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Brief paper

H_∞ almost state synchronization for homogeneous networks of non-introspective agents: A scale-free protocol design ☆

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ABSTRACT

This paper studies scale-free protocol design for H_∞ almost state synchronization of homogeneous networks of non-introspective agents in the presence of external disturbances. A linear dynamic protocol is developed based on localized information exchange over the same communication network, which does not need any knowledge of the directed network topology and the spectrum of the associated Laplacian matrix. Moreover, the proposed protocol is scalable and achieves H_∞ almost synchronization with a given arbitrary degree of accuracy for any arbitrary number of agents.

1. Introduction

In recent decades, the synchronization problem for multi-agent systems (MAS) has attracted substantial attention due to the wide potential for applications in several areas such as automotive vehicle control, satellites/robots formation, and sensor networks. See for instance the books (Ren & Cao, 2011) and Wu (2007) or the survey paper (Olfati-Saber, Fax, & Murray, 2007).

State synchronization inherently requires homogeneous networks (i.e. agents which have identical models). Therefore, in this paper we focus on homogeneous networks. State synchronization based on diffusive full-state coupling has been studied where the agent dynamics progress from single- and double-integrator dynamics e.g. Olfati-Saber and Murray (2004), Ren (2008), Ren and Beard (2005) to more general dynamics, e.g. Scardovi and Sepulchre (2009), Shen, Wang, Wang, and Li (2020a, 2020b), Tuna (2008), Wieland, Kim, and Allgöwer (2011). State synchronization based on diffusive partial-state coupling has also been considered, including static design (Liu, Saberi, Stoorvogel, & Zhang, 2018; Liu, Zhang, Saberi, & Stoorvogel, 2018b, 2018a), dynamic design (Kim, Shim, Back, & Seo, 2013; Li, Duan, Chen, & Huang, 2010; Seo, Back, Kim, & Shim, 2012; Seo, Shim, & Back, 2009; Stoorvogel, Saberi, & Zhang, 2017; Su & Huang, 2012; Tuna, 2009). Recently, scale-free collaborative protocol designs are developed for homogeneous and heterogeneous MAS (Chowdhury & Khalil, 2018; Nojavanzadeh, Liu, Saberi, & Stoorvogel, 2020) and for MAS subject to actuator saturation (Liu, Saberi, Stoorvogel, & Nojavanzadeh, 2019). Also, there are efforts for the case that agents are introspective (i.e. the agents have absolute measurements of their own dynamics in addition to relative information from the network. Otherwise, they are called as non-introspective agents.) (Kim, Shim, & Seo, 2011; Scardovi & Sepulchre, 2009; Yang, Saberi, Stoorvogel, & Grip, 2014).

For MAS subject to external disturbances, there are some works in the literature (Li, Soh, & Xie, 2017; Li, Soh, Xie, & Lewis, 2019; Li et al., 2010; Li, Duan, & Chen, 2011; Saboori & Khorasani, 2014; Wang, Wen, Yu, Yu, & Lv, 2019; Yang, Zhang, Feng, Yan, & Wang, 2019; Zhao, Duan, Wen, & Chen, 2015). On the other hand, Peymani, Grip, and Saberi (2015) introduced the notion of H_∞ *almost synchronization*[1] for homogeneous networks, where the goal is to reduce the impact of external disturbances to any arbitrary desired level, which is described by an H_∞ norm from disturbance to the synchronization error. This

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[1] The term “almost synchronization” has been selected in connection with the concept of almost disturbance decoupling (see e.g. Ozcetin, Saberi, and Sannuti (1992)) where the problem is to find a family of controllers to reduce the noise sensitivity to any arbitrary degree.

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work is also extended to some other studies (Peymani, Grip, Saberi, Wang, & Fossen, 2014; Zhang, Saberi, Grip, & Stoorvogel, 2015; Zhang, Saberi, Stoorvogel, & Sannuti, 2015) for almost output synchronization. In Zhang, Stoorvogel, and Saberi (2015), H_2 almost synchronization of heterogeneous non-introspective agents under time-varying communication networks has been studied. The H_∞ and H_2 almost state synchronization are studied later in Stoorvogel, Saberi, Zhang, and Acciani (2017), Stoorvogel, Saberi, Zhang, and Liu (2019). Solvability condition for H_∞ and H_2 almost state synchronization of homogeneous MAS is provided in Stoorvogel et al. (2019). Recently, H_∞ and H_2 almost synchronization via static protocols are studied in Stoorvogel, Saberi, Liu, and Nojavanzadeh (2019), Stoorvogel, Nojavanzadeh, Liu, and Saberi (2018) for MAS with passive and passifiable agents. In all the above mentioned papers on almost synchronization, the protocol design needs to know at least some information about the communication network and the number of agents.

In this paper, we develop scale-free design for H_∞ almost state synchronization for a MAS in the presence of external disturbances. The necessary and sufficient synchronization results are obtained by using a class of linear collaborative parameterized dynamic protocols with localized information exchange for both networks with full- and partial-state coupling. The linear dynamic protocol can work for any number of agents with any communication network which contains a spanning tree. The main contribution of this work is that the protocol design does not require any information of the communication network such as a lower bound of non-zero eigenvalue of associated Laplacian matrix. Moreover, the linear protocol is scalable and achieves almost state synchronization with a given arbitrary degree of accuracy for MAS with any number of agents.

