# Integration of Statistical and Administrative Agricultural Data from Namibia

by

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*Abstract*: Statistical and administrative agencies often collect information on related parameters. Discrepancies between estimates from distinct data sources can arise due to differences in definitions, reference periods, and data collection protocols. Integrating statistical data with administrative data is appealing for saving data collection costs, reducing respondent burden, and improving the coherence of estimates produced by statistical and administrative agencies. Model based techniques, such as small area estimation and measurement error models, for combining multiple data sources have benefits of transparency, reproducibility, and the ability to provide an estimated uncertainty. Issues associated with integrating statistical data with administrative data are discussed in the context of data from Namibia. The national statistical agency in Namibia produces estimates of crop area using data from probability samples. Simultaneously, the Namibia Ministry of Agriculture, Water, and Forestry obtains crop area estimates through extension programs. We illustrate the use of a structural measurement error model for the purpose of synthesizing the administrative and survey data to form a unified estimate of crop area. Limitations on the available data preclude us from conducting a genuine, thorough application. Nonetheless, our illustration of methodology holds potential use for a general practitioner.

*Keywords*: Administrative data, small area estimation, Africa

**1. Introduction**

Agricultural statistical systems in both developed and developing countries make use of data arising from administrative processes. Broadly, administrative data consist of information gathered for purposes other than production of statistics. In agriculture, examples of such processes include farm assistance and regulatory programs. We provide further examples and a definition of administrative data in Section 2.1. Using administrative data can reduce costs and respondent burden, improve small area estimates, and enable more timely publications. The benefits of administrative data are attained in a variety of ways, as discussed in more detail in Section 2.2. Administrative data are typically not immediately ready for use in official statistics because the objectives of the administrative agency often differ from the objectives of the statistical office. Logistical difficulties in using administrative data for statistical purposes include a need to coordinate multiple administrative agencies and confidentiality restrictions. The administrative process may lack formal quality control protocols, resulting in missing data or inconsistencies over time. We discuss ways to overcome challenges of using administrative data in Section 2.3.

Incorporating administrative data in the process of producing agricultural statistics is particularly appealing for developing countries, where resources for surveys or censuses are often scarce. Namibia, a country that gained independence from adjacent South Africa in 1990, has a history of systematically collecting administrative data on agriculture. Namibia has also conducted less frequent surveys or censuses when resources are available. We conducted a study on combining survey and administrative data for the purpose of improving estimates of crop area in 6 regions of Namibia where communal agriculture is predominant. Namibia is well-suited to an investigation of the role of administrative data in producing official statistics on agriculture for several reasons. Namibia has well-developed administrative systems, recently conducted a national agricultural survey, and has a memorandum of understanding between administrative and statistical agencies. This excellent collaboration between the administrative and statistical agencies of Namibia was invaluable in facilitating the data acquisition process for our project.

Two model structures that are capable of integrating survey and administrative data are the Fay-Herriot model and the structural measurement error model. The key difference between the Fay-Herriot and structural models is the nature of the assumption about the administrative data source. The administrative data source is treated as a fixed covariate in the Fay-Herriot model, but the structural model assumes that the administrative data are stochastic. When using the structural model, one must take care to ensure that the model parameters are identifiable. Global Strategy (2015b) discusses identification conditions.

In Section 4, we argue that a Bayesian analysis of a structural measurement error model is well-suited to the myriad of objectives surrounding this data analysis. Using the data from Namibia as a platform, we illustrate how to use a structural measurement error model to integrate survey and administrative data. We explain how to interpret the parameters of the structural model in terms of the bias of the estimator based on the administrative data source. We also illustrate the process of evaluating the extent to which the administrative data improve upon the efficiency of the survey-based estimators.

This paper synthesizes work completed as part of the Global Strategy research project on using administrative data for agricultural statistics. In Section 2, we provide a broad overview of the role of administrative data for production of agricultural statistics. In Section 3, we describe statistical and survey data sources for Namibia and explain how the general issues outlined in Section 2 relate to the specific data sources for agriculture in Namibia. In Section 4, we illustrate the use of the structural measurement error model for the purpose of estimating crop area for six administrative regions in Namibia. In Section 5, we compare the Fay-Herriot model to the structural model through simulation. Section 6 summarizes and discusses the primary implications of our data analysis.

## 2. Overview of the Role of Administrative Data for Production of Agricultural Statistics

This section introduces major themes in the discourse on uses of administrative data for official statistics. We define administrative data, discuss benefits attained through a variety of uses of administrative data, and overview approaches to addressing challenges with administrative data quality. References include Beckler (2013), Brackstone (1987), Carfagna and Carfagna (2010), Sarndal and Lundstrom (2005), United Nations (2007), Wallgren and Wallgren (2007), Wallgren and Wallgren (2010), and Zhang (2012).

### 2.1 Defining Administrative Data

Administrative data come from diverse sources, including government records, subjective reporting systems, and private organizations. Therefore, many definitions of administrative data exist. We define administrative data as in Global Strategy (2015):

*Definition 1: Administrative data are data holdings containing information collected primarily for administrative (not statistical) purposes by government departments and other organizations usually during the delivery of a service or for the purpose of registration, record keeping, or documentation of a transaction (ADLS, 2015). In the context of agriculture, these sources include (i) registers, for example, registers of farms, livestock, farmers, lists of farmers associations, and registers of parastatals/institutions that generate administrative data for commercial or cash crops; (ii) transaction data, for example, imports and exports and data generated during transactions at customs border posts; and (iii) routine data collected by agricultural extension workers.*

Definition 1 separates an agricultural routine data collection system (ARDS) (item *iii*) from traditional administrative data (TAD) (items *i-ii*). In a typical ARDS, an agricultural extension officer or local expert makes a determination based on his/her observations and interactions with agricultural producers. Many developing countries have an ARDS. In contrast to an ARDS, traditional administrative data (TAD) refers to measurements of well-defined farm entities obtained through participation in government programs or private organizations. Examples of TAD for agriculture include data collected through taxation, regulatory processes (i.e., farm inspections, vaccination campaigns), farm assistance programs (i.e., subsidy, insurance), and monitoring programs (i.e., livestock tracing systems).

Statistical and administrative data are collected for inherently different reasons. Statistical data are collected from processes, such as surveys or censuses, that intend to estimate a population parameter. Administrative data are a natural byproduct of program administration. This fundamental difference between the objectives of administrative and statistical data collections renders administrative data useful as a complement to survey data but also generates challenges when using administrative data for production of official statistics.

### 2.2 Uses and Benefits of Administrative Data

Administrative data can improve the operational efficiency of the survey process and the statistical efficiency of an estimator. Because the data are already collected for an administrative purpose, using administrative data has the potential to lower data collection costs for the statistical office and reduce respondent burden. Using administrative data can reduce biases due to non-sampling errors and can lower the variances of estimators. Administrative data collected more frequently or at more granular levels of geographic detail than survey or census data can enable small area estimates or forecasts.

The benefits of administrative data are attained in many ways. The specific benefit of administrative data depends on the use. Global Strategy (2015) classifies uses of administrative data into two broad categories: direct use and indirect use.

When used directly, administrative data are a substitute for survey or census data. The simplest direct use of administrative data is direct tabulation, where the published report contains numerical summaries, such as or means or totals, of an administrative data source. Direct tabulation may be appropriate if the administrative process collects the population parameter of interest. For example, a procedure to regulate markets may collect accurate information on sales or exports. This information may then be directly published for subsequent economic analysis. A more nuanced direct use of administrative data is as a substitute for a survey response. For instance, in the National Resources Inventory (NRI) of the United States, specific managements and practices for sampled crop fields are obtained from administrative records maintained by the Natural Resources Conservation Service of the United States Department of Agriculture. Obtaining answers to some survey questions from an administrative database can lower costs and reduce respondent burden.

Administrative data can also be used indirectly to support multiple stages of the survey process. Administrative data can improve the coverage of sampling frames and can improve the efficiency of a survey design, for example, by providing auxiliary information for defining strata or selection probabilities. In data collection and processing, administrative sources can provide contact information for sampled units and can help check for errors. For instance, administrative data helped identify outliers during the Italian Census of Agriculture (Reale, Torti, and Riani, 2013). In estimation, administrative data can provide auxiliary information for model-assisted calibration, model-based estimation, or imputation. In mass imputation, or micro-integration, survey responses are imputed onto a population defined by a collection of administrative files.

### 2.3 Overcoming Quality Challenges

Attaining the benefits of administrative data typically requires overcoming challenges related to numerous quality dimensions. Problems with administrative data quality typically stem from the fundamental characteristic that the data are generated for an administrative, rather than statistical, purpose. Improper coverage of the population can result from selective participation in administrative programs. Administrative processes can sometimes incentivize biased reporting. Changes over time and decentralization of procedures can result in inconsistencies in identification variables, coding systems, collected data items, and data collection methods. Conflicts between administrative and survey data are common and can arise, for example, because administrative and statistical agencies use different definitions.

Proactive involvement with administrative agencies and the public can overcome complex issues associated with data-sharing. Prell at al. (2009) propose a set of guidelines for successful data-sharing agreements that grew out of an analysis of several case studies in the US. One of these guidelines is to define objectives that are of mutual interest to statistical and administrative agencies. Formal assessments of administrative data quality prior to use can help facilitate integration of administrative data into the statistical production process (Iwig et al., 2013). Part of this assessment can involve audits to verify the accuracy of the administrative data.

Statistical procedures can also aid in overcoming deficiencies in administrative data quality. Formal editing procedures can correct errors or impute missing values in administrative data. Combining multiple data sources can help leverage their relative strengths and weaknesses. Survey data may be judged unbiased, while administrative data may provide a biased measurement for a larger number of population elements. Different administrative sources may collect different variables or cover different segments of the population. Record linkage and statistical modeling, more generally, are tools that enable data combination.

## 3. Data Sources for Namibia

The agricultural sector is a nontrivial contributor to Namibia's economy, providing the primary income for approximately 70% of the population (Republic of Namibia, 2003). Namibia's climate is arid, and variations in rainfall are largely responsible for fluctuations in agricultural output. Because the northeastern part of Namibia has the most stable rainfall, communal agriculture, which comprises 48% of the farming in Namibia, mostly takes place in the northern regions of Capri (Zambezi), Kavango, Ohangwena, Omusati, Oshana, and Oshikoto (Republic of Namibia, 2003). Commercial farming is more spread out across the country with ``pockets in various regions'' throughout Namibia (Republic of Namibia, 2009).

Statistical and administrative government agencies collect data on agriculture. The Namibia Statistics Agency (NSA) gathers statistical data through surveys and censuses. The Ministry of Agriculture, Water, and Forestry (MAWF) collects administrative data on crops and livestock.

### 3.1 Statistical Data: Namibia Statistics Agency

The NSA collaborated with the MAWF to administer a 2013/2014 national agricultural survey[1]. Namibia's 2013/2014 agricultural survey (NAS) was conducted separately for communal and commercial agriculture. A complete census of the communal sector was conducted through a mail survey. The final report for the communal sector was released in fall of 2015 (NCA, 2015). The sample for the communal sector used a stratified multi-stage design. The frame consisted of all households reporting participation in agriculture on the most recent demographic census. A stratified sample of census enumeration areas (clusters) was selected with Namibia's administrative regions as strata. A sample of households was selected from each enumeration area for in-person interviews, and a subset of plots was selected for yield determination through crop cutting experiments. Approximately 10,550 communal households were interviewed with approximately 3,109 plots identified for the crop cutting experiments. (NCA, 2015) The NAS communal sector data provide the direct survey estimator for the model in Section 4.

