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Predicting lifetime by degradation tests: A case study of ISO 10995

Guanqi Fang[1] | Steven E. Rigdon[2] | Rong Pan[1]

[1]School of Computing, Informatics and Decision Systems Engineering, Arizona State University, Tempe, Arizona, USA

[2]College for Public Health and Social Justice, Saint Louis University, St. Louis, Missouri, USA

Correspondence
Rong Pan, School of Computing, Informatics and Decision Systems Engineering, Arizona State University, Tempe, AZ 85281, USA.
Email: rong.pan@asu.edu

Abstract

ISO 10995 is the international standard for the reliability testing and archival lifetime prediction of optical media. The standard specifies the testing conditions in terms of the combinations of stress variables—temperature and relative humidity. The periodically collected data from tests are the error rate of the device, and failure is defined as the error rate exceeding a predetermined level. The standard assumes that the projected failure time is the actual failure time, and these projected failure times are then analyzed by using an Eyring or Arrhenius model. Since true failure times are often not directly observed, the uncertainties in the failure time must be taken into account. In this paper, we present a hierarchical model for degradation that can directly infer failure time at the use condition and compare this model with the International Standard Organization (ISO) standard through a simulation study. Not accounting for the uncertainty in the projected failure times leads to unjustified confidence in the estimation for the median lifetime at both the stress conditions used in the experiments and at the use condition.

KEYWORDS

degradation test, hierarchical model, median lifetime, random effects

1 | INTRODUCTION

Many electronic components possess a very long lifetime under their normal use conditions. It is often impossible to make the product fail over a reasonable testing period. Because the reliability of the product is defined to be its performance level below some threshold, one can monitor the degradation process of this performance index so as to infer the failure time of the product. For example, the quality of an optical storage device, such as computer hard drive, is defined by its reading or writing error rate. The degradation path of this error rate can be used to predict the product's failure time. However, even with a degradation test, the degradation rate could be too small to be noticeable under the product's normal use condition; therefore, accelerated degradation tests (ADTs) are commonly used to elevate the degradation process by subjecting the product to more severe environmental stress. Using ADT data to make a prediction of product lifetime, one needs two fundamental models—the degradation model and the acceleration model for certain degradation parameters or lifetime parameters.

An ADT would then involve the following steps:

- Choose proper testing conditions of temperature and humidity; typically, they are much higher than their designed or normal use conditions;
- Conduct ADTs and collect the degradation data under each testing condition;
- For each test unit, a regression analysis is performed on its degradation data, and the failure time is calculated as the time when the degradation path would be predicted to cross the performance threshold;

- Use the predicted failure times obtained under each testing condition to fit a failure distribution model;
- Use the acceleration model to extrapolate the median failure time from the accelerated testing conditions to the normal use condition.

ISO 10995[1] specifies all of these conditions, including sample sizes at each level of temperature and humidity.

There are several issues associated with the method of data analysis suggested in ISO 10995. First, the lifetimes inferred from testing data are treated as the true lifetimes, when in fact they are obtained by extrapolating the degradation regression line to the performance threshold level. The uncertainties in regression and extrapolation are ignored. Second, all test units are viewed as independent units; therefore, their data are analyzed individually. But, in reality, the test units that were tested under the same testing condition are typically put in the same test chamber. Thus, it is reasonable to assume that their degradation paths are correlated. A random effects model is good at modeling the chamber-to-chamber variation. However, the standard method, although straightforward, does not consider this variance structure. Third, the standard method uses a simple regression to obtain the median life at the normal use condition. It ignores the uncertainty in median life prediction. Note that, because the use condition can be far away from testing conditions, a small deviation in life prediction at the test stress level can cause a huge prediction bias at the normal use condition. Therefore, there is a need for investigating a heterogeneous acceleration model for lifetime prediction. In this paper, we propose a hierarchical modeling approach that can incorporate random effects into the degradation model to predict the lifetime under normal use condition directly.

The remainder of the paper is organized as follows. In Section 2, a literature review on accelerated degradation modeling, random-effects models and their application to optical storage devices is given. The hierarchical model is described in Section 3, followed by the formulation of the log-likelihood function and the parameter estimation method. In Section 4, the predicted median lifetime under use condition will be demonstrated. Then, a simulation study is conducted to make a comparison of our method with the ISO standard. Finally, we summarize the findings and contributions of this study in Section 6.