Notations and definitions

Given a matrix $A \in \mathbb{R}^{m\times n}$ and A^{T} denotes transpose of A while $\|A\|$ denotes the induced 2-norm, and $\operatorname{im} A$ denotes the image of A. For a signal v, we denote the L_2 norm by $\|v\|_2$. Finally, for a transfer function matrix $T(s)$ we denote the H_∞ norm by $\|T\|_\infty$. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half complex plane. $A \otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context. A *weighted graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \dots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighting matrix. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \dots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \dots, k-1$. A *directed tree* with root r is a subgraph of the graph $\mathcal{G}$ in which there exists a unique path from node r to each node in this subgraph. A *directed spanning tree* is a directed tree containing all the nodes of the graph. For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$, i.e. a vector with all entries equal to 1. When graph contains a spanning tree, then it follows from Ren and Beard (2005, Lemma 3.3) that the Laplacian matrix L has a simple eigenvalue at the origin, with the corresponding right eigenvector $\mathbf{1}$, and all the other eigenvalues are in the open right-half complex plane.

2. Problem formulation

Consider a MAS composed of N identical linear time-invariant agents of the form

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i + E\omega_i, \\ y_i &= Cx_i, \end{aligned} \qquad (i = 1, \dots, N) \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, $y_i \in \mathbb{R}^p$ are respectively the state, input, and output vectors of agent i, and $\omega_i \in \mathbb{R}^w$ are the external disturbances.

The communication network provides each agent with a linear combination of its own outputs relative to that of other neighboring agents. In particular, each agent $i \in \{1, \dots, N\}$ has access to the quantity,

$$\zeta_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) = \sum_{j=1}^{N} \ell_{ij} y_j, \tag{2}$$

where $a_{ij} \geq 0$ and $a_{ii} = 0$ indicate the communication among agents while ℓ_{ij} denote the coefficients of the associated Laplacian matrix L. This communication topology of the network can be described by a weighted and directed graph $\mathcal{G}$ with nodes corresponding to the agents in the network and the weight of edges given by the coefficient a_{ij}.

The MAS (1) and (2) is referred to as MAS with full-state coupling when $C = I$, otherwise it is called MAS with partial-state coupling.

Let N be any positive number and define $\bar{x}_i = x_i - x_N$, while

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_{N-1} \end{pmatrix} \quad \text{and} \quad \omega = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_N \end{pmatrix}.$$

We denote by $T_{\omega\bar{x}}$ the transfer function from ω to $\bar{x}$.

In this paper, we introduce a localized exchange of information among protocols. In particular, each agent $i = 1, \dots, N$ has access to localized information, denoted by $\hat{\zeta}_i$, of the form

$$\hat{\zeta}_i = \sum_{j=1}^{N} a_{ij}(\xi_i - \xi_j) \tag{3}$$

where $\xi_j \in \mathbb{R}^n$ is a variable produced internally by agent j which will be appropriately chosen in the coming sections.

First, we introduce H_∞ almost state synchronization problem with localized information exchange as following.

Definition 1. Consider a MAS described by (1) and (2) with associated communication graph $\mathcal{G}$. The H_∞ almost state synchronization problem of a MAS with localized information exchange (H_∞-ASSWLIE) is to find, if possible, for any given $\gamma > 0$ a distributed collaborative linear time-invariant dynamic protocol of the form

$$\begin{cases} \dot{x}_{i,c} &= A_c x_{i,c} + B_c \zeta_i + C_c \hat{\zeta}_i, \\ u_i &= F_c x_{i,c} \end{cases} \tag{4}$$

where $\hat{\zeta}_i$ is defined by (3), with $\xi_i = G_c x_{i,c}$ with $x_{i,c} \in \mathbb{R}^{n_c}$ such that in the absence of disturbance ω, state synchronization

$$\lim_{t\to\infty} (x_i - x_j) = 0 \quad \text{for all } i, j \in 1, \dots, N. \tag{5}$$

is achieved for all initial conditions while in the presence of disturbance ω, the H_∞ norm from ω to $x_i - x_j$ is less than γ, for all $i, j \in \{1, \dots, N\}$.

The objective of this paper includes (i) state synchronization (5) is accomplished in the absence of disturbances, (ii) the impact of disturbances on the state synchronization error dynamics is attenuated to any arbitrarily small value in the sense of the H_∞ norm of the transfer function.

Now we formulate the following problem.

Problem 1. The **scale-free H_∞ almost state synchronization problem with localized information exchange (scale-free H_∞-ASSWLIE)** for MAS (1) and (2) is to find, if possible, a fixed collaborative linear protocol parameterized in terms of a scalar parameter ρ of the form:

$$\begin{cases} \dot{\chi}_i = A_c(\rho)\chi_i + B_c(\rho)\zeta_i + C_c(\rho)\hat{\zeta}_i \\ u_i = F_c(\rho)\chi_i \end{cases} \tag{6}$$

where $\hat{\zeta}_i$ is defined by (3), with $\xi_i = H_c\chi_i$ with $\chi_i \in \mathbb{R}^{n_c}$ such that for any number of agents N, and any communication graph $\mathcal{G}$ we have:

- in the absence of the disturbance ω, for all initial conditions state synchronization (5) is achieved for any $\rho \geq 1$.
- in the presence of the disturbance ω, for any $\gamma > 0$, one can render the H_∞ norm from ω to $x_i - x_j$ less than γ for all $i, j \in \{1, \ldots, N\}$ by choosing ρ sufficiently large.

