

EM-based Radar Signal Processing and Tracking

1st Alan Nussbaum
Georgia Tech Research Institute
Georgia Institute of Technology
Atlanta, GA
alan.nussbaum@gtri.gatech.edu
&
2nd Byron Keel
Georgia Tech Research Institute
Atlanta, Ga
byron.keel@gtri.gatech.edu

3rd William Dale Blair
Georgia Tech Research Institute
Georgia Institute of Technology
Atlanta, GA
dale.blair@gtri.gatech.edu
&
4rd Umakishore Ramachandran
Georgia Institute of Technology
Atlanta, GA
rama@gatech.edu

Abstract—Maximizing the signal-to-noise ratio (SNR) associated with a measurement augmenting the radar system with an iterative Expectation-Maximization (EM) algorithm versus traditional signal processing algorithms. Maximizing the achievable SNR minimizes measurement precision affected by thermal noise, and track precision is enhanced given precise, unbiased, range measurements. While the radar is tracking the kinematic state (i.e., position, velocity, and acceleration) of the target, optimal signal processing requires knowledge of the target's signature, range rate, and range acceleration. Increasing the SNR of individual measurements by enhanced signal processing results in reduced settling time for the track filter and less valuable radar resources required to achieve a specified track quality. Moving targets introduce range-walk (RW), which degrades coherent processing leading to a reduction in SNR and blurring the point target response in range-Doppler Matrix (RD), resulting in poor Range estimate to the Tracker. Traditional radar architectures require excessive iterations of the track loop with the associated measurements to achieve a specified track quality threshold. In this paper, the Expectation-Maximization (EM) algorithm, an iterative and inference approach, is employed to provide from the Tracker estimates for velocity and acceleration to the Signal Processor the unobserved variables to maximize the measurement's SNR.

Index Terms—Signal Processing, Tracking, Expectation-Maximization Algorithm, Accelerating Targets, Sensor Processing and Architectures

I. INTRODUCTION

A moving target exhibits range walk (RW) over a coherent processing interval(CPI). If the range walk and nonlinear slow-time phase changes are not compensated for, prior to coherent processing over slow time, a loss in signal-to-noise ratio (SNR) is observed. Compensating for RW requires knowledge or an estimate of the target's range rate and acceleration. In a tracking radar, measurement precision is inversely related to SNR and directly impacts track performance. In addition, over short time intervals, estimates of abrupt changes in the target's velocity and acceleration are needed to maintain track.

Uncompensated RW produces a blurred (expanded) point target response in both range and Doppler. The blurred response degrades one's ability to estimate the target's range under reduced SNR. In this paper, we examine the use of an

iterative Expectation-Maximization (EM) algorithm [1], resident in the Signal Processor (SP) [1], to estimate changes in the target's kinematics and to correct for RW leading to enhanced measurement precision in both range and angle. The algorithm's expectation component incorporates a Kalman filter resident within the SP to estimate kinematic changes from CPI to CPI [2]. Traditional approaches to RW correction use predicted tracker states which don't account for acceleration changes between measurements. The updated target states are then applied in reprocessing the current CPI to improve the measurement precision. The approach is intended to aid the Tracker and SP in responding to kinematic changes occurring between track updates, thus increasing a track's quality performance and reduce the required track's lifecycle timeline while optimizing valuable radar resources usages [3].

In many practical sensor learning settings, only a subset of relevant features or variables might be observable [4]. The current research state to improve tracking for moving targets (e.g., maneuvers) relies upon predefined models, based on knowledge, that assumes feature state and an understanding of the expected target acceleration performance [5]. There has been extensive research and development in signal processing and tracking algorithms to increase object tracking fidelity for accelerating and maneuvering targets

