Optimal Wireless Scheduling for Remote Sensing through Brownian Approximation

Daojing Guo*, Ping-Chun Hsieh†, and I-Hong Hou*
*Electrical and Computer Engineering Department, Texas A&M University,
†Computer Science Department, National Chiao Tung University
Email: *{daojing_guo, ihou}@tamu.edu, †pinghsieh@cs.nctu.edu.tw

***Abstract*—This paper studies a remote sensing system where multiple wireless sensors generate possibly noisy information updates of various surveillance fields and delivering these updates to a control center over a wireless network. The control center needs a sufficient number of recently generated information updates to have an accurate estimate of the current system status, which is critical for the control center to make appropriate control decisions. The goal of this work is then to design the optimal policy for scheduling the transmissions of information updates. Through Brownian approximation, we demonstrate that the control center's ability to make accurate real-time estimates depends on the averages and temporal variances of the delivery processes. We then formulate a constrained optimization problem to find the optimal means and variances. We also develop a simple online scheduling policy that employs the optimal means and variances to achieve the optimal system-wide performance. Simulation results show that our scheduling policy enjoys fast convergence speed and better performance when compared to other state-of-the-art policies.**

I. Introduction

Remote sensing has recently attracted significant research interests due to its critical role in many emerging applications, such as industrial Internet-of-Things (IoT), autonomous and connected vehicles, and cyber-physical systems (CPS). In remote sensing, there are multiple sensors generating information updates about their respective surveillance fields and sending these information updates to a control center. The control center then uses its received information for real-time estimation of the current system state, so as to, in the example of industrial IoT, determine the appropriate control actions. The control center needs to be able to make accurate estimates of the system states at all times to ensure the safety and the efficiency of the system.

In this paper, we study the problem of scheduling the transmissions of information updates when the sensors and the control center communicate over a shared wireless band. We propose a new model to capture how the network behavior impacts the ability of the control center to make accurate real-time estimates. Our model is based on two important features of remote sensing: First, because the surveillance fields evolve with time, recent information updates are much more useful than stale ones. Second, because the sensors and the surveillance fields are subject to noises, the control center may need multiple recent information updates to make an accurate estimate.

Motivated by these observations, we propose a model where each sensor sets a threshold for the *freshness* of its information updates. At any given point of time, the instantaneous estimation accuracy for the sensor's surveillance field depends on the current *quantity* of *fresh* information updates at the control center. Compared to Age-of-Information (AoI), which is a popular metric that measures the freshness of the most recent information update, our model can provide a richer characterization by considering both the quantity and the freshness of data. We further address the challenge that sensors are located at different locations and are monitoring different fields by explicitly considering that different sensors can have different thresholds for freshness, different mappings between the quantify of fresh data and the estimation accuracy, and different channel conditions.

In order to analyze this model, we first demonstrate that the quantify of fresh information updates that the control center has at a given time can be expresses as a closed-form function involving the processes of update generations and update deliveries. We then show that, by applying a Brownian approximation to the update delivery process, the quantity of fresh information updates can be characterized as a random variable whose distribution only depends on the mean and temporal variance of the delivery process. The dependency on temporal variances makes it infeasible to apply traditional network optimization techniques that only consider the means of delivery processes. To take temporal variances into account, we analytically establish the fundamental constraints on the means and variances of the delivery processes for all sensors, given the limitations of the wireless bandwith and channel conditions. Thus, the problem of the optimal wireless scheduling can be transformed into a constrained optimization problem of finding the optimal means and variances, subject to the constraints imposed by the wireless channels.

After finding the optimal means and variances of the delivery processes, it remains to develop a scheduling policy that actually achieves them. To this end, we propose a simple scheduling policy and theoretically prove that its resulting means and variances are indeed the optimal ones. Thus, this scheduling policy is the one that enables the control center to have the most accurate real-time estimation. An important and surprising feature of our proposed scheduling policy is that it does not require any knowledge about the freshness of each individual information update, despite the fact that the accuracy of real-time estimation depends on data freshness.

We also conduct comprehensive simulations to evaluate the performance of our proposed scheduling policy. Simulation results show that the means and variances under our scheduling policy indeed converges to the optimal values very fast. In addition, our scheduling policy significantly outperforms other policies, including one that aims to optimize AoI.

The rest of this paper is as following: Section II describes our system model. Section III derives the closed-form expression of the quantity of fresh information updates at the control center. Section IV uses Brownian approximation to derive the distribution of the quantity of fresh information updates. Section V formulates the problem of optimal scheduling as a constrained optimization problem of finding the optimal means and variances of delivery processes. Section VI proposes an online scheduling policy and analyzes its performance. Section VII provides our simulation results. Section VIII reviews some related work. Finally, Section IX concludes the paper.

II. System Model

We consider the following network model: there is one Access Point (AP) and multiple flows, numbered as $1, 2, 3, \ldots, N$, each of which is monitoring an independent and time-varying stochastic field. Time is slotted and denoted by $t = 1, 2, 3, \ldots$. Each flow i generates one time-stamped information update about its monitored field every m_i slots. The AP schedules all transmissions. When the AP schedules a flow i to transmit, the AP first sends a POLL packet to flow i, and, upon receiving the POLL packet, flow i sends one of its information updates to the AP. The duration of a time slot is hence chosen to be sufficient for the transmission of one POLL packet and one information update, along with any necessary overheads. The AP then uses all the information updates that it has ever received to estimate the current status of each monitored field. We further consider the effects of shadowing, multi-paths, fading, and interference by assuming that each transmission for flow i is successful, that is, a status update is received after sending a POLL packet, with probability p_i.

Since the AP needs to make real-time estimation about each stochastic field, the performance of the network should be measured by the accuracy of the estimation. We need a model to express the accuracy of the estimation in terms of network behaviors. Our model is based on two observations of most estimation problems: First, recent information is much more useful than stale information; Second, the more recent information that the AP has, the more accurate its estimation can be. Hence, we model the accuracy of the estimation by assuming that it depends on the number of recent information updates that the AP has received. Compared to Age-of-Information (AoI), which measures the performance based on the freshness of the most recent data, our model offers a richer characterization as it considers both the freshness of data and the quantity of fresh data.

Specifically, we assume that each information update generated by flow i is only *useful* to the AP's estimation algorithm for T_i time slots. Afterwards, the information update becomes *stale* and is no longer useful. Thus, at time slot t, only information updates generated after time slot $t-T_i$ are useful. We use $U_i(t)$ to denote the number of useful packets that the AP has received from flow i at time t. For example, Fig. 1 illustrates the packet arrivals and deliveries histogram of a flow with $T_i = 15$. At time 40, only information updates generated after time 25 are useful, and hence we have $U_i(40) = 2$. Note that while packet 1 was delivered after time 25, it was generated before time 25 and hence is not useful at time 40.

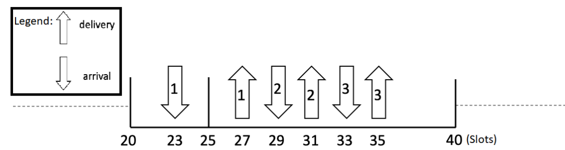


Fig. 1. An Example for Useful Packets and Deliveries

In order to make an accurate estimation of the stochastic field of flow i at time t, the AP needs to have a sufficient number of useful information updates. This requirement is described by a threshold q_i, and we say that the AP needs at least q_iT_i useful information updates, that is, $U_i(t) \geq q_iT_i$, to make an accurate estimation. If $U_i(t) < q_iT_i$ at some time t, then the estimation is inaccurate and results in a large confidence interval. In this case, we say that the AP suffers from a *Loss-of-Confidence* (LoC) of $C_i(q_iT_i - U_i(t))$ at time t, where $C_i(\cdot)$ is a strictly increasing, convex and differentiable function over $[0, +\infty)$ with $C_i(0) = 0$ and $C_i'(0) = 0$. The goal of this paper is to minimize the long-time average LoC for the entire system of all flows, which can be written as $\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i(q_iT_i - U_i(t))}{k}$.

