

Multi-Robot Information Gathering Subject to Resource Constraints

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Abstract—This paper addresses the multi-robot planning problem to generate trajectories for maximum information gathering from an area of interest. To solve this problem, the proposed framework first leverages a Gaussian Mixture Model (GMM) as prior knowledge for modeling informative regions in the target area. Taking samples from the GMM, informative robot paths are then computed that optimize the travel costs subject to energy budgets. Decomposing these paths into waypoints, robot trajectories are planned while respecting kinematic constraints and subsequently replanned online to avoid collisions among themselves. The GMM model is incrementally updated by incorporating the gathered information. We demonstrate that our framework can achieve a significant amount of information gain with the optimal travel distance. We also provide a realistic simulation with a team of mobile robots in a port infrastructure monitoring setting.

I. INTRODUCTION

Mobile robots’ capabilities to gather important information from a large geographical area are the key to their use in a broad range of applications. Some applications include critical infrastructure monitoring [1], surveillance [2], target tracking [3], search and rescue [4], source seeking [5], precision agriculture [6], and nuclear tank inspection [7]. In such applications, the objective is to maximize information gathering by planning robot trajectories while respecting their resource (limited battery life or finite time) and kinematic constraints.

The maximization of information gathering is more efficient and advantageous with multiple robots due to resiliency, coverage, and potential heterogeneity. However, this *multi-robot informative planning* problem also poses some challenges such as scalability, collision avoidance, and coordination. Furthermore, the problem of finding a number of optimal trajectories that maximize the amount of information collected while satisfying constraints is NP-hard [8]. Previous research efforts have presented approximation algorithms [8] and branch and bound solvers [9]



Fig. 1: **Information gathering from port infrastructure monitoring:** Two mobile robots gather information from blue elliptical locations within two critical regions (dotted green lines) of the Port of New Orleans, LA, USA, starting from the red depot location.

to develop informative trajectories for mobile robots. These efforts have a scalability issue under resource constraints, which we address in our work in addition to online trajectory planning for avoiding inter-robot collisions.

To solve the multi-robot informative planning problem, this paper presents a multi-robot trajectory planning framework. The framework leverages a Gaussian Mixture Model (GMM) to characterize critical regions of an area that must be monitored frequently due to their importance and/or utilization. The GMM model is used as prior knowledge and allows us to obtain samples from a mixture of a finite number of Gaussian components. The advantages of this GMM-based information gathering characterization are three-fold: 1) The GMM is a statistical model that can be built from an unknown probability distribution of gathered information; 2) Multiple robots can travel to more informative locations by sampling them from prior knowledge intelligently; and 3) This model can easily scale to higher dimensions if needed.

To maximize information acquisition from an area, we make the following contributions through our framework:

- First, informative paths for robots are calculated based on a set of sample locations from the GMM subject to energy budgets.
- Second, a coordination method is presented for planning multi-robot trajectories respecting their kinematic constraints while avoiding collisions by online replanning during their execution.
- Third, the GMM is incrementally updated by incorporating gathered information in multiple stages.
- Finally, our framework validation is demonstrated in a realistic information gathering task depicted in Fig. 1.

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The remainder of the paper is laid out as follows. To begin with, a relevant literature review on informative motion planning is provided in Section II. All the required definitions and a formulation of our tackled problem are then given in Section III. Afterward, different modules of our framework to solve the formulated problem are described in Section IV. Additionally, realistic simulation results are demonstrated in Section V to validate our framework. Lastly, we conclude and discuss directions for future work in Section VI.

II. RELATED WORK

The problem of informative motion planning has been studied extensively in robotics. For information gathering from a large exploration space that makes a comprehensive search difficult, random graph search [10], heuristic search [11], and sampling-based approaches [12], [13] have been applied. For instance, a sampling-based strategy called a rapidly-exploring information gathering (RIG) algorithm for generating informative robot trajectories is presented subject to a budget constraint [12] to maximize an information quality metric such as information gain or mutual information. Built on this RIG algorithm, an incremental sampling-based algorithm with a convergence criterion is presented [13] for online applications. This algorithm incorporates partially observable dense map representations and state uncertainty of robots into the planning process.