The EM algorithm provides an innovative approach for learning in the presence of unobserved variables to enhance algorithm performance [1]. EM is an iterative method for finding the maximum likelihood estimates of parameters [6] in a statistical model, where the model depends on the unobserved variable. In this case, the unobserved variables are range rate and radial acceleration. The EM algorithm's potential drawbacks are, in some cases, 1) an inability to converge to a reliable answer for the unobserved variable(s) and 2) the time duration to achieve convergence. EM's creation of the unobserved variables may not consistently support the sensor's near-real-time timelines [7] and its decision-making ability to attain convergence [8] [9]. However, we will demonstrate that the proposed EM algorithm enabling RW correction to meet our objectives for enhancing measurement precision and

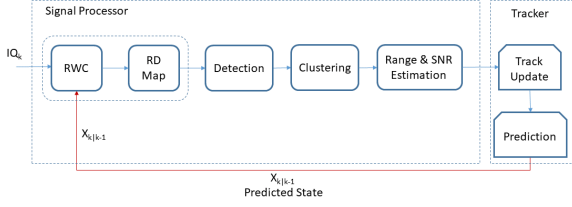


Fig. 1: Traditional sensor's pipeline the Signal Processor creates a RD Map for a range measurement to the Tracker for each scheduling period suffers an SNR loss given the uncertainty in the Tracker's predicted states used to correct for RW.

timeliness.

The traditional signal and data processor approach is depicted in Figure 1. The Tracker provides a prediction of the radial velocity and acceleration components for the $k+1$ state given the k measurement. The SP applies the predictions to correct for RW in the $k+1$ CPI. The RW correction consists of a linear phase ramp applied on a per pulse basis in the frequency domain based on the appropriate delay and a quadratic phase correction. The RW algorithm aligns the target returns to the center of the CPI. The RD map consists of pulse compression and a discrete Fourier transform (DFT) across slow time. The challenge occurs when the acceleration and associated dependent velocity change between track updates. Thus, the RW correction is degraded by incorrect kinematic predictions.

We propose augmenting the SP to support an iterative EM algorithm to enhance measurement and tracker performance. The modifications to the SP are provided in Figure 2 and are highlighted below.

- Expectation (E-step) – Given the current range measurement extracted from the RD map, a Kalman filter is applied to update the target's kinematic state to include unobserved variables, range rate $\hat{v}$, and radial acceleration $\hat{a}$.
- Maximum likelihood – Given the new state estimate (provided by the expectation step - Kalman filter), the updated velocity and acceleration states are applied to the RW correction module, and the CPI data is reprocessed to yield an improved range and SNR estimate.
- The EM algorithm is repeated until the target SNR reaches a maximum value.

As noted previously, uncompensated RW produces a blurred response across range and Doppler. An example of the blurred response for a single point target is depicted in Figure 3(a). In this instance, we are showing a portion of the range-Doppler (RD) space. The color-coding is intended to show detections post coherent processing. A detection within a RD cell is denoted by a "1" or a yellow coloring. Blue denotes

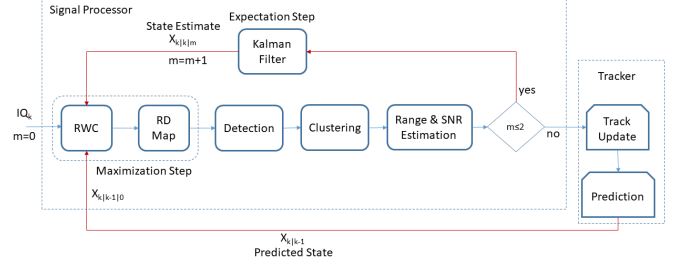


Fig. 2: Signal Processor creates an RD Map and provides a range measurement Z , $\hat{R}$ and SNR for Z to the Tracker. The first EM iteration is initiated on the 3rd measurement resulting from the Tracker and Kalman Filter processing initial velocity $\hat{v}$ and acceleration $\hat{a}$ estimates. For each new measurement, the next EM iteration E- and M-steps is executed.