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[1] Formally, the 2013/2014 agricultural survey is called the Namibia Census of Agriculture (NCA); however we prefer not to use the term census because the data collection is not a complete enumeration.

### 3.2 Administrative Data: Ministry of Agriculture Water and Forestry

Several administrative agencies under the umbrella of the Ministry of Agriculture Water and Forestry (MAWF) collect quantitative information related to communal and commercial crops and livestock. The veterinary department of the MAWF collects administrative data on livestock through programs to monitor and control animal diseases. The Meat Board of Namibia and the Namibia Agriculture Union gather data on the number of livestock marketed, slaughtered and exported, as well as the total value of all meat produced. The Namibia Agronomic Board collects information on commercial crop production through its mission to regulate agronomic markets.

Information on communal crop production is collected through a form of an ARDS. Twice a year, a non-probability sample of farmers receives a ``Crop Assessment Checklist'' with questions on crop production, food security, and availability of government assistance programs. An important set of questions asks farmers to report the annual change in area and production of the three main staples, maize, millet, and sorghum. An extension officer uses his/her expertise to subjectively aggregate the farmer-level changes to the regional level. Annual crop area and production values for the 6 communal regions are then defined by combining the estimate of change from the Crop Assessment Checklist with the estimate of level from the previous year, where the initial estimate of level was established after the 1994/1995 Census of Agriculture. These MAWF crop area estimates are covariates in the model of Section 4.

### 3.3 Discussion of Namibia's Data Sources

The statistical and administrative data for Namibia exemplify many of the general issues over-viewed in Section 2. To overcome challenges associated with data-sharing and collaboration, the NSA and the MAWF formed a Memorandum of Understanding (MOU). The MAWF crop area estimates are produced frequently (i.e., annually) but are liable to bias. The Crop Assessment Checklist is administered to a non-probability sample of farms, and the procedure to aggregate farmer reports to the regional level involves subjective judgements. In contrast, estimators from the 2013/2014 NAS are considered unbiased because they are based on a probability sample with well-defined data collection procedures. The models presented in the next section appropriately represent the survey data as unbiased and allow for a bias in the administrative data.

## 4. A Structural Model for Estimating Crop Area in Namibia's Communal Regions

We illustrate the use of administrative data in combination with survey data to obtain sub-national estimates of crop area in Namibia. Restrictions on the available data preclude us from conducting a thorough and realistic application. We consider the data analysis of this section to be an illustration of the use of modeling techniques for the purpose of combining administrative and survey data.

The parameters of interest are the total area planted to subsistence crops in the six northern regions of Namibia, where communal agriculture is predominant. Subsistence crops include Maize, Millet, and Sorghum. The six communal regions are Capri (Zambezi), Kavango, Ohangwena, Omusati, Oshana, and Oshikoto. We let $Y_i$ denote the area in subsistence crops in region (or area) $i$, where $i = 1, \dots, 6$. The $\{Y_i : i = 1, \dots, 6\}$ are the parameters to estimate.

### 4.1 Properties of Data and Analysis Objectives that Motivate Modeling Decisions

The NAS and MAWF provide two distinct estimates of crop area in Namibia's communal regions. A single, unified estimate is desired. Many modeling frameworks are capable of combining survey and administrative data to form a unified crop area estimate. In this section, we explain why the properties of the data and analysis objectives motivate us to adopt a structural model.

### 4.1.1 Direct Survey Estimators and Administrative Data

The NAS provides direct estimators of $Y_i$ for $i = 1, \ldots, 6$ and corresponding direct variance estimators. A detailed description of the unit-level data that we use to construct the direct estimates is provided in Section 4.1.1 of Global Strategy (2016). The crop area for a sampled household is measured using clock-wise and counter-clockwise GPS measurements. We use the average of the clock-wise and counter-clockwise GPS measurements to define the crop area value for a sampled household.

The unit-level data available to us are rawer than the data used to construct the published estimates. We understand from NCA publications that editing and imputation are applied before publication of the estimates. We do not have access to the final data. As a consequence, direct estimates of crop area based on our data differ from the published values. Section 4.1.1 of Global Strategy (2016) details an analysis aimed at improving the alignment between our data and the published estimates. Ultimately, we delete households with crop area values that exceed 250ha. Such large crop area values are unrealistic and are likely a consequence of measurement error. Omitting these erroneous crop area values improves the consistency between the direct estimates used for the analysis in this manuscript and the published estimates. We refer the reader to Section 4.1.1 of Global Strategy (2016) for further detail on the analysis of the unit-level data and the comparison to published estimates.

We describe the procedure to construct direct estimates from the unit-level survey data available to us, after omitting households with crop areas in excess of 250ha. Let $y_{ij}$ denote the crop area (average of clock-wise and counter-clock-wise GPS measurements) for household $j$ in area $i$, where $j = 1, \ldots, n_i$. As discussed in Section 3.1, households are nested in enumeration areas that are PSUs in the NAS design. A sampling weight for household $j$ in area $i$, denoted by $w_{ij}$, reflects the complex characteristics of the NAS design. The direct estimator of crop area in region $i$ has the form

$$\hat{Y}_i = \sum_{j=1}^{n_i} w_{ij}\, y_{ij}.$$

We use a jackknife procedure to estimate the variance of $\hat{Y}_i$. The jackknife estimator of the variance of $\hat{Y}_i$ is

$$\hat{\sigma}_{ei}^2 = \frac{L_i - 1}{L_i} \sum_{k=1}^{L_i} (\hat{Y}_i^{(k)} - \hat{Y}_i)^2,$$

where $\hat{Y}_i^{(k)}$ is an estimate of crop area for region $i$ constructed with the data for PSU $k$ deleted and $L_i$ is the number of PSU's in region $i$. The replicate estimate $\hat{Y}_i^{(k)}$ is of the form

$$\hat{Y}_i^{(k)} = \sum_{j=1}^{n_i} w_{ij}^{(k)} y_{ij},$$

where

$$w_{ij}^{(k)} = \begin{cases} \dfrac{w_i}{w_i^{(k)}} w_{ij} & \text{if } (ij) \text{ is not in PSU } k \\ 0 & \text{otherwise,} \end{cases}$$

$w_i^{(k)} = \sum_{\{j \text{ not in PSU } k\}} w_{ij}$ and $w_i = \sum_{j=1}^{n_i} w_{ij}$.

The Namibian Ministry of Agriculture, Water, and Forestry (MAWF) provides a different estimate of the area planted to subsistence crops in the 6 communal regions. To construct the MAWF estimate, a local expert aggregates farmer-reported data. The aggregation method is not based on a statistical procedure, and the data are collected from a non-probability sample of crop fields. As such, the MAWF estimator is subject to bias. We let $X_i$ denote the MAWF estimator of communal crop area in region $i$, where $i = 1, \dots, 6$.

**4.1.2 Objectives of Analysis**

The analysis aims to serve a general practitioner as well as the specific needs of the Namibia Statistics Agency. The primary purpose of our data analysis is to demonstrate an appropriate application of methods to a general practitioner. As we discuss in more detail below, limitations on the available data preclude us from conducting a thorough analysis that one would regard as a genuine application. As such, the analysis presented in Section 4.2 should be considered an illustration of techniques. Despite the limitations on the available data, the analysis presented below does have intrinsic interest. The analysis provides a unified estimate based on disparate data sources and also allows an assessment of the quality of the MAWF administrative data. The modeling activity permits us to assess the extent to which the MAWF administrative data can improve upon the efficiency of the direct survey estimators. In this sub-section, we elaborate on these analysis goals.

As discussed in Section 4.1.1, we lack complete information on the unit-level data. This incomplete information prevents us from perfectly reproducing the estimates in the NAS publication. Our lack of full information also leads us to have a degree of uncertainty regarding the design-based variance estimators that we compute. Further, we do not have any unit-level covariates. These data limitations preclude us from conducting a genuine data application. As such, the analysis presented below should be interpreted as an illustration of methods. Our primary goal then is to use the data for Namibia to construct a demonstration of methods that holds potential use for a general practitioner. A desire to integrate survey and administrative data arises frequently in production of official statistics on agriculture. We aim to demonstrate best practices associated with the use of statistical models to synthesize survey and administrative data. We formulate an example from the Namibia data that a general practitioner, interested in conducting a similar analysis, can follow as guide. We strive to present an accessible demonstration of the statistically defensible practices of model selection, estimation, and assessment. In the direction of making our analysis as useful as possible, we identify a model form that is relatively uncommon in the literature.

Although the primary role of the data analysis is illustrative, the analysis presented below does have loose connections to the genuine needs of the Namibia Statistics Agency and the MAWF. The connections to the real problem help guide our choice of model form. The Namibia Statistics Agency and the MAWF are responsible for producing official statistics on agriculture in Namibia. Developing a unified and interpretable estimate is important. Additionally, an understanding of the nature of the relationship between the two sets of estimates is of interest. Use of administrative data also has potential to improve the quality of the NAS estimate.

The MAWF and NAS provide two different estimates of crop area in Namibia's communal regions. The existence of two estimates of the same quantity can confuse a data user. One role of the modeling analysis is to provide a unified estimate based on these two data sources.

The modeling activity also allows a formal assessment of the nature of the relationship between the MAWF and NAS estimates. The NAS estimates are based on a probability sample and are therefore unbiased for the true crop area by design. Further, the survey design supports estimation of the design-variance of the survey-based estimator. In contrast, the MAWF estimates are constructed from a non-probability sample. As a consequence, the MAWF estimates are subject to bias, and a direct estimate of the variance of the MAWF estimate does not exist. Quantifying the extent of the bias of the MAWF estimators of crop area is of interest. An estimate of the variability of the MAWF estimator has potential to help the MAWF interpret the estimates based on the crop assessment checklist.

A final objective is to utilize the administrative data to improve upon the efficiency of the direct NAS estimators. The publication on the Namibian Census of Agriculture includes estimates of crop area by specific crop at the national level but does not publish regional estimates of crop area. One possible role of the administrative data is to provide the necessary additional information for improving upon the efficiency of the direct NAS estimators. We will assess the extent to which the administrative data are capable of improving upon the efficiency of the direct survey estimators.

In summary, the analysis has four objectives. The primary goal is to illustrate for a general practitioner statistically defensible practices of combining survey and administrative data. The remaining three are tied to the intrinsic needs of the MAWF and the NSA. One is to provide a unified estimate based on survey and administrative data sources. A second is to quantify of the bias and variance of the MAWF estimator. The third is to assess the extent to which use of the administrative data can improve upon the quality of the survey-based estimator.

### 4.1.3 Basic Modeling Decisions and Assumptions

Many reasonable model forms exist for constructing a unified estimate of crop area using the NAS survey data and the MAWF administrative data. After identifying a model form, one could conduct either a Bayesian analysis or a frequentist analysis. In this section, we discuss why the objectives discussed in Section 4.1.2 and the properties of the data described in Section 4.1.1 motivate us to implement a Bayesian analysis of a structural measurement error model.

