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## r-Process Sites, their Ejecta Composition, and their Imprint in Galactic Chemical Evolution

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# r-Process Sites, their Ejecta Composition, and their Imprint in Galactic Chemical Evolution

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**Abstract.** From low metallicity stars and the presence of radioactive isotopes in deep-sea sediments we know that the main r-process, producing the heaviest elements, is a rare event. The question remains whether neutron star mergers, via GW170817 the only observed r-process site, are the only contributors or also (a rare class of) supernovae, hypernovae/collapsars, as well as neutron star - black hole mergers qualify as candidates. Early galactic evolution as well as variations in nucleosynthesis signatures, e.g. actinide boost stars, might indicate the need for such other sites. We discuss and present the possible options (a) with respect to possible differences in ejecta amount and composition, and (b) in terms of their timing (onset and frequency) during galactic evolution.

## 1. Introduction

A number of contributions to this conference (see e.g. Aprahamian, Coté, Eichler, Holmbeck, Nishimura, Obergaulinger, Piran, Reichert, and others) have summarized the status of abundance observations of neutron-capture (especially r-process) elements and discussed the nucleosynthesis working, the possible astrophysical sites, and the related abundance predictions. Here we address their features, in addition to abundance



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predictions their occurrence frequency and its time evolution throughout galactic history, with the aim to provide an understanding of the impact of these individual sites on the evolution of the Galaxy. The goal is to identify the astrophysical site(s) responsible for (a) the over all total solar r-process abundances as well as (b) their individual features responsible for the variations in observed abundance patterns during galactic evolution.

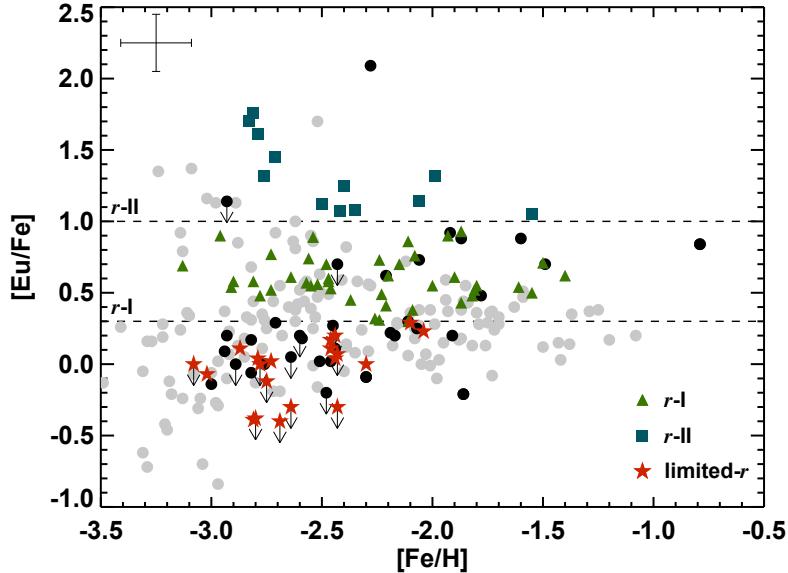
## 2. General Trends in Galactic Evolution and Observations of Low-Metallicity Stars

Based on the nucleosynthesis predictions for (regular) core-collapse (CCSNe) and for type Ia supernovae (SNeIa), plus their occurrence rates, one finds that the early phase of the evolution of galaxies is dominated by the ejecta of (fast evolving) massive stars, i.e. those leading to CCSNe. While there might exist differences for the ejecta composition of supernovae from different progenitor masses, average production ratios in the interstellar gas will be found after some time delay when many such explosions and the mixing of their ejecta with the interstellar medium have taken place. These averaged abundance ratios reflect integrated ejecta yields over the initial mass function of stars. SNeIa originate from exploding white dwarfs in binary systems, i.e. (i) from slowly evolving stars with initially less than  $8 M_{\odot}$  in order to become a white dwarf and (ii) requiring time delaying mass transfer in a binary system before the type Ia supernova explosion. Thus, such events are delayed in comparison to CCSNe from massive single stars. Therefore, SNeIa, with large amounts of Fe and Ni ejecta (typically  $0.5\text{--}0.6 M_{\odot}$  per event), are only important at later phases in galactic evolution. As CCSNe produce larger amounts of  $\alpha$ -elements (from O to Ti) than Fe-group nuclei like Fe and Ni (the latter only of the order  $0.1 M_{\odot}$ ), their average ratio of  $\alpha/\text{Fe}$  is larger than the corresponding solar ratio.

This is reflected in surface abundances of stars, representing the composition of the interstellar gas out of which they formed. If plotted as a function of metallicity  $[\text{Fe}/\text{H}]$  for stars in our Galaxy, Mg (a typical  $\alpha$ -element) shows a relatively small scatter around an average value of  $[\text{Mg}/\text{Fe}]$  between 0.3 and 0.5 up to  $[\text{Fe}/\text{H}] \leq -1$ , decreasing to solar values  $[\text{Mg}/\text{Fe}] = 0$  at  $[\text{Fe}/\text{H}] = 0$  (1). The reason is the early appearance of CCSNe from fast evolving massive, single stars, producing on average  $[\text{Mg}/\text{Fe}] = 0.4$ , see e.g. (2; 3; 4) before SNeIa set in; for their nucleosynthesis patterns see (5; 6; 7). These basic features of galactic evolution have been understood reasonably well for a majority of elements (8; 9; 10), while still many open questions exist in stellar evolution and supernova nucleosynthesis (e.g. 11; 12; 13; 14; 15; 16).

The solar abundance of Eu is to more than 90% dominated by those isotopes which are produced in the r-process (19; 20), thus playing the role of a major r-process indicator. The ratio  $[\text{Eu}/\text{Fe}]$ , displayed in the recent r-process alliance publication (see Fig. 1 above from 17), shows a huge scatter by more than two orders of magnitude at low metallicities, corresponding to very early galactic evolution. While the evolution of the average ratio resembles that of the alpha elements, being of a CCSN origin, also

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**Figure 1.** Derived [Eu/Fe] abundances as a function of metallicity (17): r-I stars (green triangles), r-II stars (blue squares), limited-r (red stars), and non r-process-enhanced stars (black dots), see classifications defined in the original article; upper limits are shown with black arrows. Grey dots refer to an earlier overview (18); reprinted with permission from the *Astrophysical Journal*.