no detections. In the absence of the RW, the normalized peak response would have a value of 0 dB; however, in this example, the normalized peak is -8.98 dB representing a loss in SNR over the no RW case. Applying RW correction with reasonable but imperfect knowledge of the target kinematics is depicted in Figure 3(b). One observes an obvious tighter response in RD, and the normalized peak is -1.598. With RW correction, the SNR has improved by more than 6dB. The Cramer Rao Lower Bound (CRLB) on measurement precision is inversely proportional to the square root of SNR as noted below

$$\sigma = k \frac{\delta}{\sqrt{2SNR}} \quad (1)$$

where k is a constant and δ is the resolution in a given dimension [4]. By increasing the SNR, we have improved the measurement precision in range as well as other dimensions as applicable (e.g., monopulse angle estimation).

This paper introduces enhancements to the Signal Processor's and Tracker's architecture that strengthens the radar system's ability to track accelerating targets within a shorter timeline and with less demand on valuable radar resources. The remainder of the paper is organized as follows. Section II provides an overview of the radar system model using in our analysis and provides additional detail regarding the EM algorithm. Section III provides a maneuvering target experimental results demonstrating the potential performance improvement, and Section IV summarizes our conclusions and proposed future work.

II. RADAR APPLICATION'S EM MODEL FOR MOVING TARGETS

This section provides a more detailed discussion of the model and the SP and Tracker employing the EM algorithm for RW correction and parameter estimation of the unobserved variables.

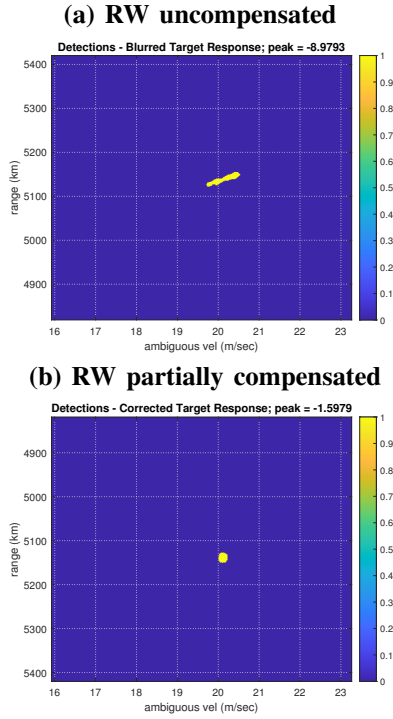


Fig. 3: (a) The blurred detection response across RD and associated SNR loss when not compensating for RW. (b) A more compact response when applying RW compensation and an associated reduction in SNR loss.

A. Radar System Model

In our work, we are modeling a pulse-Doppler X-Band radar [10]. The radar employs a linear frequency modulated (LFM) pulse. The pulse width is 10 ms, and the swept bandwidth is 10 MHz. A CPI consists of 1024 pulses. The long CPI is needed to achieve the desired SNR. A pulse repetition frequency of 10 kHz is employed. For our examples, the target is intentionally positioned to be range unambiguous; however, the target's Doppler is ambiguous. The duration of CPI is 0.1024 seconds, and the track update rate is 1 Hz. The radar system parameters are summarized in Table I.

Parameter	Value	Description
Frequency	10E9	Hz
Pulse Width	10E-6	microseconds
Bandwidth	10E6	MHz
Pulse Repetition Freq	10E3	Hz
Number of Pulses	1024	
CPI Duration	0.1024	seconds
Range Resolution	15	meters
Unambiguous Range	15,000	meters
Unambiguous Velocity	150	meters/sec
Track Update Rate	1	Hz

TABLE I: Matlab model of a pulse-Doppler X-Band radar and schedules Linear Frequency Modulation (LFM) waveforms scheduling 1024 pulses at each Coherent Processing Interval (CPI)

B. Target Scenario

To demonstrate the utility of the EM algorithm, we define a scenario where the radar initially encounters a target with

constant velocity, 300 meters/sec, and an acceleration 9.8 meters/sec^2 ($1g$), which it acquires and places in track. In Section III we created an experiment for a target exhibits a $0.5g$ maneuver [11] that initiates in the time interval between two intervening track updates and is maintained over several track update periods. The EM algorithm enables an immediate estimate of the acceleration employing the next CPI's data and compensates for the additional RW and SNR loss.

