

**Proceedings of the
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Research in Undergraduate
Mathematics Education**

Editors:

Shiv Smith Karunakaran

Zackery Reed

Abigail Higgins

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(SIGMAA) for Research in Undergraduate Mathematics Education

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Preface

As part of its on-going activities to foster research in undergraduate mathematics education and the dissemination of such research, the Special Interest Group of the Mathematics Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME) held its twenty-third annual Conference on Research in Undergraduate Mathematics Education in Boston, Massachusetts from February 27 - February 29, 2020.

The program included plenary addresses by Dr. Gregory Larnell, Dr. Eric Knuth, and Dr. Elise Lockwood, and the presentation of 148 contributed, preliminary, and theoretical research reports and 74 posters.

The conference was organized around the following themes: results of current research, contemporary theoretical perspectives and research paradigms, and innovative methodologies and analytic approaches as they pertain to the study of undergraduate mathematics education.

The proceedings include several types of papers that represent current work in the field of undergraduate mathematics education, each of which underwent a rigorous review by two or more reviewers:

- Contributed Research Reports describe completed research studies
- Preliminary Research Reports describe ongoing research projects in early stages of analysis
- Theoretical Research Reports describe new theoretical perspectives for research
- Posters are 1-page summaries of work that was presented in poster format

The conference was hosted by the Wheelock College at Boston University.

Many members of the RUME community volunteered to review submissions before the conference and during the review of the conference papers. We sincerely appreciate all of their hard work by the 165 reviewers.

We wish to acknowledge the conference program committee for their substantial contributions to RUME and our institutions. Without their support, the conference would not exist.

Finally, we wish to express our deep appreciation for Wheelock College and Boston University. Their support enabled us to have our conference and continually support our community.

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Shifting Pedagogical Beliefs into Action through Teaching for Mathematical Creativity

Gail Tang
University of La Verne

Miloš Savić
University of Oklahoma

Emily Cilli-Turner
University of La Verne

Paul Regier
University of Oklahoma

Gülden Karakök
University of Northern Colorado

Houssein El Turkey
University of New Haven

Through participation in a research project on fostering creativity in calculus, two instructors showed shifts in their beliefs on teaching. Participation in the project entailed creating mathematical tasks designed to elicit creative responses from students. Support for task development included participation in weekly online professional development sessions. In this paper, we share one instructor's shifts in beliefs as well as alignment of her pre-existing beliefs with pedagogical actions. Preliminary analysis of her entrance tickets to the professional development sessions and her exit interview indicates that this instructor a) shifted her previous beliefs about a perceived time pressure and b) manifested her existing beliefs into actions regarding multiple-approach tasks.

Keywords: mathematical creativity, calculus, pedagogical change, beliefs

Due to the ever-changing landscape of economies and jobs, creativity is a skill that is sought after by employers in STEM fields (Wilson, Lennox, Hughes, & Brown, 2017). As mathematical researchers and instructors, our roles in preparing students depend in our interactions with them, as they comprise the future STEM task-force. Since “the process [of promoting creativity] is influenced by teacher beliefs and biases” (Hershkovitz, Peled, & Littler, 2009, p. 265), positive instructor beliefs regarding the role of creativity in a course are vital to students’ experiences of creativity. Thus, instructors should be encouraged to adopt the belief that fostering creativity in mathematics is a vital part of the curriculum. However, one’s beliefs may not always align with their actions (Hutner & Markman, 2016) given that beliefs exist within complex social and political systems and in some cases acting on those beliefs may be perceived as a threat to job security. Supported by the model of constructive alignment (Biggs, 1996), we further claim that if instructors believe that fostering creativity is an instructional goal, the next step is alignment of that belief with their own pedagogical actions in the classroom.

Background Literature

One of the most cited definitions of belief is Richardson’s (1996): “Psychologically held understandings, premises, or propositions about the world that are felt to be true” (p. 103). A belief is individualistic, meaning that a person with a belief can respect another person’s contradicting belief if they find that the opposing belief can be explained in a reasonable and intelligent manner (Philipp, 2007).

