \frac{\bar{X}_i^*}{p_i} - \frac{X_i(t+1) - X_i(t)}{p_i}$ and $D(t) := \frac{\sum_{i=1}^N d_i(t)}{N}$.

We consider the Lyapunov function $L(t) = \frac{1}{2} \sum_{i=1}^N [d_i(t) - D(t)]^2$. The drift of the Lyapunov function is $\Delta L(t) := E[L(t+1) - L(t) | d_i(t)]$.

Given $[d_i(t)]$, we have, under any scheduling policy,

$$\begin{aligned}
\Delta L(t) &= E[L(t+1) - L(t)] \\
&= E\left[\frac{1}{2} \sum_{i=1}^N (d_i(t+1) - D(t+1))^2 - \frac{1}{2} \sum_{i=1}^N (d_i(t) - D(t))^2\right] \\
&= E\left[\frac{1}{2} \sum_{i=1}^N (d_i(t) - D(t) + \Delta d_i(t) - \frac{\sum_{i=1}^N \Delta d_i(t)}{N})^2\right] \\
&\quad - E\left[\frac{1}{2} \sum_{i=1}^N (d_i(t) - D(t))^2\right] \\
&= E\left[\frac{1}{2} \sum_{i=1}^N (\Delta d_i(t) - \frac{\sum_{i=1}^N \Delta d_i(t)}{N})^2\right] \\
&\quad + \sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - D(t)) \\
&\quad - E\left[\frac{\sum_{i=1}^N \Delta d_i(t)}{N}\right] \sum_{i=1}^N (d_i(t) - D(t)) \\
&\leq \beta + \sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - D(t)), \tag{16}
\end{aligned}$$

where β is a bounded positive number. The last inequality holds since $\Delta d_i(t)$ is bounded by $\frac{\bar{X}_i^* - 1}{p_i} \leq \Delta d_i(t) \leq \frac{\bar{X}_i^*}{p_i}$ and $\sum_{i=1}^N d_i(t) = ND(t)$.

Our scheduling algorithm is based on the Drift-Plus-Penalty approach [6]. Let

$$\begin{aligned}
B(t) &:= \sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - D(t)) \\
&\quad + \epsilon E\left[\left(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i}\right)^2\right], \tag{17}
\end{aligned}$$

where ϵ is a positive number whose value can be arbitrary determined by the system designer. We then have

$$\Delta L(t) + \epsilon E\left[\left(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i}\right)^2\right] \leq \beta + B(t). \tag{18}$$

We aim to design an online scheduling algorithm that minimizes $B(t)$. Note that the value of $B(t)$ depends on the scheduling decisions on all time slots within the interval t , which consists of τ time slots. Minimizing an objective function over a finite horizon of τ time slots typically requires the usage of dynamic programming. However, we will show that there exists a simple online scheduling algorithm that minimizes $B(t)$.

Our algorithm is called the *Minimum-Drift-and-Variance-First* (MDVF) policy. Under the MDVF policy, the AP calculates the value of $e_i(t) := \epsilon \frac{1}{p_i} - d_i(t)$ at the beginning of each interval t . In each time slot within the interval, the AP finds the undelivered packet with the smallest $e_i(t)$ and transmits that packet, as long as there is at least one packet to be transmitted. Alg. 1 provides a detailed description of the algorithm, where we streamline some of the steps to simplify the implementation.

Algorithm 1: The MDVF Policy

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Initialization:  $t = 0, d_i = 0, \forall i$ ;
while each new interval do
  for each  $i$  do
     $e_i = \epsilon \frac{1}{p_i} - d_i$ ;
     $d_i = d_i + \frac{\bar{X}_i^*}{p_i}$ ;
  end
  Sort all flows such that  $e_1(t) \leq e_2(t) \leq \dots$ ;
   $i = 1$ ; for each time slot do
    Transmit packet  $i$ ;
    if transmission is successful then
       $d_i = d_i - \frac{1}{p_i}$ ;
       $i = i + 1$ ;
    end
  end
   $t = t + 1$ ;
end

```

We now show that the MDVF policy indeed minimizes $B(t)$.

Lemma 1. *The MDVF policy minimizes $B(t)$.*

Proof. We prove this lemma by induction. First, we consider the optimal scheduling decision in the last time slot of the interval. At this time, some packets have already been delivered in the previous $\tau - 1$ slots, and we use V to denote the set of clients whose packets have already been delivered. As this is the last time slot of the interval, the scheduling decision of the AP only consists of choosing one client $u \notin V$ and transmitting its packet. Given V and u , we will calculate the value of $\sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - D(t)) + \epsilon E\left[\left(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i}\right)^2\right]$.

For this chosen client u , its packet will be delivered, that is, $X_u(t+1) - X_u(t) = 1$, with probability p_u , and $X_u(t+1) - X_u(t) = 0$, with probability $1 - p_u$. Hence, we have $E[\Delta d_u(t)] = \frac{\bar{X}_u - p_u}{p_u}$.

On the other hand, for each client $i \in V$, its packet has already been delivered. We have $X_v(t) - X_v(t-1) = 1$ and $E[\Delta d_i(t)] = \frac{\bar{X}_i - 1}{p_i}$.

Finally, for each client $i \notin V \cup \{u\}$, its packet will not be delivered, and we have $X_i(t) - X_i(t-1) = 0$ and $E[\Delta d_i(t)] = \frac{\bar{X}_i}{p_i}$.