One of the analysis goals is to use the administrative data to improve upon the efficiency of the direct survey estimator. This is an example of the general class of problems called small area estimation. Rao and Molina (2015) provides a comprehensive review of small area models in the literature. The two broad classes of small area models are unit-level models and area-level models. In a unit-level model for crop area, the household level crop area measurement $y_{ij}$ would serve as the response variable. In contrast, $\hat{Y}_i$ is the response variable in an area-level model. We prefer area-level models for the Namibia crop area data for several reasons. First, the only covariates available to us are the MAWF estimates of crop area in the 6 regions. These are area-level covariates. The benefits of using unit-level models are greatest when unit-level covariates are available, and we have no unit-level covariates. Second, the direct estimator $\hat{Y}_i$ and corresponding jackknife variance estimator appropriately account for the complex survey design. Accounting for a complex survey through a unit-level model is challenging (Verret, Rao and Hidiroglou, 2015; Pfeffermann and Sverchkov, 2007). Due to the limited data and information about the survey design available to us, we lack the capacity to account for the complex design through a unit-level model in this study.

After deciding to use an area-level model, one must select a response distribution for $\hat{Y}_i$. The crop areas are positive, so a distribution with a positive support seems natural. As described in Section 4.1, the direct crop area estimate is a weighted average of individual household level crop area measurements. Therefore, the Central Limit Theorem justifies a normal approximation for the distribution of the direct estimator. In the following models, we will use a normal distribution for the conditional distribution of $\hat{Y}_i$ given $Y_i$. As described in Section 4.1.1, the NAS is based on a probability sample, and data collection follows carefully defined protocols. Therefore, we assume that $\hat{Y}_i$ is unbiased for $Y_i$. Mathematically, unbiasedness means that $E[\hat{Y}_i$ - $Y_i]$ = 0. The jackknife variance $\hat{\sigma}_{ei}^2$ is an estimate of the sampling variance defined formally as $\sigma_{ei}^2 = V\{\hat{Y}_i - Y_i\}$. As is common in small area estimation, we ignore the difference between $\hat{\sigma}_{ei}^2$ and $\sigma_{ei}^2$ and act as if $\hat{\sigma}_{ei}^2 = \sigma_{ei}^2$ for the analysis. Because $\hat{Y}_i$ is unbiased for $Y_i$, $\sigma_{ei}^2$ is also the MSE of $\hat{Y}_i$. We can now phrase the final goal discussed in Section 4.1.2 in technical terms. We aim to investigate the extent to which small area procedures are capable of producing a predictor of $Y_i$ with MSE smaller than $\sigma_{ei}^2$.

One must also assign a functional form for the relationship between $Y_i$ and $X_i$. Figure 1 contains a scatterplot with $\hat{Y}_i$ on the vertical axis and the corresponding MAWF estimate of crop area for region $i$ on the horizontal axis. The dashed line is the 45-degree line through the origin. The relationship between the direct estimates of crop area and the corresponding MAWF values is nearly linear. We consider a linear form for the relationship between $Y_i$ and $X_i$.

[ Figure 1 about here ]

One arguable limitation of the linear model with normal errors is that the model does not respect the positive support for the crop areas. The analysis presented in Global Strategy (2016, Section 4.1) uses a lognormal distribution. After closer inspection of the data, we decided that a model with a normal response distribution and a linear relationship between $\hat{Y}_i$ and $X_i$ is more appropriate.

An important issue to consider is whether to regard the covariate $X_i$ as fixed or as random. A standard area-level small area model is the Fay-Herriot model (Fay and Herriot, 1979). In the Fay-Herriot model, the covariate is treated as fixed. An alternative model form is a structural measurement error model. The structural model form allows an analyst to incorporate an administrative estimate as a second response variable in the model. In the Fay-Herriot model, $X_i$ is treated as a fixed covariate. In contrast, $X_i$ enters the structural model as a random variable. The term "structural measurement error model" is not a specific term but contains a class of models. We use the terms "structural model," "structural measurement error model," and "structural Fay-Herriot model" interchangeably. We define the structural model formally in Section 4.2 and compare the structural model to the Fay-Herriot model through simulation in section 5. The properties of the MAWF estimator and the objectives of the analysis motivate us to adopt the structural model for the analysis of the data from Namibia. In the rest of this section, we expand on why we prefer the structural model relative to the Fay-Herriot model for the purpose of our analysis.

In our application, $X_i$ is a non-probability estimate obtained by the MAWF. An immediate concern is then that $X_i$ may overwhelm the survey data in the Fay-Herriot model. One of the reasons that we prefer to treat $X_i$ as random in a structural model is to guard against this possibility. We conjecture that treating $X_i$ as random may prevent the administrative data from unduly overwhelming the survey data in the prediction.

Further, as discussed in Section 4.1.2, the goal of the analysis is not only to construct predictions but also to assess the quality of the MAWF estimator of crop area. Because $X_i$ is based on a non-probability sample and subjective interpretation, $X_i$ may be biased for $Y_i$, such that $E[X_i - Y_i] \neq 0$. As shown in Figure 1, the NAS and MAWF estimates exhibit a strong, positive, linear association. The correlation between $\hat{Y}_i$ and $X_i$ across the six regions is 0.895. If the MAWF and NAS were targeting the same parameter, then we would expect the estimated regression line based on the data to have approximately an intercept of zero and a slope of 1. A structural model is well-suited to assessing the extent of the bias of the MAWF estimator of crop area. After we introduce the models formally in Section 4.2, we discuss the theoretical advantages of the structural model relative to the Fay-Herriot model for the purpose of estimating the bias of the MAWF estimator. In the simulations of Section 5, we provide empirical support for the superiority of the structural model relative to the Fay-Herriot model for bias estimation. The structural model also produces an estimate of the variance $X_i$, whereas the Fay-Herriot model does not.

Even through $X_i$ is a non-probability estimate and is subject to bias, we may still be able to use the information contained in $X_i$ to construct a predictor of $Y_i$ that is superior to $\hat{Y}_i$. Both the Fay-Herriot model and the structural model are appropriate for the purpose of constructing predictor of $Y_i$ with smaller mean square error than $\hat{Y}_i$. We discuss the relative strengths of the Fay-Herriot and structural models for the purpose of prediction theoretically in Section 4.2 and empirically through the simulation study of Section 5.

Having chosen a linear structural model, a further decision is whether to adopt a Bayesian or frequentist inference paradigm. Namibia has 6 communal regions, so the number of areas in the structural model is 6. Due to the small number of areas, the asymptotic theory underlying frequentist mean square error estimators is questionable for this data set. Therefore, we prefer a Bayesian approach.

Finally, the Bayesian analysis of the structural model is well-suited to our primary goal of developing an illustration with potential to serve a general practitioner. Many examples of applications of Bayesian analyses of the Fay-Herriot model already exist in the literature. Cruze (2018) provides an accessible explanation of how to implement a Bayesian analysis of the Fay-Herriot model using widely available software. In an application closely related to the topic of this paper, Erciulescu, Cruze, and Nandram (2019) conduct a Bayesian analysis of a multi-level Fay-Herriot model to construct small area estimates using data from the United States Department of Agriculture. To our knowledge, a discussion of how to implement a Bayesian analysis of our parametrization of the structural Fay-Herriot model for the purpose of small area estimation does not exist in the literature. Our illustration offers an analysis option for a general practitioner that may be difficult to find elsewhere in the literature. The general problem of combining administrative and survey data for the purpose of quality assessment and small area estimation is common in practice. The analysis presented below holds potential use for a general practitioner who is interested in implementing a similar analysis using a different data set.

#### 4.3 Bayesian Analysis of Structural Fay-Herriot Model

We define a parameterization of the structural model that permits estimation of the bias and variance of the MAWF estimator. We illustrate a Bayesian analysis of the structural model using the survey and administrative data from Namibia. We define the structural model and discuss its theoretical properties in Section 4.3.1. In Sections 4.3.2-4.3.4, we implement the Bayesian analysis and discuss the results.

#### 4.3.1 Structural Model for Estimation of Crop area in Namibian Communal Regions

In this application, $X_i$ is an estimate. Consequently, we prefer a model in which $X_i$ is regarded as random. We use a particular form of the measurement error model for which the parameters are identifiable, given our limited information. We regard $X_i$ as a second response variable in the model. We define a structural model for small area estimation by

$$\hat{Y}_i = Y_i + e_i, \qquad e_h \sim (0, \sigma_{eh}^2) \tag{1}$$

$$X_i = \beta_0 + \beta_1 Y_i + u_i, \qquad u_i \sim (0, \sigma_u^2)$$

and $Y_i \sim N(\mu_y, \sigma_y^2)$. Recall that $\hat{Y}_i$ denotes the NAS survey-based estimator of crop area, and $X_i$ denotes the corresponding estimator based on the MAWF administrative data.

The assumption that $Y_i \sim N(\mu_y, \sigma_y^2)$ is strong and is required for model identification. In the most general form, any structural measurement error model contains non-identifiable parameters. The model (1) imposes several parameter restrictions that ensure identifiability. In the first line of (1), we assume that the intercept and slope relating $\hat{Y}_i$ to $Y_i$ are 0 and 1, respectively. We further assume that $e_i$ and $u_i$ are independent. As in the Fay-Herriot model, we regard $\sigma_{ei}^2$ as known. The final assumption that $Y_i \sim N(\mu_y, \sigma_y^2)$ is a strong assumption that distinguishes the structural measurement error model from its functional counterpart in which $Y_i$ is treated as fixed. With these assumptions, the structural model in (1) has only two more parameters than the Fay-Herriot model. These restrictions allow us to identify the parameters of the model (1). To further confirm that the model (1) is identifiable, note that the model (1) is a special case of the measurement error models discussed in Fuller (1987). We can use the moment-matching technique of Fuller (1987) to verify that the parameters of model (1) are identified. Model (1) has 5 unknown fixed parameters: $\beta_0$, $\beta_1, \sigma_u^2, \mu_y$, and $\sigma_y^2$. Simultaneously, the data supply a vector of 5 sufficient statistics: the sample mean of $\hat{Y}_i$, the sample mean of $X_i$, the sample variance of $\hat{Y}_i$, the sample variance of $X_i$, and the sample covariance between $\hat{Y}_i$ and $X_i$. As discussed in Fuller (1987), an ability to form a one-to-one relationship between sample moments and model parameters is a way to confirm that the model parameters are identified. We refer the reader to Global Strategy (2015b) for a more extensive discussion of identification conditions for structural models.

The structural model (1) differs from other area-level measurement error models in the literature because the model (1) does not require $X_i$ to have a known measurement error variance. Several other works in the literature consider a situation in which the covariate is from a probability-based survey (Ybarra and Lohr, 2008; Arima, Datta, and Liseo, 2015). When $X_i$ is from a probability-based survey, the sampling variance of $X_i$ is often known. One define a measurement error model that incorporates the known sampling variance of $X_i$. For the Namibia data, the sampling variance of $X_i$ is unknown. The model (1) does not require a known variance for $X_i$.