Remark 1. We would like to emphasize that in our formulation of Problem 1, the protocol (6), i.e. $A_c(\rho)$, $B_c(\rho)$, $C_c(\rho)$, $F_c(\rho)$ to be solely designed based on agent model (A, B, C, E) and independent of communication graph and the number of agents.

Remark 2. Note that in H_∞ almost synchronization, one can consider the worst case disturbance with the only constraint that the power is bounded, i.e.

$$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \omega_i^{\mathrm{T}}\omega_i \mathrm{d}t < \infty.$$

3. H_∞ almost state synchronization

In this section, we will consider scale-free H_∞-ASSWLIE problem of a MAS for both cases of full- and partial-state coupling.

3.1. Full-state coupling — solvability conditions and protocol design

For case of full-state coupling, we have the following protocol.

Protocol 1: Full-state coupling

We design collaborative protocols for agent $i \in \{1, \ldots, N\}$ as

$$\begin{cases} \dot{\chi}_i = A\chi_i + Bu_i + \rho\zeta_i - \rho\hat{\zeta}_i, \\ u_i = -\rho B^{\mathrm{T}} P\chi_i \end{cases} \tag{7}$$

where ρ is a parameter satisfying $\rho \geq 1$ while P is the unique solution of algebraic Riccati equation

$$A^{\mathrm{T}}P + PA - PBB^{\mathrm{T}}P + I = 0 \tag{8}$$

and ζ_i is defined by (2). The agents communicate $\xi_i = \chi_i$, therefore each agent has access to local information

$$\hat{\zeta}_i = \sum_{j=1}^{N} a_{ij}(\chi_i - \chi_j). \tag{9}$$

Our formal result is stated in the following theorem.

Theorem 1. *Consider a MAS described by (1) and (2), where $C = I$.*

*(i) The scale-free H_∞-ASSWLIE problem as stated in Problem 1 is solvable **if and only if***

(a) (A, B) is stabilizable.
(b) All eigenvalues of A are in the closed left half plane.
(c) The graph $\mathcal{G}$, describing the communication topology of the network, contains a directed spanning tree.
(d) $\operatorname{im} E \subseteq \operatorname{im} B$.

(ii) The collaborative linear dynamic Protocol 1 solves scale-free H_∞-ASSWLIE. In other words, for any number of agents N and any graph $\mathcal{G}$ in the absence of the disturbance ω, for any $\rho \geq 1$, the state synchronization (5) is achieved for any initial conditions and in the presence of the disturbance ω, for any $\gamma > 0$, the H_∞ norm from ω to $x_i - x_j$ is less that γ for all $i, j \in \{1, \ldots, N\}$ by choosing ρ sufficiently large.

In order to prove this theorem, we need the following lemma.

Lemma 1. *Let a Laplacian matrix $L \in \mathbb{R}^{N\times N}$ be given associated with a graph that contains a directed spanning tree. We define $\bar{L} \in \mathbb{R}^{(N-1)\times(N-1)}$ as the matrix $\bar{L} = [\bar{\ell}_{ij}]$ with $\bar{\ell}_{ij} = \ell_{ij} - \ell_{Nj}$. Then the eigenvalues of $\bar{L}$ are equal to the nonzero eigenvalues of L.*

Proof. We have

$$\bar{L} = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix} L \begin{pmatrix} I \\ 0 \end{pmatrix}$$

Assume that λ is a nonzero eigenvalue of L with eigenvector x, then

$$\bar{x} = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix} x$$

satisfies

$$\begin{pmatrix} I & -\mathbf{1} \end{pmatrix} Lx = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix} \lambda x = \lambda\bar{x}$$

for $Lx = \lambda x$ and since $L\mathbf{1} = 0$ we have $L\begin{pmatrix} I \\ 0 \end{pmatrix}\begin{pmatrix} I & -\mathbf{1} \end{pmatrix} = L$, then we find that

$$\bar{L}\bar{x} = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix} L \begin{pmatrix} I \\ 0 \end{pmatrix}\begin{pmatrix} I & -\mathbf{1} \end{pmatrix} x = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix} Lx = \lambda\bar{x}.$$

This shows that $\bar{x}$ is an eigenvector of $\bar{L}$ if $\lambda \neq 0$. It is easily seen that $\bar{x} = 0$ if and only if $\lambda = 0$. Conversely if $\bar{x}$ is an eigenvector of $\bar{L}$ with eigenvalue λ then it is easily verified that $x = \begin{pmatrix} I \\ 0 \end{pmatrix}\bar{x}$ is an eigenvector of L with eigenvalue λ. ■