We now have, given V and u ,

$$\begin{aligned} & \sum_{i=1}^N E[\Delta d_i(t)] [d_i(t) - D(t)] \\ & + \epsilon E\left[\left(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i}\right)^2\right] \\ & = \frac{\bar{X}_u - p_u}{p_u} [d_u(t) - D(t)] + \sum_{i \in V} \frac{\bar{X}_i - 1}{p_i} [d_i(t) - D(t)] \\ & + \sum_{i \notin V \cup \{u\}} \frac{\bar{X}_i}{p_i} [d_i(t) - D(t)] \\ & + \epsilon \left[p_u \left(\sum_{i \in V} \frac{1}{p_i} + \frac{1}{p_u} \right)^2 + (1 - p_u) \left(\sum_{i \in V} \frac{1}{p_i} \right)^2 \right] \\ & = \epsilon \frac{1}{p_u} - d_u(t) + \lambda(V), \end{aligned} \quad (19)$$

where $\lambda(V) := D(t) + \sum_{i=1}^N \frac{\bar{X}_i}{p_i} [d_i(t) - D(t)] - \sum_{i \in V} \frac{1}{p_i} [d_i(t) - D(t)] + \epsilon \left[\left(\sum_{i \in V} \frac{1}{p_i} \right)^2 + 2 \left(\sum_{i \in V} \frac{1}{p_i} \right) \right]$ is the same regardless of the choice of u . Therefore, it is clear that an optimal scheduling algorithm that minimizes $B(t)$ will schedule the undelivered packet u with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ in the last time slot.

Now, assume that, starting from the $(s+1)$ -th time slot in an interval, scheduling the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ in each of the remaining time slot is optimal. We will show that, even in the s -th time slot, scheduling the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ is optimal.

We prove this claim by contradiction. Let u^* be the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ in time slot s . If the claim is false, then the optimal scheduling algorithm, which we denote by $\mathbb{A}$, would schedule another undelivered packet $u' \neq u^*$ in time slot s , and the value of $B(t)$ under $\mathbb{A}$ is strictly smaller than any policy that schedules u^* in the s -th time slot. By the induction hypothesis, $\mathbb{A}$ begins to schedule the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ starting from the $(s+1)$ -th time slot. As u^* is not scheduled by $\mathbb{A}$ in the s -th time slot, $\mathbb{A}$ needs to schedule u^* in the $(s+1)$ -th time slot. In summary, $\mathbb{A}$ schedules u' in the s -th time slot, and u^* in the $(s+1)$ -th time slot.

Now, we can construct another algorithm $\mathbb{B}$ by simply swapping the transmissions in the s -th time slot and the $(s+1)$ -th time slot. In other words, $\mathbb{B}$ schedules u^* in the s -th time slot, u' in the $(s+1)$ -th time slot, and then follows $\mathbb{A}$ starting

from the $(s+2)$ -th time slot. Obviously, the value of $B(t)$ under $\mathbb{A}$ and $\mathbb{B}$ is the same, which results in a contradiction.

We have established that, even in the s -th time slot, scheduling the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ is optimal. By induction, scheduling the undelivered packet with the smallest $\epsilon \frac{1}{p_u} - d_u(t)$ in each time slot is optimal, and MDVF minimizes $B(t)$. $\square$

V. PERFORMANCE ANALYSIS OF THE MDVF POLICY

We now study the performance of the MDVF policy. We will demonstrate the surprising result that the MDVF policy satisfies both constraints on mean (12) and variance (13), and the value of σ_{TOT}^2 under the MDVF policy can be made arbitrary close to a lower bound. Throughout this section, we use $\cdot|_\eta$ to denote the value of $\cdot$ under a scheduling policy η . For example, $\Delta L(t)|_{\text{MDVF}}$ denotes the value of $\Delta L(t)$ under the MDVF policy.

We first establish the following property.

Theorem 2. *Under the MDVF policy, the Markov process with state vector $\{d_i(t) - D(t)\}$ is positive recurrent.*

Proof. We prove this theorem by establishing an upper bound of $\Delta L(t)|_{\text{MDVF}}$. To simplify notations, we let Ω be the policy that schedules the undelivered packet with the maximum value of $d_i(t)$. We also sort all clients such that $d_1(t) \geq d_2(t) \geq \dots \geq d_N(t)$. Then Ω will only transmit a packet for client i if, for each $j < i$, the packet for flow j has already been delivered. This is equivalent to the largest-debt-first policy in [4], and we have, for all $1 \leq j \leq N$:

$$\begin{aligned} \sum_{i=1}^j E[\Delta d_i(t)]|_\Omega &= \sum_{i=1}^j \frac{\bar{X}_i^*}{p_i} - E\left[\sum_{i=1}^j \frac{X_i(t+1) - X_i(t)}{p_i}\right]|_\Omega \\ &= \sum_{i=1}^j \frac{\bar{X}_i^*}{p_i} - (\tau - I_{\{1,2,\dots,j\}}). \end{aligned} \quad (20)$$

By (4), we have $\sum_{i=1}^N E[\Delta d_i(t)]|_\Omega = 0$. Further, as we assume that (5) is not tight when $S \neq \{1, 2, \dots, N\}$, there exists a positive number $\delta > 0$ such that $\sum_{i=1}^j E[\Delta d_i(t)]|_\Omega \leq -\delta$ for all $1 \leq j \leq N-1$. We now have

$$\begin{aligned} & \sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - D(t))|_\Omega \\ &= \sum_{i=1}^N E[\Delta d_i(t)] (d_i(t) - d_{i+1}(t) + d_{i+1}(t) \\ & \quad - d_{i+2}(t) + \dots - d_N(t) + d_N(t) - D(t))|_\Omega \\ &= \sum_{i=1}^N E[\Delta d_i(t)] (d_N(t) - D(t))|_\Omega \\ & \quad + \sum_{i=1}^j \sum_{j=1}^{N-1} E[\Delta d_i(t)] (d_j(t) - d_{j+1}(t))|_\Omega \\ & \leq -\delta \sum_{j=1}^{N-1} (d_j(t) - d_{j+1}(t)) = -\delta (d_1(t) - d_N(t)). \end{aligned} \quad (21)$$

Next, we study $\Delta L(t)|\text{MDVF}$. By Lemma 1, the MDVF policy minimizes $B(t)$. Hence, we have

$$\begin{aligned} & \Delta L(t)|\text{MDVF} + \epsilon E[(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\text{MDVF} \\ & \leq \beta + B(t)|\text{MDVF} \quad (\text{By (18)}) \\ & \leq \beta + B(t)|\Omega \\ & \leq \beta + \epsilon E[(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\Omega \\ & \quad - \delta(d_1(t) - d_N(t)) \quad (\text{By (17) and (21)}) \end{aligned} \quad (22)$$

Since $0 \leq X_i(t+1) - X_i(t) \leq 1$, there exists some constant M such that

$$\Delta L(t)|\text{MDVF} \leq -\delta(d_1(t) - d_N(t)) + M. \quad (23)$$

Recall that we have sorted all clients such that $d_1(t) \geq d_2(t) \geq \dots$. Hence, $(d_1(t) - d_N(t)) \geq 0$ and $(d_1(t) - d_N(t)) \geq |d_i(t) - D(t)|$, for all i . We have

$\Delta L(t)|\text{MDVF} < -\delta$, if $|d_i(t) - D(t)| > \frac{M}{\delta} + 1$, for some i , and

$$\Delta L(t)|\text{MDVF} \leq M, \text{ otherwise.} \quad (24)$$

By the Foster-Lyapunov Theorem, the Markov process with state vector $\{d_i(t) - D(t)\}$ is positive recurrent. $\square$

Now we are able to show that the MDVF policy satisfies both constraints (12) and (13).