The optimization of total average LoC consists of two parts: First, the AP decides which flow to schedule in each time slot; Second, upon receiving a POLL message, the flow decides which information update to respond. For the second part, it can be shown that the Last-In-First-Out (LIFO) strategy, where the flow always responds with the newest undelivered information update, is optimal. Intuitively, the newest information update is the one that will remain useful for the longest time, and hence sending it is optimal. Recent work [1] has shown that LIFO-type strategy is optimal or near-optimal for AoI-related metrics in the queueing system under different service time. While LoC is not AoI-related, similar arguments can be used to establish the optimality of LIFO for LoC.

III. Fundamental Properties for LIFO Systems

To optimize the long-term average LoC problem, the first challenge is to model the behavior of the useful delivery. In this section, we derive a closed form expression for $U_i(t)$. This derivation is built on the assumption of LIFO strategy when the flow selects the information update in its buffer to transmit.

Let $a_i(t)$ be the indicator function that flow i generates a new information update at time t, and $x_i(t)$ be the indicator function that flow i successfully delivers a packet at time t.

Recall that flow i generates a new information update every m_i time slots. If flow i generates the first information update at time o_i with $0 \leq o_i < m_i$, then we have $a_i(t) = 1$ if and only if $t - o_i$ is a multiple of m_i. If flow i is scheduled to transmit at time t, then we have $x_i(t) = 1$ with probability p_i, since the channel reliability of flow i is p_i. Let $A_i(t) = \sum_{\tau=0}^{t} a_i(t)$ and $X_i(t) = \sum_{\tau=0}^{t} x_i(t)$ be the accumulated number of arrivals and deliveries, respectively.

We also define $u_i(\tau, t)$ be the indicator function that flow i successfully delivers an information update at time τ that will remain useful until at least time t. For example, in Fig. 1, there are deliveries at times 27, 31, and 35, but the delivery at time 27 will not be useful at time 40. Thus, we have $u_i(27, 40) = 0$ and $u_i(31, 40) = u_i(35, 40) = 1$. By this definition of $u_i(\tau, t)$, we have $U_i(t) = \sum_{\tau=t-T_i+1}^{t} u_i(\tau, t)$.

Theorem 1. *For any $t > T_i$,*

$$U_i(t) = \sum_{\tau=t-T_i+1}^{t} x_i(\tau) - \sup_{t-T_i+1 \leq s \leq t} \Big[\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau) \Big]^{+} \tag{1}$$

Proof. We first show that:

$$\sum_{\tau=t-T_i+1}^{d} u_i(\tau, t) = \sum_{\tau=t-T_i+1}^{d} x_i(\tau) - \sup_{t-T_i+1 \leq s \leq d} \Big[\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau) \Big]^{+}, \tag{2}$$

for any $t - T_i + 1 \leq d \leq t$ by induction.

First, consider the case $d = t - T_i + 1$. Any updates generated before time d will become stale by time t. Hence, flow i can deliver an update at time d that will remain useful at time t, and hence have $u_i(d, t) = 1$, only if both of the following conditions are satisfied: flow i generates an update at time d, that is, $a_i(d) = 1$, and flow i delivers an update at time d, that is, $x_i(d) = 1$. Hence, When $d = t - T_i + 1$, (2) holds.

Next, suppose (2) holds when $d = k$, we then consider the case when $d = k + 1$. At time $k + 1$, $u_i(k + 1, t) = 1$ only if the following two conditions are satisfied: First, there is one successful delivery at time $k+1$, that is $x_i(k+1) = 1$; Second, there is at least one undelivered update that will be useful at time t. Since only information updates after time $t - T_i$ will be useful at time t, the number of useful updates that flow i has generated on or before time $k + 1$ is $\sum_{\tau=t-T_i+1}^{k+1} a_i(\tau)$. Before time $k + 1$, flow i has delivered $\sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)$ information updates. Hence, the number of undelivered updates that will be useful at time t is $\sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) - \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)$. In summary, we have: $u_i(k + 1, t) = \min\{x_i(k), \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) - \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t)\}$.

We now derive $\sum_{\tau=t-T_i+1}^{k+1} u_i(\tau, t)$ from the induction hypothesis.

$$\begin{aligned}
&\sum_{\tau=t-T_i+1}^{k+1} u_i(\tau, t) = \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t) + u_i(k + 1, t) \\
&= \min \Big\{ x_i(k + 1) + \sum_{\tau=t-T_i+1}^{k} u_i(\tau, t), \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) \Big\} \\
&= \min \Big\{ \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \sup_{t-T_i+1 \leq s \leq k} \Big[\sum_{\tau=t-T_i+1}^{s} x_i(\tau) \\
&\qquad - \sum_{\tau=t-T_i+1}^{s} a_i(\tau) \Big]^{+}, \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau) \Big\} \\
&= \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \max \Big\{ \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) - \sum_{\tau=t-T_i+1}^{k+1} a_i(\tau), \\
&\qquad \sup_{t-T_i+1 \leq s \leq k} \Big[\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau) \Big]^{+} \Big\} \\
&= \sum_{\tau=t-T_i+1}^{k+1} x_i(\tau) \\
&\qquad - \sup_{t-T_i+1 \leq s \leq k+1} \Big[\sum_{\tau=t-T_i+1}^{s} x_i(\tau) - \sum_{\tau=t-T_i+1}^{s} a_i(\tau) \Big]^{+}
\end{aligned} \tag{3}$$

Hence, by induction, (2) holds for any $t - T_i + 1 \leq d \leq t$. Since $U_i(t) = \sum_{\tau=t-T_i+1}^{t} u_i(\tau, t)$, the theorem holds. $\square$

IV. Reflected Brownian Motion Approximation

Thm. 1 has shown that $U_i(t)$ can be explicitly expressed as a function of the update arrival and delivery processes, $\{a_i(\tau)\}$ and $\{x_i(\tau)\}$. In this section, we further show that, if the employed scheduling policy is ergodic, then $U_i(t)$ can be approximated by a random variable whose distribution can be expressed in closed-form.

We first study the approximation of the accumulated number of update deliveries, $X_i(t) := \sum_{\tau=1}^{t} x_i(\tau)$. Under any ergodic scheduling policy, the delivery process $\{x_i(1), x_i(2), \dots\}$ can be modeled as a positive recurrent Markov chain with finite states. By the Law of Large Numbers, the limit $\bar{X}_i := \lim_{t\to\infty} \frac{X_i(t)}{t}$ exists. Further, by the central limit theorem of Markov chains [2], $\hat{X}_i := \lim_{t\to\infty} \frac{X_i(t) - t\bar{X}_i}{\sqrt{t}}$ is a Gaussian random variable with mean 0 and some finite variance, which we denote by σ_i^2 with $\sigma_i \geq 0$. Hence, we can approximate $X_i(t) - X_i(t - T_i) = \sum_{\tau=t-T_i+1}^{t} x_i(\tau)$ as a Gaussain random variable with mean $T_i\bar{X}_i$ and variance $T_i\sigma_i^2$ for any sufficiently large T_i. Such an approximation is called a Brownian motion process, and we denote it by $X_i(t) \approx BM(\bar{X}_i, \sigma_i^2)$.