The structural model that we defined in (1) is closely connected to the traditional Fay-Herriot model. The Fay-Herriot model is a standard area level model used for small area estimation. In the Fay-Herriot model, the administrative data are treated as a fixed covariate. One can draw a connection between the structural model and the Fay-Herriot model by considering the conditional distribution of $\hat{Y}_i$ given $X_i$. Under the structural model, the conditional distribution of $Y_i$ given $X_i$ satisfies

$$Y_i \mid X_i = \gamma_0 + \gamma_1 X_i + r_i, \tag{2}$$

where

$$\gamma_0 = \mu_y - \frac{\beta_0\beta_1\sigma_y^2 - \beta_1^2\mu_y\sigma_y^2}{\beta_1^2\sigma_y^2 + \sigma_u^2}, \quad \gamma_1 = \frac{\beta_1\sigma_y^2}{\beta_1^2\sigma_y^2 + \sigma_u^2}, \quad \sigma_r^2 = \frac{\sigma_y^2\sigma_u^2}{\beta_1^2\sigma_y^2 + \sigma_u^2},$$

and $r_i \sim N(0, \sigma_r^2)$. After conditioning on $X_i$, the relationship between $\hat{Y}_i$ and $Y_i$ remains as $\hat{Y}_i = Y_i + e_i$, where $e_i \sim N(0, \sigma_{ei}^2)$. The conditional distribution of $\hat{Y}_i$ given $X_i$ is a Fay-Herriot model in new parameters.

The transformation to the conditional distributions in (2) reveals that prediction based on the structural model is nearly equivalent to prediction based on the Fay-Herriot model. From the perspective of constructing predictions, the structural model has essentially no theoretical advantage over the Fay-Herriot model. If the structural model is true, an implementation of the Fay-Herriot model will yield estimates of estimates of $\gamma_0$, $\gamma_1$, and $\sigma_r^2$. The predictor based on the Fay-Herriot model will be a predictor of $\gamma_0 + \gamma_1 X_i + r_i$. If our only objective were to construct predictions, then use of the Fay-Herriot model would be adequate. We provide empirical support for this theoretical assertion in the simulation study of Section 5.

Recall that a second objective of the analysis is to understand if the MAWF estimator of crop area may be regarded as an unbiased estimator of crop area. For the purpose of understanding the quality of the MAWF administrative data, the structural measurement error model is more appropriate than the Fay-Herriot model. If the MAWF estimator is unbiased, the intercept and slope of the structural model are equal to 0 and 1, respectively. The form of the conditional distribution (2) lends insight into the limitations of the Fay-Herriot model for the purpose of quantifying the bias of the MAWF estimator. From equation (2), we see that the intercept and slope of the Fay-Herriot model are not necessarily 0 and 1, even if the MAWF estimator of crop area is unbiased. Specifically, the slope of the Fay-Herriot model is attenuated toward zero. If the true value for $\beta_1$ is $\beta_1 = 1$, the slope of the Fay-Herriot model is less than 1. Therefore, the Fay-Herriot model is not well-suited to an assessment of the bias of the MAWF estimator. In contrast, the parameters of the structural model have natural interpretations in terms of the bias of the MAWF estimator.

**4.3.2 Prior and Posterior**

The parameters in the structural model that require prior distributions are $(\beta_0, \beta_1, \sigma_u^2, \mu_y, \sigma_y^2)$. We assign a non-informative, diffuse prior defined as $\pi(\beta_0, \beta_1, \sigma_u^2, \mu_y, \sigma_y^2) \propto 1/(\sigma_u^2\sigma_y^2)$. This prior is based on Jeffrey's prior for the parameters of a normal distribution. With this choice of prior distribution, the posterior distribution of the model parameters has the form

$$p_m^{ME}(\beta_0, \beta_1, \sigma_u^2, \mu_y, \sigma_y^2, \boldsymbol{Y} \,\big|\, \widehat{\boldsymbol{Y}}, \boldsymbol{X}) \propto (\frac{1}{\sigma_u^2}\frac{1}{\sigma_y^2})^{\frac{D}{2}+1} \prod_{i=1}^{D} \phi\left(\frac{\widehat{Y}_i - Y_i}{\sigma_{ei}}\right) \phi\left(\frac{X_i - \beta_0 - \beta_1 Y_i}{\sigma_u}\right) \phi\left(\frac{Y_i - \mu_y}{\sigma_y}\right), \quad (3)$$

where $\boldsymbol{Y} = (Y_1, \dots, Y_D)'$, $\boldsymbol{X}$=$(X_1, \dots, X_D)'$ and $\widehat{\boldsymbol{Y}} = (\widehat{Y}_1, \dots, \widehat{Y}_D)'$. For the data analysis, $D = 6$. We use the super-script of $ME$ to emphasize that the posterior distribution corresponds to the structural measurement error model.

We use Gibbs sampling to generate a Markov Chain that converges to the posterior distribution. The steps of the Gibbs sampling algorithm are defined in Appendix A.1. We assess convergence of the Markov Chain to the stationary distribution in Appendix B.1. We define a predictor as the posterior mean, and we assess the uncertainty of the predictor through the posterior standard deviation. We define the posterior mean and standard deviation precisely for the structural model in equation (6) of Appendix A.1. In the data analysis, we combine samples from three chains with independent starting values. For each chain, we use $B = 500$ and $T = 50{,}000$. We thin by an interval of 10 iterations for each chain.

#### 4.3.3 Model Assessment

An external evaluation is one way to assess the goodness of fit of a small area model. In an external evaluation, model-based predictions are compared to published values that are considered to be credible measures of the parameters of interest. The NAS report publishes an estimate of the crop area for the entire communal region of Namibia. In our notation, the crop area for the communal region of Namibia is defined as $Y = \sum_{i=1}^{6} Y_i$ . In principal, one possible external evaluation would involve comparing the prediction of $Y_i$ from the model to the corresponding value published in the NAS report. As discussed in Section 4.1, the data used to construct the published values in the NAS report incorporate editing and imputation procedures that are unavailable to us. As a consequence, a comparison to published values is infeasible for our application.

As a substitute for a comparison to published values, we consider benchmarking to the direct estimate of the overall crop area in the communal region of Namibia. If our data were identical to the data used for the NAS report, then the direct estimate of overall crop area would equal the published value. Additionally, a good-fitting model should produce a prediction of overall crop area that is close to the direct estimate. We assess the degree to which the model-based predictions of overall crop area agree with the direct estimate using the posterior predictive p-value. We adopt a particular criterion suggested in Wang et al. (2018) that focuses on the model's ability to reproduce the direct estimates of crop area. The Wang et al. (2018) procedure examines the proportion of values representing the direct estimate of overall crop area simulated from the posterior predictive distribution that exceed the corresponding direct estimate.

The specific operation of calculating the posterior predictive p-value is as follows. As defined in Appendix A, let $Y_i^{(t,ME)}$ denote the crop area for region $i$ simulated in Gibbs iteration $t$ for the structural measurement error model. A version of the direct estimator of crop area for region $i$ simulated from the posterior predictive distribution is defined as

$$\widehat{Y}_i^{(t,ME)} = Y_i^{(t,ME)} + e_i^{(t,ME)},$$

where $e_i^{(t,ME)} \sim N(0, \sigma_{ei}^2)$. We then calculate a version of a direct estimate of the crop area for the entire communal region of Namibia as $\hat{Y}^{(t,ME)} = \sum_{i=1}^{6} \hat{Y}_i^{(t,ME)}$. The posterior predictive p-value for the structural measurement error model is defined as $p^{(ME)} = T^{-1} \sum_{t=1}^{T} I[\hat{Y}^{(t,ME)} > \hat{Y}]$, where $\hat{Y} = \sum_{i=1}^{6} \hat{Y}_i$.

We apply the procedure of calculating the posterior predictive p-value to the Namibia data. The resulting posterior predictive p-value is 0.469. The p-value is not close to zero or one. This analysis gives no reason to reject the structural Fay-Herriot model.

**4.3.4 Results**

Table 1 presents the posterior means and 95% central credible intervals for the elements of ($\beta_0$, $\beta_1, \sigma_u^2, \mu_y, \sigma_y^2$), the vector of parameters governing the distribution of the structural model. One question of interest is if the MAWF administrative data are unbiased for the true crop areas. If the MAWF estimators are unbiased, then $\beta_0 = 0$ and $\beta_1 = 1$. For the structural model, 0 and 1 are easily contained in the credible intervals for $\beta_0$ and $\beta_1$, respectively. An assertion that the MAWF estimator is unbiased appears plausible for this data set.

[ Table 1 about here ]

Table 2 contains the posterior means and 95% central credible intervals for $Y_1$, ..., $Y_6$. The posterior distributions of $Y_1$, ..., $Y_6$ are approximately unimodal and symmetric, so the central credible interval is approximately a normal theory confidence interval constructed with the posterior mean and posterior standard deviation reported in Table 2. For ease of comparison, Table 2 also contains the standard error of the direct estimator (SEDirect). The column "Ratio" contains the ratio of the posterior standard deviation to the standard error of the direct estimator. The ratios are close to 1, indicating that the structural model renders little efficiency gain for this data set. One reason for the minimal efficiency gain is that the ratio of the posterior mean of $\sigma_u^2$ to the sum of the posterior mean of $\sigma_u^2$ and the standard error of the direct estimator is close to 1. As a result, the procedure assigns a high weight to the direct estimator when forming the predictor. Another reason that the gain in efficiency is minimal for this data analysis is that Namibia only has 6 communal areas. As a consequence, we have limited data to use for estimating the 5-dimensional parameter vector of the structural model. The variance due to estimating fixed parameters makes a nontrivial contribution to the posterior standard deviation.

[ Table 2 about here ]

A general practitioner with a data set containing more than 6 areas may experience more important efficiency gains from an application of the structural model. Utilizing unit-level data in conjunction with record-level covariates is a further potential way to improve the efficiency gain from modeling. We illustrate the improvement in efficiency from increasing the sample size through simulation in Section 5.

**5. Simulation**

The limitations on the available data from Namibia restrict the extent of the analysis that we can conduct. Therefore, we utilize simulation to demonstrate properties of alternative modeling approaches that the data alone are too limited to illustrate. In Section 4.2.1, we discussed a theoretical connection between the structural model and the Fay-Herriot model. In this section, we provide empirical support for that connection. We also provide simulation-based evidence for the assertion in Section 4.2.1 that the structural model is better suited to estimation of the bias of the MAWF estimator than the Fay-Herriot model.

We simulate data from two possible true models. For one model, the Fay-Herriot model is the true model. For the second, the structural model is the true model. For each true model, we apply both the Fay-Herriot model and the structural model to the simulated data. We define the models and predictors in Section 5.1. We summarize the Monte Carlo properties of the alternative predictors in Section 5.2.

**5.1 Models for Simulation**

We simulate data from the Fay-Herriot model and from the structural model. The model for the direct estimator is the same for both models. The direct estimator for area $i$ is generated as

$$\hat{Y}_i = Y_i + e_i,$$

where $e_i \sim N(0, \sigma_{ei}^2)$ for $i = 1, \dots, D$. We choose the parameters for the by dividing moment-based estimators of means and standard deviations by 1000. Division by 1000 reduces the overall variability in the simulated data relative to the true data. We set the design variances to be proportional to the average of the observed design variances for the Namibia data, giving $\sigma_{ei}^2 = \sigma_e^2 = 24{,}236{,}438/(1000)^2$ for $i = 1, \dots, D$. To assess the effect of the number of areas, we use $D = 9$ and $D = 18$ in the simulations.