Corollary 1. $\bar{X}_i|\text{MDVF} = \bar{X}_i^*$ and $\sigma_i|\text{MDVF} = \frac{\sigma_{TOT}|\text{MDVF}}{N} p_i, \forall i$.

Proof. Recall that $d_i(t) := \frac{\bar{X}_i^* t}{p_i} - \frac{X_i(t)}{p_i}$ and $D(t) := \frac{\sum_{i=1}^N d_i(t)}{N}$. By (4), we have:

$$\begin{aligned} & \lim_{\mathbb{T} \rightarrow \infty} \frac{D(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = \lim_{\mathbb{T} \rightarrow \infty} \frac{\sum_{i=1}^N d_i(\mathbb{T})|\text{MDVF}}{N\mathbb{T}} \\ & = \frac{1}{N} \sum_{i=1}^N \lim_{\mathbb{T} \rightarrow \infty} \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{p_i \mathbb{T}} \\ & = \frac{1}{N} \sum_{i=1}^N \frac{\bar{X}_i^*}{p_i} - \frac{1}{N} \sum_{i=1}^N \frac{\bar{X}_i(\mathbb{T})|\text{MDVF}}{p_i} \\ & = \frac{\tau - I_{\{1,2,\dots,N\}}}{N} - \frac{\tau - I_{\{1,2,\dots,N\}}}{N} = 0. \end{aligned} \quad (25)$$

By Theorem 2, the vector $\{d_i(t) - D(t)\}|\text{MDVF}$ converges to a steady state distribution as $t \rightarrow \infty$. Hence, both $\lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T}) - D(\mathbb{T})}{\mathbb{T}}|\text{MDVF}$ and $\lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T}) - D(\mathbb{T})}{\sqrt{\mathbb{T}}}|\text{MDVF}$ converge to 0 in probability. We then have

$$\begin{aligned} & \lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = \frac{\bar{X}_i^*}{p_i} - \frac{\bar{X}_i|\text{MDVF}}{p_i} \\ & = \lim_{\mathbb{T} \rightarrow \infty} \frac{D(\mathbb{T})|\text{MDVF}}{\mathbb{T}} = 0, \end{aligned} \quad (26)$$

and hence $\bar{X}_i|\text{MDVF} = \bar{X}_i^*$.

Next, we study $\sigma_i|\text{MDVF}$. Recall that σ_i^2 is the variance of $\hat{X}_i := \lim_{\mathbb{T} \rightarrow \infty} \frac{X_i(\mathbb{T}) - \mathbb{T}\bar{X}_i}{\sqrt{\mathbb{T}}}$. We then have:

$$\begin{aligned} \lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}} & = \lim_{\mathbb{T} \rightarrow \infty} \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{p_i \sqrt{\mathbb{T}}} \\ & = -\frac{\hat{X}_i|\text{MDVF}}{p_i}, \end{aligned}$$

since $\bar{X}_i|\text{MDVF} = \bar{X}_i^*$. This shows that the variance of $\lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}}$ is $\frac{\sigma_i^2|\text{MDVF}}{p_i^2}$.

Also, recall that σ_{TOT}^2 is the variance of $\hat{X}_{TOT} = \sum_{i=1}^N \frac{\hat{X}_i}{p_i}$. We have

$$\begin{aligned} \lim_{\mathbb{T} \rightarrow \infty} \frac{D(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}} & = \lim_{\mathbb{T} \rightarrow \infty} \frac{\sum_{i=1}^N d_i(\mathbb{T})|\text{MDVF}}{N\sqrt{\mathbb{T}}} \\ & = \lim_{\mathbb{T} \rightarrow \infty} \sum_{i=1}^N \frac{\mathbb{T}\bar{X}_i^* - X_i(\mathbb{T})|\text{MDVF}}{N p_i \sqrt{\mathbb{T}}} = -\sum_{i=1}^N \frac{\hat{X}_i|\text{MDVF}}{N p_i}, \end{aligned}$$

and the variance of $\lim_{\mathbb{T} \rightarrow \infty} \frac{D(\mathbb{T})|\text{MDVF}}{\sqrt{\mathbb{T}}}$ is $\frac{\sigma_{TOT}^2|\text{MDVF}}{N^2}$. As $\lim_{\mathbb{T} \rightarrow \infty} \frac{d_i(\mathbb{T}) - D(\mathbb{T})}{\sqrt{\mathbb{T}}}|\text{MDVF}$ converges to 0 in probability, we have $\sigma_i|\text{MDVF} = \frac{\sigma_{TOT}|\text{MDVF}}{N} p_i$. $\square$

We have shown that the MDVF policy satisfies both constraints (12) and (13). We now show that the value of $\sigma_{TOT}^2|\text{MDVF}$ can be made arbitrarily close to a theoretical lower bound.

Consider the problem (14) – (15), which ignores the constraint on variance (13). Since this problem only involves a constraint on mean, there exists a stationary randomized policy that is optimal, which we denote by ω . Obviously, $\sigma_{TOT}^2|\omega$ is a lower bound of the problem (11) – (13). We have the following theorem.

Theorem 3. $\sigma_{TOT}^2|\text{MDVF} \leq \sigma_{TOT}^2|\omega + \frac{\beta}{\epsilon}$.

