# Anchor Selection for Topology Inference and Routing in Wireless Sensor Networks

Taha Bouchoucha, Zhi Ding

Abstract—Anchor-based ad-hoc networks utilize hop measurements to generate a virtual coordinate system for topology inference and routing applications. A common problem with such coordinate system is its sensitivity to anchor placement. We present a general formulation to the anchor node selection problem. Then, we relax the optimization problem by deriving an upper-bound of the objective function. We finally propose an iterative algorithm that consists in choosing additional anchor nodes based on the connectivity information provided by the current anchor set. Numerical simulations indicate that our anchor selection method is robust to missing measurements and improves network topology inference and routing performance.

*Keywords*—anchor selection, connectivity, hop distance, routing

## I. Introduction

Wireless ad-hoc networks are decentralized, easy to deploy and scalable networks which can be applied in several fields such as environment monitoring, health, defense, etc. However, such peer-to-peer networks are usually characterized by the highly dynamic multihops topology due to random node mobility. Therefore, it is essential to keep track of the network topology for optimal information routing between networked nodes. Most ad-hoc networks can be modeled as connected graphs where nodes represent users equipment, individuals, or sensors while edges characterize connectivity and routing paths between nodes. We consider a simple binary relationship where the network topology is completely determined by the adjacency matrix of the corresponding graph representation.

We focus in this work on anchor node selection which is a common challenge faced in node localization for network

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topology inference and packet routing purposes. There exist several localization coordinate systems that differ in terms of measuring tools, complexity and cost. Geographical coordinate system (GPS) is certainly one of the most efficient and reliable localization systems. However, due to its high cost and complex implementation requirement, it might be impractical to use in networks with scarce resources and equipment complexity constraints. In addition, location estimation quality in GPS critically depends on the transmitted signal quality and the channel disturbances such as noise, interference and multi-path fading. An alternative localization method, known as a virtual coordinate system (VCS)<sup>[1-4]</sup>, uses much simpler signals which make it easier and less expensive to implement. A VCS is based on hop measurements collected from the network using controlled flooding mechanism<sup>[5,6]</sup>. First, a subset of nodes are selected as anchors. Second, each anchor measures the number of hops separated from the remaining nodes by generating a beacon signal and collecting feedback from each node that receives it. The role of each node during this process is simply to increase and forward an integer signal to its neighboring nodes. Therefore, using this controlled flooding mechanism, we can collect hop counts from the anchors at a low cost and with a small error rate thanks to the integer nature of the transmitted signal which makes it robust to channel distortion and noise. Besides, the virtual distance<sup>[1]</sup> defined in a VCS provides more connectivity information than the physical distance obtained from GPS coordinates.

However, the accuracy of this virtual distance critically depends on the anchor set responsible of collecting the hop measurements. Our work lies in choosing the optimal anchor set and thus improving the performance of VCS-based applications such as network topology inference and routing. The first application uses dimension reduction techniques such as principal component analysis (PCA) to generate a topology that mimics the physical layout of the network using only hop measurements<sup>[1,7,8]</sup>. The authors in Refs. [1,7] propose a topology preserving map where each node is placed in a two- or three-dimensional Euclidean space using virtual coordinates. Such a visualization tool can be used for various purposes such as the boundary node and backbone identification and discovery of geographic voids. A different approach in Ref. [8] consists in exploiting hop counts from three se-

lected anchors to define zones of nodes with similar coordinates rather than assigning virtual coordinates to each node. The authors in Ref. [4] provide a theoretical proof of the PCA approach which consists in projecting the hop measurement matrix into the most significant principal components. Numerical simulations show that this dimension reduction results in a subspace that captures most of the connectivity information of the network. The low rank property of the measurement matrix is also leveraged in Ref. [4] to generate a VCS that is robust to missing and noisy hop measurements. The second application consists in using a VCS to establish logical distance between nodes which is then used to optimize the path in packet routing applications. The logical distance<sup>[9]</sup> is usually defined as the Euclidean distance between virtual coordinates and it quantifies the connectivity strength between a pair of nodes. A simple routing protocol consists in forwarding packets to the neighbor that is closest to the destination in terms of logical distance<sup>[10]</sup>. One challenge that can hinder successful delivery of the packet is the problem of loop and local minima. Ref. [2] proposes a dynamic anchor selection to ensure the convexity of the distance function between source and destination. A tree-based recovery method is proposed in Ref. [3] to avoid local minimum solutions.