Next, we consider the random process $Y_i(t) := A_i(t) - X_i(t)$. Recall that $A_i(t)$ is the accumulated number of update arrivals and that flow i generates one update every m_i slots. Thus, we have $\lfloor \frac{T_i}{m_i} \rfloor \leq A_i(t) - A_i(t - T_i) \leq \lceil \frac{T_i}{m_i} \rceil$, for any t and T_i. $Y_i(t) - Y_i(t - T_i) = \big[A_i(t) - A_i(t - T_i)\big] - \big[X_i(t) -$

$X_i(t-T_i)\big]$ can then be approximated by a Gaussian random variable with mean $T_i(\frac{1}{m_i}-\bar{X}_i)$ and variance $T_i\sigma_i^2$ for any sufficiently large T_i. We express this approximation by saying $Y_i(t) \approx BM(\frac{1}{m_i}-\bar{X}_i,\sigma_i^2)$.

From Thm. 1, we have $U_i(t) = \big[A_i(t)-A_i(t-T_i)\big] - \big[Y_i(t)-Y_i(t-T_i)\big] + \sup_{t-T_i+1\le s\le t}\big[Y_i(s)-Y_i(t-T_i)\big]^+$. When we fix d and apply the approximation $Y_i(t) \approx BM(\frac{1}{m_i}-\bar{X}_i,\sigma_i^2)$, the random process $Y_i(t+d)-Y_i(d)$ can still be approximated by $BM(\frac{1}{m_i}-\bar{X}_i,\sigma_i^2)$, and the random process $Q_i(t) := Y_i(t+d)-Y_i(d)-\sup_{0\le s\le t}\big[Y_i(s+d) - Y_i(d)\big]^+$ is called a reflected Brownian process and is denoted by $RBM(\frac{1}{m_i}-\bar{X}_i,\sigma_i^2)$.

When $\bar{X}_i > \frac{1}{m_i}$, $Q_i(t)$ has a stationary distribution of an exponential variable with mean $\frac{\sigma_i^2}{2(\bar{X}_i-\frac{1}{m_i})}$ [3], and we say $Q_i(t) \sim EXP\big(\frac{2(\bar{X}_i-\frac{1}{m_i})}{\sigma_i^2}\big)$. When $\bar{X}_i < \frac{1}{m_i}$, Chen and Yao [4] propose to approximate $Q_i(t)$ by a Brownian motion process $Q_i(t) \approx BM(\frac{1}{m_i}-\bar{X}_i,\sigma_i^2)$. In this case, when t is fixed, $Q_i(t)$ is approximated by a Gaussian random variable with mean $T_i(\frac{1}{m_i}-\bar{X}_i)$ and variance $T_i\sigma_i^2$, denoted by $\mathcal{N}\big(T_i(\frac{1}{m_i}-\bar{X}_i),T_i\sigma_i^2\big)$.

We note that $U_i(t) \sim \big[A_i(t)-A_i(t-T_i)\big] - Q_i(T_i)$. Thus, we can approximate $U_i(t)$ as the following:

$$U_i(t) \approx \begin{cases} \dfrac{T_i}{m_i} - EXP\Big(\dfrac{2(\bar{X}_i-\frac{1}{m_i})}{\sigma_i^2}\Big), & if \quad \bar{X}_i > \dfrac{1}{m_i} \\ \dfrac{T_i}{m_i} - \mathcal{N}\Big(T_i(\dfrac{1}{m_i}-\bar{X}_i),T_i\sigma_i^2\Big), & if \quad \bar{X}_i < \dfrac{1}{m_i} \end{cases} \tag{4}$$

Eq. (4) shows that $U_i(t)$ can be approximated by random variables whose distributions depend on the mean and variance of the delivery process, that is, $\bar{X}_i$ and σ_i. Thus, the long-term average LoC, which depends on the distributions of $[U_i(t)]$, can be viewed as a function of $[\bar{X}_i]$ and $[\sigma_i]$.

V. Optimization Problem Formulation

Section IV has shown that the problem of minimizing the total long-term average LoC can be viewed as an optimization problem of choosing the optimal $[\bar{X}_1,\bar{X}_2,\dots,\bar{X}_N]$ and $[\sigma_1,\sigma_2,\dots,\sigma_N]$. In this section, we first establish the fundamental constraints of $[\bar{X}_i]$ and $[\sigma_i]$. We then formulate the problem as an optimization problem and discuss finding the optimal $[\bar{X}_i]$ and $[\sigma_i]$.

A. System Constraints

We first discuss the constraints on $[\bar{X}_i]$. Hou and Kumar [5] has shown that, under any work-conserving policy that schedules a transmission in each time slot, we have for all t:

$$E[\sum_{i=1}^N \frac{X_i(t)-X_i(t-1)}{p_i}] = 1. \tag{5}$$

Thus, we have, under any work-conserving and ergodic scheduling policies,

$$\sum_{i=1}^N \frac{\bar{X}_i}{p_i} = 1. \tag{6}$$

.

Next, we derive the constraint for σ_i^2. By (5), the sequence of $\{\sum_{i=1}^N \frac{X_i(t)}{p_i} - t | t = 1,2,\dots\}$ is a martingale. By the martingale central limit theorem [6], $\hat{X}_{TOT} := \lim_{k\to\infty}\frac{\sum_{i=1}^N \frac{X_i(k)}{p_i}-k}{\sqrt{k}}$ is a Gaussian random variable with mean 0 and variance

$$\sigma_{[\bar{X}_i]}^2 := \lim_{k\to\infty}\frac{1}{k}\Big[\sum_{t=1}^k\Big(\sum_{i=1}^N\frac{X_i(t)-X_i(t-1)}{p_i}\Big)^2\Big] - 1. \tag{7}$$

Suppose the AP schedules a transmission for flow j in slot t, then $X_j(t) - X_j(t-1)$ equals 1 with probability p_j and equals 0 with probability $1-p_j$. Further, $X_i(t)-X_i(t-1) = 0$ for all other flows $i \neq j$. Hence, $\sum_{i=1}^N \frac{X_i(t)-X_i(t-1)}{p_i}$ equals $\frac{1}{p_j}$ with probability p_j, and equals 0 with probability $1-p_j$. Further, let $\gamma_i(t)$ be the probability that the system schedules flow i in slot t. Then we can derive that $\lim_{k\to\infty}\frac{1}{k}\big[\sum_{t=1}^k\big(\sum_{i=1}^N\frac{X_i(t)-X_i(t-1)}{p_i}\big)^2\big] = \lim_{k\to\infty}\frac{1}{k}\big[\sum_{t=1}^k\sum_{i=1}^N\gamma_i(t)\frac{1}{p_i}\big]$. Note that, since every transmission for flow i is successful with probability p_i, we have $\lim_{k\to\infty}\frac{\sum_{t=1}^k\gamma_i(t)}{k} = \frac{\bar{X}_i}{p_i}$. Therefore, (7) can be written as:

$$\sigma_{[\bar{X}_i]}^2 = \sum_{i=1}^N\frac{\bar{X}_i}{p_i^2} - 1. \tag{8}$$

Recall the definition of $\hat{X}_i := \lim_{t\to\infty}\frac{X_i(t)-t\bar{X}_i}{\sqrt{t}}$, we have $\hat{X}_{TOT} = \sum_{i=1}^N\frac{\hat{X}_i}{p_i}$, and the variance of $\frac{\hat{X}_i}{p_i}$ is $\big(\frac{\sigma_i}{p_i}\big)^2$. By Cauchy-Schwarz Inequality, we have:

$$\begin{aligned} &\Big(\sum_{i=1}^N\frac{\sigma_i}{p_i}\Big)^2 = \Big(\sum_{i=1}^N\sqrt{Var(\frac{\hat{X}_i}{p_i})}\Big)^2 \\ =&\sum_{i=1}^N Var(\frac{\hat{X}_i}{p_i}) \\ &\quad + 2\sum_{l=1}^N\sum_{m=l+1}^N\sqrt{Var(\frac{\hat{X}_l}{p_l})Var(\frac{\hat{X}_m}{p_m})} \\ \ge&\sum_{i=1}^N Var(\frac{\hat{X}_i}{p_i}) + 2\sum_{l=1}^N\sum_{m=l+1}^N Cov(\frac{\hat{X}_l}{p_l},\frac{\hat{X}_m}{p_m}) \\ =&Var(\sum_{i=1}^N\frac{\hat{X}_i}{p_i}) = \sigma_{[\bar{X}_i]}^2, \end{aligned} \tag{9}$$

where $Var(X)$ denotes the variance of X and $Cov(X,Y)$ denotes the covariance. Thus, we have the constraint for σ_i as:

$$\sum_i^n\frac{\sigma_i}{p_i} \ge \sigma_{[\bar{X}_i]} = \sqrt{\sum_i^n\frac{\bar{X}_i}{p_i^2} - 1}. \tag{10}$$