Though accepting another’s belief is possible, changing one’s beliefs has been shown to be a difficult process (Dunbar, Fugelsang, & Stein, 2007; Schoenfeld, 2011; Shtulman & Valcarcel, 2012) because “for an individual to change their beliefs, they need to desert premises that they hold to be true, and often this is difficult and challenging” (Grootenboer & Marshman, 2016, pp. 16-17). One possible way to induce changes in instructors’ beliefs is through professional development (PD) since those experiences are critical in influencing thinking about change and enacting change (Capps et al., 2012; Enderle et al., 2014; Woodbury & Gess-Newsome, 2002).

This is largely due to the opportunities in PDs to engage instructors in reflecting on elements of teaching such as assessment, implementation, collaboration, and problem-solving.

In this paper, we share preliminary results demonstrating that providing instructors PD support to engage in the process of fostering creativity in the classroom can both help instructors shift their beliefs and also begin to transform their beliefs into pedagogical actions.

Methods

In Spring 2019, two instructors from a South-Midwest regional university participated in a study on fostering creativity in calculus. In this paper, we focus on Jo Parker, a white female with eleven years of teaching experience.

Instructors attended an initial 2-day, 4-hour (total) PD session in December 2018. During this initial PD, the authors attempted to provide context for discussing mathematical creativity and discuss their requirements for participation, which included uploading previous semesters' calculus materials, attending weekly online PD meetings, filling out entrance tickets prior to the online meeting, and consenting to be interviewed after grades were submitted. Perhaps most importantly, instructors were to implement two creativity-based tasks created by the researchers and create at least four more creativity-based tasks to use in their classroom. The term *creativity-based tasks* describes tasks that could allow for multiple solutions (Leikin, 2014), provide opportunities for students to pose questions/problems then solve their own problems (Haylock, 1997; Silver 1997), or are ill-defined, or open-ended, such that posing questions is necessitated (Kwon, J.H. Park, J.S. Park, 2006). Task development was supported through the PD by providing a list of task features instructors could use (El Turkey et al., submitted). Each time they implemented a task, they were to video-record the class. Also, they were encouraged to incorporate the Creativity-in-Progress Rubric (CPR) on Problem Solving (modeled after the CPR on Proving; Savić, Karakök, Tang, El Turkey, & Naccarato, 2017) with each task.

The first two authors used holistic coding (Saldaña, 2015) while watching the exit interviews. They separately looked for perceived belief shifts or evidence of enacting pre-existing beliefs and met together to discuss the shifts. After agreement, the first author used narrative coding (Saldaña, 2015) to analyze the entrance tickets and end-of-semester interviews.

Results and Discussion

The three most prominent themes extracted from Jo's entrance tickets, PD sessions, and exit interviews were beliefs related to *Time Pressure*, *Multiple Approaches*, and *Posing Questions*. In this section, we show Jo's belief shift in *Time* and also her alignment of a pre-existing *Multiple Approaches* belief to her classroom practices. A cursory mention of Posing Questions is addressed in the Conclusion.

Belief Shift: Time Pressure

Prior to participating in this project, Jo perceived a lack of time that impacted her capacity to incorporate creativity into her Calculus 1 course: “[d]uring a ‘normal’¹ semester, I typically feel time pressure, so I didn't make as much of an effort to incorporate creativity activities within the classroom” (exit interview, May 2019). During the semester of her participation in this project, she initially assigned the first few creativity-based tasks outside of class, but then changed the last couple of tasks to be done in class because:

¹ Jo refers to the semesters prior to her participation in this study as “normal”.

I realized that I have time to do them in class. And I wanted them to do them in class...they worked well together so why not let them utilize their time doing these together? **I have time**. It's really the time dictated and how I did things and our Calc 1 is very packed and since we only meet for three hours a week. **I was very nervous in the beginning** (exit interview, May 2019).

By the end of participating in the grant, she had shifted her thinking about the time pressure. She explained her shift in the exit interview:

[a]t the beginning of the semester, I was very nervous about time. Very packed but, who cares—in some sense—I can spend 15 minutes once a week just saying, "hey do this problem in class" or I can give it to them outside of class and then say, "hey let's come to the board and you all put up your solutions and we can talk about them." I'm much more aware of that.

