

Stochastic Model for Planning Distributed Wind Generation using Climate Analytics

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Abstract

This paper investigates the optimal design for a distributed generation (DG) system adopting wind turbines. The paper contribution is to formulate and solve a non-linear stochastic programming model to minimize the system lifecycle cost considering the loss-of-load probability and the thermal constraints using climate data from real settings. The model is solved in three cities representing high to medium to low wind speed profiles. Data analytics on 9-years hourly wind speed records permits to estimate the probability distribution for the power generation. The model is tested in a 9-node DG system with random loads. For a total mean load of 50.1MW, New York requires the largest number of turbines at the highest annual cost (6 2MW, 2 3MW, \$3,071,149), then Rio Gallegos (3 1MW, 4 2MW, \$2,689,590) and Wellington (6 1MW, 1 2MW, \$2,509,897). If the total load increases by 6%, the system is still capable to meet the reliability criteria but installed wind capacity and annual costs in New York and Rio Gallegos end higher than in Wellington. Results from decreasing the loss-of-load probability from 0.1% to 0.01% show that the system designed using stochastic programming can be highly reliable.

Keywords

Renewable Energy, Wind Generation, Stochastic Programming, Data Analytics, Reliability

1. Introduction

Distributed generation (DG) systems, also known as onsite generation, produce electricity from many distributed energy resources (DER). DER are small modular units installed in proximity to the end consumers. Hence, DG systems can reduce energy delivery loss and lower the number of transmission lines needed for long distance hauls [1]. Wind turbines (WT) have emerged as a clean and cost-effective DG technology in the past decade. The main obstacles for implementing wind-based DG systems are the relatively high lifecycle cost and the intermittency of output power. In addition, the integration of multiple DER units also complicates the system design and management. Therefore, there is a need for optimizing the design of DG systems to ensure reliable and cost-effective operations [2].

This paper presents a stochastic programming model to find the siting (i.e. placement) and sizing (i.e. capacity) of WT unit in a DG system considering the power volatility and its effect in the system reliability and capability. The model aims to minimize the expected cost of adopting the wind-based DG system. The design criteria are the satisfaction of a pre-determined power loss condition and the limit on the electric power carried in the distribution lines (i.e. thermal constraint). Researchers in [3] have minimized the life cycle cost using heuristic methods. To the best of our knowledge, this research contributes to the very scarce literature on stochastic programming models for the optimal design of DG systems in the climate data analytic framework. The paper is organized as follows. Section 2 presents a literature review on modeling wind speed. Section 3 gives the problem definition. Section 4 presents the methodology. Section 5 describes the numerical experiment and its results. Section 6 states the conclusions.

2. Modeling Wind Speed

Authors in [4] collected hourly data for wind speed in regions of Iran and concluded that it can be approximated by the normal distribution. In [5], the normal distribution was also used to model wind speed in three Canadian regions using a 15-year meteorological database. In [6], authors analyze data obtained from the North Sea between 2003 and 2005 and conclude that the data fits a Weibull distribution. Equation (1) has been used to estimate the wind speed (y_h) as a function of the ground wind speed y_g at height h_g (typically 10m), usually measured in meters per second (m/s), the height above the ground (h) and the Hellman exponent (κ). The exponent represents a friction coefficient based on the costal location, shape of the terrain, and stability of the air and its value is often assumed in the range 0.27-0.34 [7]. Equation (1) indicates that taller turbine towers encounter higher wind speeds. Modern WT systems are typically installed on the 80-m tower or above to reap larger wind profile.

$$y_h = y_g \left(\frac{h}{h_g} \right)^\kappa \quad (1)$$

2.1 Wind Turbine (WT) Power Curve

The WT power curve defined in equation (2) below is based on [8]. It computes the generated power (P) against the wind speed (y) across the turbine blades. The parameters in equation (2) are: a factor to convert from wind power to electrical power (η_{max}) the air density (ρ), the area covered by the turbine blades (A), and the WT power capacity or rated power (P_m). The equation shows that power curve has four operating phases: standby ($0 < y < v_c$), nonlinear production ($v_c \leq y \leq v_r$), rated power region ($v_r \leq y \leq v_s$) and cut-off ($y > v_s$). In the standby phase, no power is generated due to low wind speed. In the nonlinear production phase, P is directly proportional to air density, blade area, and the cube of wind speed. In the rated power region, the power output is constant. In the last phase (i.e. cut-off phase), the generator is shut down for protection and no power is produced.