Proof. Since ω is a stationary randomized policy that satisfies (15), we have $E[\Delta d_i(t)]|\omega = 0$, for all i and t . By (17), we have

$$B(t)|\omega = \epsilon E[(\sum_{i=1}^N \frac{X_i(t+1) - X_i(t)}{p_i})^2]|\omega = \epsilon \sigma_{TOT}^2|\omega.$$

Now, recall that the MDVF policy minimizes $B(t)$. Hence, for every t , we have

$$\begin{aligned} & \Delta L(t)|\text{MDVF} + \epsilon E[(\sum_{i=1}^N \frac{X_i(t) - X_i(t-1)}{p_i})^2]|\text{MDVF} \\ & \leq B(t)|\text{MDVF} + \beta \\ & \leq B(t)|\omega + \beta = \epsilon \sigma_{TOT}^2|\omega + \beta. \end{aligned}$$

Summing the above inequality over $t = 1$ to $t = \mathbb{T}$, and then divide both sides by $\mathbb{T}$ yields

$$\begin{aligned} & \frac{E[L(\mathbb{T}+1)] - E[L(0)]}{\mathbb{T}}|\text{MDVF} + \epsilon \sigma_{TOT}^2|\text{MDVF} \\ & \leq \epsilon \sigma_{TOT}^2|\omega + \beta. \end{aligned} \quad (27)$$

By Theorem 2, we have $\lim_{T \rightarrow \infty} \frac{E[L(T+1)] - E[L(0)]}{T} |MDVF = 0$, and hence $\sigma_{TOT}^2 |MDVF \leq \sigma_{TOT}^2 |\omega + \frac{\beta}{\epsilon}$. $\square$

We note that Theorem 3 holds for all ϵ , which is a constant that can be arbitrarily chosen by the system designer. By choosing a large ϵ , one can make $\sigma_{TOT}^2 |MDVF$ arbitrarily close to the lower bound $\sigma_{TOT}^2 |\omega$.

Therefore, combining Theorem 1 that gives the form of optimal solutions and Corollary 1 that shows the MDVF policy satisfies both constraints (12) and (13), the MDVF policy solves the optimization problem (11), (12) and (13).

VI. SIMULATION RESULTS

A. Simulation Settings

We present our simulation results in this section. We have implemented and tested our policy and two other state-of-the-art policies in ns-2. All simulations are conducted using the 802.11 MAC protocol with 54Mbps data rate. Simulations show that the time needed to transmit a packet and to receive an ACK is about 0.5ms. The duration of an interval is chosen to be 10ms, or, equivalently, 20 time slots. We evaluate the simulation for LoC in two convex functions: one is chosen to be $C(\theta_i(t)) := (\frac{\theta_i(t)}{T})^2$, which we call the *quadratic LoC function*, and the other is $C(\theta_i(t)) := e^{\frac{\theta_i(t)}{T}} - 1$, which we call the *exponential LoC function*. We note that both functions normalize $\theta_i(t)$ by T . Recall that $\theta_i(t)$ is the unbiased shortage accumulated in last T intervals. Hence, $\frac{\theta_i(t)}{T}$ can be thought of as the *average* unbiased shortage occurred in the last T intervals. By normalization, we are able to compare the LoC across different T . All results presented in this section are the average of 1000 runs.

We compare our MDVF policy against two other policies. The first policy is the largest debt first (LDF) policy in [4], [7]. In each interval t , the LDF policy sorts all clients in descending order of $q_i t - X_i(t)$, and transmits packets according to this ordering. It has been shown that LDF guarantees to deliver a long-term average timely-throughput of q_i to each client i , as long as it is feasible to do so. The second policy is a Max-Weight type of policy that aims to reduce the total age-of-information (AoI) in the network while guaranteeing some average timely-throughput policy [8]. We call this policy MW-AoI. Although the problem of minimizing AoI remains an open problem, it has been shown that the MW-AoI policy is 4-optimal in terms of AoI.

As for the network topology, we consider two different settings. In the first setting, there are 12 wireless clients. The channel reliability of client i is set to be $p_i = 0.9 - 0.05i$. We set $q_i = 0.85$ for the first 6 clients and $q_i = 0.75$ for the last 6 clients. We call this setting the *high-timely-throughput system*. In the second setting, there are 18 clients with $p_i = 1 - 0.05i$. We set $q_i = 0.35$ for all 18 clients. We call this setting the *low-timely-throughput system*.

B. The Impact of ϵ

Our MDVF policy makes scheduling decisions based on the value of $e_i(t) := \epsilon \frac{1}{p_i} - d_i(t)$ for each flow i , where ϵ

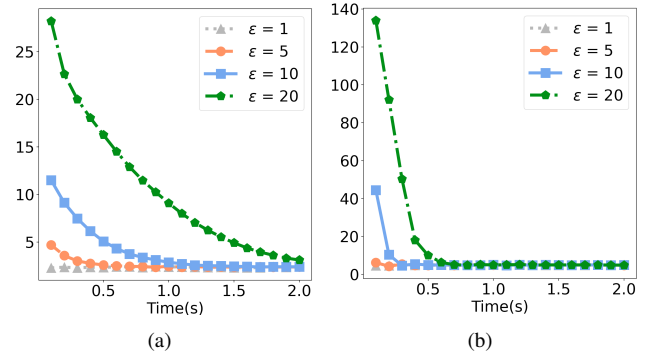


Fig. 1. The convergence of $e_{max}(t) - e_{min}(t)$ in (a) The high-timely-throughput system. (b) The low-timely-throughput system.

is a parameter determined by the system. Theorem. 3 has shown that $\sigma_{TOT}^2 |MDVF \leq \sigma_{TOT}^2 |\omega + \frac{\beta}{\epsilon}$. Therefore, larger ϵ leads to better steady-state performance. On the other hand, [9], [10] and [11] have shown that larger ϵ may lead to longer convergence time. In this section, we investigate the convergence speed of the MDVF policy under different values of ϵ .