2 | LITERATURE REVIEW

As an alternative method to the traditional life testing for assessing product reliability, degradation testing has drawn interest from both academia and industry in the past two decades. Suzuki et al[2] demonstrated the advantage of degradation testing over life testing, especially when very few failures are expected because of the high reliability of test units. Meeker and Escobar[3] gave a comprehensive discussion of degradation modeling. The general path model is one of the classic models for describing degradation processes. By setting up a proper regression model, it assumes each individual observation is the summation of a mean degradation value and a measurement error, which is given by

$$y_i(t_{ij}) = D_i(t_{ij}) + \varepsilon_{ij}, \tag{1}$$

where $y_i(t_{ij})$ is the observation of an individual test unit i at time point j, while $D_i(t_{ij})$ and ε_{ij} are the corresponding mean degradation path value and the measurement error.

To explain the effect of environmental stress, Meeker and Escobar[4] further discussed ADT modeling in presence of stress factors, such as temperature, humidity, and voltage. The knowledge of chemical kinetics of how these factors affect material properties are incorporated into the above general path model. The Arrhenius model is the one that describes the life acceleration by temperature and the Eyring model models the temperature and another factor, such as humidity. The Eyring model can be expressed as

$$\text{Acceleration Factor} = A\,T^{\alpha}\exp\left[\frac{\Delta H}{kT} + \left(B + \frac{C}{T}\right)\log RH\right] \tag{2}$$

where ΔH is the activation energy, $k = 1.3807{\times}10^{-23}$(J/molecule degree K) is Boltzmann's constant, and temperature T is in degrees Kelvin. The values α, A, B, and C are the parameters of the acceleration model.

In addition to considering the accelerating effect of environmental stress, several researchers also take into account the unit-to-unit variability, which makes some model parameters become random variables. This type of mixed-effects model can be applied to analyze many degradation phenomena. For example, Zimmerman et al,[5] Robinson et al,[6] and Lu and Meeker[7] used nonlinear parametric regression methods to analyze the crack growth data from Bogdanoff and Kozin.[8] Hausler[9] provided a nonlinear mixed-effects model for laser diodes degradation analysis. Park[10] studied the organic light-emitting diodes degradation by a nonlinear random-coefficients model with the consideration of temperature and electric current stresses. Pan and Crispin[11] proposed a hierarchical model and treated the power parameter of time as a random coefficient to account for the unit-to-unit variation. Xing et al[12] developed an ensemble model to characterize the capacity degradation and to predict the remaining useful performance of lithium-ion batteries. Bae and Kvam[13] developed a nonlinear random coefficients model to analyze the degradation path of vacuum fluorescent displays.

On optical disk media, the mixed-effects ADT modeling approach can also be applied to its failure analysis. Normally, the disk functionality of either "read" or "write" data is performed by altering the transparency of an organic dye layer on the device.[14] Because of the organic nature of the dye, degradation and breakdown of the transparent portion of dye layer will occur over a long period of time as a natural process. This process, which has its roots in chemical kinetics, can take several years in a normal environmental condition,[15] but higher temperature or humidity can accelerate this process tremendously. The effects of these stress variables can be modeled using various models including the Eyring model,[16] which is derived from the study of chemical kinetics.

The end of life of a disc can be defined as the time when the information recorded on the disc cannot be retrieved without losses. In practice, the error rate value is monitored. Its gradual change can serve as an indicator of the media stability. Among these indicators, block error rate[17] is used to monitor Compact Discs (CDs), and the parity inner (PI)[18] error rate, as summed over 8 consecutive error correction blocks (PI Sum8),[18] is used to monitor Digital Video Disc (DVDs). In both cases, these quality characteristics are used to indicate the extent of media deterioration.[19] Another data storage media, hard disk drives (HDDs), share similar features to optical disk media. A rise in temperature, voltage, relative humidity, duty cycle, or particle induction can accelerate its degradation process and shorten its lifetime. William[20] designed life testing experiments for removable HDDs and predicted their archival life. Storm et al[21] showed that the head-disk collision would increase at an elevated temperature, while spindle motor and bearings may also fail early under a high temperature. The hierarchical degradation model we develop in this paper can be applied on all these optical storage devices, including CDs, DVDs, and HDDs.

