

Chemical freeze-out of light nuclei in high energy nuclear collisions and resolution of the hyper-triton chemical freeze-out puzzle

K. A. Bugaev^{1,2}, O. V. Vitiuk^{2,3}, B. E. Grinyuk¹, N. S. Yakovenko², E. S. Zhrebtsova^{4,5}, V. V. Sagun^{1,6}, O. I. Ivanytskyi^{1,6}, D. O. Savchenko¹, L. V. Bravina³, D. B. Blaschke^{4,7,8}, G. R. Farrar⁹, S. Kabana¹⁰, S. V. Kuleshov¹¹, E. G. Nikonov¹², A. V. Taranenko⁶, E. E. Zabrodin^{3,13} and G. M. Zinovjev¹

¹ Bogolyubov Institute for Theoretical Physics, Metrologichna str. 14-B, 03680 Kyiv, Ukraine

² Department of Physics, Taras Shevchenko National University of Kyiv, 03022 Kyiv, Ukraine

³ Department of Physics, University of Oslo, PB 1048 Blindern, N-0316 Oslo, Norway

⁴ National Research Nuclear University (MEPhI), Kashirskoe Shosse 31, 115409 Moscow, Russia

⁵ Institute for Nuclear Research, Russian Academy of Science, 108840 Moscow, Russia

⁶ CFisUC, Department of Physics, University of Coimbra, 3004-516 Coimbra, Portugal

⁷ Institute of Theoretical Physics, University of Wrocław, Max Born Pl. 9, 50-204 Wrocław, Poland

⁸ Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie Str. 6, 141980 Dubna, Russia

⁹ Department of Physics, New York University, 726 Broadway, New York, NY 10003, USA

¹⁰ Instituto de Alta Investigación, Universidad de Tarapacá, Casilla 7D, Arica, Chile

¹¹ Departamento de Ciencias Físicas, Universidad Andres Bello, Sazié 2212, Santiago, Chile

¹² Laboratory for Information Technologies, JINR, Joliot-Curie str. 6, 141980 Dubna, Russia

¹³ Skobel'tzyn Institute of Nuclear Physics, Moscow State University, 119899 Moscow, Russia

E-mail: Bugaev@th.physik.uni-frankfurt.de

Abstract. We present a summary of the recent results obtained with the novel hadron resonance gas model with the multicomponent hard-core repulsion which is extended to describe the mixtures of hadrons and light (anti-, hyper-)nuclei. A very accurate description is obtained for the hadronic and the light nuclei data measured by STAR at the collision energy $\sqrt{s_{NN}} = 200$ GeV and by ALICE at $\sqrt{s_{NN}} = 2.76$ TeV. The most striking result discussed here is that for the most probable chemical freeze-out scenario for the STAR energy the found parameters allow us to reproduce the values of the experimental ratios S_3 and \bar{S}_3 without fitting.

1. Introduction

The development of the hadron resonance gas model (HRGM) with the multicomponent hard-core repulsion between the constituents [1, 2, 3, 4, 5, 6], i.e. with several hard-core radii of hadrons, converted the so-called thermal model into a powerful and convenient tool of the heavy ion physics phenomenology, but also it led to a few real breakthroughs in our understanding of the chemical freeze-out (CFO) process. Indeed, using just a few extra parameters compared to

the traditional HRGM [7], which employs a single hard-core radius for baryons R_b and the one R_m for the mesons, it was possible to reach an unprecedented accuracy in the description of hadronic yields measured in the central nuclear collisions from the low AGS BNL collision energy ($\sqrt{s_{NN}} = 2.7$ GeV) to the highest RHIC one ($\sqrt{s_{NN}} = 200$ GeV) with a quality $\chi^2/dof \simeq 1.15$ [3, 4, 5] (if one includes into the fitting the hard-core radius of pions R_π and kaons R_K) or with $\chi^2/dof \simeq 0.96$ [6] (if one includes into the fitting the hard-core radius of Λ -(anti-)hyperons R_Λ in addition to R_π and R_K).

The high accuracy achieved by the HRGM with multicomponent hard-core repulsion allowed us not only to elucidate the characteristics of the CFO of A+A collisions, but also to resolve several long-standing problems of the CFO process [2, 3, 4, 5, 6, 8, 9, 10, 11, 12]: (i) in Refs. [2, 3, 6] it was shown that the so-called (anti-) Λ puzzle [7] is the result of oversimplifying assumptions; (ii) in Refs. [2, 3] it was found a natural solution to the Strangeness Horn [13] description puzzle which troubled the heavy ion community for a decade; (iii) the concept of separate CFOs of strange and non-strange hadrons was independently suggested in Refs. [4, 14]. Moreover, the high quality of data description allowed us to find out several new irregularities of thermodynamic quantities at the CFO which helped us to formulate new and promising signals of two phase transitions [5, 8, 9, 10, 11, 12] that are expected to exist in strongly interacting matter [15, 16, 17].