Recall that our MDVF policy sorts all flows by their $e_i(t)$ and schedules packets according to the ordering in each interval t . Hence, when the system reaches steady-state, all flows should have roughly the same $e_i(t)$. Based on this observation, we evaluate the convergence speed of the MDVF policy as follows: In each simulation run and at each interval t , we find the flow with the largest $e_i(t) =: e_{max}(t)$ and the flow with the smallest $e_i(t) =: e_{min}(t)$. We then use $e_{max}(t) - e_{min}(t)$ as the indicator of convergence. Obviously, a small value of $e_{max}(t) - e_{min}(t)$ implies that the values of $e_i(t)$ are roughly the same for all flows. We then calculate the average of $e_{max}(t) - e_{min}(t)$ over 1000 simulation runs for all t .

Simulation results for different values of ϵ and for both the low-timely-throughput system and the high-timely-throughput system are shown in Fig. 1. Not surprisingly, it can be easily observed that, while $e_{max}(t) - e_{min}(t)$ converges to a small value for all settings, larger ϵ leads to longer convergence time. It can also be observed that the convergence speed of the setting with $\epsilon = 5$ is reasonably fast. By setting $\epsilon = 5$, both the high-timely-throughput system and the low-timely-throughput system converge in less than 0.5 second. Hence, in the sequel, we choose $\epsilon = 5$ for our MDVF policy.

C. The Approximation Accuracy in T

Throughout the paper, we assume the CLT approximation of Markov chain that $X_i(t) - X_i(t - T)$ can be approximated as a Gaussian random variable with mean $T\bar{X}_i$ and variance $T\sigma_i^2$ when T reasonably large as the duration of the window. Hence, in this section, we evaluate how large T needs to be for this approximation under our MDVF policy.

Recall the Corollary. 1, $\frac{\sigma_i |MDVF}{p_i}$ should have roughly the same value as $\frac{\sigma_{TOT} |MDVF}{N}$. Although $\sigma_{TOT} |MDVF$ is a near-optimal variable determined by the value of ϵ , the

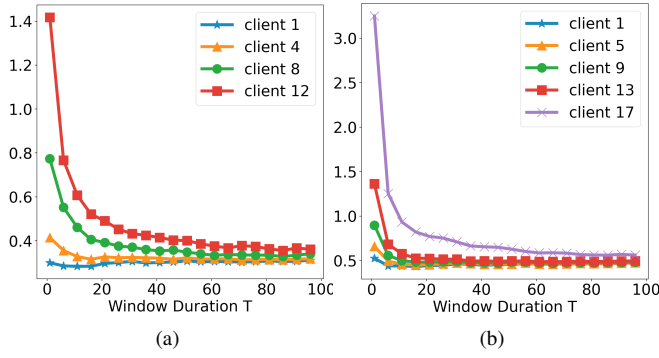


Fig. 2. The convergence of $\frac{\sigma_i}{p_i}$ in (a) The high-timely-throughput system. (b) The low-timely-throughput system.

values of $\frac{\sigma_i}{p_i}$ should converge across all clients to a same value when ϵ is fixed. Therefore, we design simulations in both the high-timely-throughput system and the low-timely-throughput system as follows: In each simulation run, t is set to be 500, and we obtain $X_i(500 + T) - X_i(500)$ for each client i , where T is set to be increasing in increments of 5, i.e. 1, 6, 11, 16, ..., 151. Consequently, we collect 1000 samples over all simulation runs, then calculate the value of $\frac{\sigma_i}{p_i}$ for all T .

Results are shown in Fig. 2. We plot the curve of client 1, 4, 8 and 12 for the high-timely-throughput system in Fig. 2(a), and the curve of client 1, 5, 9, 13 and 17 for the low-timely-throughput system in Fig. 2(b). The results show that, when T becomes larger, the values of $\frac{\sigma_i}{p_i}$ converge to the same value across clients. It can also be observed, the convergence speed is reasonably fast that, values converge from dramatic gaps into reasonably small gaps among clients after $T = 20$, and they keep converging slowly and stably along with T .

D. Performance Comparison

We now present our simulation results that evaluate the LoC performance of the three policies, namely, our MDVF policy, the LDF policy, and the MW-AoI policy. We set $\epsilon = 5$ and test the three cases for $T = 10$, $T = 20$, and $T = 30$. For each simulation run, we record the total LoC incurred in the past second for up to 10 seconds.

Simulation results of the LoC in two system are shown in Fig. 3 and Fig. 4. Since LoC can only be defined after the system has run for T intervals, the first data point is at time 2 second, which is the total LoC incurred between time 1 second and 2 second. The figures clearly show that our MDVF policy achieves the smallest LoC in all settings, including both the high-timely-throughput system and the low-timely-throughput system, both the quadratic LoC function and the exponential LoC function, and the three different choices of T .

A very surprising result is that the MW-AoI policy has higher LoC than the LDF policy, even though the LDF policy only considers long-term average timely-throughput while the MW-AoI policy considers short-term fluctuations in the form of Age-of-Information. The reason is that the MW-AoI policy focuses on optimizing AoI, which only depends on

the time of the most recent packet delivery. However, many estimation techniques require more than the most recent data to make an accurate estimation. Even basic techniques like linear extrapolation needs at least two data points to make an estimate. This simulation result highlights that AoI may fail to completely capture the reliability of estimation. On the other hand, the LDF policy only aims to optimize the long-term average timely-throughputs and ignores temporal variance. This leads it to also have suboptimal total LoC.

Some interesting observations can be made by comparing the performance of the MDVF policy between different values of T . We note that the LoC of the MDVF policy decreases as T becomes larger. Under our settings, the computed value of $\bar{X}_i^*$ is always larger than q_i for each i . Therefore, we have $\frac{\theta_i(t)}{T} \rightarrow 0$ as $T \rightarrow \infty$. Our simulation results indeed demonstrate such trends.

VII. A CASE STUDY OF REAL-TIME STATE ESTIMATION

An important motivation of this work is emerging applications that require real-time state estimation, such as industrial IoT and VR. From an end user's perspective, the perceived performance is the user's ability to make accurate estimation. In order to demonstrate the practical value of our proposed metric, LoC, and our proposed policy, MDVF, this section studies the problem of sensing and estimating several independent linear Gaussian processes, where the performance of a flow is determined by the mean square error (MSE) of the resulting estimation.