$$P(y) = \begin{cases} 0 & 0 < y < v_c, y > v_s \\ 0.5\eta_{max}\rho Ay^3 & v_c \leq y \leq v_r \\ P_m & v_r \leq y \leq v_s \end{cases} \quad (2)$$

3. Problem Definition

Figure 1 shows the interconnected DG system under study. The central node of the system (labeled as 1) has a substation that may occasionally bring electricity from the central power plant to cover marginal energy needs of the system. The remaining nodes (i.e. labeled with 2 to 9) represent small cities, companies, stores, or farms adopting onsite wind generation where L_2, L_3, \dots, L_9 represent the power demands at each node. The eight arrows correspond to the power distribution lines. The goal is to find the optimal system design by determining the siting and sizing of WT units at nodes 2-9. The trade-off is to lower the system lifecycle costs and increase its reliability because wind speed fluctuation significantly affects the power output of the WT. The design requires that the total power generated satisfies the total load (i.e. electricity demand) a high percentage of the time. For instance, it is desirable that the system shortages must be as low as one day every 365 days (i.e. a loss-of-load probability of 0.003). The proposed model will determine the capacity of the WT to install at each node and the size of the substation to satisfy the electricity demands with a high probability while abiding to the line thermal limits on the electrical power transmitted to the system nodes (i.e. thermal constraints) under all possible wind speed scenarios.

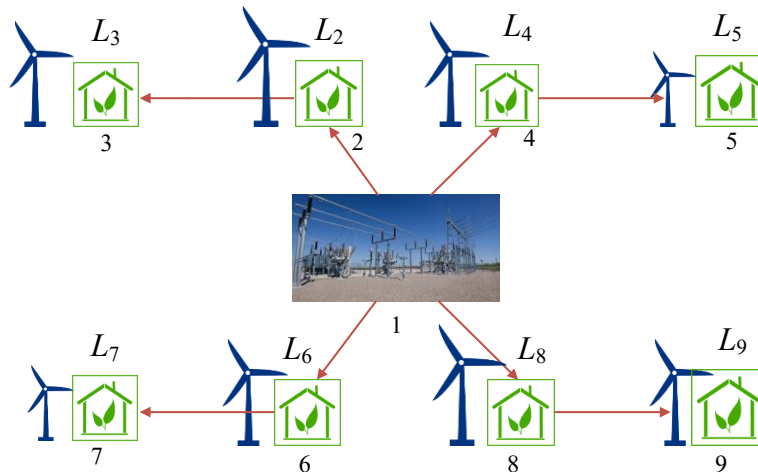


Figure 1: A renewable DG system integrated with wind technology

4. Methodology

The DG planning problem described in the previous section is modeled as a stochastic program that includes: (1) a probabilistic constraint to satisfy the system total reliability requirements; and (2) different wind speed scenarios to guarantee that the thermal constraints are not violated under any circumstance. Tables 1-3 present the notation used in the model. The model and its discussion are immediately below the tables.

Table 1: Sets

Notation	Definition
I	Types of distributed energy resources (DER), (i.e. WT and substations of varying capacity)
J	Nodes in the DG system
F	Upper nodes in the DG system excluding the central node. Distribution lines that originate at these nodes carry power to lower nodes (i.e. terminal nodes).
E	Lower nodes in the DG system
E_f	Lower nodes connected to upper node F
K	Capacity types feasible to adopt for the substation located at the central node
S	Wind speed scenarios. Each element of S is a vector of realizations for the wind speed at each node

Table 2: Decision variables and functions of the decision variables

Notation	Definition
x_{ij}	Binary decision variable. It becomes 1 if DER type i is installed in node j and 0 otherwise
P	Total power generated in the DG system, $P = \sum_{i \in I} \sum_{j \in J} x_{ij} P_{ij}$. Using the central limit theorem, $P \sim N(\sum_{i \in I} \sum_{j \in J} x_{ij} E[P_{ij}], \sum_{i \in I} \sum_{j \in J} x_{ij}^2 \sigma^2(P_{ij}))$ where $E[P_{ij}]$ and $\sigma^2(P_{ij})$ are defined in the first two entries in Table 3. Here P is a function of x_{ij} and the wind speed y_h because P_{ij} , the power output of DER i at node j , is also a function of y_h . Function arguments dropped to simplify notation
f_P	Probability density function for the total generated power (P) generated by the system
f_L	Probability density function for the total system load (L). In this paper, the term load (L) is used to refer to the system power demand