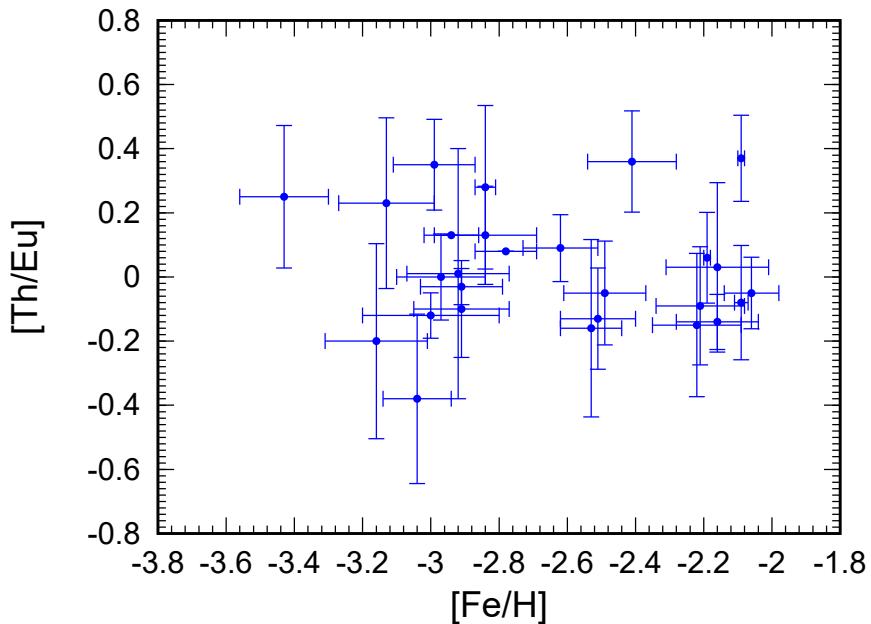
experiencing a decline to solar ratios for  $[\text{Fe}/\text{H}] \geq -1$ , it is far more complex to understand Eu than  $\alpha$ -elements like Mg.

In this context we want to discuss the suggested origins for the r-process and the possibility of their discrimination. There are several aspects to consider: In case there is a single (or at least a dominant contributor), then it has to reproduce the overall solar r-process abundances (a) in terms of the abundance pattern and (b) in terms of their total amount. The latter requires a certain combination of occurrence frequency with the total r-process ejecta from a given site (21). With a typical CCSN frequency of 1/100 yrs about  $10^{-4}$  to  $10^{-5} M_{\odot}$  of r-process matter would need to be produced; for binary merger ejecta with about  $10^{-2} M_{\odot}$  the frequency must be rarer by a factor of 100 to 1000, and in case about  $0.1 M_{\odot}$  or more of r-process matter would be ejected in specific events, the frequency must be again be lower by another factor of 10 or more. If r-process events are rare (although consistent with overall solar r-abundances), due to the fact that more frequent supernovae produce Fe, a not yet well mixed (or averaged) interstellar medium will exist for extended periods of galactic evolution with varying [Eu/Fe] abundances. At lowest metallicities one might actually see the abundance patterns of individual events, which would in case of occurring within a pristine ISM (not yet polluted with Fe from supernovae), have imprinted the Eu/Fe ratio of that event. The amount of observed scatter will vary from such early extreme ratios down to a very small scatter around the average ratio [Eu/Fe] observed at  $[\text{Fe}/\text{H}] = -1$ , when SNeIa set in (22). As can be seen from Fig.1, this happens in observations only in the interval  $-2 \leq [\text{Fe}/\text{H}] \leq -1$ . For

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$[\text{Mg}/\text{Fe}]$  (but also other alpha elements and e.g. Zn and Ge), produced by CCSNe, the approach to average values occurs already at about  $[\text{Fe}/\text{H}] = -3$  or below. Thus one could conclude that r-process events occur at a much lower rate than supernovae, possibly by a factor 100 or more.

The above would be the straight-forward interpretation if there exists only one type of r-process production site. If there exist variations in the overall abundance patterns at lowest metallicities, this could also point to a variety of r-process sites. Indications for the latter are found due to (a) observations with varying Th/Eu ratios (and otherwise close to solar r-abundance patterns?), mostly found at around  $[\text{Fe}/\text{H}] \approx -3$ , see Fig. 2, utilizing the SAGA Database (1) as well as (b) observations which show a steeper decline of the r-process pattern towards heavy nuclei (e.g. 23).



**Figure 2.** Th/Eu ratios for stars with detected thorium abundances from the SAGA Database (1). One can see that at low metallicities around  $[\text{Fe}/\text{H}] \approx -3$  (but also up to  $[\text{Fe}/\text{H}] \approx -2$ ) quite a number of so-called actinide-boost stars can be found. If utilizing initial r-process production ratios which would fit solar r-abundances (24), unreasonable, and even negative, (decay) ages of these stars are obtained (25), not at all consistent with their metallicity, which would indicate the formation of these stars in the very early Galaxy.

Thus, the question is whether the solar r-process composition is either dominated by a single production site or a superposition of ejecta compositions from different sites. While the discussion above, related to actinide-boost stars and “Honda-type” stars, points to the latter (but see also 26), the next question is whether all of these events are rare or could also be frequent. The small scatter for  $[\text{Eu}/\text{Fe}]$  around low values of 0 for the “limited-r” observations (Honda-type stars) in Fig. 1 could permit an event type as frequent as supernovae to produce such an abundance pattern.

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A further interesting aspect is related to the question whether r-process elements are correlated or not correlated with other nucleosynthesis products, in order to determine whether they were co-produced in the same nucleosynthesis site or require a different origin. When comparing the abundances of Fe, Ge, Zr, and r-process Eu in low metallicity stars. a strong correlation of Ge with Fe was found (27), indicating the same nucleosynthesis origin (CCSNe), a weak correlation of Zr with Fe, indicating that other sites than CCSNe (without or low Fe-ejection) contribute as well, and no correlation between Eu and Fe, pointing essentially to a pure r-process origin with (within the observational uncertainties) negligible Fe-ejection. More recent data from the SAGA database (1) permit a weak correlation for  $[\text{Eu}/\text{Fe}] < 0.5$ , i.e. for stars with lower than average r-process enrichment. Interpreted in a straight-forward way this would point to a negligible Fe/Eu ratio in the major r-process sources, while a noticeable co-production of Fe with Eu is possible in less strong r-process sources, e.g. possibly with a weak r-process. Such cases could again be identified with the entry limited-r in Fig. 1.

This apparently "negligible" co-production of Fe with dominant r-process sites is, however, constrained by observational limits for possibly high  $[\text{Eu}/\text{Fe}]$  ratios which would result if such sources are frequent and occur early in galactic evolution, i.e. expecting exactly the Eu/Fe ratios for lowest metallicity stars polluted only by one such event. This concern has been raised recently (28), pointing out that such events should not exceed  $[\text{Eu}/\text{Fe}] > 2$  (see Fig.1). However, it only applies for events occurring earliest in galactic evolution. Events taking place either delayed or that infrequent that regular supernovae contributed already sufficient amounts of Fe, would not be noticeable.