A. Overview of the Sensing and Estimation Problem

Consider a system with one estimator and N wireless sensors. Each sensor is monitoring an independent linear Gaussian process. We number the sensors and stochastic process so that sensor i is monitoring process i . Further, we denote $z_{i,t}$ as the value of process i in interval t . The stochastic process i evolves according to the recursion:

$$z_{i,t+1} = z_{i,t} + w_{i,t}, \quad (28)$$

where $\{w_{i,1}, w_{i,2}, \dots\}$ is a sequence of i.i.d Gaussian random variables with mean 0 and variance W_i . We also call $w_{i,t}$ the Process Noise (PN).

In each interval t , each sensor i obtains a noisy measurement of the value of process i . The value of the measurement is denoted by $m_{i,t}$, and we assume that:

$$m_{i,t} = z_{i,t} + r_{i,t}, \quad (29)$$

where $\{r_{i,1}, r_{i,2}, \dots\}$ is a sequence of i.i.d Gaussian random variables with mean 0 and variance R_i . We call $m_{i,t}$ the actual measurement or observation under noise, and $r_{i,t}$ the Measurement Noise (MN).

The network model is the same as that in Section II. In each interval t , each sensor i generates a packet containing the value of $m_{i,t}$ and the timestamp t . The packet is discarded either when it is successfully delivered to the estimator or when the sensor generates a newer packet. The estimator, which is also the AP, schedules all transmissions. Thus, the estimator has

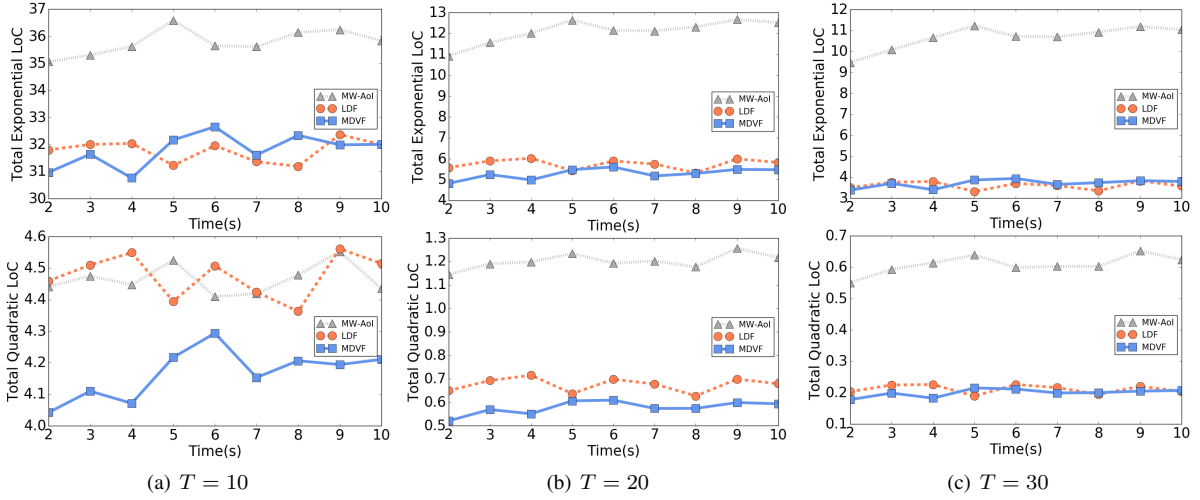


Fig. 3. Two LoC Functions in the past second of high-timely-throughput system: (a) $T = 10$. (b) $T = 20$. (c) $T = 30$.

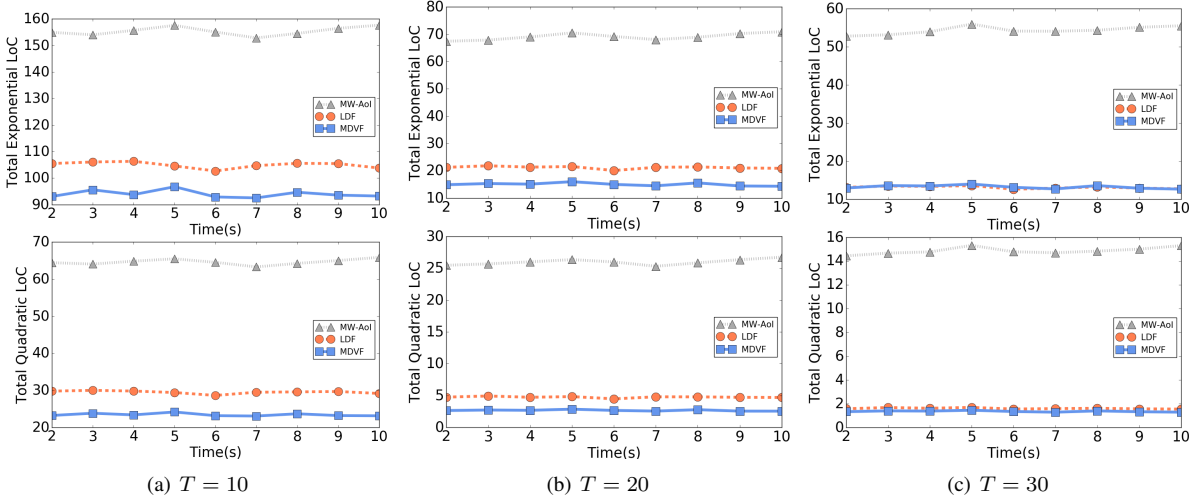


Fig. 4. Two LoC Functions in the past second of low-timely-throughput system: (a) $T = 10$. (b) $T = 20$. (c) $T = 30$.

access to the value of $m_{i,t}$ if and only if a packet is delivered for sensor i in interval t .