Proof of Theorem 1. Firstly, let $\bar{x}_i = x_i - x_N$ and $\bar{\chi}_i = \chi_i - \chi_N$. We find:

$$\begin{aligned} \dot{\bar{x}}_i &= A\bar{x}_i + B(u_i - u_N) + E(\omega_i - \omega_N), \\ \dot{\bar{\chi}}_i &= A\bar{\chi}_i + B(u_i - u_N) + \rho\textstyle\sum_{j=1}^{N-1} \bar{\ell}_{ij}(\bar{x}_j - \bar{\chi}_j), \\ u_i - u_N &= -\rho B^{\mathrm{T}}P\bar{\chi}_i. \end{aligned}$$

Next, we define

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_{N-1} \end{pmatrix}, \quad \bar{\chi} = \begin{pmatrix} \bar{\chi}_1 \\ \vdots \\ \bar{\chi}_{N-1} \end{pmatrix}, \text{ and } \omega = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_N \end{pmatrix}$$

and we obtain the following closed-loop system

$$\begin{cases} \dot{\bar{x}} = (I \otimes A)\bar{x} - \rho(I \otimes BB^{\mathrm{T}}P)\bar{\chi} + (\Pi \otimes E)\omega \\ \dot{\bar{\chi}} = (I \otimes A)\bar{\chi} - \rho(I \otimes BB^{\mathrm{T}}P)\bar{\chi} + \rho(\bar{L} \otimes I)(\bar{x} - \bar{\chi}) \end{cases}$$

where $\bar{L}$ as defined in Lemma 1 and $\Pi = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix}$. Let $e = \bar{x} - \bar{\chi}$, we can obtain

$$\dot{\bar{x}} = [I \otimes (A - \rho BB^{\mathrm{T}}P)]\bar{x} + \rho(I \otimes BB^{\mathrm{T}}P)e + (\Pi \otimes E)\omega \tag{10}$$

$$\dot{e} = (I \otimes A - \rho\bar{L} \otimes I)e + (\Pi \otimes E)\omega \tag{11}$$

According to Lemma 1, we have that the real part of the eigenvalues of $\bar{L}$ are positive. Therefore, there exists a non-singular transformation matrix T such that

$$(T \otimes I)(I \otimes A - \rho\bar{L} \otimes I)(T^{-1} \otimes I) = I \otimes A - \rho\bar{J} \otimes I, \tag{12}$$

where

$$\bar{J} = \begin{pmatrix} \lambda_2 & 0 & \cdots & 0 \\ J_{21} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ J_{N,1} & \cdots & J_{N,N-1} & \lambda_N \end{pmatrix}$$

with $\mathrm{Re}(\lambda_i) > 0$ $(i = 2, \ldots, N)$, where λ_i are the nonzero eigenvalues of L.

Then, for the stability of (12), we just need to prove the stability of $A - \rho\lambda_i I$ for $i = 1, \ldots, N-1$. Since the eigenvalues of A are in the closed left half plane and $\rho \geq 1$, $A - \rho\lambda_i I$ is asymptotically stable, i.e. the real part of its eigenvalues is all negative. Since (12) is asymptotically stable, we find that $I \otimes A - \rho\bar{L} \otimes I$ is asymptotically stable. It is verified that there exists a constant $M > 0$ such that

$$T_{\omega e}(s) = (sI - I \otimes A + \rho\bar{L} \otimes I)^{-1}(\varPi \otimes E)$$

satisfies

$$\|T_{\omega e}\|_\infty \leq \frac{M}{\rho}$$

for all $\rho \geq 1$. On the other hand, from the condition $\operatorname{im} E \subseteq \operatorname{im} B$, we know there exists a matrix X such that $E = BX$.

Then, we choose the following Lyapunov function for (10),

$$V = \bar{x}^{\mathrm{T}}(I \otimes P)\bar{x}$$

with P satisfying (8). We obtain:

$$\begin{aligned}
\dot{V} &= -\bar{x}^{\mathrm{T}}\left(I \otimes [I + (2\rho - 1)PBB^{\mathrm{T}}P]\right)\bar{x} \\
&\quad + 2\rho\bar{x}^{\mathrm{T}}(I \otimes PBB^{\mathrm{T}}P)e + 2\bar{x}^{\mathrm{T}}(\varPi \otimes PBX)\omega \\
&= -\bar{x}^{\mathrm{T}}\left(I \otimes [I + (2\rho - 1)PBB^{\mathrm{T}}P]\right)\bar{x} \\
&\quad + 2\rho\bar{x}^{\mathrm{T}}(I \otimes PB)(I \otimes B^{\mathrm{T}}P)e + 2\bar{x}^{\mathrm{T}}(I \otimes PB)(\varPi \otimes X)\omega \\
&\leq -\bar{x}^{\mathrm{T}}\left(I \otimes [I + (2\rho - 1)PBB^{\mathrm{T}}P]\right)\bar{x} + \rho\bar{x}^{\mathrm{T}}(I \otimes PBB^{\mathrm{T}}P)\bar{x} \\
&\quad + 2\rho e^{\mathrm{T}}(I \otimes PBB^{\mathrm{T}}P)e + 2\rho^{-1}\omega^{\mathrm{T}}(\varPi^{\mathrm{T}}\varPi \otimes X^{\mathrm{T}}X)\omega \\
&\leq -\alpha V - \frac{1}{2}\|\bar{x}\|_2^2 + \frac{2}{\rho}(M^2\|PB\|^2 + \|X\|^2\|\varPi\|^2)\|\omega\|^2
\end{aligned}$$