B. Optimization Problem Formulation

From (4), the distribution $U_i(t)$ is very different in two different regimes, the regime $\bar{X}_i > \frac{1}{m_i}$ and the regime $\bar{X}_i < \frac{1}{m_i}$. When $\bar{X}_i > \frac{1}{m_i}$, then the rate of delivery is larger than the rate of update arrival. Hence, we say that the system operates in the *under-sampled regime* when $\bar{X}_i > \frac{1}{m_i}$ for all i. Conversely, we say that the system operates in the *over-sampled regime* when $\bar{X}_i < \frac{1}{m_i}$ for all i.

We first discuss the under-sampled regime. From (6), we know that it is possible to operate in the under-sampled regime if and only if $\sum_{i=1}^{N} \frac{1}{p_i m_i} < 1$. In this case, $U_i(t)$ is approximated by $\frac{T_i}{m_i} - EXP(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2})$. Let $\Theta_\lambda(z) := 1 - e^{-\lambda z}$ be the Cumulative Distribution Function (CDF) of an exponential variable with mean $\frac{1}{\lambda}$. Then we have

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i\big(q_i T_i - U_i(t)\big)}{k} \tag{11}$$

$$= \lim_{k\to\infty} \sum_{i=1}^{N} E\big[C_i\big(q_i T_i - U_i(t)\big)\big] \tag{12}$$

$$\approx \sum_{i=1}^{N} E\Big[C_i\Big(q_i T_i - \frac{T_i}{m_i} + EXP\big(\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}\big)\Big)\Big] \tag{13}$$

$$= \sum_{i=1}^{N} \int_z C_i\big(z - (\frac{1}{m_i} - q_i)T_i\big) d\Theta_{\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}}(z). \tag{14}$$

The problem of minimizing the total LoC in the under-sampled regime is to find $[\bar{X}_i]$ and $[\sigma_i]$ that minimize (14), subject to (6) and (10).

Next, we discuss the over-sampled regime, which can happen when $\sum_{i=1}^{N} \frac{1}{p_i m_i} > 1$. In this case, $U_i(t)$ is approximated by $\frac{T_i}{m_i} - \mathcal{N}\big(T_i(\frac{1}{m_i} - \bar{X}_i), T_i\sigma_i^2\big)$. Let $\phi(z)$ represents the CDF of a random variable under standard Normal distribution, then the CDF of $\{\hat{U}_i(t) - T_i\bar{X}_i\}$ is $\phi(\frac{z}{\sqrt{T_i\sigma_i^2}})$. Then we have:

$$\lim_{k\to\infty} \frac{\sum_{i=1}^{N}\sum_{t=T_i+1}^{k+T_i} C_i\big(q_i T_i - U_i(t)\big)}{k} \tag{15}$$

$$= \lim_{k\to\infty} \sum_{i=1}^{N} E\big[C_i\big(q_i T_i - U_i(t)\big)\big] \tag{16}$$

$$\approx \sum_{i=1}^{N} E\Big[C_i\Big(q_i T_i - T_i\bar{X}_i + T_i\bar{X}_i - \mathcal{N}\big(T_i\bar{X}_i, T_i\sigma_i^2\big)\Big)\Big] \tag{17}$$

$$= \sum_{i=1}^{N} \int_z C_i\big(\sqrt{T_i\sigma_i^2} z - (\bar{X}_i - q_i)T_i\big) d\phi(z). \tag{18}$$

The problem of minimizing the total LoC in the over-sampled regime is to find $[\bar{X}_i]$ and $[\sigma_i]$ that minimize (18), subject to (6) and (10).

C. Obtaining the Optimal Solution

A challenge in finding the optimal $[\bar{X}_i]$ and $[\sigma_i]$ is that the objective functions (14) and (18) both involve integrals. We propose using the Monte Carlo Method (MCM) [7] to address this challenge. For the under-sampled regime, since the rate λ_i of $EXP(\lambda_i)$ of each flow i involves both the control variables $\bar{X}_i$ and σ_i, we further convert (14) into the following form to obtain $EXP(1)$:

$$\sum_{i=1}^{N} \int_z C_i\big(z - (\frac{1}{m_i} - q_i)T_i\big) d\Theta_{\frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2}}(z)$$

$$= \sum_{i=1}^{N} \int_y C_i\big(\frac{y\sigma_i^2}{2(\bar{X}_i - \frac{1}{m_i})} - (\frac{1}{m_i} - q_i)T_i\big) d\Theta_1(y), \tag{19}$$

where $y = \frac{2(\bar{X}_i - \frac{1}{m_i})}{\sigma_i^2} z$. Then we can apply the Monte Carlo Method and generate K random numbers using the exponential distribution with rate 1 for each $1 \le i \le N$, which are denoted by $y_{i,1}, y_{i,2}, \ldots, y_{i,K}$. The objective function (19) can then be approximated by $\sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i(\frac{y_{i,k}\sigma_i^2}{2(\bar{X}_i - \frac{1}{m_i})} - (\frac{1}{m_i} - q_i)T_i)$, and the problem of minimizing the total LoC can be written as

$$Min \quad \sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i\big(\frac{y_{i,k}\sigma_i^2}{2(\bar{X}_i - \frac{1}{m_i})} - (\frac{1}{m_i} - q_i)T_i\big) \tag{20}$$

$$s.t. \quad \sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = 1 \tag{21}$$

$$\sum_{i=1}^{N} \frac{\sigma_i}{p_i} \ge \sigma_{[\bar{X}_i]} = \sqrt{\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i^2} - 1} \tag{22}$$

$$\bar{X}_i \ge 0 \quad and \quad \sigma_i \ge 0, \quad \forall i. \tag{23}$$

The above problem is a well-defined optimization problem, and we can apply standard techniques to find the optimal $[\bar{X}_i]$ and $[\sigma_i]$.

Similarly, for the over-sampled regime, we generate K random numbers using the Normal distribution $\mathcal{N}(0,1)$ for each $1 \le i \le N$, which are denoted again by $z_{i,1}, z_{i,2}, \ldots, z_{i,K}$. Then we approximate (18) by $\sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i\big(\sqrt{T_i\sigma_i^2} z_{i,k} - (\bar{X}_i - q_i)T_i\big)$. The problem of minimizing the total LoC can be written as

$$Min \quad \sum_{i=1}^{N} \frac{1}{K} \sum_{k=1}^{K} C_i\big(\sqrt{T_i\sigma_i^2} z_{i,k} - (\bar{X}_i - q_i)T_i\big) \tag{24}$$

$$s.t. \quad \sum_{i=1}^{N} \frac{\bar{X}_i}{p_i} = 1 \tag{25}$$

$$\sum_{i=1}^{N} \frac{\sigma_i}{p_i} \ge \sigma_{[\bar{X}_i]} = \sqrt{\sum_{i=1}^{N} \frac{\bar{X}_i}{p_i^2} - 1} \tag{26}$$

$$\bar{X}_i \ge 0 \quad and \quad \sigma_i \ge 0, \quad \forall i. \tag{27}$$

VI. Online Scheduling Policy

Section V has shown how to find the optimal $[\bar{X}_i]$ and $[\sigma_i]$ to minimize the total LoC. Let $[\bar{X}_i^*]$ and $[\sigma_i^*]$ be the optimal solution. It remains to find a scheduling policy that ensures that the mean and the variance of the update delivery process $X_i(t)$ are indeed $\bar{X}_i^*$ and σ_i^{*2}. In this section, we propose such an online scheduling policy.