The goal of the estimator is to find the best estimate of the current value $z_{i,t}$ of each process i based on all the packets that it has received so far. Let $\mathcal{M}_i^t$ be the set of sensor readings, along with their timestamps that have been delivered to the estimator on or before interval t . Let $\hat{z}_{i,t}$ be the best estimate of $z_{i,t}$ and $\Sigma_{i,t}$ be the Mean Square Error (MSE) of the best estimate, with observation up to interval t . We then have:

$$\hat{z}_{i,t} = E[z_{i,t} | \mathcal{M}_i^t] \quad (30)$$

$$\Sigma_{i,t} = E[(z_{i,t} - \hat{z}_{i,t})^2 | \mathcal{M}_i^t]. \quad (31)$$

When all packets are successfully delivered on time, it is well-known that Kalman Filter [12] [13] [14], a recursive algorithm that calculates $\hat{z}_{i,t}$ and $\Sigma_{i,t}$ simultaneously, yields the best estimate of the underlying Gaussian linear processes. In our system, some packets may be dropped due to deadline

violation, which leads to some missing samples. This scenario has been discussed in [15] and [16], where a variation of Kalman Filter has been proposed and proved to be optimal. Alg. 2 summarizes the variation of Kalman Filter.

B. Simulation of the Estimation Problem

MSE captures the error variance that occurred in the estimate process. Thus it shows the accuracy of the estimate. We design the simulation to see the performance of MSE of three policies. For all policies, we collect the average MSE and 95 percentile MSE of all clients in the past second for 10 seconds in total. The simulation result is based on the average of 1000 runs.

An important challenge of this simulation is to make the MSEs of different processes comparable to each other. In this simulation, we set $W_i = q_i = p_i$ for each process and $R_i = 20$ for all i . This setting is chosen based on the following two reasons. First, suppose each sensor i delivers one packet every $\frac{1}{q_i}$ intervals periodically, and therefore delivers $q_i T$ packets in

Algorithm 2: Kalman Filter Recursion Rule with Missing Samples

Result: The $\hat{z}_{i,t}$ and $\Sigma_{i,t}$ based on $\mathcal{M}_i^t$.

Initialization: $\hat{z}_{i,t}$, $\Sigma_{i,t}$, when $t = 0$;

while each new interval **do**

for each i **do**

if $m_{i,t+1}$ is not delivered **then**

$\hat{z}_{i,t+1} = \hat{z}_{i,t}$;

$\Sigma_{i,t+1} = \Sigma_{i,t} + W_i$;

else

$\hat{z}_{i,t+1} = \hat{z}_{i,t} + \frac{\Sigma_{i,t+1}(m_{i,t+1} - \hat{z}_{i,t})}{(\Sigma_{i,t+1} + R_i)}$;

$\Sigma_{i,t+1} = \frac{(\Sigma_{i,t} + W_i)R_i}{(\Sigma_{i,t} + W_i + R_i)}$;

end

end

$t = t + 1$;

end

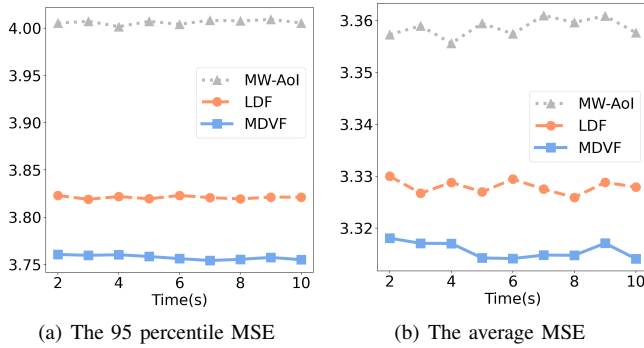


Fig. 5. The average MSE and 95 percentile MSE result of the 12-process system.

every T intervals, then it can be shown that all processes have the same MSE. Second, consider the case that each sensor i delivers one packet every $\frac{1}{q_i - \delta p_i}$ intervals. In this case, the unbiased shortage of all sensors are δT . It can be shown that, under this case, all processes still have the same steady-state MSE. In summary, setting $W_i = q_i = p_i$ for each process and $R_i = 20$ for all i ensures that the MSEs between different processes are comparable. The exact calculations for the MSEs

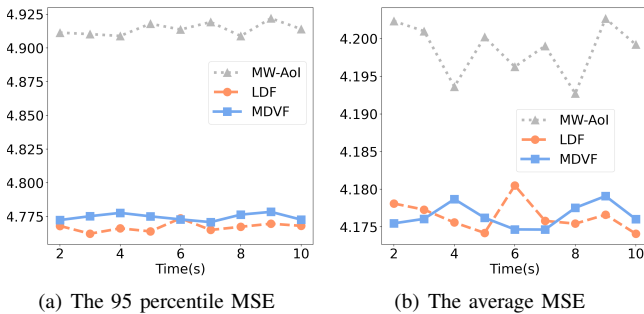


Fig. 6. The average MSE and 95 percentile MSE result of the 18-process system.

are shown in Appendix A.

We consider two different systems in our simulations. The first system has 12 processes and is called the 12-process system. The values of p_i are $\{0.5, 0.47, 0.45, 0.43, 0.4, 0.37, 0.35, 0.33, 0.3, 0.27, 0.25, 0.23\}$. The second system has 18 processes and is called the 18-process system. The values of p_i are $\{0.3, 0.29, 0.28, 0.27, 0.26, 0.25, 0.24, 0.23, 0.22, 0.21, 0.20, 0.19, 0.18, 0.17, 0.16, 0.15, 0.14, 0.13\}$.

The simulation results are shown in Fig 5 and Fig 6. In the 12-process system, all three policies have roughly the same average MSE, but our MDVF policy has a smaller 95-percentile MSE. In the 18-process system, our MDVF policy and the LDF policy have almost identical performance, and both of them perform better than the MW-AoI policy, both in terms of average MSE and 95-percentile MSE. In our network model, whenever a process delivers a packet of measurement data, the AoI of that process drops to zero. However, since the measurement is noisy, the delivery of one single packet is not sufficient to make an accurate estimation. Instead, the estimator needs to have multiple recent measurements to make an accurate estimation. This is why our MDVF policy performs better than the MW-AoI policy.

This simulation result demonstrates that our MDVF policy indeed provides superior performance for real-time estimation applications. It also suggests that AoI-based solutions may not be sufficient to capture the reliability of estimation when considering many estimate techniques.