One should, however, remember that the multicomponent versions of the HRGM [1, 2, 3, 4, 5, 6] based on the popular Van der Waals (VdW) approximation to the hard-core repulsion, i.e. which use the classical second virial coefficients, are rather complicated and take a lot of CPU time, since for N different hard-core radii for each iteration of the experimental data fitting it is necessary to solve the system of $(N + 1)$ transcendental equations containing a few hundreds of double integrals. Hence, the application of the multicomponent HRGM based on VdW approximation to cases of $N \gg 1$ is somewhat problematic [18, 19]. However, an entirely new and efficient approach to deal with the multicomponent hard-core repulsion in the grand canonical ensemble for large values of N was invented in Ref. [20].

This novel approach is based on the induced surface tension (IST) concept [20]. It has two principal advantages over the other multicomponent versions of the HRGM: (i) the number of equations which should be solved is two only and does not depend on N , and (ii) as shown in [18, 19, 21, 22] it allows one to go far beyond the usual VdW approximation and to take into account not only the second, but the third and even the fourth virial coefficients of the classical hard spheres. In Refs. [18, 19] it was recently shown that, in contrast to the oversimplified version of the HRGM like the one used in [23], there is no proton yield puzzle neither at ALICE energy $\sqrt{s_{NN}} = 2.76$ TeV, nor at RHIC energies of collisions.

Our next step was to extend the IST equation of state (EoS) to the description of the mixtures of the hadrons with light nuclear clusters, i.e. the deuterons (d), helium-3 (^3He), helium-4 (^4He) and hyper-triton ($^3_\Lambda\text{H}$) and their antiparticles, and to apply the developed EoS to the simultaneous description of the STAR $\sqrt{s_{NN}} = 200$ GeV data on the nuclear multiplicities [24, 25, 26], the ALICE $\sqrt{s_{NN}} = 2.76$ TeV data on light nuclear cluster yields [27, 28, 29] and the hadronic multiplicities measured at these collision energies. To our great surprise not only the quantum, but also the classical second virial coefficients of such nuclei and hadrons were never discussed in the literature. Therefore, we had to resolve this problem first. Since the HRGM with the classical second virial coefficients of hadrons with the hard-core repulsion, i.e. with the excluded volumes, is rather successful, we extended this approach to the classical second virial coefficients of hadrons and light nuclear clusters [30, 31, 32, 33].

In this work we summarize our very recent results [32, 33] obtained on the description of the STAR $\sqrt{s_{NN}} = 200$ GeV data on the nuclear multiplicities [24, 25, 26] and the ALICE $\sqrt{s_{NN}} = 2.76$ TeV data on light nuclear cluster yields [27, 28, 29], and discuss some findings which were not reported previously, in particular, the problematic hyper-triton ratios (PHTR)

$S_3 = \frac{3}{\Lambda} H/{}^3He \cdot p/\Lambda$ and \bar{S}_3 measured by the STAR and ALICE Collaborations which were not described until now either by the HRGM or by the coalescence model [34].

2. HRGM for the mixture of hadrons and light nuclear clusters

The HRGM based on the IST EoS has the following hard-core radii of pions $R_\pi=0.15$ fm, kaons $R_K=0.395$ fm, Λ -(anti-)hyperons $R_\Lambda=0.085$ fm, other baryons $R_b=0.365$ fm and other mesons $R_m=0.42$ fm [18, 19, 11] which only slightly differ from the our previous results found within the VdW approximation [4, 6]. Since all the details of the IST EoS and the fitting procedure of the hadronic data are well documented in Refs. [18, 19, 11], here we do not discuss them.

To account for the classical excluded volumes of light nuclear clusters and hadrons we use two approaches worked out in [30, 31, 32] with one exception, namely we consider the hyper-triton (HTR) differently as it is suggested in [33]. Both of these approaches employ the classical excluded volumes of light nuclei of $A \in \{2, 3, 4\}$ baryonic constituents and hadron h [32, 33]

$$b_{Ah} = b_{hA} = A \cdot \frac{2}{3}\pi(R_b + R_h)^3, \quad \text{except the HTR}, \quad (1)$$

$$b_{HTRh} = b_{hHTR} = 2 \cdot \frac{2}{3}\pi(R_b + R_h)^3 + \frac{2}{3}\pi(R_\Lambda + R_h)^3, \quad \text{for the HTR}. \quad (2)$$

The equations above can be found from the fact that all light nuclear clusters analyzed here are roomy clusters. The mean distances among the baryons inside of such clusters are rather large [32, 33] and, hence, it is possible to freely translate any hadron h with the hard-core radius R_h around each constituent of a nucleus without touching any other constituent of this nucleus.