Table 3: Parameters

Notation	Definition
$E[P_{ij}]$	Mean of the power output for DER i at node j for $i \in I$ and $j \in J$
$\sigma^2(P_{ij})$	Variance of the power output for DER i at node j
p_{sj}	Probability for wind speed scenario s at node j
P_{ijs}	Power output when DER type i is installed at node j and the wind speed scenario is $s \in S$
$P_i^{(c)}$	Capacity of DER type i
a_{ij}	Present cost per MW for installing DER type i at node j
\emptyset	Factor to convert a present value to annuity. It is a function of the annual interest rate and the number of years to pay off the amount
b_i	Annual operation and maintenance cost per MW for DER type i
c_i	Tax penalty or subsidy per MW for installing DER type i
L_f	Mean load (i.e. demand) at an upper node f . To simplify the notation, L_f is used instead of $E[L_f]$
L_e	Mean load (i.e. demand) at lower node e
L	System total load (i.e. demand). Using the central limit theorem, $L \sim N(\sum_{i \in I} E[L_i], \sum_{i \in I} \sigma^2(L_i))$ where $E[L_i]$ and $\sigma^2(L_i)$ represent the mean load and its variance at node i
I_f	Maximum allowed current flow at the distribution line running from substation to an upper node f
I_e	Maximum allowed current flow at the distribution line reaching lower node e
V_{DG}	Normal voltage of the DG system
α	loss-of-load probability

Stochastic DG Planning Model:

Minimize:

$$g(x) = \sum_{i \in I} \sum_{j \in J} (a_i \emptyset P_i^{(c)}) x_{ij} + \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} p_{sj} P_{ijs} (b_i + c_i) x_{ij} \quad (3)$$

Subject to:

$$\int_0^\infty \left(\int_y^\infty f_p(z) dz \right) f_L(y) dy \geq 1 - \alpha \quad (4)$$

$$\sum_{e \in E_f} (L_e - \sum_{i \in I} x_{ie} P_{ies}) + L_f - \sum_{i \in I} x_{if} P_{ifs} \leq V_{DG} I_f \quad f \in F, s \in S \quad (5)$$

$$L_e - \sum_{i \in I} x_{ie} P_{ies} \leq V_{DG} I_e \quad \forall e \in E, s \in S \quad (6)$$

$$\sum_{i \in I} x_{ij} \leq 1 \quad \forall j \in J \quad (7)$$

$$\sum_{k \in K} x_{k1} \leq 1 \quad (8)$$

Objective function (3) minimizes the expected total annual cost of adopting the DG system. It is assumed that the annualized installation cost ($\emptyset a_{ij}$) and the annual operations and maintenance costs per MW (b_i) for DER equipment of the same size are independent from the place where it is installed. The term c_i represents a tax incentive or subsidy if a WT is installed or a penalty cost due to emissions of greenhouse gases primarily associated with generation equipment using fossil fuels.

Constraint (4) ensures that the system power quality is guaranteed for a high percentage of the time in a year. It can be written as $\Pr\{P > L\} \geq 1 - \alpha$. Systems operating with small α values are very reliable. For example, if the system is allowed for one day power shortage in a year, α should be less than 0.003 (i.e. 1/365). In practice, when uncertainty in the wind generation increases, extra capacities from substations may be used to compensate the renewable power shortage, but it is not recommended for long time periods. Given a solution, x_{ij} , the left-side of constraint (4) can be computed by a solver assuming the total power (P) and the total load (L) are normally distributed according to the central limit theorem (CLT). Parameters for the distributions of P and L are given in Tables 2 and 3, respectively.

Thermal constraints (5) are for nodes that may provide power to other nodes (i.e. upper nodes) excluding the central node. The constraints ensure that the power carried by the distribution line (DL) serving an upper node and all its lower nodes does not exceed the maximal power limits for such DL and consider that WT installed on the nodes will mitigate some of the requirements. Thermal constraints (6) have the same purpose as the ones in (5) but are for the lower nodes. In (6), the loads (L_e) can be mitigated only by WT installed at those nodes. Thermal constraints (5) and (6) must be satisfied for each wind speed scenario. Constraint (7) specifies that at most one DER is installed at each node. This condition can be relaxed if needed in real applications. Equation (8) requires that the substation be installed at the central node to facilitate the bulk power supply. This constraint is reasonable as most of the electricity is provided by the substation [4]. It is impossible to have a perfect forecast of random wind speed behavior. The stochastic program solved in this paper provides a more realistic solution than the one in [9] where the number of scenarios was reduced to one and the power at the nodes (P_{ij}) was assumed equal to its mean value $E[P_{ij}]$.