3 | HIERARCHICAL DEGRADATION MODEL

3.1 | ISO/IEC approach

By the ISO/IEC 10995:2011 standard, the effects of two stress variables—temperature and relative humidity—on the lifetime of optical media are to be investigated. Four stress conditions are specified, and they are listed in Table 1.

For each specimen, a linear regression model of log-transformed error rate is fitted, and the time-to-failure is predicted. The obtained failure times are assumed to follow a lognormal distribution, and only the location parameter of the lognormal distribution will be affected by stress variables. Then, a reduced Eyring model (from Equation 2) is used to carry out a least squares fit to the log failure times across all specimens and stress conditions. Using this fitted Eyring model, the survival probability and confidence interval (CI) at the normal use condition can be calculated.

TABLE 1 Testing conditions

Test Cell	Temperature, ^0C	Relative Humidity, *%RH*	Number of Specimens
1	85	85	20
2	85	70	20
3	65	85	20
4	70	75	30

In the example provided by the standard, the specimens tested in test cells 1 and 2 are measured at 0, 250, 500, 750, and 1000 hours of testing; 0, 500, 1000, 1500, and 2000 hours in test cell 3; and 0, 625, 1250, 1875, and 2500 hours in test cell 4. The raw data table is shown in the Appendix. Figure 1 presents the degradation path of measurements in log scale versus time. After fitting a linear regression model for each specimen's measurements and predicting their failure times (ie, the time when the error rate exceeds the established threshold value that defines failure), the log median lifetimes under the 4 stress conditions are obtained, and they are used to estimate the parameters of the reduced Eyring model by least squares. Note that this is a two-step approach. First, the failure time is predicted for each test specimen, and then, these predicted failure times are treated as real failure time observations for establishing the acceleration model. The prediction error in the first step is clearly ignored in the model parameter estimation of the second step.

3.2 | Hierarchical model description

We build a hierarchical degradation model to describe the heterogeneous degradation paths presented in the ISO

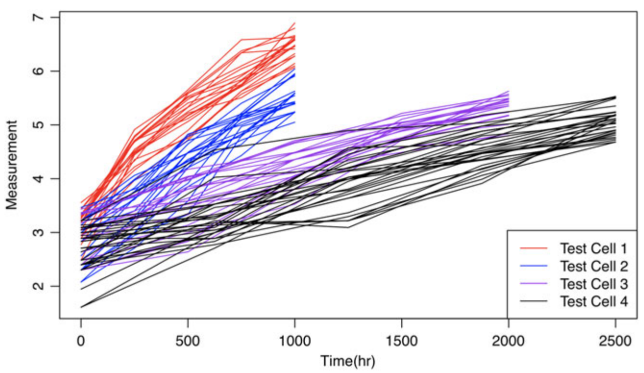

FIGURE 1 Degradation path [Colour figure can be viewed at wileyonlinelibrary.com]

example. This model has a two-level structure. On the first level, the log scale of response variable for unit i under condition j at time point k, $\log y_{ijk}$, is defined to be the sum of degradation level (which is assumed to be linear in the transformed time variable $t_{ijk}^{\gamma_i}$ and measurement error, ie,

$$\begin{aligned}\log y_{ijk} &= D_{ijk} + \varepsilon_{ijk} \\ &= \beta_{0i} + \beta_{1j} t_{ijk}^{\gamma_i} + \varepsilon_{ijk},\end{aligned} \tag{3}$$

where $i=\{1, ..., 90\}$, $j=\{1,...,4\}$, and $k=\{1,...,5\}$. The time scale transformation on t_{ijk} is applied to ensure linearity between the response and the (transformed) predictor. We assume that the measurement errors are i.i.d. normally distributed with zero mean, ie, $\varepsilon_{ijk} \sim N(0, \sigma^2)$.