Because of the advantages and efficiency of multi-robot systems, a stream of prior research has been conducted on multi-robot informative planning. A coordination algorithm [14] plans sequences of waypoints for a team of robots in a decentralized fashion for their information gathering in an infrastructure inspection scenario. For solving exploration and mapping problems, a distributed algorithm based on the sequential greedy assignment has been presented to maximize mutual information gain [15]. Several multi-robot information gathering algorithms have utilized Gaussian processes (GPs) [16], [7] to model environmental phenomena. The main limitation of these algorithms is that they do not scale well with the increase of sensor measurements, leading to high computation time. Some such algorithms have employed Partially Observable Markov Decision Processes (POMDPs) [17] to model the quality of possible paths and Deep Reinforcement Learning (Deep RL) [18] to tackle model-free information-gathering tasks when existing models cannot describe the information of interest.

In addition, the GMM model has been applied to information gathering [12] and information-theoretic exploration [19] by mobile robots due to its good computational performance. Hence, we employ a multi-stage GMM model for multi-robot informative planning to incorporate gathered information after each stage. Unlike many of existing information gathering algorithms, our framework accounts for important regions within an environment that are likely to contain locations of high information and thus reduces the overall coverage area for robots. We also generate collision-free robot trajectories from a single depot location for information gathering.

III. PROBLEM FORMULATION

We model a two-dimensional area of interest as a workspace denoted by $\mathcal{W} \subset \mathbb{R}^2$. A set of p polygonal obstacles is denoted as $\mathcal{O}$. The free navigable space for robots is characterized as $\mathcal{E}$, where $\mathcal{E} = \mathcal{W} \setminus \mathcal{O}$. The state space of a single robot is $X = \mathcal{E} \times S^1$, where S^1 represents robot's orientations. Let $x \in X$ be the state of a robot. A robot trajectory in the free space for a fixed time interval that ends at T is represented by $\tau : [0, T] \rightarrow X$. We consider n homogeneous robots that gather information from $\mathcal{E}$ through their range sensors. Each robot has an energy budget B that models a limited battery life.

We model the phenomenon of our interest as a GMM because it has been utilized before for encoding sensor observations and maintaining a local occupancy map [20]. Furthermore, the GMM has several benefits such as scalability, the availability of inference methods, and computational efficiency. We assume that GMM parameters are unknown to our framework and need to be estimated.

Given GMM model parameters Θ , n number of robots, m number of information gathering stages, the informative trajectory planning for n robots under energy budgets and kinematic constraints needs solving the following maximization problem:

$$\arg \max_{\tau_{i,j}} \sum_{j=1}^m \sum_{i=1}^n I(\tau_{i,j} | \Theta) \quad (1)$$

subject to

$$\begin{aligned} x_{t+1} &= f(x_t, u_t), \\ \forall t, \forall i : c(\tau_{i,j}) &\leq B \text{ where } c : \Psi \rightarrow \mathbb{R}^+, \\ \forall t, \forall (i, k), \forall j : \|\tau_{i,j}(t) - \tau_{k,j}(t)\|_2 &\geq \delta \text{ and } i \neq k, \\ \forall p, \forall t : \|\tau_{i,j}(t) - \mathcal{O}_p\|_2 &\geq \delta, \end{aligned}$$

where Ψ is the space of all possible trajectories for n robots, τ is a single robot trajectory, f is a state transition function that estimates a new state x_{t+1} from a state x_t with an action u_t at time t , c is a trajectory cost function, B is energy budget constant (e.g., travel time or battery life), δ is a constant that represents the safety distance between two robots or one robot and an obstacle, and I is a function that corresponds to the gathered information quality.

IV. PROPOSED FRAMEWORK

The pipeline of our proposed framework for solving the multi-robot informative planning problem is illustrated in Fig 2. This section describes each module of the pipeline in detail.