with $\alpha = \frac{1}{2}\|P\|^{-1}$ and $\rho \geq 1$, which implies (10) is asymptotically stable and H_∞ norm from ω to $\bar{x}$ is less than $\frac{\bar{M}}{\sqrt{\rho}}$ with $\bar{M} = 2\sqrt{M^2\|PB\|^2 + \|X\|^2\|\varPi\|^2}$. We find for zero initial conditions that

$$\|T_{\omega(x_i - x_j)}\|_\infty = \sup_{\omega \neq 0} \frac{\|x_i - x_j\|_2}{\|\omega\|_2} \leq \frac{\bar{M}}{\sqrt{\rho}}$$

for $\rho \geq 1$. Also note that the above analysis implies that in the absence of the disturbance ω, we have $\lim_{t\to\infty} \bar{x}_i \to 0$ i.e. $\lim_{t\to\infty} x_i - x_j \to 0$ for arbitrary initial conditions.

Now, we will prove the necessity. Assume we have a protocol of the form (6) that achieves synchronization for any possible graph in the absence of disturbances. It is easily seen that this requires that condition (a) is satisfied. On the other hand, we have that

$$\begin{cases} \dot{\bar{x}} = (I \otimes A)\bar{x} + (\bar{L} \otimes BF_c(\rho))\bar{\chi} \\ \dot{\bar{\chi}} = (I \otimes A_c(\rho))\bar{\chi} + (\bar{L} \otimes B_c(\rho)C)\bar{x} + (\bar{L} \otimes C_c(\rho)H_c)\bar{\chi} \end{cases}$$

must be asymptotically stable for all possible Laplacian matrices. By letting $\bar{L} \to 0$ we see that in the limit the system must have all eigenvalues in the closed left half plane which yields that condition (b) must be satisfied. It is well-known that state synchronization is impossible to achieve if the network does not have a directed spanning tree. Finally, from the result on H_∞ almost disturbance decoupling in Saberi, Lin, and Stoorvogel (1996, Theorem 2.5), we find that (d) is also a necessary condition. ■

3.2. Partial-state coupling — solvability conditions and protocol design

For case of partial-state coupling, we have the following protocol.

Protocol 2: Partial-state coupling

We design collaborative protocols for agent $i \in \{1, \ldots, N\}$ as

$$\begin{cases} \dot{\hat{x}}_i = A\hat{x}_i - \rho BB^{\mathrm{T}}P\hat{\zeta}_i + \delta^{-2}Q_\rho C^{\mathrm{T}}(\zeta_i - C\hat{x}_i) \\ \dot{\chi}_i = A\chi_i + Bu_i + \rho\hat{x}_i - \rho\hat{\zeta}_i \\ u_i = -\rho B^{\mathrm{T}}P\chi_i, \end{cases} \tag{13}$$

where $P > 0$ is the unique solution of (8). Since $(A, E, C, 0)$ is minimum-phase and left invertible, then for any $\rho \geq 1$, there exists $\delta > 0$ small enough such that $Q_\rho > 0$ is the unique solution of

$$Q_\rho A^{\mathrm{T}} + AQ_\rho + EE^{\mathrm{T}} - \delta^{-2}Q_\rho C^{\mathrm{T}}CQ_\rho + \rho^2 Q_\rho^2 = 0. \tag{14}$$

In this protocol, agents communicate $\xi_i = \chi_i$, i.e. each agent has access to localized information (9), while ζ_i is defined by (2).

Then, we have the following theorem.

Theorem 2. *Consider a MAS described by* (1) and (2).

(i) The scale-free H_∞-ASSWLIE problem stated in Problem 1 is solvable ***if and only if***

(a) (A, B) is stabilizable and (C, A) is detectable.
(b) All eigenvalues of A are in the closed left half plane.
(c) $(A, E, C, 0)$ is minimum phase and left invertible.
(d) The graph $\mathcal{G}$, describing the communication topology of the network, contains a directed spanning tree.
(e) $\operatorname{im} E \subseteq \operatorname{im} B$.

(ii) The collaborative linear dynamic Protocol 2 solves scale-free H_∞-ASSWLIE, for any number of agents N and any graph $\mathcal{G}$ such that in the absence of disturbance ω, for any $\rho \geq 1$, the state synchronization (5) is achieved for any initial conditions and in the presence of disturbance ω, for any $\gamma > 0$, the H_∞ norm from ω to $x_i - x_j$ is less than γ for all $i, j \in \{1, \ldots, N\}$ by choosing ρ sufficiently large.