Hence, even if we have a sufficient number of anchors, if they are not properly placed in the network, the generated VCS will suffer from identical coordinates and local minima problems. In fact, when anchors are randomly placed, there is a possibility that two arbitrary nodes have the exact same coordinates in the VCS because they are separated by the same hop distance from those anchors. This results in an ambiguous coordinate system and a poor routing performance. Therefore, it is necessary to optimize the choice of anchor nodes.

In most of the aforementioned works, the anchor nodes are randomly selected. Anchors selection is still an open problem. To the best of our knowledge, this work is the first attempt to investigate the problem of anchor placement for network topology inference and routing purposes. Few recent attempts have proposed an anchor placement for outlier detection<sup>[11]</sup> as well as an energy efficient anchor deployment<sup>[12]</sup>. In this work, we develop an anchor selection method that improves network connectivity inference and results in an accurate and reliable VCS for routing. We start by formalizing the problem in function of the probability of topology inference error. We then break down the optimization problem into an iterative process and relax the objective function by providing a tight upper-bound. After investigating different selection criteria, we propose an iterative anchor selection algorithm which provides a near-optimal performance in terms of network connectivity inference. We then test our method for routing applications.

This manuscript is organized as follows. Section II presents the system model and the anchor selection problem formulation. In section III, we detail our approach to solve the problem and provide an iterative algorithm. Section IV is dedicated to VCS-based routing applications. We finally provide numerical results in section V to demonstrate the benefits and robustness of our anchor selection method through several simulation test examples before concluding the paper in section VI.

## II. SYSTEM MODEL

We consider a network of an unknown topology consisting of N nodes given by the set  $\mathcal{N} = \{n_1, n_2, \dots, n_N\}$ . We denote the network topology by its  $N \times N$  connectivity (adjacency) matrix A in which A(i, j) = 1 denotes the existence of a link between node  $n_i$  and node  $n_j$  whereas A(i, j) = 0, otherwise.

We consider in this work networks that are represented by connected and undirected graphs, i.e., we can always find a path connecting any pair of distinct nodes and the adjacency matrix  $\boldsymbol{A}$  is symmetric. Let  $\mathscr E$  denote the space of all possible solutions of the adjacency matrix  $\boldsymbol{A}$ . Since A(i,j)=A(j,i) and A(i,i)=0, it is clear that  $\mathscr E\subset\{0,1\}^{\frac{N(N-1)}{2}}$ .

We then designate a subset of M < N network nodes  $\mathcal{A} =$  $\{A_1, A_2, \cdots, A_M\} \subset \mathcal{N}$  as anchors which use a controlled network flooding method<sup>[5]</sup> to collect hop measurements from the remaining nodes. More specifically, anchor nodes start by transmitting probing packets to other nodes within the network. Each node records and forwards the number of hops it has taken from the originating anchor node while keeping the smallest value in case a packet is received multiple times from the same anchor. At the end of this network flooding process, anchor nodes collect the hop distance reports from the nodes and generate a hop distance matrix of dimension  $N \times M$  denoted as  $H(\mathscr{A})$ . More details about this flooding technique are provided in Ref. [6]. Note that H(i, j) records the shortest hop distance between node  $n_i$  and anchor  $A_i$ . Since each node  $n_i$  collects a hop vector  $h_i$  consisting of hop distances to the M anchors, we can write the hop measurements matrix relative to anchor set  $\mathcal{A}$  as

$$\boldsymbol{H}(\mathscr{A}) = [\boldsymbol{h}_1^{\mathrm{T}}, \boldsymbol{h}_2^{\mathrm{T}}, \cdots, \boldsymbol{h}_N^{\mathrm{T}}], \tag{1}$$

where  $h_i$  is the raw virtual coordinate vector associated with node  $n_i$ . Fig. 1 is a simple example showing the graphical representation of a network composed of 9 nodes as well as the hop measurement matrix relative to the anchor set  $\mathscr{A} = \{n_1, n_6\}$ .