We first introduce some notations before we propose and analyze our policy. Let $d_i(t) := \frac{t\bar{X_i}^* - X_i(t)}{p_i}$ denote the *deficit* of flow i in slot t. Consequently, we define $\Delta d_i(t) := d_i(t+1) - d_i(t) = \frac{\bar{X}_i^*}{p_i} - \frac{X_i(t+1) - X_i(t)}{p_i}$ as the change of the deficit in a slot.

We are now ready to propose our policy, which is called *Variance-Weighted-Deficit-First* (VWDF) policy. The VWDF policy assigns a weight of $v_i := \frac{p_i}{\sigma_i^*}$ to each flow i. In each time slot t, the VWDF policy schedules the client with the largest $v_i d_i(t)$ for transmission.

Let $D(t) := \frac{\sum_{i=1}^N d_i(t)}{\sum_{i=1}^N 1/v_i}$ be the weighted average of $v_i d_i(t)$. We first establish the following theorem.

Theorem 2. *Under VWDF policy, the Markov process with state vector $\{v_i d_i(t) - D(t)\}$ is positive recurrent.*

Proof. By the design of our VWDF policy, at the beginning of each time slot t, the flow with largest $v_i d_i(t)$ will be transmitted at this slot. We use r_t to represent the flow that has the largest $v_i d_i(t)$ at time t, hence this flow r_t has $v_{r_t} d_{r_t}(t) \geq v_i d_i(t)$ for all i. Since a transmission for flow r_t is successful with probability p_{r_t}, we have $\Delta d_{r_t}(t) = \frac{\bar{X}_{r_t}^*}{p_{r_t}} - \frac{1}{p_{r_t}}$ with probability p_{r_t} and $\Delta d_{r_t}(t) = \frac{\bar{X}_{r_t}^*}{p_{r_t}}$ with probability $1-p_{r_t}$. For all other flows $i \neq r_t$, $\Delta d_i(t) = \frac{\bar{X}_i^*}{p_i}$. Then we have the expectation as $E[\Delta d_{r_t}(t)] = \frac{\bar{X}_{r_t}^*}{p_{r_t}} - 1$ and $E[\Delta d_i(t)] = \frac{\bar{X}_i^*}{p_i}$ for $i \neq r_t$. This also gives:

$$\sum_{i=1}^N E\big[\Delta d_i(t)\big] = \sum_{i=1}^N \frac{\bar{X}_i^*}{p_i} - 1 = 0. \tag{28}$$

Similarly, let $\Delta D(t) := D(t+1) - D(t)$. Following the above result, we also have:

$$E\big[\Delta D(t)\big] = E\big[\frac{\sum_{i=1}^N (d_i(t+1) - d_i(t))}{\sum_{i=1}^N \frac{1}{v_i}}\big] = 0. \tag{29}$$

Define the Lyapunov function $L(t) = \frac{1}{2}\sum_{i=1}^N \frac{1}{v_i}(v_i d_i(t) - D(t))^2$, and we have the derivation for the Lyapunov drift when given the state at t:

$$\begin{aligned}
\Delta L(t) &= E[L(t+1) - L(t)] \\
&= E\Big[\frac{1}{2}\sum_{i=1}^N \frac{1}{v_i}\big(v_i d_i(t+1) - D(t+1)\big)^2 \\
&\quad - \frac{1}{2}\sum_{i=1}^N \frac{1}{v_i}\big(v_i d_i(t) - D(t)\big)^2\Big] \\
&= \beta + E\big[\sum_{i=1}^N v_i d_i(t)\Delta d_i(t) - \sum_{i=1}^N D(t)\Delta d_i(t)\big] \\
&\quad - E\big[\Delta D(t)\sum_{i=1}^N d_i(t) - \Delta D(t)\sum_{i=1}^N d_i(t)\big]
\end{aligned} \tag{30}$$

where β is a bounded positive number, and (30) is from the definition of $D(t)$.

Since we have $E[\Delta d_{r_t}(t)] = \frac{\bar{X}_{r_t}^*}{p_{r_t}} - 1$, $E[\Delta d_i(t)] = \frac{\bar{X}_i^*}{p_i}$ for $i \neq r_t$, $\sum_{i=1}^N E[\Delta d_i(t)] = 0$, and $E[\Delta d_i(t)] = 0$, we further have:

$$\Delta L(t) \leq \beta + \sum_{i=1}^N \frac{\bar{X}_i^*}{p_i}\big(v_i d_i(t) - v_{r_t} d_{r_t}(t)\big). \tag{31}$$

By the design of VWDF policy, $v_{r_t} d_{r_t}(t) \geq v_i d_i(t)$, for all $i \neq r_t$. Suppose, at time t, $\max_{1\leq i\leq N} |v_i d_i(t) - D(t)| > \delta$, for some positive δ. Then, there exists a flow i_t' with $v_{i_t'} d_{i_t'}(t) - v_{r_t} d_{r_t}(t) < -\delta$, and hence $\Delta L(t) < \beta - \delta\frac{\bar{X}_{i_t'}^*}{p_{i_t'}}$. By choosing δ to be larger than $\frac{2\beta p_i}{\bar{X}_i^*}$, we have $\Delta L(t) < -\beta$ if $\max_{1\leq i\leq N} |v_i d_i(t) - D(t)| > \delta$. Therefore, by Foster-Lyapunov Theorem, we have $\{v_i d_i(t) - D(t)\}$ is positive recurrent. □

Since the Markov process $\{v_i d_i(t) - D(t)\}$ is positive recurrent, it has a stationary distribution. Hence, $\lim_{k\to\infty} \frac{v_i d_i(k) - D(k)}{k} \to 0$, and $\lim_{k\to\infty} \frac{v_i d_i(k) - D(k)}{\sqrt{k}} \to 0$, for all i.

Moreover,

$$\begin{aligned}
\lim_{k\to\infty} \frac{D(k)}{k} &= \lim_{k\to\infty} \frac{\sum_{i=1}^N \frac{k\bar{X}_i^* - X_i(k)}{p_i}}{k\sum_{i=1}^N \frac{1}{v_i}} \\
&= \lim_{k\to\infty} \frac{\sum_{i=1}^N \frac{\bar{X}_i^*}{p_i} - \sum_{i=1}^N \frac{X_i(k)}{kp_i}}{\sum_{i=1}^N \frac{1}{v_i}} = 0.
\end{aligned} \tag{32}$$

Then, we have the following:

$$\begin{aligned}
\lim_{k\to\infty} \frac{v_i d_i(k)}{k} &= \lim_{k\to\infty} v_i \frac{k\bar{X}_i^* - X_i(k)}{kp_i} \\
&= v_i \frac{\bar{X}_i^*}{p_i} - v_i \lim_{k\to\infty} \frac{X_i(k)}{kp_i} = 0,
\end{aligned} \tag{33}$$

and hence, $\bar{X}_i = \bar{X}_i^*$.