### 3. Possible r-Process Sites

Combining these considerations, we want to pass through the possible sites suggested in the literature:

- (a) Electron-capture supernovae can possibly produce a weak r-process (e.g. 29; 30), not a strong one, and they are probably not rare, if containing stars from the interval of 8 to 10  $M_{\odot}$  of the initial mass function. They could be candidates for "limited-r" observations. But see also recent results related to the final fate of 8 to 10  $M_{\odot}$  stars (31; 6; 32)
- (b) The neutrino-induced processes in He-shells of low-metallicity massive stars (33; 34; 35) would be frequent events at low metallicities, and not lead to a large scatter of e.g.  $[\text{Eu}/\text{Fe}]$ . In addition, the location of the related peaks would not be consistent with the solar r-process pattern.
- (c) The regular neutrino-driven CCSNe which produce Fe, but at most a weak r-process (e.g. 29; 36; 37) are excluded as site of a strong r-process, because they do not produce the correct abundance pattern and would also be too frequent, not permitting a large scatter in  $[\text{Eu}/\text{Fe}]$  at low metallicities. They could, however, be candidates for "limited-r" observations.

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- (d) The frequency of quark deconfinement supernovae, if existent, is still an open question. Present predictions display not a full strong r-process, but the production of elements up to the actinides is possible, however, with the heaviest elements strongly reduced (38).
- (e) Magneto-rotational (MHD jet) supernovae, leading to magnetars (i.e. neutron stars with magnetic fields of  $10^{15}$  G) and neutron-rich polar jet ejection, rely still on parameter studies and depend on assumptions on rotation rates and magnetic field strengths (with pre-collapse magnetic fields of the order  $10^{12}$ - $10^{13}$  Gauss and fast rotation, for which the stellar evolution circumstances have still to be investigated). Due to these somewhat extreme initial conditions before collapse (39; 40; 41; 42; 43; 44; 45), such events, displaying a full and strong r-process, will be rare, possibly as infrequent as 1 in 100 to 1 in 1000 of regular CCSNe. These sites could produce  $10^{-2}$   $M_{\odot}$  of r-process matter and  $10^{-5}$   $M_{\odot}$  of Eu, i.e. lead to a large [Eu/Fe] scatter, with [Eu/Fe] as high as 3.5 in their remnants (28). However, due to their rareness, earlier supernovae producing Fe, would reduce this ratio (see next section). Within the occurrence frequency constraints discussed above, their ejecta would be consistent with the required total r-process production (and possibly the solar r-process abundance pattern). However, this site still requires observational confirmation. Less extreme initial magnetic fields can be enhanced via magneto-rotational MRI instabilities and also lead to explosions (see e.g. 46). Depending on the delay of the explosion, varying degrees of r-processing will be obtained from no, over a weak, to a strong r-process.
- (f) Collapsars, with large initial masses, resulting in black hole formation after core collapse, will occur if fast rotation causes strong magnetic fields, black hole accretion disks, polar jet ejection, and Gamma-ray bursts GRBs (e.g. 47; 48; 49; 50; 51; 52; 53; 54). Their nucleosynthesis has been discussed in a number of publications (55; 56; 57; 58; 59). r-process ejecta of the order  $>0.1 M_{\odot}$  (59) would be consistent with the required total amount of solar r-process matter, if they would occur even rarer than magneto-rotational supernovae by a factor of 10 or more. Until present, there exists no fully consistent model with respect to the amount and composition of jet ejecta vs. black hole accretion disk outflows (and thus also whether long-duration GRBs, hypernovae, and collapsars are a homogeneous class of objects). If combined with hypernova models (49), the [Eu/Fe] ratio in their remnants would be higher than 3, i.e. beyond observed values for lowest metallicity stars. It was concluded that this would speak against collapsars as main r-process sources (28), but we want to point again to the above discussion on the uncertain (and not yet consistently modeled) jet vs. black hole accretion disk outflows. In any case, such ratios would be consistent with the finding that r-process sites should not be correlated with Fe (i.e. not co-produce Fe - in comparison to solar ratios within observational uncertainties, 27).
- (g) Compact binary mergers (NS-NS mergers) have been suggested as r-process sites

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since the 1970s (60) with very first ejecta mass (61) and nucleosynthesis (62) predictions in 1999. Since then (and especially after GW170817 63) research in this field has been exponentially growing, we just mention here a few recent articles, mainly reviews (64; 65; 66; 67; 21; 68; 69; 70; 71; 72; 73; 74). Nucleosynthesis ejecta are composed of dynamic ejecta, consisting of tidal arm (e.g. 75; 76) and prompt collisional ejecta (e.g. 77; 78; 79; 80), a neutrino wind during the intermediate phase of a hypermassive neutron star (if the combined mass is permitting such an intermediate phase, rather than directly leading to black hole formation, e.g. 81), and finally black hole accretion disk outflows (e.g. 82; 83; 84; 85). Light curve predictions vs. observations measure the impact of radioactive decay (e.g. 86; 87; 88; 89; 90; 91; 92), optical, infrared as well as  $\gamma$ -ray spectra give a clue to elemental abundance patterns (e.g. 93; 94; 95; 96), with a first direct detection of element lines (Sr, 97). Based on present observations, a ratio  $>1$  of the accretion disk outflow to dynamical ejecta is expected in GW170817. This suggests an overall abundance pattern close to solar r-abundances and a total amount of about  $0.01 M_{\odot}$  r-process ejecta with an Eu mass of close to  $10^{-5} M_{\odot}$ , combined with a (rare) occurrence frequency, as discussed above. At present there exists one multi-messenger observation with gravitational waves, a short duration GRB, and an electromagnetic counterpart (kilonova) for GW170817. Before LIGO/Virgo was sensitive enough to detect gravitational waves, three kilonova events were observed associated with an sGRB, pointing also to neutron star mergers. In 2019 during the LIGO/Virgo O3 run, another five gravitational wave candidate event alerts have been provided ‡, not yet accompanied by detection of an electromagnetic counterpart. But they occurred at much larger distances than GW170817 which was as close as 40 Mpc.

- (h) Neutron Star - Black Hole Mergers were actually the first suggested site among compact binary mergers (60; 98), leading to the disruption of the neutron star by the black hole. In 2019 it seems that within the gravitational wave candidate events of the LIGO/Virgo O3 five candidates have been identified, but no electromagnetic counterpart has been detected, yet. This could depend on sensitivity limits for the related distances (being all much further away than GW170817) or that the black hole mass and spin did not permit the ejection of matter after disruption of the neutron star (99). Similar (or larger) tidal ejecta r-process masses as for neutron star mergers have been predicted (100; 75; 73), the neutrino wind and black hole accretion disk outflows depend strongly on the BH/NS mass ratio and BH spin (101; 102).