Alignment of Belief to Pedagogical Practices: Multiple Approaches

The PD entrance ticket on 12/11/18 had the question: *What are some “ways” to foster students’ mathematical creativity? Please be as specific as you can.* In Jo’s answer, we coded four themes, which are given in bold below.

I think failure can foster creativity. **[Failure]** You attempt a problem using "standard" procedures and it doesn't lead to anything fruitful. So, you think about other ways to approach the problem. **[Multiple Approaches]** What additional information do you have? Is that information useful? etc. **[Evaluation]** I think making the students the teachers can also foster creativity. Outside of the classroom, they tend to teach one another in alternative ways to how they were taught in class. Why not foster that inside the classroom as well? **[Students as Teachers]**

Her answer shows that prior to participating, she already held the belief that Multiple Approaches is important to foster creativity. However, when asked, *Looking at the ways you listed above, can they be implemented/used in Calculus I course? Why or why not?*, Jo only referred to the ideas related to the Students as Teachers theme and did not mention Multiple Approaches as a pedagogical action that can be implemented: “I think it would be difficult, but not impossible, to get students to "teach" during class. (It is a matter of creating a safe environment.) I think well thought out assignments can work but creating the assignments can be tricky.” It is worth mentioning that she acknowledges intentional assignment development is needed to help her enact this pedagogical method of teaching.

Though Jo believed having students approach problems in more than one way can foster creativity, she acknowledged that she does not explicitly model this behavior in her class. In her 2/5/19 entrance ticket, when asked to *Describe a moment in the classroom where a student surprised you with their work or other discourse. Reflecting on that same moment, what aspects of the course or your teaching do you think contributed to that moment?*

I was actually surprised with...the Limit Task [Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$ in as many ways as possible]. It appears that most found the limit one way and didn't bother to find it another way. It was like the question just read “Evaluate (blah).” **I think a lot of it has to do with the problems we do in class. We usually only approach them one way and we call it good.** Sometimes I approach a problem [in] multiple ways, but I rarely show my work. (I talk through the alternative method(s).)

Here, Jo referred to a misalignment between instructor’s practices in the classroom (i.e. approaching problems only one way) and how students are assessed, in this case through HW.

Constructive alignment between the instructional methods and assessment methods is important because it supports students in reaching instructional goals (Biggs & Tang, 2011), in this case mathematical creativity.

Referring to the Limit Task again during the exit interview (May 2019), the interviewer asked Jo why she called it “cute”:

I love the fact that you can **approach it in many different** ways...I think the traditional way that students think of it is multiply by the conjugate, but I mean, they can factor it... It's just a little trick. And they don't think to factor linear terms. And so, I like that. I think it's cute. **It's a little bit outside the box but it's still within their realm of knowledge.**

Jo created a review assignment for her students that included a question similar to this Limit Task. When she said it didn't go as well as she had hoped, the interviewer asked her to describe what “going well” would mean to her. She responded focusing on multiple approaches:

It'd be nice if just another student popped up and say “hey I approached it by factoring or I used the T chart” which I know is not the best method but it gives me intuition as to what's going on. So...in my mind that's what's going well [means].

What is demonstrated in these two quotes is that not only did Jo include more creativity-based tasks on her final review, she is now using multiple approaches as a metric of success in terms of a class session. Additionally, when she says the T chart method may not be the best, but gives her insight into her students' thinking, she is valuing a student-centered approach rather than a content-centered one.

El Turkey et al. (submitted) coded this review assignment and found that there were 5 creativity-based tasks with several features that have been reported to foster creativity, one of the features being “different approaches.” This is in stark contrast to final exam review created prior to her participation in the grant where no questions were coded with creativity fostering features.