Proof of Theorem 2. Similar to Theorem 1 and by defining $\tilde{x}_i = \hat{x}_i - \hat{x}_N$, we have

$$\begin{cases} \dot{\bar{x}}_i = A\bar{x}_i + B(u_i - u_N) + E(\omega_i - \omega_N) \\ \dot{\tilde{x}}_i = A\tilde{x}_i - \rho BB^{\mathrm{T}}P\sum_{j=1}^{N-1}\bar{\ell}_{ij}\bar{\chi}_j + \delta^{-2}Q_\rho C^{\mathrm{T}}C\sum_{j=1}^{N-1}\bar{\ell}_{ij}(\bar{x}_j - \tilde{x}_i) \\ \dot{\bar{\chi}}_i = A\bar{\chi}_i + B(u_i - u_N) + \rho\tilde{x}_i - \rho\sum_{j=1}^{N-1}\bar{\ell}_{ij}\bar{\chi}_j \end{cases}$$

We define

$$\bar{x} = \begin{pmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_{N-1} \end{pmatrix}, \quad \tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_{N-1} \end{pmatrix}, \quad \bar{\chi} = \begin{pmatrix} \bar{\chi}_1 \\ \vdots \\ \bar{\chi}_{N-1} \end{pmatrix}, \text{ and } \omega = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_N \end{pmatrix}$$

then we have the following closed-loop system

$$\begin{aligned}\dot{\bar{x}} &= (I\otimes A)\bar{x} - \rho[I\otimes BB^{\mathrm{T}}P]\bar{\chi} + (\Pi\otimes E)\omega\\ \dot{\tilde{x}} &= [I\otimes(A-\tfrac{1}{\delta^2}Q_\rho C^{\mathrm{T}}C)]\tilde{x} - \rho(\bar{L}\otimes BB^{\mathrm{T}}P)\bar{\chi} + \tfrac{1}{\delta^2}(\bar{L}\otimes Q_\rho C^{\mathrm{T}}C)\bar{x}\\ \dot{\bar{\chi}} &= (I\otimes A - \rho\bar{L}\otimes I)\bar{\chi} - \rho(I\otimes BB^{\mathrm{T}}P)\bar{\chi} + \rho\tilde{x}\end{aligned}$$

By defining $e = \bar{x} - \bar{\chi}$ and $\bar{e} = (\bar{L}\otimes I)\bar{x} - \tilde{x}$, we can obtain

$$\begin{aligned}\dot{\bar{x}} &= [I\otimes(A-\rho BB^{\mathrm{T}}P)]\bar{x} + \rho(I\otimes BB^{\mathrm{T}}P)e + (\Pi\otimes E)\omega\\ \dot{\bar{e}} &= [I\otimes(A-\delta^{-2}Q_\rho C^{\mathrm{T}}C)]\bar{e} + (\bar{L}\Pi\otimes E)\omega\\ \dot{e} &= (I\otimes A - \rho\bar{L}\otimes I)e + \rho\bar{e} + (\Pi\otimes E)\omega\end{aligned}$$

Thus, we can obtain the following transfer function

$$\begin{aligned}T_{\omega e} &= \begin{pmatrix}0 & I\end{pmatrix}\begin{pmatrix}T_1 & 0\\ -\rho I & T_2\end{pmatrix}^{-1}\begin{pmatrix}\bar{L}\Pi\otimes E\\ \Pi\otimes E\end{pmatrix}\\ &= T_2^{-1}\left[\Pi\otimes E + \rho T_1^{-1}(\bar{L}\Pi\otimes E)\right]\end{aligned}$$

where $T_1 = sI - I\otimes(A-\delta^{-2}Q_\rho C^{\mathrm{T}}C)$ and $T_2 = sI - (I\otimes A - \rho\bar{L}\otimes I)$. Meanwhile, we have

$$\begin{aligned}\|\Pi\otimes E + \rho T_1^{-1}(\bar{L}\Pi\otimes E)\|_\infty &= \|\Pi\otimes E + \rho T_{\omega\bar{e}}\|_\infty\\ &\le \|\Pi\otimes E\| + \rho\|T_{\omega\bar{e}}\|_\infty\end{aligned}$$

We choose the following Lyapunov function for $\bar{e}$

$$V_0 = \bar{e}^{\mathrm{T}}(I\otimes Q_\rho^{-1})\bar{e}$$

with Q_ρ satisfying (14). Then we have

$$\begin{aligned}\dot{V}_0 =& \bar{e}^{\mathrm{T}}[I\otimes(Q_\rho^{-1}A + A^{\mathrm{T}}Q_\rho^{-1} - 2\delta^{-2}C^{\mathrm{T}}C)]\bar{e} + 2\bar{e}(\bar{L}\Pi\otimes Q_\rho^{-1}E)\omega\\ \le& -\bar{e}^{\mathrm{T}}[I\otimes(\rho^2 I + Q_\rho^{-1}EE^{\mathrm{T}}Q_\rho^{-1})]\bar{e}\\ &+ \bar{e}^{\mathrm{T}}(I\otimes Q_\rho^{-1}EE^{\mathrm{T}}Q_\rho^{-1})\bar{e} + \omega^{\mathrm{T}}(\Pi^{\mathrm{T}}\bar{L}^{\mathrm{T}}\bar{L}\Pi\otimes I)\omega\\ \le& -\rho^2\|\bar{e}\|^2 + \|\bar{L}\|^2\|\Pi\|^2\|\omega\|^2\end{aligned}$$

with $\rho \ge 1$. By integrating the above inequality, we have

$$\int_0^\infty -\rho^2\|\bar{e}(t)\|^2 + \|\bar{L}\|^2\|\Pi\|^2\|\omega(t)\|^2 \mathrm{d}t \ge 0$$

for zero initial conditions and hence

$$\rho^2\|\bar{e}\|_2^2 \le \|\bar{L}\|^2\|\Pi\|^2\|\omega\|_2^2$$

i.e.