On the second level, the two parameters in Model (3), β_{0i} and γ_i, are treated as random effects, thus accounting for the unit-to-unit variability. Specifically, the intercept, β_{0i}, represents the initial error rate (in log scale) that was measured prior to accelerated aging, and the scale parameter, γ_i, varies among units. Moreover, the degradation rate β_{1j} is assumed to be a function of 2 environmental stress variables. Thus, a reduced Eyring function is used here, where β_{0i}, γ_i, and β_{1j} are given by

$$\beta_{0i} = \mu_0 + \varepsilon_{0i}, \tag{4}$$

$$\gamma_i = \gamma_0 + \varepsilon_{1i}, \tag{5}$$

$$\beta_{1j} = \exp\left(\log A + B \log RH_j + \Delta H \frac{11605}{T_j + 273.15}\right), \tag{6}$$

where μ_0, γ_0, $\log A$, B, and ΔH in Equation 5 are model parameters to be estimated. The two random terms, ε_{0i} and ε_{1i}, have a bivariate normal distribution with 0 means, that is,

$$\begin{bmatrix}\varepsilon_{0i} \\ \varepsilon_{1i}\end{bmatrix} \sim N\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \begin{bmatrix}\sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2\end{bmatrix} \right).$$

3.3 | Log-likelihood function

The above two-level hierarchical model is a nonlinear mixed effects model. To infer the embedded parameters by the maximum likelihood estimation method, the log-likelihood function for the whole model needs to be specified. By decomposition, the contribution by individual observation to the total likelihood, conditioning on degradation path values, can be written as

$$L_{ijk} | D_{ijk} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\log y_{ijk} - D_{ijk})^2}{2\sigma^2}\right). \tag{7}$$

The degradation path, D_{ijk}, can be further developed as a function of random coefficients, β_{0i} and γ_i, so we have

$$L_{ijk} | \beta_{0i}, \gamma_i = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(\log y_{ijk} - \beta_{0i} - \beta_{1j} t_{ijk}^{\gamma_i}\right)^2}{2\sigma^2}\right). \tag{8}$$

By integrating out the random effects β_{0i} and γ_i, the marginal likelihood is found to be

$$L_{ijk} = \int\int \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}\left(\log y_{ijk} - \beta_{0i} - \beta_{1j} t_{ijk}^{\gamma_i}\right)^2\right) f(\beta_{0i}, \gamma_i) \quad d\beta_{0i} \quad d\gamma_i. \tag{9}$$

Here, $f(\beta_{0i}, \gamma_i)$ is the Probability Density Function (PDF) of the bivariate normal distribution such that

$$f(\beta_{0i}, \gamma_i) = \frac{1}{2\pi|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\boldsymbol{\beta} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\boldsymbol{\beta} - \boldsymbol{\mu})\right), \tag{10}$$

where

$$\boldsymbol{\beta} = \begin{bmatrix}\beta_{0i} \\ \gamma_i\end{bmatrix}$$
$$\boldsymbol{\mu} = \begin{bmatrix}\mu_0 \\ \gamma_0\end{bmatrix}$$

and

$$\boldsymbol{\Sigma} = \begin{bmatrix}\sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2\end{bmatrix}.$$

Therefore, the total log-likelihood function is given by

$$\begin{aligned}&\log L(\mu_0, \gamma_0, \log A, B, \Delta H, \sigma, \sigma_0, \sigma_1, \sigma_{01}) \\ &= \sum_i \sum_j \sum_k \log L_{ijk}.\end{aligned} \tag{11}$$

3.4 | Parameter estimation

The integrals in the above log-likelihood function cannot be evaluated analytically so some approximation methods are needed. Based on the Newton-Raphson method, Lindstrom and Bates[22] provided a two-step algorithm, which iterates between a penalized nonlinear least squares (PNLS) step and a linear mixed effects (LME) step. In the PNLS step, the estimate of variance-covariance matrix of random effects is fixed, and the conditional modes of the variance components and the conditional estimates of fixed effects are obtained by minimizing a PNLS objective function. The LME step updates the estimate of the variance-covariance matrix based on a first-order Taylor expansion of the model function around the current modes and estimates. In short, this algorithm is called the LME approximation since the second step creates an approximation to the log likelihood. In our case, the PNLS step is initiated by starting values obtained by fitting the model with only fixed effects. Next, successive iterations will be carried out until the convergence criterion is met. The computation is implemented in R, and our code is shown in the Appendix.