Summarizing the sites above, we have three possible sites for a weak r-process (limited-r of Fig. 1), being (a), (c), possibly (d) and also (e) in the transition to low magnetic fields which lead to a variation of r-process strength. All of them (might) come with a high occurrence frequency, permitting already at low metallicities a small scatter

‡ see [https://en.wikipedia.org/wiki/List\\_of\\_gravitational\\_wave\\_observations](https://en.wikipedia.org/wiki/List_of_gravitational_wave_observations)

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as seen for the limited-r sample. (e) would be rare for a full, main r-process but could be increasingly frequent towards lower magnetic fields, thus possibly covering a whole range of [Eu/Fe] ratios from a strong to a weak r-process. The sites responsible for the main r-process (i.e. producing also the heaviest elements), which are or can be consistent with the observational constraints on total r-process ejecta masses for explaining the solar r-abundance and also reproducing a solar r-process pattern, the following ones are remaining: (e) MHD jet, magneto-rotational supernovae (in case of strong magnetic fields, magnetars), (f) collapsars, and compact binary mergers (g, h). Of these (e) and (f) would belong to massive stars, i.e. occurring during the earliest instances of galactic evolution. (g) is related to the coalescence of compact objects, produced via the collapse of massive stars, and would experience a delay in their occurrence. Such differences could be recognizable in galactic evolution modeling. (h) includes the formation of one black hole (with a more massive stellar progenitor than neutron stars and a possibly faster inspiral). This leads to smaller delay times.

#### 4. Galactic Chemical Evolution of r-Process Events

How can rare or frequent nucleosynthesis events be modeled consistently in galactic chemical evolution? Which role plays the mixing of ejecta with the local interstellar medium and how should global or turbulent mixing be treated? In addition, there exist indications that (ultra-faint) dwarf galaxies are the earliest building blocks of galactic evolution and their merging will finally lead to the evolution of the early Galaxy as a whole. Due to different gas densities such galactic substructures might experience different star formation efficiencies and due to a low gravitational pull they might lose explosive ejecta more easily. This can have an effect on the point in time (and metallicity) when the first imprints of explosive ejecta can be observed. But before discussing such complexities, we start with the simple models, assuming instantaneous mixing of new nucleosynthesis ejecta throughout the whole galaxy.

##### 4.1. Homogeneous evolution models

Early evolution models go back to (103) and (104). More advanced approaches took into account that (explosive) stellar ejecta enter the interstellar medium (ISM) delayed with respect to the birth of a star by the duration of its stellar evolution. The understanding from light elements up to the Fe-group, based the evolution and death of single stars as well as SNeIa, came with approaches employing the instantaneous mixing approximation IMA, i.e. mixing ejecta instantaneously throughout the galaxy (e.g. 8; 9). Applications towards the enrichment of heavy elements (including r-process contributions) as a function of time or metallicity [Fe/H] followed (see e.g. 105; 106; 107; 108; 109; 110; 111; 112; 113; 114; 115; 116; 117).

The IMA simplification encounters the problem that all stars at a given time inherit the same abundance patterns of elements. This means (a) that a unique relation between

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time and  $[\text{Fe}/\text{H}]$  is established, (b) it is also impossible to reproduce a scatter in the galactic abundances  $[\text{X}/\text{Fe}]$  as a function of  $[\text{Fe}/\text{H}]$ , and (c) due to extended mixing  $[\text{X}/\text{Fe}]$  imprints are already seen at very low metallicities, where they would not be existent in realistic simulations. Thus, instead of a spread of abundance distributions, curves with a single value for each  $[\text{Fe}/\text{H}]$ , and extending possibly to too low metallicities are obtained. Nevertheless, when in realistic evolution models sufficient star formation and stellar deaths occur, to sample a superposition over the IMF or also r-process events, this approach is applicable. It permits to get a quick overview of the trends in chemical evolution with a considerably lower computational effort and is probably approximately valid also in case of rare r-process events for  $[\text{Fe}/\text{H}] > -2$ .

Early investigations utilized coalescence delay times for neutron star mergers after the formation of the binary neutron star system with a narrow spread. Population synthesis studies, consistent with the occurrence of short-duration gamma-ray bursts (sGRBs, related to compact binary mergers) indicate that the possible delay times follow a distribution with a large spread, ranging over orders of magnitude with a  $t^{-1}$  behavior. Based on such a behavior, studies with the simpler IMA modeling of chemical evolution (112; 113; 115) come to the conclusion that mergers would not be able to reproduce the galactic evolution for metallicities  $[\text{Fe}/\text{H}] > -2$ , including the decline of  $[\text{Eu}/\text{Fe}]$  at  $[\text{Fe}/\text{H}] = -1$ . This would require either a different delay time distribution (118) or an additional source for the main, strong, r-process. Another solution was suggested (116): star formation takes only place in cooled regions of the ISM, i.e. not all recently ejected matter can already be incorporated and stars contain lower metallicities  $[\text{Fe}/\text{H}]$  than the overall ISM at the time of their birth. This shifts e.g.  $[\text{Eu}/\text{Fe}]$  ratios to lower  $[\text{Fe}/\text{H}]$  and has a similar effect as a steeper delay-time distribution.

This subsection has, however, not discussed the behavior and possible problems at very low metallicities. For a detailed study of especially early chemical evolution, including the reproduction of spreads in abundance ratios due to local inhomogeneities, a more complex inhomogeneous chemical evolution treatment is required.

#### *4.2. Inhomogeneous galactic chemical evolution at low metallicities*

The above subsection has shown, that homogeneous galactic evolution models, approximately applicable for metallicities  $[\text{Fe}/\text{H}] > -2$ , indicate some problems for neutron star mergers as the sole main, strong r-process source, but that these can possibly be resolved with variations of the time delay distribution of mergers after the second neutron star is formed or when introducing that star formation only takes place in a cold ISM. The present subsection is dedicated to the challenges of explaining the r/Fe scatter at lower metallicities and which sample of r-process sites are required for explaining these observations. This task can only be tackled with an inhomogeneous approach.

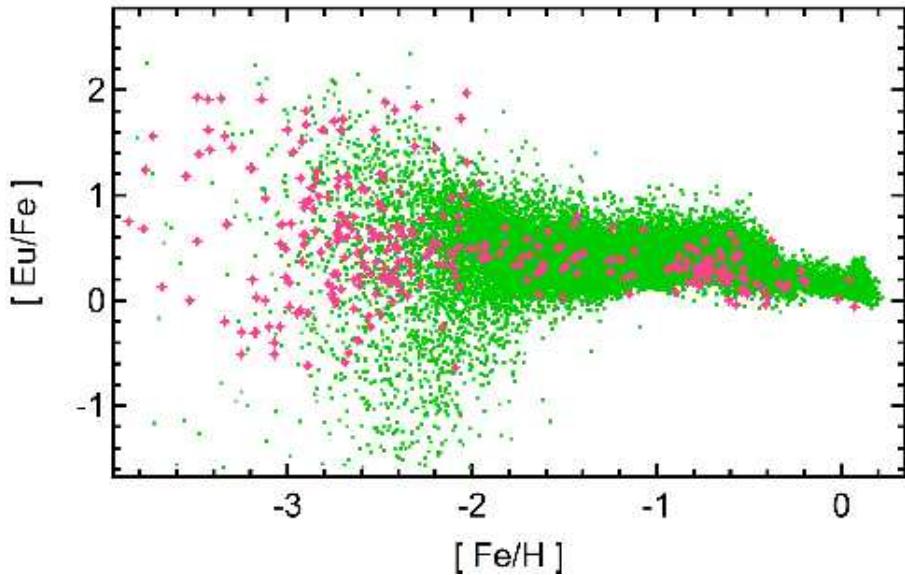
Local inhomogeneities can only be produced if only limited amounts of ISM are polluted by and mixed with the ejecta of each event. The latter effect is of essential