Using this virtual coordinate system, we define the logical distance  $d_{\mathscr{A}}$  relative to anchor set  $\mathscr{A}$ 

$$d_{\mathscr{A}}(n_i, n_j) = \|\boldsymbol{h}_i - \boldsymbol{h}_j\|_2. \tag{2}$$

We study hereafter the joint problem of adjacency matrix inference and anchor node selection. Let us denote by

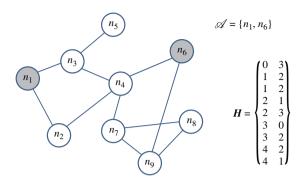


Figure 1 System model: controlled flooding and generation of hop measurements matrix  $\boldsymbol{H}$ 

 $P(e_k|\mathscr{A})$  the probability that  $e_k \in \mathscr{E}$  is the solution to the adjacency matrix, given the hop measurements relative to the anchor set  $\mathscr{A}$  which is provided by  $H(\mathscr{A})$ . We denote by  $\hat{e}_k$  the estimated adjacency matrix. The probability of adjacency inference error is then given by

$$P_{e}(\mathscr{A}) = \sum_{e_{k} \in \mathscr{E}} \sum_{j \neq k} P(e_{k}|\mathscr{A}) P(\hat{e}_{k} = e_{j}|e_{k};\mathscr{A}), \quad (3)$$

where  $P(\hat{e}_k = e_j | e_k; \mathcal{A})$  is the probability that the inferred adjacency matrix  $\hat{e}_k$  is equal to an adjacency matrix  $e_j$  that is different from the ground truth  $e_k$ .

The problem of anchor node selection with a given anchor budget M consists in finding the optimal anchor set  $\mathscr{A}^*$  that minimizes the probability of error

$$\min_{\mathscr{A}} \quad P_e(\mathscr{A}),$$
 s.t. 
$$|\mathscr{A}| \leqslant M.$$
 (4)

Therefore, given the probability of adjacency inference error  $P_e$ , the goal of problem (4) is to choose a subset  $\mathscr{A}^*$  from the set of nodes  $\mathscr{N}$  to be anchors. However, the estimation of  $P_e$  can be challenging since the network topology is unknown. In the next section, we approximate the probability  $P_e$  and propose an iterative approach to find a near-optimal solution.

## III. PROPOSED ANCHOR SELECTION

We notice that the optimization problem (4) is equivalent to the NP-complete subset sum problem<sup>[13]</sup>. Therefore, it requires exponential running time as well as the assumption that hop measurements from all the networked nodes would be known. However, in practice, we have limited computational power and we can only obtain hop measurements for a limited set of anchor nodes.

We therefore propose to solve the problem iteratively where, at each iteration, the connectivity knowledge acquired in the previous iteration is used to select the next anchor. Let  $\mathscr{A}^{(i)}$  denote the anchor set at the *i*th iteration. We assume that we have no prior knowledge about the network topology, so