A. Modeling and Sampling Informative Locations

Our framework first utilizes a GMM to represent critical regions of an area $\mathcal{E}$ as prior knowledge for information gathering. We then sample informative locations from the GMM for maximum information collection. The GMM formally represents a weighted sum of r Gaussian probability density functions (PDFs). For a set of sensor measurements $\mathbf{z} \in$

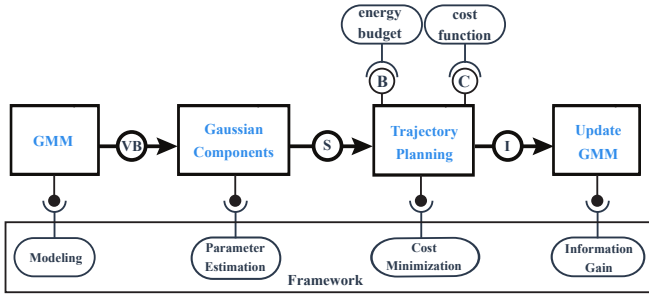


Fig. 2: **The pipeline of our proposed framework:** Information collection from critical regions of an area is modeled as an estimation problem of GMM parameters; The Variational Bayesian (VB) inference algorithm estimates the GMM with r number of Gaussian components; To collect information, a set of samples (S) are taken from these components; For covering these sample locations, multiple robot trajectories under energy and kinematic constraints are planned while minimizing the travel cost; Finally, GMM parameters are updated by computing information gain (I) from gathered measurements.

$\mathbb{R}^d$, we model the information gathering problem using the probability density of the parametric GMM as follows [19]

$$p(\mathbf{z}|\Theta) = \sum_{i=1}^r \alpha_i \mathcal{N}(\mathbf{z}|\mu_i, \Lambda_i), \quad (2)$$

where $\Theta = \{\alpha_i, \mu_i, \Lambda_i\}_{i=1}^r$ represents the parameters of the GMM. $\alpha_i \in \mathbb{R}^1$ is a weight such that $\sum_{i=1}^r \alpha_i = 1$ and $0 \leq \alpha_i \leq 1$, μ_i is a mean, and Λ_i is a covariance matrix for the i^{th} d -dimensional Gaussian PDF of the distribution. The multivariate probability density for $\mathbf{z}$ is expressed as

$$\mathcal{N}(\mathbf{z}|\mu_i, \Lambda_i) = \frac{|\Lambda_i|^{-1/2}}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2}(\mathbf{z} - \mu_i)^T \Lambda_i^{-1}(\mathbf{z} - \mu_i)\right).$$

One way to solve this problem is to utilize a non-adaptive sampling strategy that suffers from sampling sparsity at the maximum. To increase the number of informative samples, we use the Variational Bayesian (VB) algorithm to infer the number of components (Gaussians) from prior knowledge. Afterward, we take random samples uniformly from these components (multivariate Gaussian distributions). Formally, we create a set of samples $S \sim \mathcal{N}$ to find the most informative locations of the area. This way we can plan to have multiple robots synergistically collect measurements from these locations.

B. Finding Informative Paths under Energy Constraints

The main idea of our path planning is to optimally travel all the informative locations in the target area by individual robots while considering their energy constraints. We consider that all robots are initially located at a depot location and that each robot can get recharged at the depot location after following an informative path. Thus, each robot can follow multiple informative paths. We choose a subset S with k samples from S such that $S \subset \mathcal{S}$, where $S = \{1, 2, \dots, k\}$. All the informative paths start from and end to the same depot location while exploring S sample locations.

Next, we need to calculate the lower bounds on informative path costs. Since S are in a 2-D workspace with polygonal obstacles, we calculate the shortest path between two samples that avoids these obstacles. To achieve this, we apply the shortest-path roadmap algorithm [21]. Formally, the roadmap is considered a weighted graph $G = (V, E, w)$, where V is a set of nodes, E is a set of edges, and $w : E \rightarrow \mathbb{R}^{\geq 0}$ are the edge costs. Once the roadmap is computed, we can efficiently find the shortest paths and their costs between sample locations. For a given pair of samples, the shortest path is computed by connecting them with the roadmap.