C. Signal Processing and Tracker

Referring to Figure 2, an RW correction is applied to the in-phase and quadrature (I/Q) samples comprising a CPI based on the Tracker's predicted state for measurement $k+1$. Once RW correction has been applied, a matched filter is applied in range, and a DFT is applied across slow time to form the RD map. A detection threshold is then applied to the map based on the desired probability of false alarm. A clustering algorithm [12] [13] is then applied to the detections to associate them with a given target. The specific clustering algorithm is a "density-based spatial clustering of applications with noise" (DBSCAN) [14] [15]. The RD cells associated with the target cluster are used to estimate the range to the target. The range estimate is based on a power weighted centroiding algorithm. An estimate of the SNR is obtained based on the peak amplitude within the target cluster. When employing the EM algorithm, the range estimate and estimate measurement variance are passed to the Kalman filter. The Kalman filter returns an updated target statement, which is then passed to the RW correction (RWC) step, and the RD map processing is repeated. The second pass through RD map results is then passed to the Kalman filter for a second state update [16]. The final updated stated is passed to the RWC, and RD processing and the estimated range and measurement variance are then sent to the Tracker.

The results will analyze the iterative algorithm's ability to adjust rapidly to create high-fidelity measurements and track. The target's initial configuration are defined as constant $.5g$, $a(4.9 \text{ meters/sec}^2)$, with 30 dB SNR settings for an X-band radar experiment. The target is within the radar range, and the initial range is 4500 meters, with the target's trajectory increasing in the range from the radar. Target sensitivity is decreasing, for each range measurement ($\hat{R}$), because the object's trajectory is increasing in range. Its initial velocity estimate is 300 meters/sec, which increases during the maneuver with each subsequent measurement ($\hat{v}$) resulting from the object's acceleration effects.

The accelerating target adverse effects on the range-Doppler Matrix, for Signal Processor we elected for detection processing to utilize the DBSCAN clustering algorithm. The algorithm creates clusters from the detections above the calculated SNR threshold. The Signal Processor selects the cluster with the maximum number of detections and then calculate the average of all the detection ranges to generate $\hat{R}$ measurement. The Tracker will process the first measurement and use $\hat{R}$ from the Signal Processor to initiate the track and create the initial covariance matrix based on the measurement's SNR,

the variance of the measurement errors (R1), and default parameters. The second measurement will calculate an estimate for $\hat{v}$ based on range estimates from both measurements and update the covariance. The third measurement will start the EM algorithm because the Tracker will use all three measurements to calculate an estimate for the unobserved variables for velocity $\hat{v}$ and acceleration $\hat{a}$.

Table II demonstrates for the 1g target starting at 30 dB SNR, the Tracker, and Kalman filter predictions for a state vector (X1, X2, and X3) and Covariance Matrix state based on each measurement processing described in Figure 2. SNR decreases per measurement because the target's trajectory is increasing in range. This section demonstrates the EM-based algorithm accomplishing our goals to reduce the need for valuable radar resources while increasing SNR, resulting in higher confidence in track quality in a compressed timeline.

Meas	SNR	Rhat	Det	PeakdB	X1	X2	X3
1	30.0	4517.7	2323	-7.46	4517.7	0.0	0.00
2	28.9	4821.4	1050	-7.44	4821.4	303.7	0.00
3	27.8	5136.4	808	-7.67	5136.4	320.6	11.27
3-EM	27.8	5137.4	434	-0.73	5137.2	321.8	12.12
4-EM	26.7	5462.4	433	-0.66	5462.4	330.0	9.57
5-EM	25.7	5797.1	259	0.24	5797.6	340.2	10.02