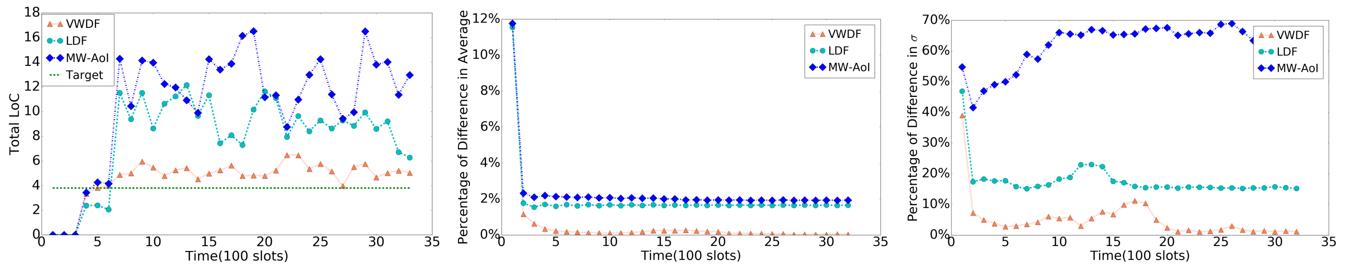
Next, recall the definition $\hat{X}_i := \lim_{k\to\infty} \frac{X_i(k) - k\bar{X}_i}{\sqrt{k}}$ and $\hat{X}_{TOT} := \lim_{k\to\infty} \frac{\sum_{i=1}^N \frac{X_i(k)}{p_i} - k}{\sqrt{k}}$. Hence, we have:

$$\lim_{k\to\infty} \frac{v_i d_i(k)}{\sqrt{k}} = \lim_{k\to\infty} \frac{v_i\big(k\bar{X}_i^* - X_i(k)\big)}{p_i\sqrt{k}} = -\frac{v_i}{p_i}\hat{X}_i, \tag{34}$$

and

$$\begin{aligned}
\lim_{k\to\infty} \frac{D(k)}{\sqrt{k}} &= \lim_{k\to\infty} \frac{\sum_{i=1}^N d_i(k)}{\sqrt{k}\sum_{i=1}^N \frac{1}{v_i}} \\
&= \lim_{k\to\infty} \frac{\sum_{i=1}^N \frac{k\bar{X}_i^*}{p_i} - \sum_{i=1}^N \frac{X_i(k)}{p_i}}{\sqrt{k}\sum_{i=1}^N \frac{1}{v_i}} \\
&= -\lim_{k\to\infty} \frac{\sum_{i=1}^N \frac{X_i(k)}{p_i} - k}{\sqrt{k}\sum_{i=1}^N \frac{1}{v_i}} = -\frac{\hat{X}_{TOT}}{\sum_{i=1}^N \frac{1}{v_i}}.
\end{aligned} \tag{35}$$

By definition, the variance of (34) is $\frac{v_i^2}{p_i^2}\sigma_i^2 = \frac{\sigma_i^2}{\sigma_i^{*2}}$ and the variance of (35) is $\frac{\sigma_{[\bar{X}_i]}^2}{(\sum_{i=1}^N \frac{1}{v_i})^2} = \frac{\sigma_{[\bar{X}_i]}^2}{(\sum_{i=1}^N \frac{\sigma_i^*}{p_i})^2}$. Since $[X_i^*]$ and $[\sigma_i^*]$ is the optimal solution to the optimization problem of



(a) The Total LoC (b) The Total Error from $\bar{X}_i^*$ in Percentage (c) The Total Error from σ_i^* in Percentage

Fig. 2. The over-sampled and heavily-loaded system

minimizing either (14) or (18), subject to (6) and (10) and the objective function is increasing in $[\sigma_i^*]$, we have $\sum_{i=1}^{N} \frac{\sigma_i^*}{p_i} = \sigma_{[\bar{X}_i]}$, and the variance of (35) is 1. As (34) and (35) have the same variance, we have $\sigma_i = \sigma_i^*$, for all i.

In summary, we have:

Theorem 3. *Under the VWDF policy, $\bar{X}_i = \bar{X}_i^*$ and $\sigma_i = \sigma_i^*$, for all i.* □

Since $[\bar{X}_i^*]$ and $[\sigma_i^*]$ are the optimal vectors that minimize system-wide total LoC under the Brownian approximation, Theorem 3 implies that the VWDF policy is the optimal scheduling policy. We note that the VWDF policy makes scheduling decisions only based on the deficit of each flow. In particular, the VWDF policy does not keep track of the number of undelivered useful updates that each flow has. Such a feature makes it very easy to implement the VWDF policy. It is also surprising that the VWDF policy is able to minimize the total LoC, which depends on the number of useful information updates, without any knowledge about the usefulness of individual updates.

VII. Simulation Results

We present our simulation results in this section. We have tested our VWDF policy and compared it with with two other state-of-the-art policies in NS-2 simulation. All simulations are performed under 802.11 MAC protocol with 54Mbps data rate. Simulations show that the time needed for the AP to schedule a transmission and receive an information update is 813μs. All results presented in this paper are average in 100 runs.

We compare our VWDF policy against two other policies. The first policy is the Largest Debt First (LDF) policy from [5], [8], which schedules the flow withe largest $q_i t - X_i(t)$ in each time slot. The main difference between the LDF policy and our VWDF policy is that the LDF policy does not weigh the deficit of each flow by its variance. The second policy is a policy aiming to minimize AoI under some throughput constraints [9]. Under our model, the policy schedules the flow with the largest sum of $q_i t - X_i(t)$ and AoI in each time slot. Hence, we call this policy MW-AoI in the following simulations. In all policies, each flow sends status updates using LIFO.

In our simulations, there are eight wireless flows, which are divided into two groups. The first four flows are in the first group with $C_i(q_iT_i - U_i(t)) = (q_iT_i - U_i(t))^2$. The other four flows are in the second group with $C_i(q_iT_i - U_i(t)) = e^{q_iT_i - U_i(t)} - (q_iT_i - U_i(t)) - 1$.

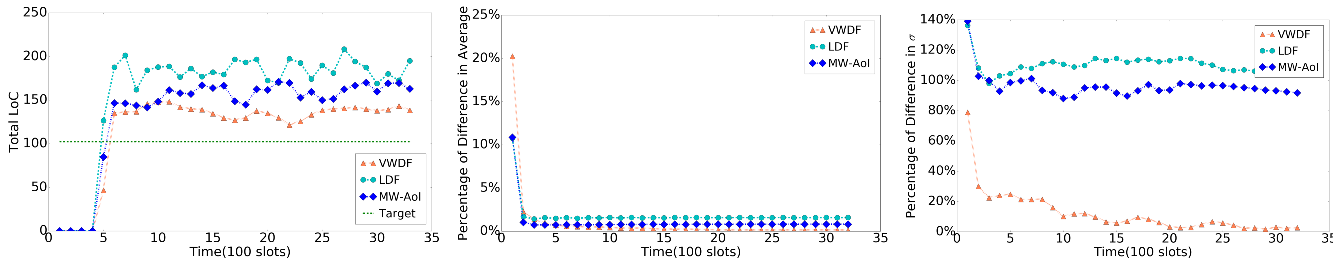
We evaluate the performance under four scenarios operating in very different regimes:

- **Over-sampled and heavily-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 1$. Specifically, we choose $[p_i] = [0.65, 0.65, 0.7, 0.7, 0.75, 0.75, 0.8, 0.8]$, $[q_i] = [0.13, 0.065, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, $[m_i] = [5, 5, 5, 5, 8, 8, 8, 8]$, and $[T_i] = [200, 200, 200, 200, 500, 500, 500, 500]$.
- **Over-sampled and over-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 1.1 > 1$. Specifically, we choose $[p_i]$ and $[m_i]$ to be the same as the first system, $[q_i] = [0.1625, 0.0975, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, and $[T_i] = [400, 400, 400, 400, 300, 300, 300, 300]$.
- **Over-sampled and under-loaded system**: This setting has $\bar{X}_i < \frac{1}{m_i}$, for all i, and $\sum_{i=1}^{N} \frac{q_i}{p_i} = 0.95 < 1$. Specifically, we choose $[p_i]$ and $[m_i]$ to be the same as the first system, $[q_i] = [0.0975, 0.065, 0.14, 0.07, 0.075, 0.075, 0.08, 0.08]$, and $[T_i] = [400, 400, 400, 400, 300, 300, 300, 300]$.
- **Under-sampled system**: This setting has $\bar{X}_i > \frac{1}{m_i}$, for all i. Specifically, we choose $p_i = 0.52$, $q_i = 0.0625$, $m_i = 16$, and $T_i = 400$, for all i.