Finding the informative shortest paths at the minimum cost for n robots with an energy budget B can be considered as a capacitated vehicle routing problem [22]. To solve this problem, we utilize Mixed Integer Linear Programming (MILP). Formally, we define a set of locations $\mathcal{V}$ that incorporates both S and the depot location 0 such that $\mathcal{V} = S \cup \{0\}$. Let A be a set of paths, with $A = \{(i, j) \in \mathcal{V}^2 : i \neq j\}$. Let b_i be the amount of energy that is required to travel to the i^{th} location and c_{ij} be the travel cost between the i^{th} and j^{th} locations. The MILP solver uses binary variables λ_{ij} to optimally assign an informative path to a robot minimizing the sum of linear shortest path costs while respecting its energy constraint as follows

$$\text{minimize} \quad \sum_{i,j \in A} c_{ij} \lambda_{ij}, \quad (3)$$

subject to

$$\sum_{i \in \mathcal{V}, i \neq j} \lambda_{ij} = 1 \quad i \in S, \quad (4)$$

$$\sum_{j \in \mathcal{V}, j \neq i} \lambda_{ij} = 1 \quad j \in S, \quad (5)$$

$$u_j = u_i + b_j \quad i, j \in A, i, j \neq 0, \lambda_{ij} \neq 0, \quad (6)$$

$$b_i \leq u_i \leq B \quad i \in S. \quad (7)$$

This formulation includes a continuous variable u_i for every $i \in \mathcal{V} \setminus \{0\}$ that represents the energy depletion by the robot after it visits the i^{th} location. Eq. (4) and (5) are path constraints that indicate each location can be visited at most once. Eq. (6) and (7) impose constraints on the energy budget for all feasible informative paths.

C. Online Trajectory Planning Respecting Kinematic Constraints

We plan a robot trajectory τ employing the rapidly exploring random tree (RRT) algorithm [23] to follow an informative path. At the high level, we first plan initial trajectories for robots while avoiding static obstacles in the target area and then using online replanning to adjust their corresponding trajectories when facing unexpected obstacles.

In particular, we start with decomposing the informative path obtained from Eq. (3) into a set of waypoints in $\mathcal{E}$. Given a waypoint, we then plan an initial feasible trajectory for a robot while respecting its kinematic constraints. Since this trajectory does not consider other robots as dynamic obstacles, a further step is necessary to handle collisions with dynamic obstacles. To address such situations, we replan a

robot's trajectory by sensing the presence of dynamically changing obstacles. This online replanning allows a robot to navigate in initially unknown environments without prior knowledge of other robots. Generating such trajectories is computationally very efficient since they do not require exact information about other robots' trajectories. However, during replanning, the robot might slightly adjust an intermediate goal waypoint to find feasible and collision-free trajectories. Thus, a robot completes an assigned path by repeating these steps for all waypoints in its trajectory.

D. Update the Model from Gathered Measurements

As a robot explores the informative locations, it gathers measurements through its range sensor and collates them along a trajectory $\tau \in \Psi$. Let $R(x) \subset \mathcal{E}$ be the informative region in $\mathcal{E}$ around a robot state $x \in \tau$. The range sensor reading is

$$h(z, \mathcal{E}) = \begin{cases} z & \text{if } z \in R(x) \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