Though Jo has put into practice her belief that assigning multiple-approach problems can foster creativity, she extended this belief into her teaching practices. Beliefs of mathematical creativity held by mathematicians have been reported to align with actions in their research (Borwein, Liljedahl, & Zhai, 2014), but Jo mentions that those beliefs may not necessarily align with actions in their teaching:

Obviously, I think [creativity is] beneficial for teaching, but I mean we do it all the time in mathematics when we're conducting research...But I mean **it's encouraging me to think in different ways** necessarily than I normally do, yeah, and also ideally, hopefully **teach in different ways too that are more beneficial.**

Jo uses creativity while conducting mathematics research, but she expressed a desire to incorporate creativity into her teaching more often, particularly with respect to teaching in multiple ways. This intention also aligns with research showing that differentiated (multiple) ways of teaching are more beneficial for student engagement, as well as social and academic inclusion (Katz, 2013).

Conclusion

Consistent with Enderle et al. (2014), this paper shows that teaching beliefs and practices have a reciprocal nature; not only can beliefs influence the promotion of creativity in the classroom, but the converse can be true: promoting creativity can shift teaching beliefs, biases, and practices. In the previous section, we showed that Jo shifted her belief in not having enough time to cover everything in Calculus 1. Through analysis of her final exam review, we showed that her pre-existing belief (i.e. multiple-approach tasks can foster creativity) was transformed

into action. While this provides preliminary evidence of shifting of beliefs and alignment into actions, further analysis of the classroom videos is needed to triangulate this finding.

This quote from Jo on Week 10 of the professional development encapsulates all three themes: Time Pressure, Multiple Approaches, and Posing Questions.

Admittedly on Monday, my students—I'm now behind—but they were asking **phenomenal questions**. We were talking about increasing and decreasing and then all of a sudden one of the students goes “well doesn't that show you that you have an absolute min at this point?” “Heck yes, it does.” ...**So, they're saying and thinking great things that I don't normally get out of my students...So they're thinking about things a different way.**”

The quotes above show that the students' reactions are influencing Jo's thinking on pedagogical shifts. Additionally, Jo commented in her exit interview that she had fewer students come to her office and complain about non-routine tasks when compared to a “normal” semester. In other words, “[t]he ways in which teachers perceive, interpret, and act on students' reactions to their attempts to make change affects their sense of what is ‘working’ in their classrooms” (Woodbury & Gess-Newsome, 2002, p. 768). We believe that the reactions and successes of the classroom were reflected on, discussed, and reinforced through the professional development of fostering mathematical creativity.

As we observed these shifts in beliefs and enacting beliefs, we examined our PD actions using nine critical features of effective PDs (Capps et al. 2012), which are italicized in the next few sentences. The initial PD (*Total Time*) followed by semester-long participation in the PD (*Extended Support*) provided opportunities for instructors to reflect (*Reflection*) on their teaching practices through developing creativity-based tasks (*Developed Lessons*). The research group provided two tasks as models (*Modelling*). Once the tasks were developed with the content in mind (*Content Knowledge*), instructors presented them to the rest of the participants in PD and received feedback on the task itself as well as implementation (*Transference*). Going through this process of taking risks and creating new ideas is parallel to the creative process mathematicians engage in during their own problem-solving processes (*Authentic Experiences*). The participants were presented with the importance of mathematical creativity as posited by mathematicians (*Coherency*). Although the PD sessions touched on all nine of these features, the degrees of engagement in these features varied. For example, our PD was strong in the areas of Total Time, Extended Support, Reflection, Developed Lessons, Content Knowledge, Transference, and Authentic Experiences. Coherence and Modelling are two areas in which our future PDs will seek improvement. For example, the importance of mathematical creativity will be aligned to goals of NCTM, MAA IP guide, and research results (e.g. Omar, Karakök, Savić, El Turkey, & Tang, 2019)

Shifting beliefs is the first step to change, but not the final. Beliefs do not always translate to action, but non-action does not imply that the belief is not held (Hutner & Markman, 2016); there are complex systemic or cultural issues that may be preventing the instructor from acting on their beliefs. In our future research, we will be examining more belief-in-action shifts (Philipp, 2007; Bishop et al., 2003), as we believe “Reflection without action is...[an] armchair revolution” (Friere, 1970, p.149).

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