$$\|T_{\omega\bar{e}}\|_\infty = \frac{\|\bar{e}\|}{\|\omega\|} \le \frac{\|\bar{L}\|\|\Pi\|}{\rho}$$

for $\rho \ge 1$. Thus we have

$$\|\Pi\otimes E - \rho T_1^{-1}(\bar{L}\Pi\otimes E)\|_\infty \le \|\Pi\otimes E\| + \rho\|T_{\omega\bar{e}}\|_\infty \le W_1$$

for $W_1 = \|\Pi\otimes E\| + \|\bar{L}\|\|\Pi\|$. Meanwhile, since $\bar{L}$ is invertible, there exists a constant $W_2 > 0$ such that $\|T_2^{-1}\|_\infty \le \frac{W_2}{\rho}$ for $\rho \ge 1$. It means that we have

$$\|T_{\omega e}\|_\infty \le \frac{W_1 W_2}{\rho}.$$

Let X such that $E = BX$. Similar to Theorem 1, we choose the following Lyapunov function,

$$V = \bar{x}^{\mathrm{T}}(I\otimes P)\bar{x}$$

with P satisfying (8) Thus, we have

$$\dot{V} \le -\alpha V - \frac{1}{2}\|\bar{x}\|^2 + \frac{2}{\rho}(W_1^2 W_2^2\|PB\|^2 + \|X\|^2\|\Pi\|^2)\|\omega\|^2$$

with $\alpha = \frac{1}{2}\|P\|^{-1}$ for $\rho \ge 1$. We obtain:

$$\|T_{\omega(x_i-x_j)}\| = \sup_{\omega\neq 0}\frac{\|x_i - x_j\|_2}{\|\omega\|_2} \le \frac{\bar{W}}{\sqrt{\rho}} \qquad (15)$$

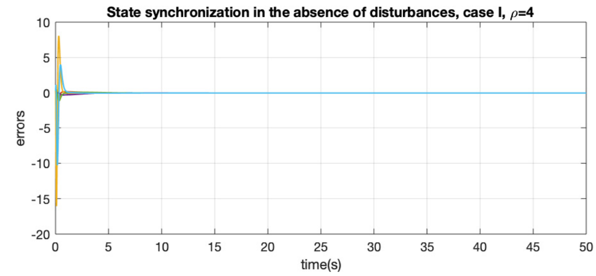

Fig. 1. State synchronization in the absence of disturbance and with the choice of $\rho = 4$ for the MAS with $N = 3$.

with $\bar{W} = 2\sqrt{W_1^2 W_2^2\|PB\|^2 + \|X\|^2\|\Pi\|^2}$ for $\rho \ge 1$. Moreover, the above analysis in the absence of disturbances yields: Thus we have

$$\lim_{t\to\infty}\bar{x}_i \to 0$$

i.e. $\lim_{t\to\infty} x_i - x_j \to 0$ for arbitrary initial conditions.

As the next step, we will prove the necessity. Similar to the proof of Theorem 1, we have that (a), (b) and (d) are necessary conditions. From the result on H_∞ almost disturbance decoupling in Saberi et al. (1996, Theorem 2.5), we find that (c) and (e) are also necessary conditions in case of partial-state coupling. ∎

4. Numerical example

In this section we will illustrate the effectiveness of our protocol design with numerical examples for H_∞ state synchronization of MAS with partial-state coupling.

Consider agent models (1) with

$$A = \begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix},\quad B = \begin{pmatrix}0\\0\\1\end{pmatrix},\quad C = \begin{pmatrix}1&0&0\end{pmatrix},\quad E = B.$$

For this agent model, we obtain Protocol 2, by solving algebraic Riccati equations (8) and (14) for two values of $\rho = 4$ and $\rho = 10$ (The Riccati equation (14) is solved with $\delta = 0.0004$). We create two homogeneous MAS with different number of agents and different communication topologies to show that the designed protocol is scale-free, i.e. it is independent of communication network and the number of agents N.

- *Case I*: In this case, we consider MAS with 3 agents and communication topology with $\mathcal{A}_1$, with $a_{21} = a_{32} = 1$. The result of state synchronization in the absence of disturbance is shown in Fig. 1. The H_∞ almost state synchronization results for this MAS are shown in Fig. 2 ($\rho = 4$) and Fig. 3 ($\rho = 10$) with disturbances $\omega_1 = 0, \omega_2 = \omega_3 = \cos(t)$. The results show that by increasing ρ to 10, one can decrease the impact of disturbances on disagreement dynamics.
- *Case II*: Next, we consider a MAS with 20 agents and associated adjacency matrix $\mathcal{A}_2$, with $a_{16} = a_{21} = a_{32} = a_{43} = a_{54} = a_{65} = a_{76} = a_{87} = a_{98} = a_{10,9} = a_{11,10} = a_{12,11} = a_{13,12} = a_{13,20} = a_{14,13} = a_{15,14} = a_{15,6} = a_{16,15} = a_{17,16} = a_{18,17} = a_{19,18} = a_{20,18} = 1$. Fig. 4 shows the result for state synchronization in the absence of disturbance with $\rho = 4$. The H_∞ almost state synchronization results are shown in Fig. 5 ($\rho = 4$) and Fig. 6 ($\rho = 10$) with the disturbances $\omega_1 = \omega_7 = \omega_{11} = \omega_{17} = 0, \omega_2 = \omega_{12} = \omega_8 = \omega_{18} = \cos(t), \omega_3 = \omega_{13} = 0.5, \omega_4 = \omega_{10} = \omega_{14} = \omega_{20} = \sin(2t), \omega_5 = \omega_{15} = \cos(3t), \omega_6 = \omega_{16} = \sin(t), \omega_{13} = 1$, and $\omega_{19} = 1.5$.