where $z \in \mathbb{R}^2$. The collated measurements along a trajectory for a robot are represented as $\mathbf{z} = \{z|\tau, h\}$. Given all gathered measurements $\mathbf{Z}$ from n robots, where $\mathbf{Z}$ is the set of n observed variables such that $\mathbf{Z} = \{\mathbf{z}_1, \dots, \mathbf{z}_n\}$ and a GMM configuration from Eq. (2), our objective is to estimate the GMM parameters Θ which approximately match the distribution of sensor measurements. We then employ the Variational Bayesian (VB) algorithm to find an approximation for the posterior distribution $p(\mathbf{Z}|\bar{\Theta})$. The key idea of the VB algorithm is to begin with the prior parameters Θ and then estimate the posterior parameters $\bar{\Theta}$, such that $p(\mathbf{Z}|\bar{\Theta}) \geq p(\mathbf{Z}|\Theta)$. The prior parameters are updated with the posterior parameters for the next stage. This process is repeated for multiple stages until a convergence threshold is reached. The distance between the estimated and true distributions for two consecutive stages is calculated by the Kullback-Leibler (KL) divergence [24], which is also interpreted as information gain.

It is important to note that if we collect sensor measurements from all possible locations over the distribution, then the estimated GMM converges to the true distribution. Since measurements are collected at the expense of the robots' exploration with range sensors that requires energy consumption, our framework minimizes the exploration time while maximizing information gain.

V. EXPERIMENTS

Our experiments are carried out in a port infrastructure monitoring setting, which simplifies the problem of information gathering. Fig. 3 shows our simulated port environment, where the red square box represents the depot location and the gray boxes are static obstacles. All robots are initially located at the depot, and they are required to periodically visit informative regions to collect measurements with their range sensors. We set a one meter sensing radius for the range sensors. We adopt a GMM on a 2-D environment

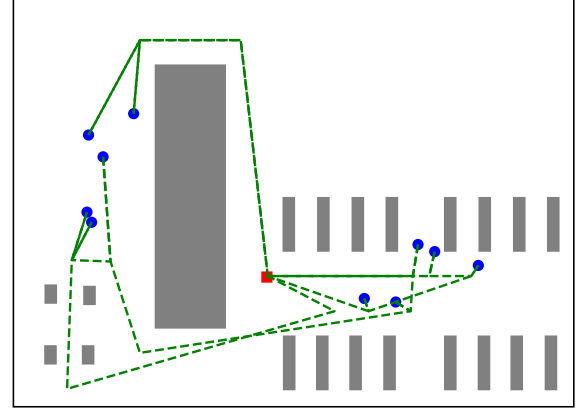
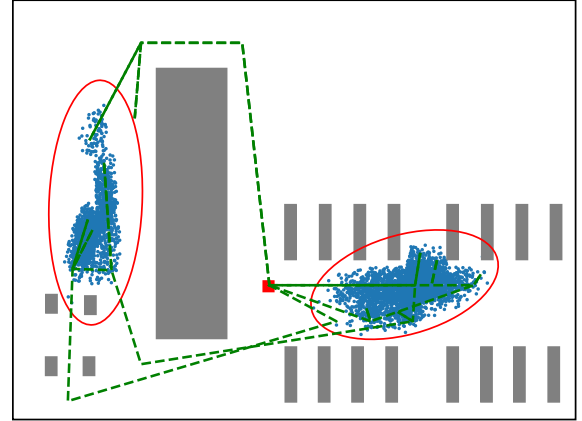
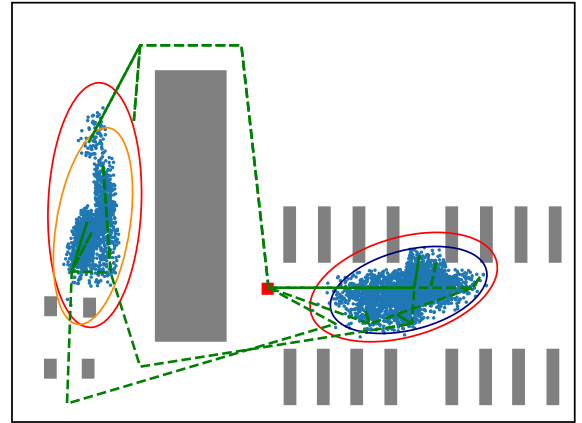


Fig. 3: **Assigned paths for the robots:** The green dotted lines represent paths that are generated based on the robots' energy budget (battery capacity). The red square represents the depot location where robots are initially located and blue circles are sampling locations where robots need to travel to accumulate measurements. The gray boxes represent static obstacles.