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Algorithm 1 Iterative anchor selection
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\begin{aligned} \textbf{Data:} & \, \mathcal{N} \\ \textbf{Result:} & \, \mathcal{A}^{(M)} \\ & \, \mathcal{A}^{(1)} \leftarrow \text{random node from } \mathcal{N} \; ; \\ \textbf{for } i \leftarrow 2 \textbf{ to } M \textbf{ do} \\ & \, n^{(i)} = \underset{n \in \mathcal{N} \setminus \mathcal{A}^{(i-1)}}{\operatorname{argmin}} \, \mu(\mathcal{A}^{(i-1)} \cup n); \\ & \, \mathcal{A}^{(i)} \leftarrow \mathcal{A}^{(i-1)} \cup n^{(i)}; \\ \textbf{end} \end{aligned}
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we initialize  $\mathscr{A}^{(1)}$  with a random node. At the *i*th iteration, we increase the previous anchor set  $\mathscr{A}^{(i-1)}$  with an optimal anchor  $n^{(i)}$  that minimizes the probability of error  $P_e$ 

$$n^{(i)} = \underset{n \in \mathcal{N} \setminus \mathcal{A}^{(i-1)}}{\operatorname{argmin}} \quad P_e(\mathcal{A}^{(i-1)} \cup n). \tag{5}$$

Unfortunately, the probability of error  $P_e(\cdot)$  is hard to formulate or estimate due to a limited amount of hop measurements. Therefore, we propose to relax it by providing an upper bound that is easily derived in each iteration using the hop measurements provided by the current set of anchors. We derive our upper bound using a previous network connectivity study<sup>[4]</sup> in which we provided a quantitative measure of how informative the hop distance matrix  $H(\mathscr{A})$  is about the adjacency matrix A. The idea consists in organizing the nodes into layers. Then, based on a search strategy and a number of nodes in each layer, the connectivity relationships between each pair of nodes are classified into connected, disconnected, or ambiguous nodes. This classification results in a partial adjacency matrix inference and allows us to reduce the size of the set of connectivity solutions  $\mathscr{E}$ . It is also shown in Ref. [4] that the non-ambiguous inferred edges are perfectly determined with zero error probability. Further details and examples about adjacency matrix inference from anchor-based hop measurements can be found in section V-A of Ref. [4]. Therefore, if we denote by  $\mu(\mathscr{A})$  the connectivity inference ambiguity relative to the set of anchors A given by the percentage of ambiguous entries in the partial inferred adjacency matrix, we can easily conclude that  $P_{e}(\mathscr{A}) < \mu(\mathscr{A})$ . Using this upper-bound, the problem of anchor node selection with a given anchor budget M is then relaxed as

$$n^{(i)} = \underset{n \in \mathcal{N} \setminus \mathcal{A}^{(i-1)}}{\operatorname{argmin}} \quad \mu(\mathcal{A}^{(i-1)} \cup n). \tag{6}$$

The iterative approach is summarized in Algorithm 1 where we start by initializing the anchor set with a random node. Next, we choose anchors based on their marginal gain in terms of ambiguity function according to problem (6).

However, it is challenging to solve problem (6) because it requires the knowledge of connectivity information relative to the nodes in  $\mathcal{N} \setminus \mathcal{A}^{(i-1)}$  which is not available at the *i*th iteration. To overcome this problem, we adopt a greedy approach

to estimate  $\mu$  that only requires available connectivity information relative to  $\mathscr{A}^{(i-1)}$ .

At the *i*th iteration, we propose the following anchor selection criteria.

•  $\mathcal{S}_{DFS}$ . Choose the furthest node, in terms of logical distance  $d_{\mathscr{A}^{(i-1)}}(\cdot)$ , of the current set of anchors  $\mathscr{A}^{(i-1)}$ . The intuition behind this criterion is to choose an anchor that is logically far from the current anchor set to explore network depth similarly to a depth-first search (DFS) algorithm

$$n^{(i)} = \underset{n \in \mathcal{N} \setminus \mathcal{A}^{(i-1)}}{\operatorname{argmax}} \left( \min_{m \in \mathcal{A}^{(i-1)}} d_{\mathcal{A}^{(i-1)}}(n, m) \right).$$

•  $\mathcal{S}_{BFS}$ . Choose the node with the maximum number of ambiguous edges based on the adjacency matrix inferred from the set of anchors  $\mathcal{A}^{(i-1)}$ . The idea of this criterion is to select the anchor node that minimizes the local ambiguity by revealing as much neighboring connectivity links as possible similarly to a breadth-first search (BFS) algorithm

$$n^{(i)} = \underset{n \in \mathcal{N} \setminus \mathcal{A}^{(i-1)}}{\operatorname{argmax}} \left( \mu^{(i-1)}(n) \right),$$

where  $\mu^{(i-1)}(n)$  is the local ambiguity of node n at the (i-1)th iteration given by the number of ambiguous entries in the column of the inferred adjacency matrix relative to node n.