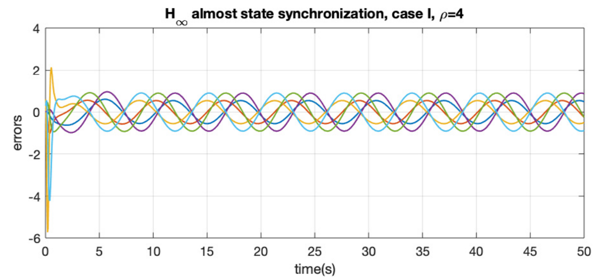

Fig. 2. H_∞ almost state synchronization with the choice of $\rho = 4$ for the MAS with $N = 3$.

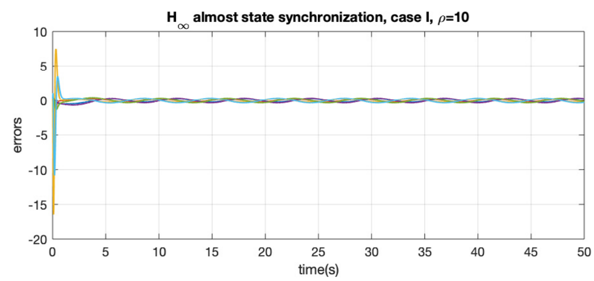

Fig. 3. H_∞ almost state synchronization with the choice of $\rho = 10$ for the MAS with $N = 3$.

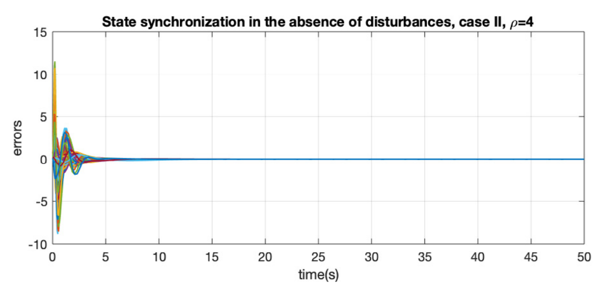

Fig. 4. State synchronization in the absence of disturbance with the choice of $\rho = 4$ for the MAS with $N = 20$.

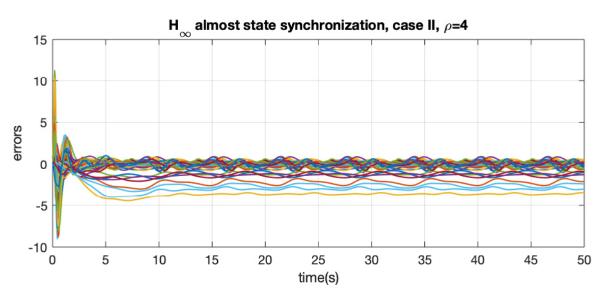

Fig. 5. H_∞ almost state synchronization with the choice of $\rho = 4$ for the MAS with $N = 20$.

The simulation results show that the protocol design is independent of the communication graph and is scale free so that we can achieve H_∞ almost state synchronization with one-shot protocol design, for any graph with any number of agents. The simulation results also show that by increasing the value of ρ, almost state synchronization is achieved with higher degree of accuracy.

5. Conclusion

In this paper, we proposed scale-free design for H_∞ almost state synchronization of homogeneous networks of non-introspective agents. A parameterized scalable linear collaborative dynamic protocol, parameterized in scalar ρ, was developed using localized information exchange among neighbors over the same communication network. We achieved almost synchronization for a given arbitrary degree of accuracy by choosing ρ sufficiently large. It should be emphasized that the proposed protocols were designed solely based on agent models, i.e., despite all the existing results, our design methodology was scale-free so that we did not need any information about the communication network such as bounds on the associated Laplacian matrix and the number of agents. As our future work, we aim to extend the scale-free designs proposed in this paper to the broader classes of agent models.

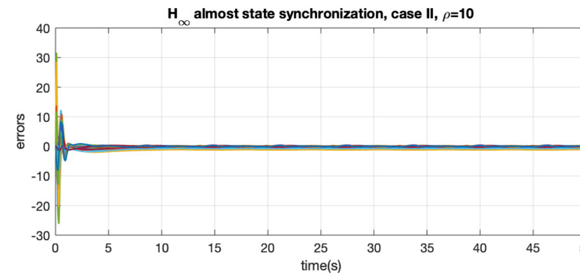

Fig. 6. H_∞ almost state synchronization with the choice of $\rho = 10$ for the MAS with $N = 20$.

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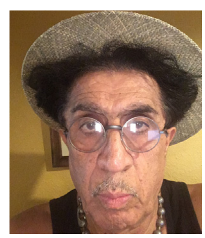

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