(a) Collection of measurements



(b) Estimation of informative regions

Fig. 4: **Informative regions:** The red ellipses (ground truth or given distributions) are bounded regions where measurements are available. The blue dots represent measurements that are gathered using range sensors with a 1 m sensing radius. The orange and blue ellipses are estimated informative regions using the VB algorithm.

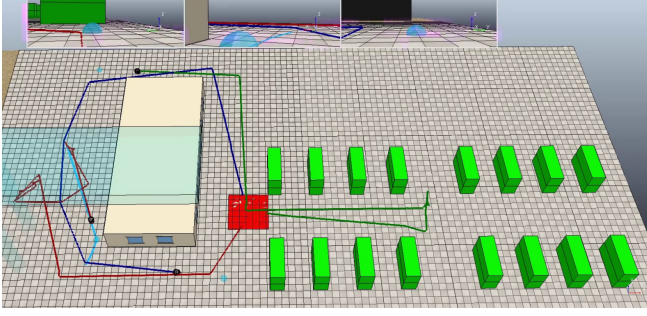


Fig. 5: Each robot executes its assigned informative trajectory from its initial location at the depot, represented by the red square. Each robot’s trajectory is represented by different colored lines.

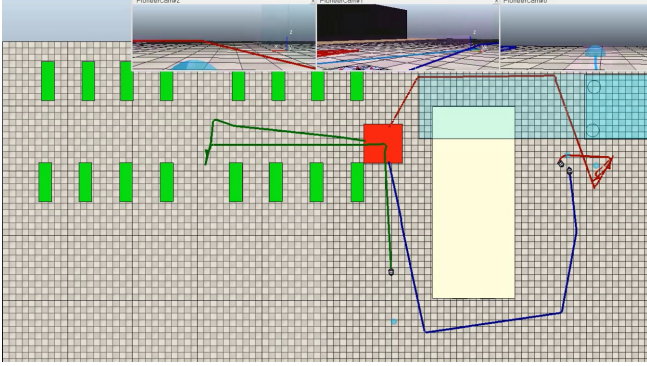


Fig. 6: A collision scenario between two robots that are unaware of each other’s locations. The robots with the blue and red trajectory lines sense each other as dynamic obstacles and execute their online replanning procedures.

to represent events associated with the port infrastructure monitoring. To begin with the task of navigating informative locations using multiple mobile robots, our sampling strategy selects a set of informative locations based on the GMM prior. These informative locations are shown with blue circles in Fig. 3. Then, we plan a set of informative paths for multiple robots while considering collision avoidance and energy consumption. Our path planning module integrates the multi-query visibility roadmap algorithm [21] to efficiently compute the informative paths and their corresponding costs between two sample locations. We use the Gurobi [25] optimizer to find the informative paths for robots based on Mixed Integer-Linear Programming (MILP). The dotted green lines in Fig. 3 are the resultant informative paths.

To facilitate information gathering with multi-robot exploration, we implement decentralized waypoint-based navigation with online replanning. Fig. 5 shows the execution of robot trajectories starting at the red depot location within our simulated port environment built in CoppeliaSim (V-REP) [26]. A video of our experiment can be found at <https://youtu.be/E7uSwGWUVnk>. A team of Pioneer mobile robots decompose their assigned paths into waypoints and then use the RRT algorithm [23] to generate intermediate trajectories. While the robots are navigating, they utilize ultrasonic sensors to dynamically detect obstacles and avoid collisions using their online replanning procedures. Once a robot detects a dynamic obstacle, it moves a short distance