5. Numerical Experiment and Results

5.1 System Topology and DG System Costs

The model is tested on a 9-node DG system as shown in Figure 1. This network topology was originally given in [10]. A substation with capacity of 40MW or 50MW can be sited only at the central node. A WT with capacity 1MW, 2MW or 3MW may be installed at each of the remaining nodes. Table 4 presents the related DER costs.

Table 4: Costs for the DER units (Note: O&M=Operations and Maintenance)

i	DER Unit	DER Capacity (MW) $P_i^{(c)}$	Equipment Cost (\$/MW) a_i	Annual O&M Cost (\$/MW) b_i	Annual Penalty Cost (\$/MW) c_i
1	WT 1	1	1,400,000	15,000	0
2	WT 2	2	1,250,000	12,750	0
3	WT 3	3	1,100,000	10,500	0
4	Substation	40	273,000	22,500	5,000
5	Substation	50	227,500	18,750	7,500

5.2 Probability Distribution for the Power Output by DER type i at node j (P_{ij})

The numerical experiment solves the model in three cities: Wellington, New Zealand, Rio Gallegos, Argentina, and New York, USA. Wellington has high winds as well as Rio Gallegos while New York is not as windy. Climate data analytics is done on large samples (about 8,760 observations per year) of wind speed collected hourly for 9 years (2006-2014). The data allows to characterize the probability distribution of the wind speed at each city and compute the power output of each WT unit considered to install at each node. The computational procedure is shown in Figure 2. The defined wind speed (m/s) scenarios are in Table 5. They reflect very well the operational values for a WT with $v_c=2\text{m/s}$, $v_r=12\text{m/s}$ and $v_s = 25\text{m/s}$. For the substation the power output is fixed at its capacity (40 or 50MW).

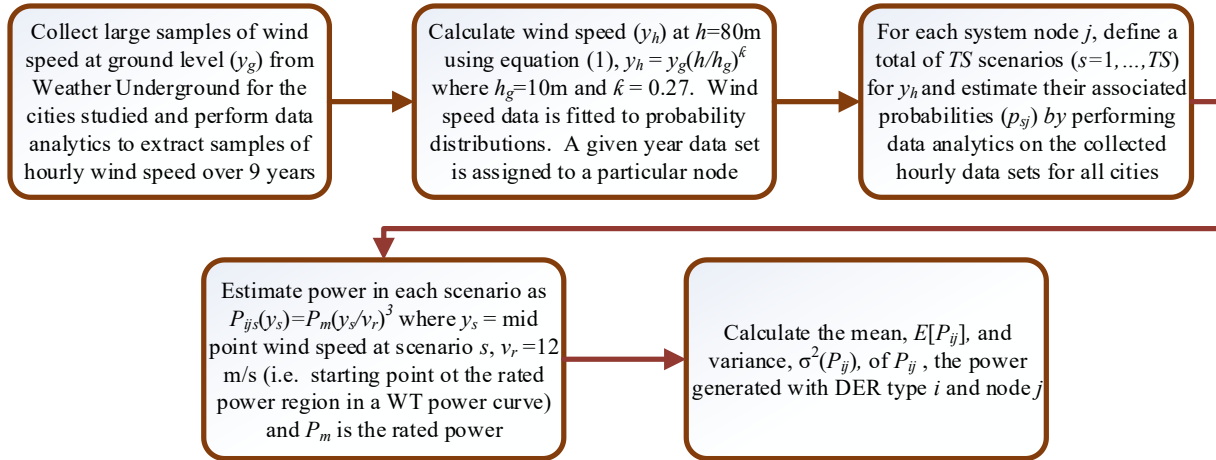


Figure 2: Procedure to compute the probability distribution for the power of each type of WT unit at each node

Table 5: Wind speed scenarios for the numerical experiment based on the wind turbine power curve

Scenario No.	1	2	3	4	5	6	7	8	9	10	11	12	13
Range (m/s)	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	>12
Midpoint (y_s)	0	0	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	18.5

5.3 System Loads, Maximum Current at the Distribution Lines and other Parameter Values

Table 6 presents the mean values for the load at the nodes (MW) and their variances (MW²) and the mean and variance for the total load, $E(L)$, and $\sigma^2(L)$, respectively. Table 6 also shows the maximum allowed current flowing towards each node I_i . The voltage of the distribution lines (V_{DG}) is 33KV and the loss-of-load probability (α) is 0.01.