•  $\mathscr{S}_{Prop}$  (our proposed criterion). After testing the two previous criteria, we noticed that  $\mathscr{S}_{BFS}$  does not guarantee a unique solution and  $\mathscr{S}_{DFS}$  does not perform as well as  $\mathscr{S}_{BFS}$ . So our proposed criterion is that if we obtain multiple candidates from criterion  $\mathscr{S}_{BFS}$ , we use  $\mathscr{S}_{DFS}$  to decide on a unique solution.

In the numerical simulation section, we show that the performance of the proposed criterion is very close to the optimal solution.

## IV. ROUTING APPLICATION

A VCS is a useful coordinate system for routing and finding the shortest path<sup>[3,9,10]</sup>. Most of the previous works rely on the logical distance  $d_{\mathcal{A}}$  to optimize the path for packet forwarding. For example, in Ref. [9], the authors defined the logical distance using the K < M most important principal components of the virtual coordinates. Reducing coordinates dimension from M to K lowers redundancy and improves routing performance. The routing protocol<sup>[9]</sup> simply lets each node forward its packets to its neighbor node that is closest to the packet destination node in terms of logical distance. More specifically, consider a source node S with a reduced dimension virtual coordinate vector  $g_S = [s_1, s_2, \cdots, s_K]^T$  in a K-dimensional space after PCA. Similarly, let T be the destination node with coordinate vector  $g_t = [t_1, t_2, \cdots, t_K]^T$ . Based on logical distance, node S searches among its neighbors to

find the next intermediate hop that has minimum logical distance to the destination node T. In case such an intermediate node is not unique, a fallback mechanism is activated. It consists in incrementally reducing the dimension of the virtual coordinates space until an intermediate node with a unique minimum logical distance to T is found. Despite its simplicity, this method has some drawbacks such as falling in local minimum or infinite loop.

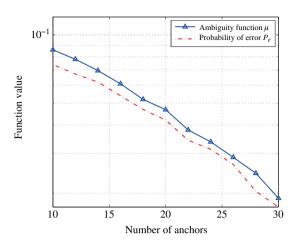
We propose another approach that leverages the ability of our anchor selection method to infer the adjacency matrix of the network. When the recovered adjacency matrix is obtained without edge errors, the shortest path is guaranteed to exist and can be found using a simple algorithm such as Dijkstra. So, the only source of delivery failure in our case is an error in the recovered adjacency matrix which is less likely to happen as the number of anchors increases, as it is shown in the next numerical simulation section.

#### V. NUMERICAL SIMULATIONS

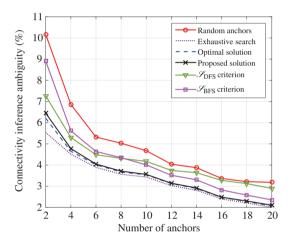
We test hereafter the efficiency of our anchor selection method in terms of network topology inference and routing performance. We also test its robustness to missing hop measurements. We generate random wireless networks by deploying N nodes in a two-dimensional area. Next, we randomly assign edges between pairs of nodes such as the generated graph remains connected. We denote by  $\gamma = |E|/(N(N-1)/2)$  the edge density or the percentage of non-zero entries in the adjacency matrix A. After deploying this network with the unknown topology, we select a subset of anchor nodes according to different methods discussed in section III. Next, we gather the hop measurements from the anchors using the controlled flooding method presented in section II. We finally compare the quality of the measurements provided by each anchor selection method for network connectivity and routing applications. Unless otherwise stated, all simulation results are averaged over 100 independent Monte Carlo runs with the network nodes and edges independently generated for each run according to a uniform distribution. We also fix N = 100 and  $\gamma = 15\%$  and vary the number of anchors M.