The first approach is the ISTA EoS and it uses exactly the excluded volumes (1) and (2). It is rigorously derived using a self-consistent treatment of classical excluded volumes of light nuclear clusters and hadrons [32] with the help of the methods developed in [21, 22]. In contrast, the IST EoS which employs (1) for the HRT is called the IST EoS.

The second approach is approximate and complementary to the exact one. It is based on an approximate, but the rather accurate treatment of the equivalent hard-core radius of roomy nuclear cluster and pions which are the dominating component of the HRG at the energy range of our interest. In the latter approach one can find an effective hard-core radius of nuclei of A baryons as $R_A \simeq A^{1/3}R_b$, since the hard-core radius of pions is very small and, hence, it generates a negligible correction to R_A [30, 31, 32]. Since the hard-core radius of light nuclear clusters defined in this way is similar to the expression of the Bag Model [35], it is called the BMR EoS. A more accurate expression for the HTR hard-core radius $R_{HTR} \simeq 2^{1/3}R_b$ is derived in [33] and such a model is called the BMRA EoS. The main reason to compare the results of these two approaches is that, despite the difference in the equations, they should reproduce the data with the same quality by construction. Hence, finding the region of parameters which provide a similar quality of the data description one can remove the ambiguity in choosing the appropriate CFO parameters by analyzing the wide and shallow minima χ_A^2 of light nuclei.

Following our ideology outlined in [30], we verify two different scenarios of the CFO of nuclei clusters, namely a single CFO together with the hadrons and their separate CFO from the hadrons. The major reason for such an analysis is that the mechanisms of the hadron production and production of nuclei in collisions can be rather different. One can clearly see from the left panels of Figs 1 and 2 that the minimum of the light nuclear clusters $\chi_A^2(T_A)$ as a function of their CFO temperature T_A is located far away from the minimum of $\chi_h^2(T_h)$ of hadrons as the function of the hadronic CFO temperature T_h . The total $\chi_{\text{tot}}^2(V)$ is defined as

$$\chi_{\text{tot}}^2(T_h, T_A, V) = \chi_R^2 + \chi_Y^2(V) = \sum_{k \neq l \in R} \left[\frac{\mathcal{R}_{kl}^{\text{theo}} - \mathcal{R}_{kl}^{\text{exp}}}{\delta \mathcal{R}_{kl}^{\text{exp}}} \right]^2 + \sum_{k \in Y} \left[\frac{\rho_k V - N_k^{\text{exp}}}{\delta N_k^{\text{exp}}} \right]^2, \quad (3)$$