Table 6: Mean and variances for the loads and maximum current flowing toward each node (N/A=not applicable)

Node	1	2	3	4	5	6	7	8	9	Total
L_i	0.00	7.64	7.72	4.58	4.00	7.64	7.27	6.11	5.14	50.1
$\sigma^2(L_i)$	0.000	0.146	0.210	0.052	0.04	0.146	0.132	0.093	0.066	0.886
I_i	N/A	500	250	450	210	500	250	450	210	N/A

5.4 Numerical Results

The model is coded in AMPL, solved with Knitro and further validated with Analytic Solver Platform. It solves for all the cities with the parameters given in Section 4. Due to the variation of wind speeds, for a total load (L) of 50.1MW (base load), New York requires the largest number of WT at the highest total annual cost (6 2MW, 2 3MW, \$3,071,149), then Rio Gallegos (3 1MW, 4 2MW, \$2,689,590) and Wellington (6 1MW, 1 2MW, \$2,509,897). The costs for the DG system are helpful to guide a decision maker on choosing the optimal place to install the system.

Sensitivity analysis for the behavior of the total cost to increases in the total load (L) ranging from -6% to 6% of the base load, i.e. [47.10MW, 53.10MW], are in Figure 3. In all cases and cities, the system meets the load by adopting WT units. This result is very satisfactory considering that in practice the power demands randomly fluctuate. At 53.10MW, Wellington and Rio Gallegos install 8 WT but the 3MW capacity used in Wellington is much less. Wellington requires (7 2MW, 1 3MW, \$3,071,573) and generates total power (P) of 61.49MW while Rio Gallegos

requires (1 2MW, 7 3MW, \$3,323,156) and generates 64.22MW. New York uses all 3MW turbines at each node and generates 61.28MW with the highest cost of \$3,329,629. Figure 4 presents how the total cost varies with loss-of-load probability (α). If α decreases to $\alpha=0.0001$ (a value close to 0 in the x-axis), the system is feasible but costly because all cities need 3MW turbines except Wellington that requires (1 2MW, 7 3MW). This result gives confidence to the decision maker because it shows that the system designed using stochastic programming is highly reliable.

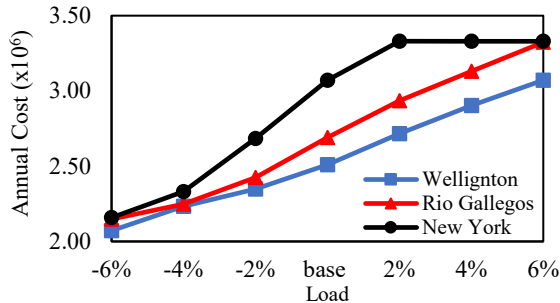
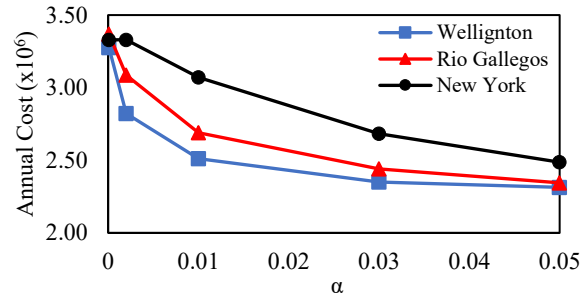


Figure 3: System cost vs. total load

Figure 4: System cost vs. loss-of-load probability (α)

6. Conclusions

This paper demonstrates the benefits of using a non-linear stochastic programming model to find the optimal sizing and siting of variable generators in a DG system. The problem is formulated to keep the system's loss-of-load probability below a pre-specified threshold and to satisfy the thermal constraints under all wind speed scenarios. By leveraging climate data analytics, this research contributes to the very scarce literature on stochastic programming models for the optimal design of DG systems and shows that the proposed model is suitable to renewable integration for a wide range of wind profiles. In addition, the wind-based DG system is able to achieve a loss-of-load probability as low as 0.0001 with affordable cost. Further research is to contrast the results with the ones from a simulation-optimization model, integrate other DER units such as solar photovoltaics and develop operational models subject to uncertain load growth over a multi-year horizon.

Acknowledgements

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