For the structural model, we generate $Y_i \sim N(\mu_y, \sigma_y^2)$, with $\mu_y = 61119.67/(1000)^2$ and $\sigma_y^2 = 1{,}081{,}270{,}560/(1000)^2$. We then generate $X_i = Y_i + u_i$, where $u_i \sim N(0, \sigma_u^2)$, and $\sigma_u^2 = c\sigma_e^2$. We assess the effect of the ratio of $\sigma_u^2$ to $\sigma_e^2$ by considering three choices for $c$ of $c = 0.5, 1$, and 4. For the structural model, we generate a different set $\{\left(X_i^{(m)}, Y_i^{(m)}, \hat{Y}_i^{(m)}\right) : i = 1, \dots, D\}$ in each Monte Carlo sample $m$, where $m = 1, \dots, M$.

For the Fay-Herriot model, we generate a single set of covariates that we hold fixed across Monte Carlo iterations. We generate the fixed covariates for the Fay-Herriot model as $X_i \sim N(\mu_x, \sigma_x^2)$. In each Monte Carlo iteration, we then generate $Y_i$ as $Y_i = X_i + b_i$, where $b_i \sim N(0, \sigma_b^2)$, and $\sigma_b^2 = c\sigma_e^2$. We generate direct estimators as for the structural model. The data generated from the Fay-Herriot model in Monte Carlo sample $m$ are of the form $\{\left(X_i, Y_i^{(m)}, \hat{Y}_i^{(m)}\right) : i = 1, \dots, D\}$, where $m = 1, \dots, M$.

For each generated data set, we implement a Bayesian analysis of both the Fay-Herriot model and the structural measurement error model. The priors for the structural model are the same as those defined in Section 4.3.2 for the Namibia data analysis. We use the same procedure to generate samples from the posterior distribution of the structural model that we used for the data analysis.

The parameters of the Fay-Herriot model that require priors are $(\beta_0, \beta_1, \sigma_b^2)$. We assign a flat prior defined as $\pi(\beta_0, \beta_1, \sigma_b^2) \propto 1/(\sigma_b^2)$. The posterior distribution of the parameters then has the form

$$p_m^{FH}\left(\beta_0, \beta_1, \sigma_b^2, \boldsymbol{Y} \mid \hat{\boldsymbol{Y}}, \boldsymbol{X}\right) \propto \left(\frac{1}{\sigma_b^2}\right)^{\frac{D}{2}+1} \prod_{i=1}^{D} \phi\left(\frac{\hat{Y}_i - Y_i}{\sigma_{ei}}\right) \phi\left(\frac{Y_i - \beta_0 - \beta_1 X_i}{\sigma_b}\right). \qquad (4)$$

For the Fay-Herriot model, direct simulation from the posterior distribution, without use of Markov Chain Monte Carlo, is possible. We describe a direct simulation method for the Fay-Herriot model in Appendix C. We adopt Gibbs sampling for this simulation study. The Gibbs sampling procedure used to simulate from the posterior distribution of the Fay-Herriot model is defined in Appendix A.2.

**5.2 Comparison of Efficiency of Predictors in Simulation**

We assess the Monte Carlo bias and root mean square error of predictors of $Y_i$ based on the Fay-Herriot and structural models. We define a predictor of $Y_i$ as the posterior mean. Let $l$ denote the model type, where $l = FH$ for the Fay-Herriot model, and $l = ME$ for the structural model. Let $t$ denote the iteration of the Gibbs sampler, where $t = B + 1, \ldots, T$, and $B$ denotes the burn-in period. Then, the predictor of $Y_i$ for model type $l$ is the posterior mean defined as

$$\tilde{Y}_i^{(l)} = \frac{1}{T}\sum_{t=B+1}^{T} Y_i^{(t,l)} , \qquad (5)$$

where $Y_i^{(t,l)}$ denotes the parameter generated in Monte Carlo iteration $t$ for model $l$ using the Gibbs sampling procedure defined in Appendix A. We use a burn-in period of $B = 100$ and subsequently retain $T = 500$ iterations for each model type and Monte Carlo sample.

We denote the predictor (5) for model type $l$ in each Monte Carlo sample $m$ by $\tilde{Y}_i^{(l,m)}$, where $m = 1, \ldots, M$ and $l \in \{ME, FH\}$. We define the Monte Carlo root mean square error for model type $l$ by

$$RMSE_l = \sqrt{\frac{1}{D}\sum_{i=1}^{D}\frac{1}{M}\sum_{m=1}^{M}(\tilde{Y}_i^{(l,m)} - Y_i^{(l,m)})^2},$$

where $Y_i^{(l,m)}$ denotes the version of $Y_i$ generated from model $l$ in Monte Carlo sample $m$. Similarly, we define the Monte Carlo bias for model type $l$ by

$$Bias_l = \frac{1}{D}\sum_{i=1}^{D}\frac{1}{M}\sum_{m=1}^{M}\{\tilde{Y}_i^{(l,m)} - Y_i^{(l,m)}\} .$$

Table 3 summarizes the properties of the predictors for the 12 configurations defined by all combinations of the 2 model types, the 3 values of $c$, and the 2 values of $D$. The true model used to generate the data is labeled in the column "True Model." The headings "Est-ME" or "Est-FH" indicate which model was assumed for the purpose of defining the predictors. The columns $c$ and $D$ give the values for $c$ and the number of areas, respectively. As a basis for comparison to the variance of the direct estimator, the column labeled ``Ratio'' contains the ratios defined as $RMSE_l^2/\sigma_e^2$. For all but two simulation configurations ($ME, D = 9, c = 0.5$ and $FH, D = 18, c = 4$), the Fay-Herriot model yields predictors with smaller RMSE than predictors based on the measurement error model. The Fay-Herriot model produces efficient predictors of $Y_i$, even if the measurement error model is the true model. This result is consistent with the theory of Section 4.2 that the conditional distribution of $Y_i$ given $X_i$ satisfies a Fay-Herriot model in new parameters. We expect that the reduction in RMSE from applying the Fay-Herriot model relative to applying the measurement error model results because the Fay-Herriot model requires estimating two fewer parameters than the structural model.

As expected, increasing $D$ improves the efficiency of the predictors. Increasing the number of areas from 9 to 18 reduces the RMSE of each type of predictor for each combination of model type and $c$. For instance, when the measurement error model is the true model and $c = 0.5$, the RMSE of the structural model predictor is 4.947 when $D = 9$ and is 4.625 when $D = 18$. The improvement in efficiency as the number of areas increases supports our previous statement that one of the limitations of the data analysis is that we are using an area-level model with only 6 data points.

The relationship between $c$ and the efficiency of the predictor is more clear when the Fay-Herriot model is the true model than when the measurement error model is the true model. For a fixed value of $D$, increasing $c$ increases the RMSE of the Fay-Herriot predictor when the Fay-Herriot model is the true model. The relationship between $c$ and the RMSE of the structural model predictor when the structural model is the true model is less clear. The reason is that the efficiency of the structural model depends on the variance of $r_i$ of equation (2). The parameter $c$ enters both the numerator and denominator of the ratio defining $\sigma_r^2$. For the structural model, both $c$ and $\sigma_y^2$ impact the weight assigned to the direct estimator when forming the predictor. Therefore, the effect of $c$ on the efficiency of the structural model predictor is difficult to disentangle.

[Table 3 about here ]

Table 4 summarizes the MC mean and standard deviation of the estimators of the intercept and slope. We define estimators of the intercept and slope in each Monte Carlo iteration as the posterior mean of the appropriate parameter. Specifically, the estimators of $(\beta_0, \beta_1)$ obtained from model type $l$ in Monte Carlo sample $m$ are of the form

$$\left(\tilde{\beta}_0^{(l,m)}, \tilde{\beta}_1^{(l,m)}\right) = \frac{1}{T}\sum_{t=B+1}^{T}\left(\beta_0^{(t,l,m)}, \beta_1^{(t,l,m)}\right),$$

where $(\beta_0^{(t,l,m)}, \beta_1^{(t,l,m)})$ are simulated from the posterior distributions, as defined in Appendix A. The Monte Carlo mean and standard deviation of the intercept and slope are defined, respectively, as the sample mean and standard deviation of $\{\left(\tilde{\beta}_0^{(l,m)}, \tilde{\beta}_1^{(l,m)}\right): m = 1, \ldots, M\}$.

The results in Table 4 are consistent with the theory that the structural model is better suited than the Fay-Herriot model to an investigation of the bias of the MAWF estimator. When the structural model is the true model, the Fay-Herriot model produces a bias for the intercept and slope. The structural measurement error model produces approximately unbiased estimators of the intercept and slope, when the Fay-Herriot model is the true model and $c \in \{0.5, 1\}$. The bias of the structural model estimators of the intercept and slope under the Fay-Herriot model with $c = 4$ is less severe than the bias of the Fay-Herriot model estimators under the structural model.

[ Table 4 about here ]

**6. Discussion**

Administrative and statistical data are collected for inherently different purposes. Statistical data are collected with the direct intention of making inference for the parameters of a target population. Often, statistical data are collected through probability samples. In probability sampling, a subset of population elements is selected in a way that permits scientifically defensible inferences for the parameters of the target population. Probability samples also permit variance estimators and confidence intervals that reflect the characteristics of the survey design. In contrast to statistical data, administrative data are a natural byproduct of the administration of government programs. Administrative data are collected for the purpose of implementing the operational requirements of a program. The purpose of the administrative program is typically not to conduct inferences for target population parameters.

The distinct characteristics of administrative and statistical data have implications for their fitness for use in production of official statistics. The designer of a statistical operation has control over how samples are selected and how data are collected. As a consequence, estimators based on probability samples are often regarded as unbiased for corresponding population parameters. Consequently, estimators based on controlled probability samples offer scientifically defensible statistics for official publications. When a survey-based estimator is accompanied with a standard error or confidence interval, a data user can make statistically defensible inferences about plausible values of parameters of a target population. Collecting statistical data through controlled probability samples is relatively costly. Administrative data are inexpensive in the sense that the data have already been collected as a natural byproduct of program administration. Depending on the nature of the administrative program, administrative data may be collected more frequently or at more granular levels of geographic detail than survey data. A trade-off of the ease of collecting administrative data is that the quality of the data is beyond the control of the statistical agency. The needs of the administrative program determine the target populations and the data collection protocols. Estimates resulting from an administrative source are subject to bias for the parameters of interest to the statistical office. As a result of the quality issues, administrative data are seldom immediately ready for use as an official estimator of a parameter of interest.

When used in combination with survey estimators, administrative data can enhance the final statistical product. Administrative and statistical data often collect information on related parameters of interest. Procedures that integrate survey and administrative data can benefit from the opposing strengths and weaknesses of these distinct yet related data sources. Incorporating additional information from an administrative data source has potential to improve the quality of an estimator based on a probability survey. Conversely, a probability-based survey can inform on the quality of administrative data. Consequently, integration of statistical and administrative data is a problem of general interest. One approach to combining administrative and survey data is to use statistical models that describe relationships between survey and administrative data.