For all these four systems, we evaluate the average total LoC incurred in every 100 time slots. We also plot the target optimal value obtained by solving the optimization problems. Moreover, to evaluate whether our VWDF converges to the desirable $\bar{X}_i^*$ and σ_i^*, we also evaluate the total deviation from the desirable values, namely, $\frac{1}{N}\sum_{i=1}^{N} |\frac{\bar{X}_i - \bar{X}_i^*}{\bar{X}_i^*}|$ and $\frac{1}{N}\sum_{i=1}^{N} |\frac{\sigma_i - \sigma_i^*}{\sigma_i^*}|$.

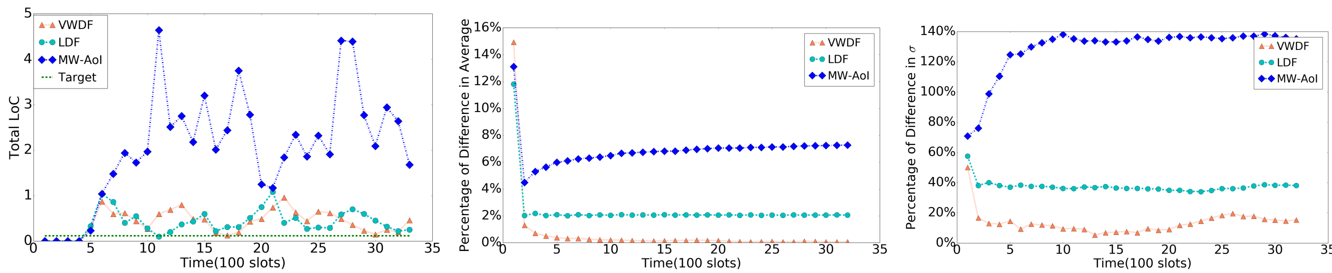
The simulation results for the four systems are shown in Fig. 2, 3, 4, and 5. It can be observed that our VWDF policy achieves the smallest total LoC among all three evaluated policies in all systems. The total LoC of VWDF is also very close to the target optimal value. Moreover, it can be observed that the empirical values of $\bar{X}_i$ and σ_i converge to the target values $\bar{X}_i^*$ and σ_i^* typically within 500 time slots, which, under our network setting, is less than 0.5 second. These result suggest that VWDF not only has good performance but also fast convergence rate.

Next, we evaluate the performance of VWDF under differ-



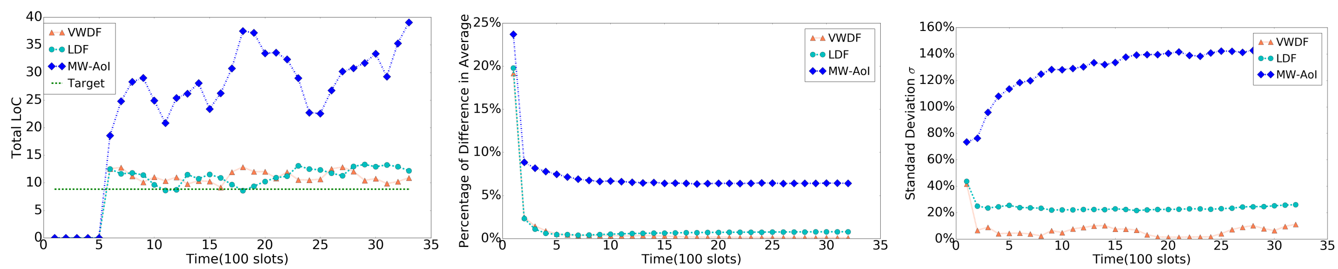
(a) The Total LoC (b) The Total Error from $\bar{X}_i^*$ in Percentage (c) The Total Error from σ_i^* in Percentage

Fig. 3. The over-sampled and over-loaded system



(a) The Total LoC (b) The Total Error from $\bar{X}_i^*$ in Percentage (c) The Total Error from σ_i^* in Percentage

Fig. 4. The over-sampled and under-loaded system



(a) The Total LoC (b) The Total Error from $\bar{X}_i^*$ in Percentage (c) The Total Error from σ_i^* in Percentage

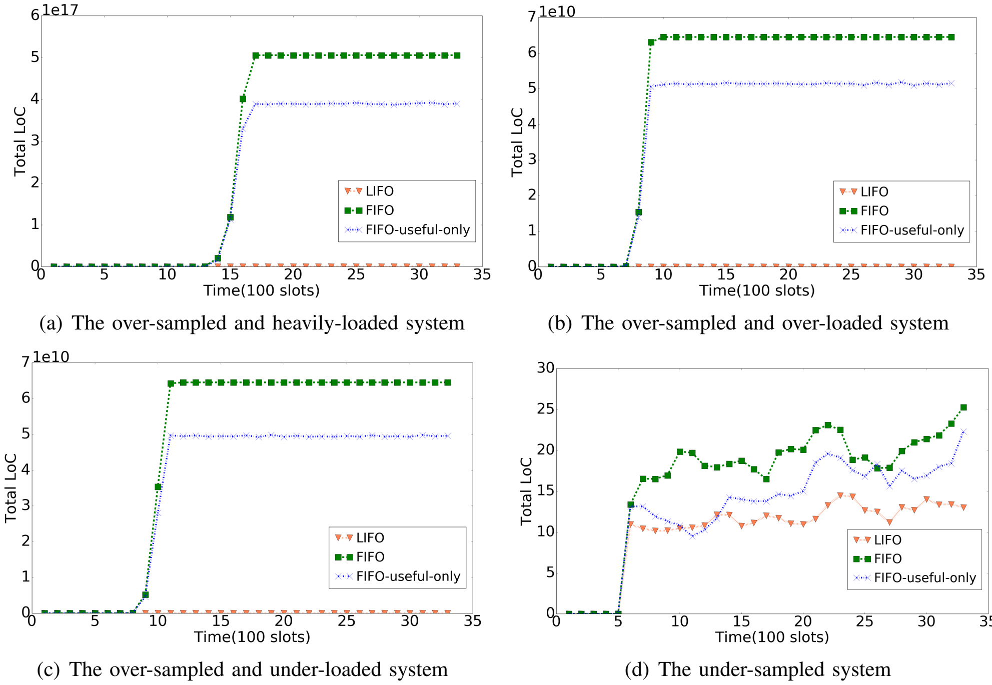
Fig. 5. The Under-sampled System

ent queueing disciplines. In addition to LIFO, we also evaluate two other queueing discipline. The first one is First-In-First-Out (FIFO), where each flow sends the oldest undelivered status update every time it receives a POLL message. The second is a variation of FIFO where each flow drops status updates that have become stale, and sends the oldest useful status update every time it receives a POLL message. This discipline, which we call FIFO-useful-only, is effectively the same as the Earliest-Deadline-First (EDF) policy.

We evaluate these queueing discipline under the four systems describe above. The results are shown in Fig. 6. Clearly, LIFO significantly outperforms the other two queueing discipline. It is well-known that the EDF policy is optimal when the goal is to maximize the number of timely deliveries in real-time wireless networks. The result that FIFO-useful-only performs so poorly also highlight that there are fundamental differences between real-time wireless networks and information update systems.