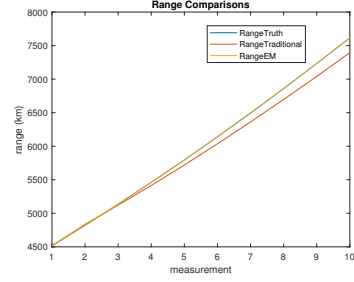
TABLE II: The table demonstrates the 1) Tracker building up the State Vector (X1, X2, and X3) for its traditional processing and initiating and seeding the iterative EM steps with $\hat{v}$ and $\hat{a}$ for acceleration and velocity estimates. 2) Maximum likelihood algorithm (3-EM, 4-EM and 5-EM) in the Signal Processor reprocessing the receive data using the calculated unobserved variables from the E-step to increase the measurement SNR and provide a quality input the Kalman Filter to create a higher confidences track state [17] and Covariance Matrix.

Figure 4 demonstrates the radar model's performance and the differences for the 1)traditional radar tracking closed-loop, 2)new EM iterative control loop, and 3) target's truth. Figure 4 4 plots are the difference for range, velocity, acceleration, and normalize peakdB (zero is maximum peakdB). The data validates that the EM-Based approach increases the accuracy of the Signal Processor's generated measurement and Tracker and Kalman filter's processing for a 1g accelerating object. The next step for this research to demonstrate that the EM iterative control loop can be implemented in a real-time architecture.

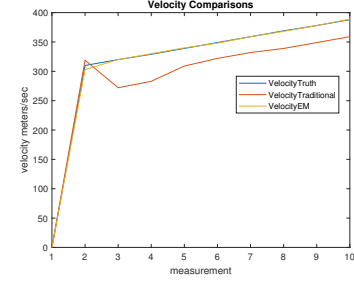
III. EXPERIMENT, RESULTS, AND ANALYSIS

In this section, we perform an experiment with a constant velocity target observed on the 4th track update in the previous section. During the interval between the 4th and 5th track update, the target implements a 0.5 g maneuver. Table III contains measurement and track metrics on the 4th through 7th measurements. The last steps on the 4th CPI include the tracker state update and the target's prediction for the 5th track update. In this instance, the predicted velocity is 300.2 m/sec, and the predicted acceleration is 0.03 meters/sec²). Employing

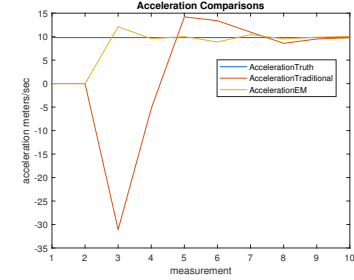
(a) Range Comparisons Observations



(b) Velocity Comparisons Observations



(c) Acceleration Comparisons Observations



(d) SNR Comparisons Observations

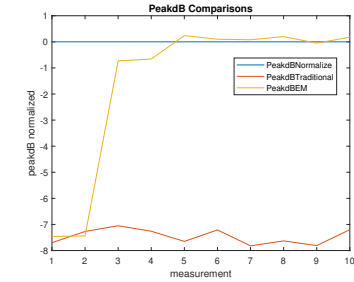


Fig. 4: 1g constant acceleration target (a) RangeEM estimate converge on 1st EM-based measurement (3rd) and Traditional grow on each measurement, (b) VelocityEM converge on first EM measurement, (c) AccelerationEM converge on the 2nd EM-based measurement (4th measurement), and (d) PeakdBEM on average per EM measurement increase by at least 6.7 dB