A common source of administrative data on agriculture in developing countries is a routine data collection system, in which extension officers record information on agriculture. The routine data are often based on non-probability samples and involve a degree of subjective interpretation. As the routine data are essentially non-probability estimates, they are not immediately suitable as official statistics. Integrating information from routine data collection systems for the purpose of improving agricultural statistics in developing countries is of interest.

The data collected through government-sponsored programs in Namibia exemplify many of the general issues associated with integration of administrative data into the agricultural statistical system. The national statistical agency in Namibia conducted a probability-based survey of the agricultural population over the period 2013/2014. Simultaneously, Namibia’s ministry of agriculture has a long history of collecting data through a routine data collection system. The survey and administrative programs produce two different estimates of the crop area in the six regions of Namibia where communal agriculture is predominant. A desire to integrate the routine data with the data from the 2013/2014 survey exists.

We illustrate the use of a structural model for the purpose of combining survey data collected by the Namibia Statistics Agency with administrative data from the routine data collection system. Limitations on the available data preclude us from conducting a genuine application. The main purpose of our data analysis is to illustrate modeling techniques that combine administrative and survey data. The techniques demonstrated hold potential use for general survey practice.

The efficiency gain from our models is small – typically, less than 5%. The only region for which the gain efficiency exceeds 5% is Oshana, where the gain in efficiency from prediction is about 10%. For this data set, the direct estimators are fairly reliable, having estimated coefficients of variation consistently below 12%. That the improvement from modeling is only slight raises the question of whether the modeling activity has any value at all.

One practical benefit of the structural model is to produce a unified estimate based on two different data sources. The MAWF and NAS data provide two different estimates of the same parameter. Existence of two different estimators of the same parameter can confuse a data user. The models illustrated in this data analysis allow a user to construct a unified estimate that integrates both data sources. For our analysis, the gain in efficiency from incorporating the administrative data source is trivial, at best. The main benefit of the modeling activity, from a practical perspective, may be to provide a single estimate that incorporates the information from the two data sources.

Another practical benefit of our modeling activities is to gain insight into the relationship between the NAS and MAWF estimates. Information on this relationship is useful for planning purposes. If the slope and the intercept are 0 and 1, respectively, then the MAWF would provide an unbiased estimator of crop area. In our analysis, 0 and 1 are contained in 95% credible intervals for the intercept and slope, respectively. Therefore, an statement that the MAWF estimates are unbiased appears plausible. We are only able to assess the bias of the MAWF estimate because we have access to data from a probability-based survey sample. The existence of the probability-based sample enables us to formally assess the quality of the administrative data source. Without the information from the survey, such an assessment would be impossible.

The administrative data should not simply replace the survey data. The administrative data are essentially non-probability data. One cannot obtain an understanding of the bias or variance of the administrative data source in the absence of information from a probability-based survey. The methods illustrated in our data analysis demonstrate how to utilize statistical modeling techniques to combine administrative and survey data for the purpose of constructing a unified estimate.

For the purpose of informing a general practitioner, a note on the computational requirements is worthwhile. For our analysis, the data set is small, so computation is not an issue. Computation becomes more important as the sample size increases. In our experience preparing this paper, the Gibbs sampling algorithms converge quickly to the posterior distribution. We do not expect computation to be an issue for the models considered here.

Two critical aspects enabled our analysis of the data from Namibia. The first was excellent coordination between the administrative and statistical agencies of Namibia. The cooperation between these agencies, as facilitated through their MOU, enabled us to obtain the required data for our analysis. The second critical aspect of the data for Namibia that enabled direct application of small area estimation is the existence of an estimated variance for the survey estimators. When conducting survey estimation, we advise statistical offices to invest energy in variance estimation as well as point estimation. Doing so will enhance options for using administrative data in a statistically defensible fashion.

**Appendix: Details of Approximating Posteriors and Convergence Diagnostics**

**Appendix A: Gibbs Sampling Algorithms**

*A.1 Gibbs Sampling for the Structural Model*

The starting value for the vector of regression coefficients, $\left(\beta_0^{(0)}, \beta_1^{(0)}\right)'$, is obtained from the simple regression of $X_i$ on $\hat{Y}_i$ for $i = 1, \dots, D$. The starting value for $\sigma_u^2$ is the estimate of the variance from the regression model defined specifically as

$$\sigma_u^{(0)2} = \frac{1}{D-2}\sum_{i=1}^{D}(X_i - \beta_0^{(0)} - \beta_1^{(0)}\hat{Y}_i)^2.$$

The starting values for the mean and variance of the distribution of $Y_i$ are $\mu_y^{(0)} = D^{-1}\sum_{i=1}^{D}\hat{Y}_i$ and $\sigma_y^{(0)2} = (D-1)^{-1}\sum_{i=1}^{D}\left(\hat{Y}_i - \mu_y^{(0)}\right)^2$. Define an initial predictor by $\mu_{i,num}^{(0,ME)}/\mu_{i,den}^{(0,ME)}$, where

$$\mu_{i,num}^{(0,ME)} = \frac{\hat{Y}_i}{\sigma_{ei}^2} + \frac{(X_i - \beta_0^{(0)})/\beta_1^{(0)}}{\sigma_u^{(0)2}} + \frac{\mu_y^{(0)}}{\sigma_y^{(0)2}},$$

and

$$\mu_{i,den}^{(0,ME)} = \frac{1}{\sigma_{ei}^2} + \frac{1}{\sigma_u^{(0)2}} + \frac{1}{\sigma_y^{(0)2}}.$$

We define the Gibbs sampling algorithm for a general case in which the priors for $\beta_s$, $s = 0,1$ are independent normal distributions with mean 0 and variance $\delta_s^2$. Likewise, the algorithm allows a general prior for $\mu_y$, where $\mu_y \sim N\left(0, \delta_\mu^2\right)$. For the flat prior used for this analysis, $\delta_0^{-2} = \delta_1^{-2} = \delta_\mu^{-2} = 0$. We index the sample $t$ for the structural measurement error model by $(t, ME)$. Gibbs sampling involves repeating the four steps below for $t = 1, 2, \dots, T + B$, where $B$ is a specified burn-in period.

1. Generate $\boldsymbol{\beta}^{(t,ME)} = \left(\beta_0^{(t,ME)}, \beta_1^{(t,ME)}\right)' \sim BVN(\boldsymbol{m}_\beta^{(t,ME)}, \boldsymbol{\Sigma}_\beta^{(t,ME)})$, where $\boldsymbol{\mu}_\beta^{(t)} = \left(\sum_{i=1}^{D}\boldsymbol{y}_i^{(t-1,ME)}\boldsymbol{y}_i^{(t-1,ME)\prime}(\sigma_u^{(t-1,ME)2})^{-1} + diag(\delta_0^{-2}, \delta_1^{-2})\right)^{-1}\sum_{i=1}^{D}\boldsymbol{y}_i^{(t-1,ME)}(\sigma_u^{(t-1,ME)2} + \sigma_{ei}^2)^{-1}x_i$, $\boldsymbol{V}_\beta^{(t)} = \left(\sum_{i=1}^{D}\boldsymbol{y}_i^{(t-1,ME)}\boldsymbol{y}_i^{(t-1,ME)\prime}(\sigma_u^{(t-1,ME)2} + \sigma_{ei}^2)^{-1} + diag(\delta_0^{-2}, \delta_1^{-2})\right)^{-1}$, and $\boldsymbol{y}_i^{(t-1,ME)} = (1, Y_i^{(t-1,ME)})'$.

2. Generate $\sigma_u^{(t,ME)2} = 1/\tau^{(t,ME)2}$, where $\tau^{(t,ME)2} \sim Gamma(D/2, 0.5\sum_{i=1}^{D}(X_i - \beta_0^{(t,ME)} - \beta_1^{(t,ME)}Y_i^{(t-1,ME)})^2)$, and $Gamma(a, b)$ denotes a gamma distribution with shape $a$ and rate $b$.

3. Generate

$$\mu_y^{(t,ME)} \sim N\{ \ (\tfrac{D}{\sigma_y^{(t-1,ME)}} + \ \delta_\mu^{-2}\ )^{-1} \ (\textstyle\sum_{i=1}^{D} y_i) \tfrac{1}{\sigma_y^{(t-1,ME)}}, \quad (\tfrac{D}{\sigma_y^{(t-1,ME)}} + \ \delta_\mu^{-2}\ )^{-1}\}$$

and $\sigma_y^{(t,ME)2} = 1/\tau_y^{(t,ME)2}$, where

$$\tau_y^{(t,ME)2} \sim Gamma\left\{\frac{D}{2}, 0.5\sum_{i=1}^{D}\left(\ Y_i^{(t,ME)} - \ \mu_y^{(t,ME)}\ \right)^2\right\}.$$

4. Generate $Y_i^{(t,ME)}$ independently as

$$Y_i^{(t,ME)} \sim N\left(\frac{\mu_{i,num}^{(t,ME)}}{\mu_{i,den}^{(t,ME)}}, \frac{1}{\mu_{i,den}^{(t,ME)}}\right),$$

where

$$\mu_{i,num}^{(t,ME)} = \frac{\hat{Y}_i}{\sigma_{ei}^2} + \frac{(X_i - \beta_0^{(t,ME)})/\beta_1^{(t,ME)}}{\sigma_u^{(t,ME)2}} + \frac{\mu_y^{(t,ME)}}{\sigma_y^{(t,ME)2}},$$

and

$$\mu_{i,den} = \frac{1}{\sigma_{ei}^2} + \frac{1}{\sigma_u^{(t,ME)2}} + \frac{1}{\sigma_y^{(t,ME)2}}.$$

The posterior mean and standard deviation of the parameters representing the regional crop areas for the measurement error model are defined, respectively, as

$$\tilde{Y}_i^{pred,ME} = \ T^{-1}\sum_{t=B+1}^{T} Y_i^{(t,ME)}$$

and

$$sd_i^{pred,ME} = \sqrt{(T-1)^{-1}\sum_{t=B+1}^{T}(Y_i^{(t,ME)} - \tilde{Y}_i^{pred,ME})^2}\ . \qquad (6)$$

The posterior mean and standard deviation of the model parameters are defined analogously as the sample mean and standard deviation of $\{\left(\sigma_u^{(t,ME)2},\ \beta_0^{(t,ME)}\ ,\ \beta_1^{(t,ME)}, \sigma_y^{(t,ME)2}, \mu_y^{(t,ME)}\right) : t = B + 1, \dots, T\}$.

*A.2 Gibbs Sampling for the Fay-Herriot Model*

We use as starting values the REML estimates of the parameters in the Fay-Herriot model. We use the full model in which both the slope and the intercept are estimated. Denote the vector of initial values by $(\sigma_u^{(0)2},\ \beta_0^{(0)}\ ,\ \beta_1^{(0)})$. We use a burn-in period of $B$ and a total of $B + T$ iterations of the chain. We index the sample $t$ from the Markov Chain by $(t, FH)$, where the index of $FH$ separates the samples for the Fay-Herriot model from the samples for the structural measurement error model. The $\delta_1^{-2}$ and $\delta_0^{-2}$ are the same as for the measurement error model. The Gibbs sampling algorithm involves repeating the four steps below for $t = 1,2, \dots, B + T$.