VIII. RELATED WORK

Age-of-information (AoI) is another metric that aims to capture the short-term performance of information flows that has gained a lot of research interests. For the wireless information-update system, Kadota et.al have built the model of AoI optimization problem in [17] when considering unreliable channel and multiple information flows. Further, Kadota, Sinha and Modiano in [8] have proposed the optimal scheduling policy for the AoI minimization problem when providing the minimum throughput requirement. Both Hsu et.al in [18] and Kadota et.al in [19] have recently proposed the scheduling algorithm when considering random arrivals to minimize the average age. Kam et.al in [20] and [21] first introduce the idea of “effective AoI” to capture the estimation error extended from AoI for a Markov source. Some extended area are also researched with AoI, such as [22] in vehicular network, [23] in multi-hops wireless network, and [24] for link-scheduling optimization in wireless system. The major difference of our work to the AoI approach is that our work cares not only the “timely” status update but also the amount of timely information. When regarding the remote-estimate problems, AoI-based policies normally consider the Markov-style estimate problems, rather than other estimate problems requiring multiple data points, which is our main concentration.

Real-time wireless networks have gained a lot of research interests. Hou, Borkar, and Kumar [4] have proposed a frame-based model to describe delay requirements of real-time flows. Under this model, the performance of each flow is determined

by its timely-throughput, which is the long-term average number of timely deliveries. Jaramillo, Srikant, and Ying [25] have studied wireless flows with heterogeneous delay and timely-throughput requirements. Kang et. al. [26] have studied the performance of timely-throughputs in ad hoc wireless networks with stochastic packet arrivals. Meko and Seid [27] have proposed a randomized scheduling algorithm for real-time flows. Zhang et. al. [28] have studied timely-throughputs in heterogeneous cellular networks with mobile nodes. Lashgari and Avestimehr [29] have looked for the additive gap of maximal timely throughput in a relaxed problem under the time-varying channel states. However, all these studies focus on the long-term average timely-throughput of each flow. As demonstrated in this paper, the temporal variance of timely-throughput can have significant impact on the credibility of an information flow. Singh, Hou, and Kumar [30] have studied the fluctuation of timely-throughput, but its results only hold for a limiting scaled workloads. Hou [2] has proposed a scheduling policy to optimize the short-term performance of real-time flows, but the policy only applies to wireless networks where all links have the same quality.

IX. CONCLUSION

We have studied the problem of minimizing the total Loss-of-Confidence (LoC) in real-time wireless networks, where the LoC of each flow only depends on the timely deliveries in a window of the recent past. We have shown that, unlike most existing network utility maximization (NUM) problem, the problem of minimizing total LoC requires the precise control of the temporal variance of timely deliveries. To solve this problem, we have proposed a simple online algorithm called the MDVF policy, and have proved that the MDVF policy is near-optimal. Simulation results have demonstrated that the MDVF policy outperforms other state-of-the-art policies. Further, we have studied the application of real-time estimation of multiple independent linear Gaussian processes, where an estimator aims to make the best estimate of the current states based on all the measurements that it has received. We evaluate the performance of our policy and others by their resulting estimation error. Simulation results show that our MDVF policy achieves both the smallest average estimation error as well as the smallest 95-percentile of estimation error. This case study suggests that AoI solutions may fail to capture the performance for real-time remote estimation problems.

There are several important directions for future work: First, this work relies on the knowledge of channel reliability p_i . It is of interest to study other channel models, including fading channels where p_i can change overtime and not known in advance and the usage the rate adaptation. Second, although our simulations showed that LoC-based policies outperform AoI-based policies, more comprehensive comparisons will be involved in future work. Third, we have studied one case of estimation in Section. VII and showed that LoC-based policy leads to smaller MSE. However, the connection between MSE and LoC can be further explored in the future.

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APPENDIX A

THE STEADY-STATE MSE OF REAL-TIME FLOWS WITH PERIODIC DELIVERIES

In this section, we consider the real-time estimation problem as described in Section VII. We calculate the MSE of a real-time flow when the corresponding sensor i delivers one packet every $\frac{1}{q_i - \delta p_i}$ intervals periodically.

Let t_0 be the first interval in which sensor i delivers a packet. Sensor i therefore delivers one packet in each of the intervals $t_0, t_0 + \frac{1}{q_i - \delta p_i}, t_0 + 2\frac{1}{q_i - \delta p_i}, \dots$. Recall that $\Sigma_{i,t}$ is the MSE of sensor i in interval t . We then define the *steady-state MSE* of sensor i as $\Sigma_i := \lim_{k \rightarrow \infty} \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}}$.

To calculate Σ_i , we note that, by Alg. 2, we have:

$$\begin{aligned} & \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} \\ &= (\Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} - 1 + W_i)R_i / (\Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} - 1 + W_i + R_i), \end{aligned}$$

since there is a packet delivery in interval $t_0 + k\frac{1}{q_i - \delta p_i}$. In addition, we also have

$$\begin{aligned} \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} - 1 &= \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} - 2 + W_i \\ &= \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} - 3 + 2W_i \\ &\dots \\ &= \Sigma_{i, t_0 + (k-1)\frac{1}{q_i - \delta p_i}} + \left(\frac{1}{q_i - \delta p_i} - 1\right)W_i. \end{aligned}$$

Combining these two equations and setting $W_i = p_i = q_i$ and $R_i = 20$, as used in Section VII, yield

$$\begin{aligned} \Sigma_i &= \lim_{k \rightarrow \infty} \Sigma_{i, t_0 + k\frac{1}{q_i - \delta p_i}} \\ &= \lim_{k \rightarrow \infty} \frac{20(\Sigma_{i, t_0 + (k-1)\frac{1}{q_i - \delta p_i}} + \frac{W_i}{1-\delta})}{(\Sigma_{i, t_0 + (k-1)\frac{1}{q_i - \delta p_i}} + \frac{W_i}{1-\delta} + 20)} = \frac{20(\Sigma_i + \frac{W_i}{1-\delta})}{(\Sigma_i + \frac{W_i}{1-\delta} + 20)}, \end{aligned}$$

and hence

$$\Sigma_i = \frac{-\frac{1}{1-\delta} + \sqrt{\frac{1}{(1-\delta)^2} + \frac{80}{1-\delta}}}{2},$$

for all i . In particular, when $\delta = 0$, we have $\Sigma_i = 4$. This justifies the simulation settings in Section VII.

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