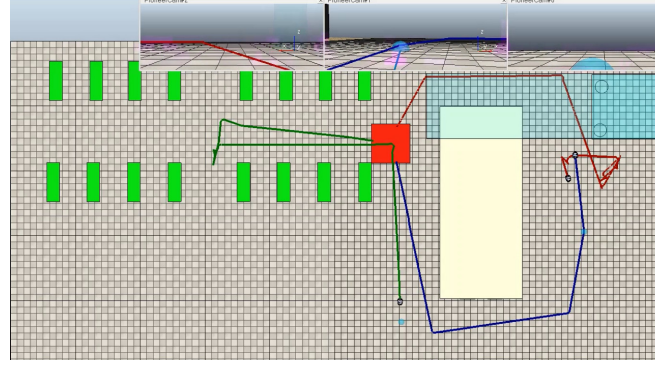


Fig. 7: Two robots replan their trajectories to avoid the collision. During replanning, the robots that detected each other in Fig. 6 move a short distance away and replan around one another.

away from the obstacle and then replans a new trajectory around the obstacle. Fig. 6 shows a collision scenario between two robots, which is avoided in Fig. 7 by online replanning. Each robot fully recharges its battery upon arrival at the depot location after completing a trajectory and then continues on to another trajectory afterward.

The robots receive sensor readings upon arriving at informative regions. In Fig. 4(a), the red ellipses represent the ground truth informative regions and the blue dots are readings that are captured by the range sensors. Finally, utilizing these sensor readings, the VB algorithm updates the informative regions, shown with orange and blue ellipses in Fig. 4(b). We use the Python Scikit-learn VB library [27] for this purpose.

Experiment	Energy Budget	Path Length	Information Gain
1	35 min	2.30 km	0.855
2	35 min	2.33 km	0.838
3	35 min	2.38 km	0.846

TABLE I: Three different experiments with three mobile robots under an energy budget are conducted to compute the path lengths and information gains which are normalized between 0 ~ 1.

Table I shows a qualitative performance evaluation of the proposed framework. We perform three different experiments with three mobile robots with the same energy budget to gather information from the informative regions in Fig. 4. We select a workspace of 400 m × 200 m with an energy budget of 35 minutes operation time for each robot. We observe a consistent cumulative path length of around 2.34 km for these experiments due to the use of an optimal path planner that utilizes the MILP and the visibility graph. We also observe that our exploration strategy can gather a higher information gain of around 0.846. The information gain scores are normalized between 0 and 1, where 0 represents no information gain and 1 represents no loss of information with respect to the ground truth. We use the Python Scikit-learn normalized mutual information score library [27] to compute the information gain. Additionally, Fig. 4(b) validates these results as we can observe that our estimated informative regions (shown in orange and blue ellipse) are very close to the ground truth informative regions (shown in red ellipses).

VI. CONCLUSION AND FUTURE DIRECTIONS

In this paper, we present a multi-robot information gathering framework under resource and kinematic constraints. First, we utilize a GMM as prior knowledge to model critical regions in an area of interest for information gathering. Second, informative robot paths are generated based on a set of sample locations from the GMM while considering the robots' energy budgets and avoiding static obstacles. Third, we plan robot trajectories taking kinematic constraints into account and then replan them for avoiding inter-robot collisions. Fourth, we update the GMM by adding collected information to solve the problem in multiple stages. Our realistic simulations in a port infrastructure monitoring setting show the applicability of our framework for information gathering utilizing the robots' range sensors. There are still several important avenues for future work.

One of our motivating applications for our ideas is monitoring coastal infrastructure [1]. For this application, it is beneficial to consider heterogeneous teams composed of aerial (UAV), ground (UGV), surface (ASV), and underwater (AUV) assets that complement each other in terms of sensor payload, workspace reachability, and energy constraints. We believe that parts of our methodology can be extended to this scenario such as the GMM for modeling the phenomena of interest and online trajectory planning.

Specifically, there are two extensions to our ideas that we would like to explore in the short term. First, we would like to explore in detail other study cases and applications in oceanic and environmental monitoring, where the Gaussian mixtures can model concentrations of physical and chemical quantities of interest (e.g., salinity, chlorophyll, turbidity). Second, we would like to include spatial and temporal variabilities in our modeling. A good alternative for this can be using a mixture of Gaussian Processes (GP) for the models which have been used in the context of multi-robot informative path planning [28].

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