1. Generate $\boldsymbol{\beta}^{(t,FH)} = \left(\beta_0^{(t,FH)}, \beta_1^{(t,FH)}\right)' \sim BVN(\boldsymbol{\mu}_\beta^{(t,FH)}, \boldsymbol{V}_\beta^{(t,FH)})$, where

$\boldsymbol{\mu}_{\beta}^{(t,FH)} = \left(\sum_{i=1}^{D} \boldsymbol{x}_i \boldsymbol{x}_i' (\sigma_u^{(t-1,FH)2} + \sigma_{ei}^2)^{-1} + diag(\delta_0^{-2}, \delta_1^{-2})\right)^{-1} \sum_{i=1}^{D} \boldsymbol{x}_i (\sigma_u^{(t-1,FH)2} + \sigma_{ei}^2)^{-1} y_i$, and $\boldsymbol{V}_{\beta}^{(t,FH)} = \left(\sum_{i=1}^{D} \boldsymbol{x}_i \boldsymbol{x}_i' (\sigma_u^{(t-1,FH)2} + \sigma_{ei}^2)^{-1} + diag(\delta_0^{-2}, \delta_1^{-2})\right)^{-1}$.

2. Generate $\sigma_u^{(t,FH)2} = 1/\tau^{(t,FH)2}$, where $\tau^{(t,FH)2} \sim Gamma(\frac{D}{2}, 0.5 \sum_{i=1}^{D} \left(Y_i^{(t-1,FH)} - \beta_0^{(t,FH)} - \beta_1^{(t,FH)} x_i\right)^2)$.

3. Generate $Y_i^{(t,FH)}$ independently as
$$Y_i^{(t,FH)} \sim N(\mu_i^{(t,FH)}, V_i^{(t,FH)}),$$
where
$$\mu_i^{(t,FH)} = \frac{\frac{\hat{Y}_i}{\sigma_{ei}^2} + \frac{\beta_0^{(t,FH)} - \beta_1^{(t,FH)} x_i}{\sigma_u^{(t,FH)2}}}{\frac{1}{\sigma_{ei}^2} + \frac{1}{\sigma_u^{(t,FH)2}}},$$
and
$$V_i^{(t,FH)} = \frac{1}{\frac{1}{\sigma_{ei}^2} + \frac{1}{\sigma_u^{(t,FH)2}}}.$$

We discard the first $B$ iterations as burn-in and calculate posterior means and posterior standard deviations based on the remaining $T$ iterations. The posterior mean and standard deviation are defined, respectively, as

$$\tilde{Y}_i^{pred,FH} = T^{-1} \sum_{t=B+1}^{T} Y_i^{(t,FH)}$$

and

$$sd_i^{pred,FH} = \sqrt{(T-1)^{-1} \sum_{t=B+1}^{T} (Y_i^{(t,FH)} - \tilde{Y}_i^{pred,FH})^2} \ . \qquad (7)$$

The posterior mean and standard deviation of the model parameters are defined analogously as the sample mean and standard deviation of $\{\left(\sigma_u^{(t,FH)2}, \ \beta_0^{(t,FH)}, \ \beta_1^{(t,FH)}\right) : t = B+1, \dots, T\}$.

**Appendix B: Convergence Diagnostics**

We assess convergence of the Gibbs sampling algorithm for the structural model applied to the Namibia data. We run three chains with dispersed starting values. The starting values for the FH model in chain $q$ are defined as $\beta_0^{(0)} + Z_{q,10}$, $\beta_1^{(0)} + Z_{q,1}$, and $\sigma_u^{(0)2} + U_{q,10}$, where $Z_{q,10} \sim N(0,10)$, $Z_{q,1} \sim N(0,1)$, and $U_{q,10} \sim |N(0, 10)|$ independently for $q = 1, 2, 3$. For the structural measurement error model, the starting values in chain $q$ are defined as $\beta_0^{(0)} + Z_{q,10,1}$, $\beta_1^{(0)} + Z_{q,1}$, $\sigma_u^{(0)2} + U_{q,10,1}$, $\mu_y^{(0)} + Z_{q,10,2}$, and $\sigma_y^{(0)2} + Z_{q,10,2}$ where $Z_{q,10,j} \sim N(0,10)$, $Z_{q,1} \sim N(0,1)$, and $U_{q,10,j} \sim |N(0, 10)|$ independently for $j = 1,2$ and $q = 1,2,3$. For each chain, we retain 50,000 iterations of the Markov chain after a burn-in period of

500. We thin each chain by an interval of 10. Each chain then has 5000 samples. We use three chains instead of one to reduce the autocorrelation and thereby increase the effective sample size. Long chains are needed for two reasons. One is to ensure that we sample from the entire posterior distribution. The other is to reduce the variance of Monte Carlo approximations for summaries of the posterior distribution.

We calculate Geweke statistics, scale reduction factors, and the effective sample size, using R functions provided in the Coda R package. We also construct trace plots of model parameters for the three chains. Appendix B.1 presents convergence diagnostics for the structural measurement error model, applied to the Namibia data. For both models, the convergence diagnostics give no reason for concern. The Geweke Z-statistics are all less than 2, providing no reason to reject a null hypothesis that the mean of the first 10% of the chain is the same as the mean of the last 50% of the chain. The fractions of 10% and 50% are the default values in the R function geweke.diag. The potential scale reduction factors (PSRF) and trace plots indicate that the Markov Chain is mixing appropriately. The effective sample sizes are based on a total of 150,000 iterations. The effective sample sizes are all above 40,000. For regression coefficients and mean parameters, the effective sample sizes are above 130,000. Thinning removes the effect of the auto-correlation on the effective sample size. After thinning and burn-in the effective sample size for the intercept and slope equal the total sample size of 15,003.

[ Table B.1 about here ]

[ Figure B.1 about here ]

**C. Direct simulation method for Fay-Herriot model**

The Fay-Herriot model permits a direct simulation method of sampling from the posterior distribution that does not require Markov Chain Monte Carlo. For the Fay-Herriot model, one can determine that the marginal posterior distribution for $\sigma_u^2$ is of the form

$$p(\sigma_u^2 \mid \hat{\boldsymbol{y}}) \propto \tilde{c}(\sigma_u^2)\det\left(V(\sigma_u^2)\right),$$

where $V(\sigma_u^2) = \left(\sum_{i=1}^{D}(1,x_i)'(1,x_i)\left(\sigma_u^2+\sigma_{ei}^2\right)^{-1}\right)^{-1}$, and

$$\tilde{c}(\sigma_u^2) = c(\sigma_u^2)\exp\left(0.5\mu(\sigma_u^2)'V(\sigma_u^2)^{-1}\mu(\sigma_u^2)\right)\prod_{i=1}^{D}\frac{1}{\sqrt{\sigma_u^2+\sigma_{ei}^2}}\exp\left(-0.5\sum_{i=1}^{6}\frac{\hat{y}_i^2}{\sigma_u^2+\sigma_{ei}^2}\right).$$

One can similarly deduce that the full posterior distribution of $\beta = (\beta_0, \beta_1)'$ is a normal distribution. Specifically, $\beta \mid (\sigma_u^2, \hat{\boldsymbol{y}}) \sim BVN\left(V(\sigma_u^2)^{-1}\sum_{i=1}^{D}(1,x_i)'\left(\sigma_u^2+\sigma_{ei}^2\right)^{-1}\hat{y}_i \quad, V(\sigma_u^2)^{-1}\right)$.

Simulating from the posterior of $\sigma_u^2$ is difficult because the parameter space for $\sigma_u^2$ is unbounded. We transform $\sigma_u^2$ to $\theta = \bar{\sigma}_e^2(\bar{\sigma}_e^2+\sigma_u^2)^{-1}$, where $\bar{\sigma}_e^2 = 6^{-1}\sum_{i=1}^{6}\sigma_{ei}^2$. The parameter space for $\theta$ is [0,1]. We assign a uniform prior for $\theta$ defined by $\theta \sim Unif(0,1)$. The marginal posterior for $\theta$ is the function of $\theta$ defined by $\tilde{p}(\theta) = p(\sigma_u^2(\theta))$, where $\sigma_u^2(\theta) = \theta^{-1}\bar{\sigma}_e^2 - \bar{\sigma}_e^2$.

We define a three-step procedure to simulate from the joint posterior distribution, $p(\beta, \sigma_u^2 \mid \hat{\boldsymbol{y}})$. In the first step, we simulate from a discrete approximation for $\tilde{p}(\theta)$ defined by a grid on $[0,1]$. In the second step, we transform $\theta$ to $\sigma_u^2$ by $\sigma_u^2(\theta) = \theta^{-1}\bar{\sigma}_e^2 - \bar{\sigma}_e^2$. Finally, we generate $\beta$ from the normal distribution defining the full conditional posterior distribution for $\beta$. The procedure of approximating the posterior distribution for $(\beta, \sigma_u^2)$ involves repeating those three steps for a large number of iterations.

The details of the algorithm are defined as follows. For Monte Carlo samples $m = 1, \dots, M$, repeat the two steps below:

1. Set $\theta_m = k/(K+1)$ for $k = 1, \dots, K$ with $K = 100$ with probability $p_k \propto \tilde{p}(k/(K+1))$ such that $\sum_{k=1}^{K} p_k = 1$.
2. Set $\sigma_{u,m}^2 = \theta_m^{-1}\bar{\sigma}_e^2 - \bar{\sigma}_e^2$.
3. Generate $\beta_m \sim BVN\left(V(\sigma_{u,m}^2)^{-1}\sum_{i=1}^{D}(1, x_i)'(\sigma_{u,m}^2 + \sigma_{ei}^2)^{-1}\hat{y}_i \quad, V(\sigma_{u,m}^2)^{-1}\right)$.

These two steps generate *M iid* pairs $(\beta_m, \sigma_{u,m}^2)$. We use standard Monte Carlo procedures to approximate the posterior distribution using the *M iid* pairs.

We apply the method to the Namibia data. Table C.1 contains 95% credible intervals for the parameters of the Fay-Herriot model. The credible intervals for the parameters of the Fay-Herriot model are reasonable, given the results of the structural model.