Func	meas	SNR dB	Peak dB	Range	Vel	Acc
Tracker	1	30.0 dB	-4.21	4516.0	0.0	0.0
Tracker	2	28.9 dB	-5.03	4816.5	0.04	1.09
Tracker	3	27.8 dB	-4.45	5116.4	299.5	-0.68
Kalman	3	27.8 dB	0.16	5116.7	300	-0.36
Kalman	3	27.8 dB	0.16	5116.7	300	-0.35
Predict	4	n/a	n/a	5416.6	299.7	-0.35
Kalman	4	25.8 dB	0.22	5417.0	300.3	-0.08
Kalman	4	25.8 dB	0.20	5417.0	300.3	-0.08
Tracker	4	25.8 dB	-0.20	5416.9	300.2	0.03
Predict	5	n/a	n/a	5717.1	300.2	0.03
Kalman	5	25.9	-2.46	5719.9	304.2	2.98
Kalman	5	25.9	-0.45	5719.8	304.2	2.98
Tracker	5	25.9	-0.45	5719.7	304.1	2.94
Predict	6	n/a	n/a	6025.3	307.1	2.94
Kalman	6	25.0	-0.37	6026.9	309.4	4.68
Kalman	6	25.0	0.11	6026.9	309.4	4.68
Tracker	6	25.0	0.11	6027.0	309.4	4.70
Predict	7	n/a	n/a	6338.8	314.1	2.94
Kalman	7	24.1	0.00	6339.6	315.3	5.54
Kalman	7	24.1	-0.03	6339.6	315.3	5.54
Tracker	7	24.1	-0.03	6339.6	315.3	5.53

TABLE III: Maneuver started between CPI 4 5. The truth for measurements 4, 5, 6 7 a) Range truth 5415.4, 5718.1, 6025.7, 6338.2 b) Velocity truth 300.0, 305.1, 310.0, 314.9 c) Acceleration truth 0, 4.9, 4.9, 4.9. Table demonstrates EM-based algorithm converges on measurement 6

the predicted target state in the initial RWC resulted in an SNR loss of 2.46 dB. The first pass through the Kalman filter yields an estimated velocity of 304.2 m/sec and an estimated $2.98 \text{ meters/sec}^2$ acceleration. Employing these estimates, the RWC improved the SNR results in less than 0.5 dB of loss. The second pass through the Kalman and RWC yields similar results. Employing the EM algorithm within a single CPI, we have improved velocity and acceleration estimates and increased the measurement SNR. An additional improvement is observed on the next track update.

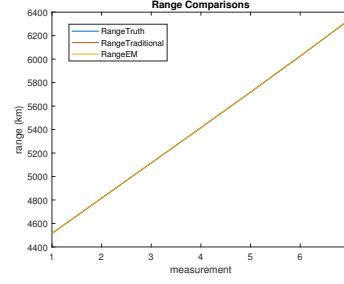
The plots in Figure 5 show the comparisons for Tracker's states (range, velocity, and acceleration) and the SP measurement SNR as a function of each CPI or measurement index over 7 CPIs. Each plot contains truth, the state estimates employing traditional processing, and the state estimates employing the EM-based algorithm. The rapid convergence of the EM algorithm provides tracking stability for accelerating targets and a direct benefit in reducing the need to maintain the object's track, shorten the needed track lifecycle, and reducing the need for essential radar sensor resources. The EM-based algorithm creates for .5g target processing produced an unblurred RDM and cluster of detections that results in higher SNR >2 dB, Figure 6.

IV. PROCESSOR ARCHITECTURE

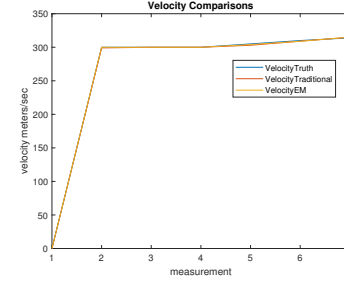
As demonstrated, an EM-based approach to RWC and parameter estimation can improve track performance by responding to acceleration changes between track updates within a single measurement period. The additional benefits include a reduction in radar resources and the timeline needed to address a maneuvering target [5].