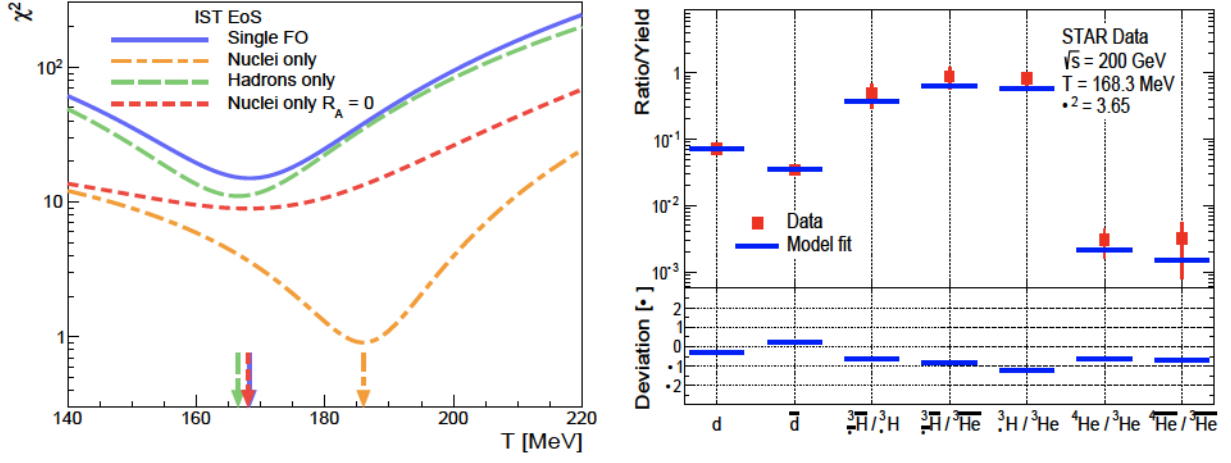


Figure 1. Left panel: Temperature dependence of χ^2_{tot} , χ^2_h and χ^2_A for fit of the STAR data measured at $\sqrt{s_{NN}} = 200$ GeV found for the ISTA EoS. **Right panel:** The yields of nuclear clusters measured at $\sqrt{s_{NN}} = 200$ GeV by STAR vs. theoretical description with ISTA EoS. Insertion shows the deviation of theory from data in the units of experimental error.

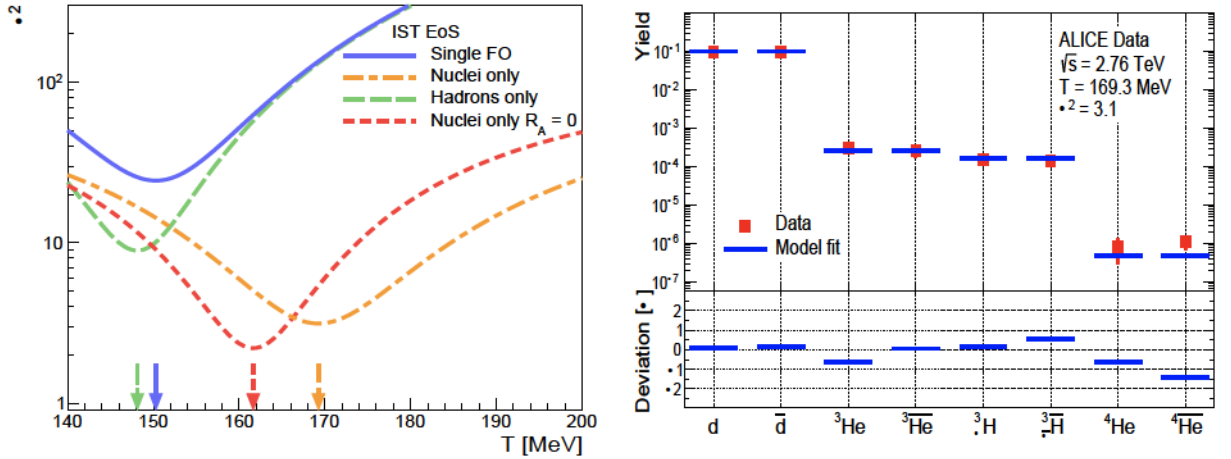


Figure 2. Left panel: Same as in Fig. 1, but for the fit of the ALICE data measured at $\sqrt{s_{NN}} = 2.76$ TeV.

where χ^2_R and χ^2_Y denote, respectively, the mean squared deviation for the ratios and for the yields, while V is the CFO volume of nuclei and ρ_k is the particle number density of the k -th sort of particles. A combined fit of particle yields and ratios is dictated by the available data and by numerical convenience. It is interesting that for the vanishing hard-core radii of all nuclei the minimum of $\chi^2_A(T)$ is close to the minimum of $\chi^2_h(T)$ for the STAR data, but still it is far away for the ALICE one (see the short dashed curves in the left panels of Figs 1 and 2).

From Fig. 1 one can see that for the separate CFO of light nuclei ISTA EoS provides the CFO temperature of nuclei T_A above 186 MeV (a similar result is found for IST EoS but with a larger χ^2/dof value [33]). Note, however, that according to the lattice version of QCD at vanishing values of the baryonic chemical potential [36] it is rather problematic to use the hadronic EoS for such CFO temperatures since this region is located above the cross-over to the quark-gluon plasma. Although for the separate CFO scenario all the light nuclei data are reproduced by