VIII. Related Work

Information update systems have gained a lot of research interests in the recent years, as many emerging wireless applications require real-time status updates. One state-of-the-art performance metric is Age-of-Information (AoI), which focuses on the elapsed time of last delivery. There have been a lot of studies on the optimizaiton of AoI. Kadota et.al [10] consider the multiple real-time flows with unreliable channel. Further, Kadota, Sinha and Modiano [9] propose a scheduling policy to optimize the AoI when guaranteeing some throughput requirements. Zheng, Zhou and Niu [11] model the estimation error along timeliness and propose the Urgency of Information (UoI) as a new performance metric that can be viewed as the non-linear AoI problems. Li, Li and Hou [12] analyze different sampling behavior and propose a guideline for AoI minimizing policy in general sampling and remote estimation problems. Kam et al. in [13] and [14] propose the concept of *Effective AoI* to capture the estimation error extended from AoI for a Markov source. Yin et al.



(a) The over-sampled and heavily-loaded system

(b) The over-sampled and over-loaded system

(c) The over-sampled and under-loaded system

(d) The under-sampled system

Fig. 6. The LoC of Three Buffer Strategies

[15] propose a proactive scheduling policy that takes some user‘s request patterns into account. Tsai and Wang [16] propose a framework for controller side AoI problem and sensor side remote estimation problem under random 2-way delay. However, the AoI only focuses on the information from the last delivery. On the contrary, our work considers not only the freshness of the information but also the quantity of fresh information updates.

Another related performance metric is timely-throughput, which is defined as the long-term average of timely deliveries. Hou, Borkar and Kumar [5] first propose a frame-based model for the real-time wireless networks and captures the delay constraints of wireless flows. This model has been extended into many directions. Tsanikidis and Ghaderi [17] recently propose a randomized policy to improve the deliver ratio in the frame-based model. Chen and Huang [18] derive a Markov Decision Process solution for optimizing the timely-throughput and quantify the improvement when applying the predictive scheduling policy. An important limitation of these studies is that they only consider the long-term average of timely deliveries and ignore short-term fluctuations. Capturing short-term fluctuations is in particular relevant to information update systems. Singh, Hou and Kumar [19] study the fluctuation of the timely-throughput. Hou [20] and Guo and Hou [21] consider systems where the instantaneous performance of a flow depends on the number of recent timely deliveries, and propose scheduling policies that aim to optimize the system.

IX. Conclusion

We have studied a remote sensing problem and built a model to catch the estimation accuracy when the control center needs to make a estimation of current status then make a appropriate decision. This model considers both the freshness of the information update and the quantity requirements of real-time wireless flows. Through Brownian motion approximation, we approximate the process of the fresh information update as a Reflected Brownian motion. Moreover, the model of the real-time estimation accuracy is described as an optimization problem with the constrains in the averages and temporal variances of the delivery process. We then propose a simple online scheduling policy that employs the optimal averages and variances to achieve the optimal system-wide performance. We also perform comprehensive simulations to show that our scheduling policy converges fast to the optimal averages and variances and outperforms the other two state-of-the-art policies: the LDF policy and the MW-AoI policy. Moreover, our policy does not require any knowledge about the freshness of each information update, and is shown to successfully capture the estimation performance depends on data freshness.

X. Acknowledgement

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REFERENCES

[1] A. M. Bedewy, Y. Sun, and N. B. Shroff, “Minimizing the age of information through queues,” *IEEE Transactions on Information Theory*, vol. 65, no. 8, pp. 5215–5232, 2019.

[2] G. L. Jones, “On the markov chain central limit theorem,” *Probab. Surveys*, vol. 1, pp. 299–320, 2004. [Online]. Available: https://doi.org/10.1214/154957804100000051

[3] J. Harrison, *Brownian Motion and Stochastic Flow Systems*. Robert E. Krieger Publishing Company, 1990. [Online]. Available: https://books.google.com/books?id=6g1tQgAACAAJ

[4] H. Chen and D. D.Yao, *Fundamentals of Queueing Networks*. Spriniger, 2001.

[5] I.-H. Hou, V. Borkar, and P. R. Kumar, “A theory of qos for wireless,” in *INFOCOM 2009, IEEE*, 2009, pp. 486–494.

[6] B. M. Brown, “Martingale central limit theorems,” *The Annals of Mathematical Statistics*, vol. 42, no. 1, pp. 59–66, 1971.

[7] R. E. Caflisch, “Monte carlo and quasi-monte carlo methods,” *Acta Numerica*, vol. 7, p. 1–49, 1998.

[8] I. Hou, “Scheduling heterogeneous real-time traffic over fading wireless channels,” *IEEE/ACM Transactions on Networking*, vol. 22, no. 5, pp. 1631–1644, Oct 2014.

[9] I. Kadota, A. Sinha, and E. Modiano, “Optimizing age of information in wireless networks with throughput constraints,” in *IEEE INFOCOM 2018 - IEEE Conference on Computer Communications*, 2018, pp. 1844–1852.

[10] I. Kadota, A. Sinha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, “Scheduling policies for minimizing age of information in broadcast wireless networks,” *IEEE/ACM Transactions on Networking*, vol. 26, no. 6, pp. 2637–2650, 2018.

[11] X. Zheng, S. Zhou, and Z. Niu, “Beyond age: Urgency of information for timeliness guarantee in status update systems,” in *2020 2nd 6G Wireless Summit (6G SUMMIT)*, 2020, pp. 1–5.

[12] C. Li, S. Li, and Y. T. Hou, “A general model for minimizing age of information at network edge,” in *IEEE INFOCOM 2019 - IEEE Conference on Computer Communications*, 2019, pp. 118–126.

[13] C. Kam, S. Kompella, G. D. Nguyen, J. E. Wieselthier, and A. Ephremides, “Towards an “effective age” concept,” in *2018 IEEE 19th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 2018, pp. 1–5.

[14] ——, “Towards an effective age of information: Remote estimation of a markov source,” in *IEEE INFOCOM 2018 - IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS)*, 2018, pp. 367–372.

[15] B. Yin, S. Zhang, Y. Cheng, L. X. Cai, Z. Jiang, S. Zhou, and Z. Niu, “Only those requested count: Proactive scheduling policies for minimizing effective age-of-information,” in *IEEE INFOCOM 2019 - IEEE Conference on Computer Communications*, 2019, pp. 109–117.

[16] C.-H. Tsai and C.-C. Wang, “Unifying aoi minimization and remote estimation — optimal sensor/controller coordination with random two-way delay,” in *IEEE INFOCOM 2020 - IEEE Conference on Computer Communications*, 2020.

[17] C. Tsanikidis and J. Ghaderi, “On the power of randomization for scheduling real-time traffic in wireless networks,” in *IEEE INFOCOM 2020 - IEEE Conference on Computer Communications*, 2020.

[18] K. Chen and L. Huang, “Timely-throughput optimal scheduling with prediction,” in *IEEE INFOCOM 2018 - IEEE Conference on Computer Communications*, 2018.

[19] R. Singh, I. Hou, and P. R. Kumar, “Fluctuation analysis of debt based policies for wireless networks with hard delay constraints,” in *IEEE INFOCOM 2014 - IEEE Conference on Computer Communications*, 2014, pp. 2400–2408.

[20] I. Hou, “On the modeling and optimization of short-term performance for real-time wireless networks,” in *IEEE INFOCOM 2016 - The 35th Annual IEEE International Conference on Computer Communications*, 2016, pp. 1–9.

[21] D. Guo and I.-H. Hou, “On the credibility of information flows in real-time wireless networks,” in *2019 17th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, 2019.