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importance especially at low metallicities, where portions of the ISM are already polluted by stellar winds and supernovae, and others are not. Inhomogeneous mixing could produce larger element ratios in strongly polluted areas and smaller values in still less polluted ones. This means that the scatter in [X/Fe] at low metallicities can be a helpful asset in hinting to the origin of element X. Inhomogeneous mixing can experience similar [Fe/H] values in different locations of the Galaxy at different times, or different [Fe/H] values at the same time. In addition, different portions of the ISM are polluted by different types of events, leading to a scatter at the same metallicity, which can in fact be utilized as a constraint for these different stellar ejecta. Therefore, more advanced chemical evolution studies revoked the instantaneous mixing approximation (e.g. 119; 120; 121; 122; 123; 124; 125; 126; 127; 128; 129; 130; 131; 132). For the reasons summarized above, specially for the origin of r-process elements like Eu at lowest metallicities, only such inhomogeneous chemical evolution models should be utilized. To explore this approach fully, also a related resolutions is required. A  $10^{51}$  erg supernova explosion mixes via a Sedov blast wave only with about  $5 \times 10^4 M_{\odot}$  of interstellar medium (see 28). Although different explosion energies and ejecta geometries are encountered, detailed simulations (133) come to similar results for neutron star mergers. In order to follow the evolution of the ISM correctly, equivalent resolutions are required, lower resolutions lead to an artificial mixing of ejecta with larger amounts of the ISM, altering results into the direction of the IMA.

Inhomogeneous chem(odynam)ical evolution models for r-process elements, like Eu, have been provided (121), comparing neutron star mergers and core-collapse supernovae, (125; 126) comparing MHD jet-SNe and regular core-collapse supernovae, (124; 127; 128; 129) only utilizing neutron star mergers, (132) comparing neutron star mergers and neutron star - black hole mergers, and (130; 131) addressing how early, single star related rare r-process ejecta could complement the shortcomings of neutron star mergers as the only main, strong r-process site. One of the main questions here is related to the problem of reproducing [Eu/Fe] at low(est) metallicities. (126) and (132) treated the galactic chemical evolution of europium (Eu), iron (Fe) and  $\alpha$ -elements, like e.g. oxygen (O), still utilizing a more classical stochastic approach which neglects large scale turbulent mixing effects (e.g. spiral arm mixing) and includes only those introduced by stellar explosions and the so initiated mixing with the surrounding ISM according to a Sedov-Taylor blast wave. This stochastic approach grasps the main features of the impact of the first stars and their (explosive) ejecta on the evolution of the heavy element enrichment. On the other hand, more sophisticated SPH models, being more realistic on galactic infall, outflow, and turbulent mixing, have the disadvantage that often the size of the SPH particles and/or the smoothing length utilized is too large, thus automatically mixing ejecta with unrealistically large amounts of ISM. This leads to results closer to the IMA and thus moves [Eu/Fe] features incorrectly to lower metallicities, apparently solving the problems NS mergers alone experience at lowest metallicities. (130) and (131) did probably perform the most extensive simulations of all these approaches, both coming to the conclusion that at lowest metallicities another

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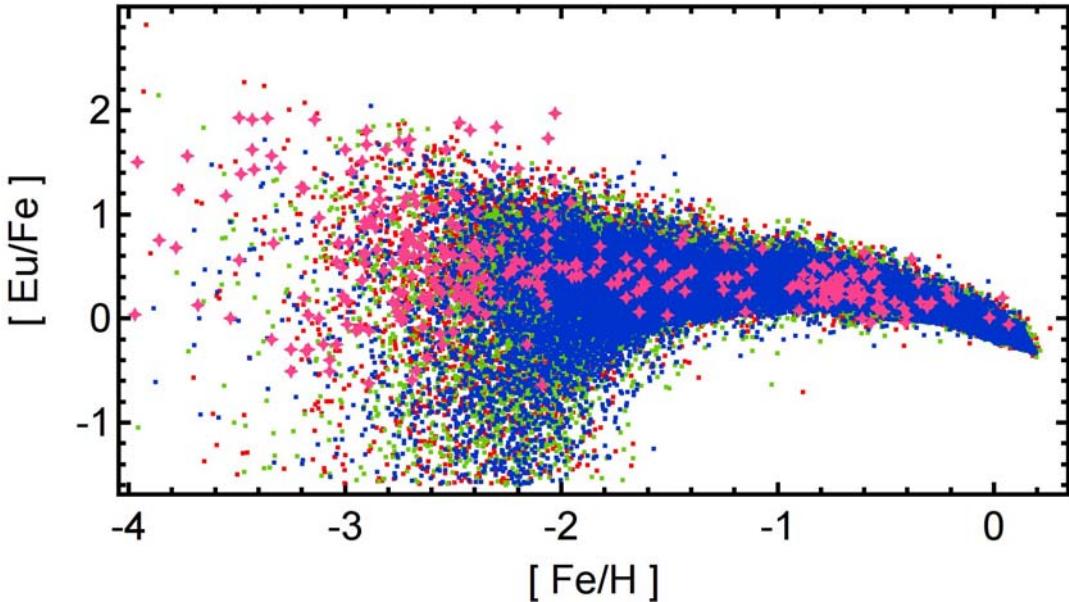
**Figure 3.** Evolution of Eu-abundances in galactic chemical evolution models (126) including both magneto-rotational supernovae and neutron star mergers as r-process sites. Magenta stars represent observations whereas green dots represent model stars. The combination of magneto-rotational MHD-jet supernovae - being of strong importance at low metallicities - early in the evolution of the Galaxy, and neutron star mergers permits a perfect fit with observations.

r-process site than neutron star mergers is required to remedy these shortcomings.

(126) showed how the inclusion of MHD jet supernovae can avoid the problems at lowest metallicities. As mergers experience first the local Fe pollution by two supernovae, producing the later merging neutron stars, the merger event takes place in a medium already moved to higher metallicities ( $0.2 M_{\odot}$  of Fe mixed with  $5 \times 10^4 M_{\odot}$  of ISM, dominated by H, leads to an [Fe/H] ratio of -2.6). An MHD jet supernovae, when exploding in a pristine ISM, will only result in the [Fe/H] of this event, which can be as low as -3.9. Fig. 3 shows such a combination of MHD jet supernovae, contributing already at lowest metallicities, and NS mergers, setting in below [Fe/H]=-2 (both assumed to take place with similar occurrence frequencies).