In Fig. 2, we estimate the probability of error  $P_e(\cdot)$  and compare it to the upper-bound provided by the connectivity inference ambiguity  $\mu(\cdot)$ . We notice that upper-bound is tight which allows us to provide near-optimal solution by solving the relaxed iterative version (6) of the original optimization problem (4).

Fig. 3 shows the performance of different anchor selection methods in terms of network connectivity inference. We plot the connectivity inference ambiguity in function of the number of anchors for different selection criteria and we compare them with the optimal solution of the iterative problem (5) as well as the solution of the original NP-complete problem



**Figure 2** Upper-bound of the probability of error  $P_e$ , N = 100,  $\gamma = 15\%$ 

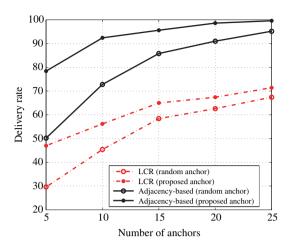


**Figure 3** Anchor selection methods comparison, N = 100,  $\gamma = 15\%$ 

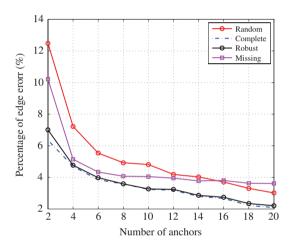
(4) obtained through exhaustive search. We notice that the performance of our proposed greedy criterion overlaps with the optimal solution. A higher performance gap between our proposed method and the exhaustive search solution is only noticed for a low number of anchors due to the lack of previous connectivity information. In fact, in Algorithm 1, the first anchor  $\mathscr{A}^{(1)}$  is randomly chosen.

Fig. 4 is dedicated to routing performance comparison. We randomly choose a source and a destination node and apply logical distance-based routing, a.k.a, LCR as well as our proposed adjacency matrix-based routing. We plot for each method the successful delivery rate. We notice that the proposed anchor selection method improves the performance of both routing algorithms by at least 20%. In addition, our proposed routing outperforms the logical distance-based routing. In fact, by choosing the appropriate set of anchors, the delivery failure risk decreases (especially for larger sets of anchor nodes), which results in a low percentage of ambiguous edges.

Hop measurements are collected using controlled network flooding where nodes transmit back to the anchors a simple



**Figure 4** Routing performance comparison, N = 100,  $\gamma = 15\%$ 



**Figure 5** Robustness of anchor selection method to missing hop measurements, N = 100,  $\gamma = 15\%$ 

integer value in response to a received beacon signal. Even though transmitted signals in a VCS are noise-resilient and more reliable compared to GPS, we still have to deal with missing measurements due to random failure of report channels, power outage, or hacking of certain anchor nodes. The work in Ref. [4] proposes a robust VCS based on low rank matrix completion. We investigate in Fig. 5 the robustness of our anchor selection method to missing hop measurements using the robust VCS proposed in Ref. [4]. We label 20% of the hop measurements as missing. Then we compare the network inference performance using complete, incomplete and robust VCSs. We notice that the performance of our robust method is close to the performance of complete data especially for a higher number of anchors.

# VI. CONCLUSION

We have proposed a novel solution to the anchor selection problem in generative ad-hoc networks to capture the underlying structure from multihops observations. We use a simple ambiguity function that exploits available connectivity information to approximate the probability of network inference error. We then adopt an iterative approach to construct the optimal anchor set based on the selection criterion derived from the ambiguity function. Our anchor selection method generates an accurate VCS and results in a percentage of edge error comparable to the exhaustive search solution. Various experiments on synthetic networks show that our anchor set selection method improves both network connectivity inference and routing performance. For future work, this anchor selection method could be generalized to other types of networks and it can possibly be adapted to other applications such as virus blocking and disease outbreak detection.

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