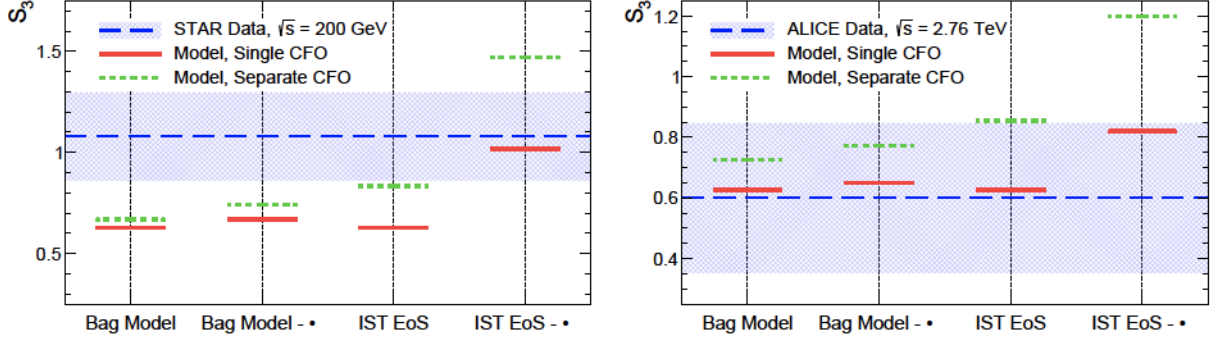


Figure 3. Left panel: S_3 ratio measured at $\sqrt{s_{NN}} = 200$ GeV by STAR Collaboration vs. theoretical description obtained for different CFO scenarios and EoS. Right panel: The same as in the left panel, but for the data of ALICE Collaboration measured at $\sqrt{s_{NN}} = 2.76$ TeV.

the ISTA EoS with the deviation smaller than 1σ , this scenario can be ruled out by requiring consistency with the lattice QCD results. The single CFO scenario of light nuclear clusters and hadrons corresponds to a CFO temperature $T_A = T_h \simeq 168.30 \pm 3.85$ with $\chi^2_{tot}/dof \simeq 1.069$.

As an independent benchmark in favor of the single CFO scenario for the STAR energies the S_3 and \bar{S}_3 ratios

$$S_3 = \frac{{}^3\Lambda\text{H}}{{}^3\text{He}} \times \frac{p}{\Lambda}, \quad \bar{S}_3 = \frac{{}^3\bar{\Lambda}\text{H}}{{}^3\bar{\text{He}}} \times \frac{\bar{p}}{\bar{\Lambda}}, \quad (4)$$

can be used [33]. From the left panel of Fig. 3 one can see that the ratio S_3 provided by the STAR Collaboration [24] is accurately reproduced only for the single CFO scenario found by the ISTA EoS. It is remarkable that the data on the S_3 and \bar{S}_3 ratios, which were not used in our fits, are reproduced by the most advanced version of the HRGM for the single CFO scenario. The quality of the light nuclei STAR data description for this scenario is shown in the right panel of Fig. 1.

From the analysis of the ALICE $\sqrt{s_{NN}} = 2.76$ TeV data [27, 28, 29] we obtained the opposite results. In other words, the separate CFO scenario with $T_h \simeq 148.12 \pm 2.03$, $T_A \simeq 169.25 \pm 5.57$ and $\chi^2_{tot}/dof \simeq 0.753$ looks more preferable than the single CFO scenario with $T_h = T_A \simeq 150.29 \pm 1.92$ and $\chi^2_{tot}/dof \simeq 1.433$. The details of χ^2_{tot}/dof behavior and its parts are shown in the left panel of Fig. 2, while the right panel of this figure demonstrates the high quality of the nuclear data description achieved by the ISTA EoS. In contrast to the STAR data, the ALICE data on the S_3 ratio are inconclusive, since six points out of eight ones found in our analysis are located within the large error bars of this quantity (see the right panel of Fig. 3).

3. Conclusions

In this work we discussed a very accurate description of the hadronic and light nuclear clusters data measured by the STAR Collaboration at $\sqrt{s_{NN}} = 200$ GeV and by the ALICE LHC at $\sqrt{s_{NN}} = 2.76$ TeV with the combined value $\chi^2/dof \simeq \frac{26.261}{18-3+17-3} \simeq 0.91$ for two best fits of both data sets. Such a high quality of data description is achieved by applying the new strategy of analyzing the light nuclear clusters data and by using the small value of the hard-core radius of the Λ -(anti)-hyperons $R_\Lambda = 0.085$ fm found in [18, 19] in the expressions for the classical second virial coefficients of HTR and for the equivalent hard-core radius of HTR.

It is remarkable that the small value of the hard-core radius of the Λ -(anti)-hyperons R_Λ found in our previous works allowed us, for the first time, to accurately describe the PHTR

ratios measured by the STAR Collaboration. The observed high sensitivity of the HTR data to the classical hard-core radius of Λ -(anti-)hyperons allows us to hope that in the future one can measure the hard-core radii of other hyperons with high precision, if they form the hyper-nuclei.

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