Another possible solution, or at least improvement for the lowest metallicities can be obtained when including NS-BH merger. They experience only the Fe pollution of one supernova and occur earlier than NS mergers, because black holes are resulting from failed supernovae, i.e. massive stars which experience a black hole rather than neutron star formation at the end of their evolution (an important feature at low metallicities). Fig.4 shows the effect of the lower limit for black hole formation on galactic evolution modeling. It has, however, to be considered that the full ejecta from NS BH mergers (taken from 75) is dependent on the size and spin of the black hole, and for the more massive black holes all of the neutron star matter can be swallowed by the black hole, rather than resulting in a disruption of the neutron star and ejection of matter.

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**Figure 4.** Effect of the different choices of the prescriptions for failed SN at low metallicities on the chemical evolution of [Eu/Fe] from (author?) (132): **Magenta crosses** represent observations. Red (green, blue) squares represent GCE models where all stars  $> 20 M_{\odot}$  ( $25 M_{\odot}$ ,  $30 M_{\odot}$ ) at metallicities  $Z \leq 10^{-2} Z_{\odot}$  are forming failed SNe at the end of their life.

(59) and (117) included also collapsars in a simpler (IMA) chemical evolution modeling. They could show that, within the assumptions discussed above in item (f), collapsars could also solve the problems experienced at lowest metallicities. This involves, however the uncertainties related to the fact that up to now no complete and consistent modeling of IGRB jet ejecta combined with the black hole accretion disk outflow and the accompanying nucleosynthesis has taken place, yet.

#### 4.3. Utilizing long-lived radioactive isotopes

A complete list of isotopes with half-lives in the range  $10^7 - 10^{11}$  yr is given in Table 1. They cover a time span from a lower limit in excess of the evolution time of massive stars up to (and beyond) the age of the Universe. Such nuclei can be utilized as “chronometers” for nucleosynthesis processes in galactic evolution and also serve as a measure for the age of the Galaxy, if these processes contributed early (see e.g, 134; 135; 136; 137; 24; 138; 139; 140). The list is not long. Two of the nuclei require predictions for the production of the ground and isomeric states ( $^{92}\text{Nb}$ ,  $^{176}\text{Lu}$ ). With the exception of  $^{40}\text{K}$ , all of the remaining nuclei are heavier than the “Fe-group” and can only be made via neutron capture.

The nuclei with half-lives comparable to the age of the Galaxy/Universe,  $^{232}\text{Th}$  and  $^{238}\text{U}$ , are made in a single nucleosynthesis process, the main, strong r-process (as well as all other actinide isotopes listed here). The question is how to predict reliable

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**Table 1.** Isotopes with half-lives in the range  $10^7$ - $10^{11}$  yr

Isootope	Half-Life	Isootope	Half-Life
$^{40}\text{K}$	$1.3 \times 10^9$ yr	$^{205}\text{Pb}$	$1.5 \times 10^7$ yr
$^{87}\text{Rb}$	$4.8 \times 10^{10}$ yr	$^{232}\text{Th}$	$1.4 \times 10^{10}$ yr
$^{92}\text{Nb}$	$3.5 \times 10^7$ yr	$^{235}\text{U}$	$7 \times 10^8$ yr
$^{129}\text{I}$	$1.6 \times 10^7$ yr	$^{236}\text{U}$	$2.3 \times 10^7$ yr
$^{147}\text{Sm}$	$1.1 \times 10^{11}$ yr	$^{238}\text{U}$	$4.5 \times 10^9$ yr
$^{176}\text{Lu}$	$3.7 \times 10^{10}$ yr	$^{244}\text{Pu}$	$8 \times 10^7$ yr
$^{187}\text{Re}$	$4.4 \times 10^{10}$ yr	$^{247}\text{Cm}$	$1.6 \times 10^7$ yr

production ratios for these long-lived isotopes, if (a) not even the site is completely clear, and (b) even for a given site nuclear uncertainties enter. The abundances of especially these heaviest nuclei produced in the r-process depend on mass models,  $\beta$ -decay properties (half-lives, delayed fission, delayed neutron emission), fission barriers and fragment distributions, and last but not least neutron captures, especially their rates during the r-process freeze-out (65; 76; 141).

Nevertheless, parametrized, so-called site-independent fits, based on a superposition of neutron densities, have been utilized to reproduce all solar r-process abundances from  $A = 130$  through the actinides. These were then applied to predict the production ratios of long-lived isotopes, which can be compared to meteoritic ratios for these long-lived actinide isotopes like  $^{232}\text{Th}$  and  $^{235,238}\text{U}$ , indicating for ratios, indicating the abundances at the formation of the solar system. This permits conclusions on overall galactic evolution (see e.g. 134; 135; 136; 140). When utilizing instead observations in individual old stars, and making use of e.g. detected elemental Th/Eu and U/Eu ratios, it is possible to obtain age estimates for these stars. If the latest pollution before their birth had a solar-type pattern, the change in abundance ratios due to decay can give an indication for the age of the star. This resulted in typical ages in the range 12-14 Gyr (137; 24; 138; 139) for the lowest metallicity (and oldest) stars in the Galaxy. This use of low-metallicity stars has the advantage that one can avoid uncertainties introduced by chemical evolution modeling.

As discussed above, among the stars with observed Th and U, there exist a number of actinide-boost stars with an enhanced ratio of Th/Eu (see Fig. 2) and U/Eu in comparison to the other r-process enhanced stars (see e.g. 139; 25). These are observed especially at low metallicities around  $[\text{Fe}/\text{H}] \approx -3$  (142; 26). When utilizing production ratios from the parametrized fits, discussed above, to estimate the age of those stars, unrealistically low to negative ages result. Although it appears that most of their element abundances, up to the third r-process peak, are close to solar-system r-abundances, one should further investigate possible correlations between the actinide boost and other elemental abundance features. The question is whether this points to a different site than the one responsible for the solar-type r-process abundances or variations of conditions

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within the same site

The actinide to Eu ratios is related to the path of the r-process and the timing (a) when the actinides are reached via the r-process flow and (b) when fission plays a role after their production and continuation on to heavier nuclei. Due to this the r-process results are dependent on the proton/nucleon ratio  $Y_e$  in expanding matter, determining the neutron to seed ratio. Intuitively one could expect that the lowest (most neutron-rich)  $Y_e$ 's would lead to the highest actinide production and thus actinide to Eu ratios. (142) showed, with their nuclear physics input, that the actinide to Eu ratio is highest for a  $Y_e$  in the range 0.1-0.15 (see their Figs. 16 and 17), with the highest ratio around  $Y_e=0.125$ . Higher  $Y_e$ -values (i.e. less neutron-rich conditions) lead to a smaller actinide production, because of a less strong r-process which did not produce, yet, large amounts of actinides. Lower  $Y_e$ -values (i.e. more or very neutron-rich conditions) lead also to smaller actinide to Eu ratios. This is due to the fact that an initially higher actinide production is reduced later by fission cycling, which can be very effective in destroying the actinides. The details depend on mass models and related fission barriers.