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**Tables**

*Table 1: Posterior means and 95% central credible intervals for parameters of structural model.*

| Parameter | Posterior Mean | Central Credible Interval |
|---|---|---|
| $\beta_0$ | 2.4056E+03 | [-3.2186E+04, 3.7046E+04] |
| $\beta_1$ | 7.6027E-01 | [2.4503E-01, 1.2775E+00] |
| $\sigma_u^2$ | 3.4217E+08 | [5.2342E+07, 1.4544E+09] |
| $\mu_y$ | 6.0948E+04 | [2.6933E+04, 9.5062E+04] |
| $\sigma_y$ | 1.7446E+09 | [3.9378E+08, 6.2637E+09] |

*Table 2: Posterior means, 95% central credible intervals, and posterior standard deviations of predictors for structural model, compared to standard error of direct estimator (SEDirect). Ratio is SEDirect/(Posterior SD).*

| Region ($i$) | Posterior Mean | 95% Central Credible Interval | Posterior SD | SEDirect | Ratio |
|---|---|---|---|---|---|
| Zambezi (1) | 16210.66 | [12241.46, 20193.57] | 2031.603 | 2035.712 | 0.997982 |
| Kavango (2) | 49773.23 | [40704.17, 58844.94] | 4621.376 | 4617.180 | 1.000909 |
| Omusati (3) | 107172.85 | [96350.18, 118135.6] | 5561.844 | 5593.323 | 0.994372 |
| Ohangwena (4) | 82067.61 | [74010.88, 90090.48] | 4109.607 | 4148.187 | 0.990700 |
| Oshana (5) | 41620.74 | [27285.83, 55762.91] | 7248.338 | 7924.158 | 0.914714 |
| Oshikoto (6) | 68546.29 | [62907.18, 74241.28] | 2898.269 | 2944.683 | 0.984238 |

*Table 3: Properties of predictors of $Y_i$ in simulation. For the column "ME," the measurement error model is the fitted model. For the column "FH," the Fay-Herriot model is the fitted model.*

| | | | RMSE | | Bias | | Ratio | |
|---|---|---|---|---|---|---|---|---|
| **True Model ($l$)** | $D$ | $c$ | **Est-ME** | **Est-FH** | **Est-ME** | **Est-FH** | **Est-ME** | **Est-FH** |
| ME | 9 | 0.5 | 4.947 | 4.970 | 0.121 | 0.054 | 1.010 | 1.019 |
| ME | 18 | 0.5 | 4.625 | 4.621 | 0.172 | 0.186 | 0.882 | 0.881 |
| ME | 9 | 1 | 4.875 | 4.856 | 0.178 | 0.174 | 0.981 | 0.973 |
| ME | 18 | 1 | 4.855 | 4.857 | 0.210 | 0.200 | 0.973 | 0.974 |
| ME | 9 | 4 | 4.856 | 4.833 | 0.017 | 0.066 | 0.973 | 0.964 |
| ME | 18 | 4 | 4.784 | 4.776 | -0.021 | -0.033 | 0.944 | 0.941 |
| FH | 9 | 0.5 | 4.314 | 3.984 | -0.167 | 0.019 | 0.768 | 0.655 |
| FH | 18 | 0.5 | 3.526 | 3.422 | -0.214 | -0.214 | 0.513 | 0.483 |
| FH | 9 | 1 | 4.275 | 4.264 | -0.146 | -0.147 | 0.754 | 0.750 |
| FH | 18 | 1 | 4.162 | 4.236 | -0.237 | -0.197 | 0.715 | 0.740 |
| FH | 9 | 4 | 5.012 | 4.984 | -0.110 | -0.079 | 1.036 | 1.025 |
| FH | 18 | 4 | 4.471 | 4.429 | -0.174 | -0.190 | 0.825 | 0.809 |

*Table 4: MC mean and MC standard deviation of estimators of slope and intercept. True values are $\beta_0 = 0$ and $\beta_1 = 1$.*

| | | | Measurement Error Model Estimates | | | | Fay-Herriot Model Estimates | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | MC Mean | | MC SD | | MC Mean | | MC SD | |
| **True Model ($l$)** | $D$ | $c$ | $\beta_0$ | $\beta_1$ | $\beta_0$ | $\beta_1$ | $\beta_0$ | $\beta_1$ | $\beta_0$ | $\beta_1$ |
| ME | 9 | 0.5 | -3.189 | 1.044 | 14.974 | 0.245 | 11.149 | 0.823 | 11.640 | 0.185 |
| ME | 18 | 0.5 | 0.507 | 0.989 | 6.787 | 0.102 | 9.225 | 0.856 | 6.146 | 0.092 |
| ME | 9 | 1 | -0.802 | 1.014 | 11.601 | 0.157 | 7.850 | 0.869 | 9.884 | 0.139 |
| ME | 18 | 1 | -1.110 | 1.007 | 8.899 | 0.125 | 10.150 | 0.843 | 6.333 | 0.088 |
| ME | 9 | 4 | -0.065 | 1.010 | 13.234 | 0.172 | 7.766 | 0.870 | 10.085 | 0.131 |
| ME | 18 | 4 | -0.270 | 1.005 | 7.685 | 0.123 | 8.343 | 0.860 | 6.937 | 0.107 |
| FH | 9 | 0.5 | -0.420 | 1.016 | 6.569 | 0.116 | 0.052 | 0.995 | 5.923 | 0.100 |
| FH | 18 | 0.5 | 0.661 | 0.992 | 2.788 | 0.048 | -0.403 | 1.003 | 2.408 | 0.044 |
| FH | 9 | 1 | -0.158 | 1.003 | 5.050 | 0.102 | 0.145 | 0.997 | 4.861 | 0.094 |
| FH | 18 | 1 | 0.699 | 0.990 | 3.053 | 0.046 | -0.411 | 1.005 | 2.917 | 0.046 |
| FH | 9 | 4 | 5.783 | 0.872 | 10.221 | 0.191 | 0.098 | 1.006 | 12.852 | 0.243 |
| FH | 18 | 4 | 5.486 | 0.906 | 4.713 | 0.074 | -0.663 | 1.004 | 5.412 | 0.086 |

*B.1 Convergence Diagnostics for Structural Measurement Error Model*

| | Geweke Statistics | | | Potential Scale Reduction Factor | | Effective Sample Size |
|---|---|---|---|---|---|---|
| **Parameter** | **Chain 1** | **Chain 2** | **Chain 3** | **Statistic** | **Upper C.I.** | **(Univariate)** |
| $\beta_0$ | -1.1697 | 0.5966 | -0.6516 | 1.0000 | 1.0000 | 141,334.71 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\beta_1$ | 1.2611 | -0.8389 | 0.3119 | 1.0000 | 1.0001 | 139,045.58 |
| $\sigma_u^2$ | -0.6622 | -0.6675 | 0.2245 | 1.0044 | 1.0044 | 58,166.32 |
| $\mu_y$ | -1.0469 | -0.0890 | -0.1458 | 1.0002 | 1.0002 | 145,105.00 |
| $\sigma_y$ | -0.2835 | 1.0329 | 0.5790 | 1.0082 | 1.0084 | 93,750.77 |

*Table C.1: Credible intervals for Fay-Herriot model parameters using direct simulation method of Appendix C.*

| $\beta_0$ | $\beta_1$ | $\sigma_u^2$ |
|---|---|---|
| $[-30639.05, \; 47573.16]$ | $[0.352, \quad 1.799]$ | $[82193139, 2423643834]$ |

**Figures Captions**

*Figure 1: Scatterplot of NAS estimates on y-axis and MAWF estimates on x-axis. Dashed line is 45-degree line through the origin.*

*Figure B.1: Trace plots of parameters of structural model generated from posterior distribution through Gibbs sampling. Three colors are for three independent chains.*

**Figures (with captions)**

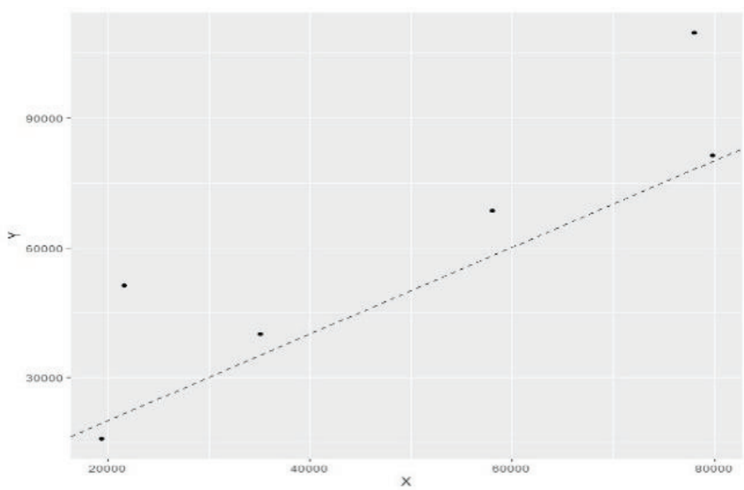


*Figure 2: Scatterplot of NAS estimates on y-axis and MAWF estimates on x-axis. Dashed line is 45-degree line through the origin.*

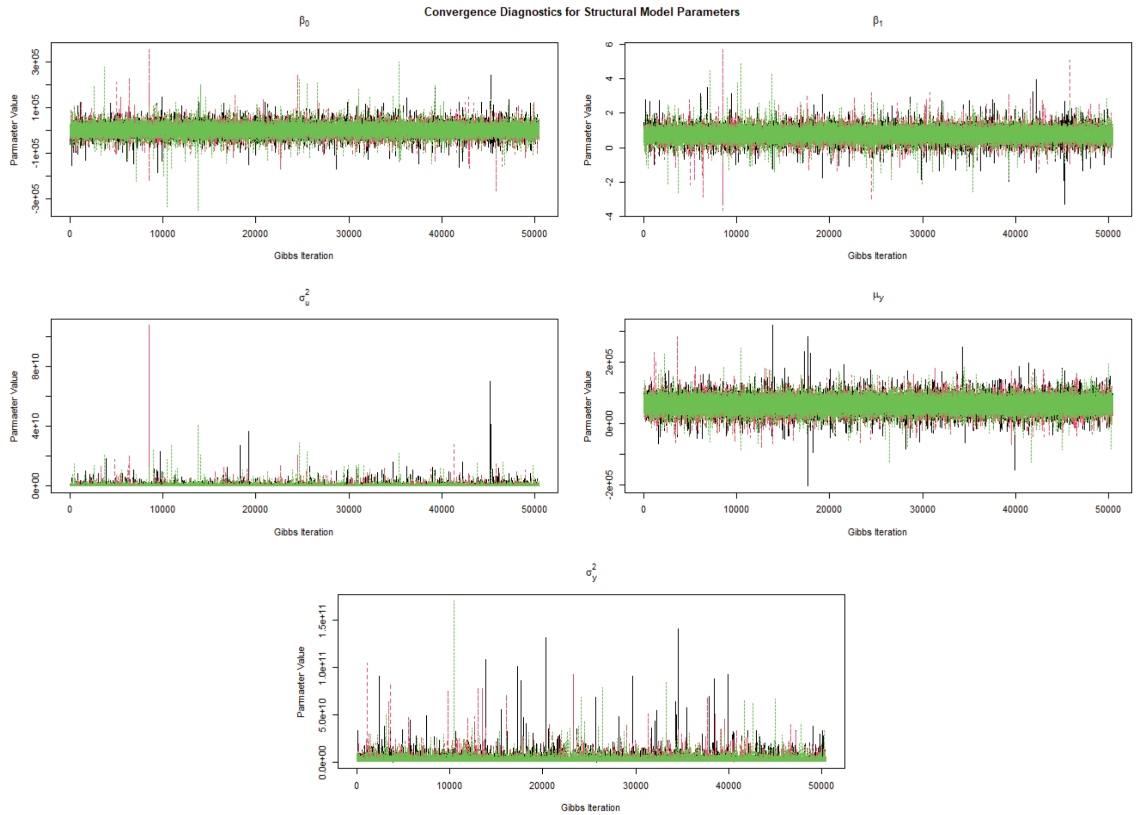


*Figure B.1: Trace plots of parameters of structural model generated from posterior distribution through Gibbs sampling. Three colors are for three independent chains.*