The EM approach is iterative and will increase the SP processing execution by a factor of three or more depending on

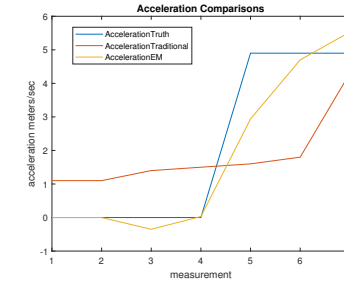
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(b) Velocity Comparisons Observations



(c) Acceleration Comparisons Observations



(d) SNR Comparisons Observations

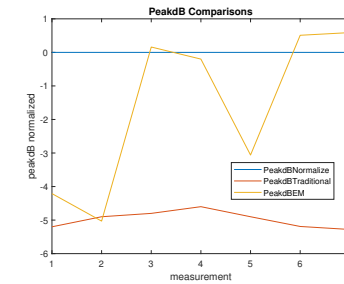


Fig. 5: .5g constant acceleration target estimate start to converge on 5th EM-based measurement and improve the critical peakdb by 2.01 dB

the number of times the RWC and RD processing are called. The proposed control loop architecture breaks away from the traditional track closed-loop pipeline approach where the SP and Tracker elements were disjointed elements. The control loop [3] EM-based includes iterations within the SP and collocated Kalman filter intended to maximize SNR.

As part of our future research, we will be developing and optimizing the signal and data processor to accommodate the proposed approach, emphasizing real-time application [18]

4th Meas RDM and Cluster prior and post to M-step

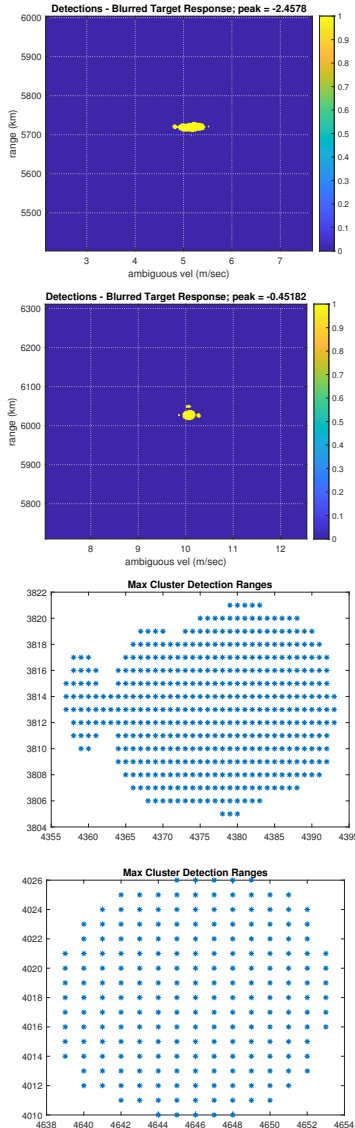


Fig. 6: .5g target maneuver I/Q reprocessing data resulting from the EM-based algorithm rapidly adjust to target RW in range and Doppler space and creation of a high quality detection cluster

V. CONCLUSION AND NEXT STEPS

We have shown that unreliable preconfigured models for airborne targets or multi-sensor configurations are not required to track maneuvering and constant accelerated targets. The research established an integrated Signal Processing and Tracking augmented with an iterative Expectation-Maximization algorithm. For the E-step, the Tracker and Kalman Filter provides the uncertain kinematic state of the accelerating target and knowledge-aided for the unobserved variables to allow the Signal Processor to perform the M-step (maximum likelihood) that results in higher SNR up to 7db. The EM research only requires no more than two iterations to converge. This modification to the traditional pipeline architecture will

benefit sensor processing for many different applications (image processing, cellular phones, communications, sonar, and collision avoidance systems).

The next research step is being implemented to the EM-based algorithm to utilize an IMM filter to support multiple maneuvering targets changing velocity and acceleration. Add direction of arrival (DOA) estimates to the EM-based radar algorithm and C++ control looped implementation to verify it can support near real-time sensor latency requirements. Future research plans will be based on the algorithm's success to false alarms, track multiple closely-spaced targets in the presence of clutter or jamming, and maneuvering targets.

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