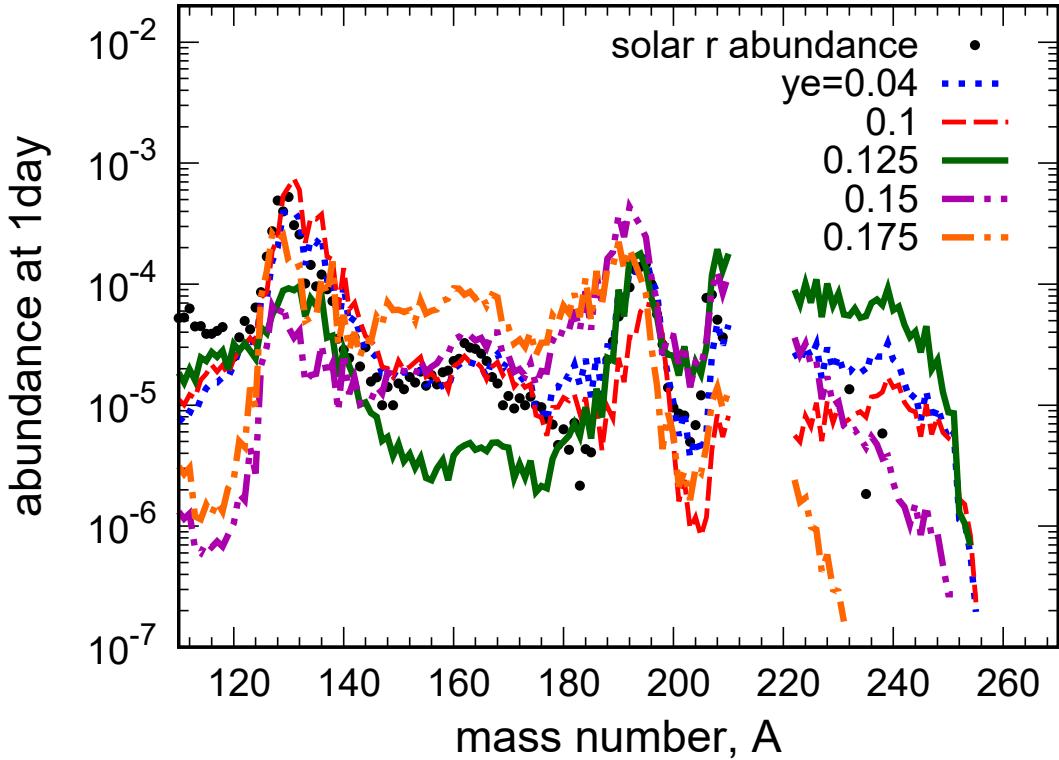
(143) could show a similar behavior as indicated in Fig. 5 using the trajectory adopted in (87). Thus, an actinide boost can be attained by having  $Y_e$ -conditions close to 0.125. Lower actinide to Eu ratios can either be attained by a superposition of conditions with  $Y_e>0.15$  or by having a larger contribution from very neutron-rich environments with  $Y_e<0.10$ . (144) did an independent study, testing in detail the influence of nuclear physics uncertainties. They find slightly higher  $Y_e$ -values of 0.15 for the maximum actinide production, but similar conclusions. In addition, the actinide decline for lower  $Y_e$ 's is examined as a function of the number of fission cycles permitted by the actual  $Y_e$ . In all these cases Th/U, both actinide nuclei close in mass numbers, are not strongly affected by a variation in  $Y_e$ .

(26) argue, that superpositions of a variety of conditions, as occurring in neutron star mergers of possibly different masses and/or mass ratios (affecting the total amount of dynamic ejecta, neutrino wind, and black hole accretion disk outflows), can be responsible for the different outcome resulting in solar-type r-process patterns or actinide boosts. Another option is that this points to different sites, containing, e.g., larger amounts of lower  $Y_e$ 's (as expected from the dynamic tidal ejecta of neutron star mergers). In this case the dominant r-process site could lead to smaller actinide to Eu ratios, as found in most r-enhanced stars.

Based on the discussion above, MHD jets with slightly higher  $Y_e$ 's, reaching only down to  $Y_e=0.15$  might result in higher actinide abundances. But this is still speculative and also strongly dependent on uncertain nuclear input physics as well as uncertainties in site specific conditions. However, it is intriguing to check how both types of abundance patterns can be observed in low-metallicity stars. Improved predictions for all the most probable main r-process sites, discussed in the previous section can hopefully lead to a one-to-one connection between responsible production sites and observations.

When discussing how a variation in the produced abundance pattern can affect kilonova lightcurves and spectra (92), with the aim to identify the exact pattern for

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**Figure 5.** From (143): Utilizing the DZ mass model permits large variations of actinide production, even at low  $Y_e$ . The highest actinide production is found at  $Y_e = 0.125$

individual observed events. (59) and (117) discuss variations expected among neutron star mergers and collapsars, the latter being dominated by less neutron-rich black hole accretion disk outflows.

In addition to the identification and possible explanation of the abundance pattern in actinide boost stars, related to long-lived unstable Th and U isotopes, short-lived radioactive isotopes have been addressed by (145; 146). Nuclei with half-lives of a few  $10^6$  to  $10^7$  yr permit to probe recent nucleosynthesis events in the vicinity of the presolar nebula. In the present context only nuclei of an r-process origin are of interest here. Of these (147) point out  $^{129}\text{I}$  and  $^{247}\text{Cm}$  with identical half-lives.

In addition to observations of long-lived radioactive species like Th and U, seen via the spectra of stars throughout galactic evolution, there have also been detections in deep-sea sediments, indicating more recent additions of these elements to the earth. While the earlier discussion points to rare strong r-process events in the early galaxy, the latter detections, suggest the same in recent history. Radioactive species can act as witness of recent additions to the solar system, dependent on their half-lives. Two specific isotopes have been utilized in recent years to measure such activities in deep sea sediments. One of them,  $^{60}\text{Fe}$ , has a half-life of  $2.6 \times 10^6$  yr and can indicate recent additions from events occurring up to several million years ago.  $^{60}\text{Fe}$  is produced during

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the evolution and explosion of massive stars, leading to supernovae (148; 149; 4; 14). It is found in deep-sea sediments which incorporated stellar debris from a nearby explosion about two million years ago (150; 151; 152; 153). Such a contribution is consistent with a supernova origin and related occurrence frequencies, witnessing the last nearby event. Another isotope utilized,  $^{244}\text{Pu}$ , has a half-life of  $8.1 \times 10^7$  yr (see table 1) and would contain a collection from quite a number of contributing events. As discussed above, strong r-process events with a frequency as high as CCSNe would require  $10^{-4}$ – $10^{-5}$   $\text{M}_\odot$  of r-process matter ejected per event in order to explain the present day solar abundances (21). The  $^{244}\text{Pu}$  detection in (154) is lower than expected from such predictions by two orders of magnitude, suggesting that considerable actinide nucleosynthesis is very rare (permitting substantial decay since the last nearby event). This indicates that CCSNe did not contribute significantly to the strong r-process in the solar neighborhood for the past few hundred million years (155), but does not exclude a weak r-process contribution with very minor Eu production (156). Very recent investigations (157), possibly indicating that  $^{60}\text{Fe}$  from the last CCSNe might have been accompanied by a (very) minor  $^{244}\text{Pu}$  contribution underline (i) the rare major actinide producing events, but (ii) possibly also a frequent weak r-process, producing very small, but not negligible amounts of actinides.

## 5. Discussion and Conclusions

The previous sections underline that observations of Honda-type or limited-r stars ask for a weak r-process, which can be frequent, causing a small scatter in [Eu/Fe] also at low metallicities, when originating from such sources. Possible candidates, although not proven yet, are (see section 3 for sources) (a) EC supernovae, (c) CCSNe, and (d) Quark Deconfinement supernovae. Detailed predictions still need to emerge, especially whether not only small amounts of Eu but also small (but not negligible) amounts of actinides can be co-produced.

The main, strong r-process, has to come from rare events, with an occurrence frequency lower by a factor 100 (or more) than that of CCSNe. This requirement can be matched by (see section 3 for sources) (g) NS mergers, but also additional rare sites like (e) MHD jet supernovae, (f) collapsars, and (h) NS-BH mergers are not excluded. In fact, a number (among them the most sophisticated) chemical evolution simulations argue for such additional candidate sites (126; 125; 112; 114; 113; 130; 59; 131; 117; 132), especially to explain deficiencies with respect to [Eu/Fe] observations encountered for low metallicities down to  $[\text{Fe}/\text{H}] = -3$  and below. This goes together with the question whether actinide boost stars require a different origin or are just a sign of varying conditions among the variety of compact binary merger events.

Thus, while there exist substantial doubts whether neutron star mergers can be the only site of the main, strong r-process, responsible for the heavy r-process elements up to the actinides (also at lowest metallicities), there might exist possible ways out of the dilemma, avoiding contradictions with observations:

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- neutron star kicks during the second supernova explosion can act in such a way that the actual neutron star merger takes place outside the initial Fe pollution by the preceding supernovae (158; 159; 160), thus moving the [Eu/Fe] imprint to lower metallicities. This could permit the ejection of r-process matter in environments with a lower [Fe/H]. If in such a way the merger event can be displaced from the original supernovae, it is found (161) that mergers could be made barely consistent with the [Eu/Fe] observations, if such displacement is taken into account and very short coalescence timescales of  $10^6$  yr are used.
- One of the major aspects for the treatment of compact binary mergers is the connection of earlier supernova events, producing a neutron star and Fe ejecta, with the later delayed merger event and its r-process ejecta. Special binary evolution aspects might apply for such close binary systems and the resulting supernovae (162; 163). It is not clear whether the evolution of a close binary system of massive stars with several mass exchange periods leads to the same type of CCSN events with substantial Fe ejection as known from single star evolution.
- In addition to reproducing galactic evolution observations of [Eu/Fe] at lowest metallicities (halo stars), which challenges the early r-process contributions by NS mergers, there exists a challenge of the Eu enrichment at high metallicities (disk stars), related to the observed decrease of [Eu/Fe] vs. metallicity at  $[Fe/H] > -1$ , (112; 113; 114; 115; 117). Possible solutions for the latter are that star formation takes only place in the cold ISM, which does not include the latest and recent enrichments (116) or that the delay-time distributions of the mergers (after formation of the two neutron stars) does not follow a simple  $t^{-1}$  power law as inferred from population synthesis studies and statistics of short duration gamma-ray bursts (118).
- Triple and multiple stellar systems can cause different delay time distributions for the neutron star mergers (164; 165) and enhance NS merger rates.
- Like other hydrodynamic calculations, large-scale SPH simulations, can suffer from resolutions problems, which overestimates the material mixing. This mixes Fe with larger amounts of interstellar medium and thus causes a decrease in the metallicity at which r-process sets in. This seemed (possibly incorrectly) to permit NS mergers as the only source of the main, strong r-process, also at lowest metallicities (124; 127; 128; 129). On the other hand, such simulations can handle substantial turbulent mixing of interstellar medium matter in the early Galaxy, not included in simpler stochastic inhomogenous chemical evolution simulations (see section 4.2).
- Another option is that early on, in galactic substructures of the size of dwarf galaxies, different star formation rates can exist combined with a loss of nucleosynthesis ejecta out of these galaxies due to smaller gravity. This can shift the behavior of the [Eu/Fe] ratio as a function of metallicity [Fe/H] to lower metallicities. When also considering a statistical distribution of (down to small) coalescence timescales in the individual substructures, the low-metallicity

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observations could possibly be matched (166), while the merging of these substructures within the early Galaxy at later times can be made consistent with the [Eu/Fe] decline (similar to alpha elements) at [Fe/H]=-1. See in this context also (167).

The discussion above underlines, that it is still inconclusive whether binary compact mergers alone can explain low metallicity observations, although they could be responsible for the dominant amount of r-process products in the solar system and present Galaxy. Independent of this uncertainty, when introducing an additional component which acts already at lowest metallicities, a perfect fit to observations can be obtained. The detection of actinide boost stars, found in particular at metallicities as low as  $[Fe/H] \approx -3$ , adds to this question, whether an additional component with different nucleosynthesis conditions, being active at such low metallicities, is required. The alternative would be that a variety in the statistical distribution of NS mergers properties with different masses and mass ratios can cover the variation in nucleosynthesis conditions.

This overview of the possible astrophysical r-process sites, from proven ones to other still more speculative options, has shown that substantial progress has been made since the process was postulated in the 1950s. But it also shows that, despite the very first observation of an r-process production site (GW170817) in 2017, confirming neutron star mergers as probably the most important site, many open questions remain and further progress on all fronts is required in a truly interdisciplinary effort. This includes nuclear physics far from stability and the nuclear equation of state, magneto-hydrodynamic modeling with high resolution to resolve the magneto-rotational instability (MRI), sophisticated inhomogeneous galactic evolution modeling with high resolution from small dwarf-galaxy size substructures to clusters, including inflows and outflows, but foremost multi-messenger astronomical observations which permit to detect and understand magnetars and other rare classes of supernovae, core-collapse supernovae and the option whether they can give rise to a weak r-process, collapsar/hypernovae/long duration GRBs, and the vast class of compact binary mergers, sampling the whole variety of possible systems and their statistical properties.

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