The estimation results are presented in Table 2. From their *P* values, one can see that all parameters are statistically significant. A normal probability plot of residuals and a plot of residuals versus predicted values, shown in Figure 2, indicate that the normality assumption of measurement error is valid.

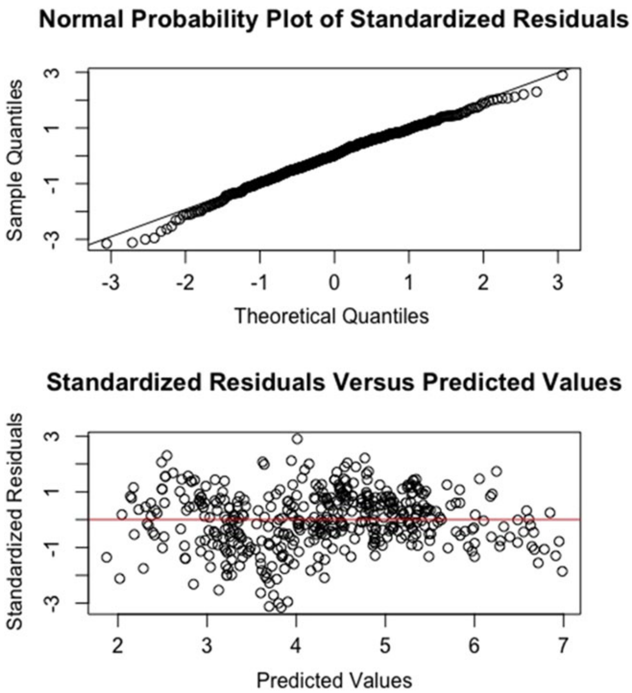

FIGURE 2 Residuals plot [Colour figure can be viewed at wileyonlinelibrary.com]

4 | MEDIAN LIFETIME PREDICTION

Because of the hierarchical nature of the model, estimating the median lifetime is a challenging problem even if all of the parameters are known. It is important to be able to estimate the median lifetime because our criterion for comparing the hierarchical model with the ISO method involves the median lifetime.

For DVD-R/-RW,+R/+RW, failure time is defined as the time when the total number of errors, PI Sum 8, reaches 280. To construct a CI for the median lifetime, we design an algorithm based on Monte Carlo simulation, which is described as follows:

1. Tables of specimen degradation paths under multiple high stress conditions using Model (1) given the parameter values in Table 2 are generated $N = 1{,}000$ times so as to simulate N experiments.
2. For each data table, the hierarchical method is performed to achieve N new sets of estimated parameters (ie, N new fitted models).
3. For each new fitted model, $M = 2{,}000$ degradation paths under use condition are simulated until the failure threshold is reached. As a result, M pseudo units are tested under the normal use condition, and their failure times are recorded. The R code to produce degradation path is shown in the Appendix.
4. Then, the median of M failure times can be found so that there are N median lifetimes being recorded. The 95% CI can be approximated by calculating the lower 2.5% quantile value and the upper 2.5% value.

TABLE 2 Parameter estimation

Parameter	Estimation	Standard Error	*P* Value
μ_0	2.846681	0.0521206	<.0001
$\log A$	4.101767	0.9697551	<.0001
B	2.722749	0.1975449	<.0001
ΔH	-0.628511	0.0225797	<.0001
γ_0	0.805419	0.0212988	<.0001
σ	0.23614788	0.004818	<.0001
σ_0	0.43406174	0.03324	<.0001
σ_1	0.02709766	0.000170	<.0001
σ_{01}	−0.0086	0.002077	<.0001

To briefly summarize the simulation algorithm, Figure 3 shown below gives a graphical explanation.

It turns out that the 95% CI of median lifetime under the normal use condition is [12.73,13.39](yr) approximately. On the contrary, the standard provides a CI to be [12.63,12.69] (yr), which is much tighter than that of the hierarchical method. This should not come as a surprise since the standard ignores the uncertainty in the projected lifetimes when treating the pseudo-failure times as actual observations. Moreover, the step of obtaining the pseudo-failure times by linear regression implies the assumption of a linear degradation path. However, according to the result of parameter estimation, the fact of $\hat{\gamma}_0$ not being unity indeed indicates the

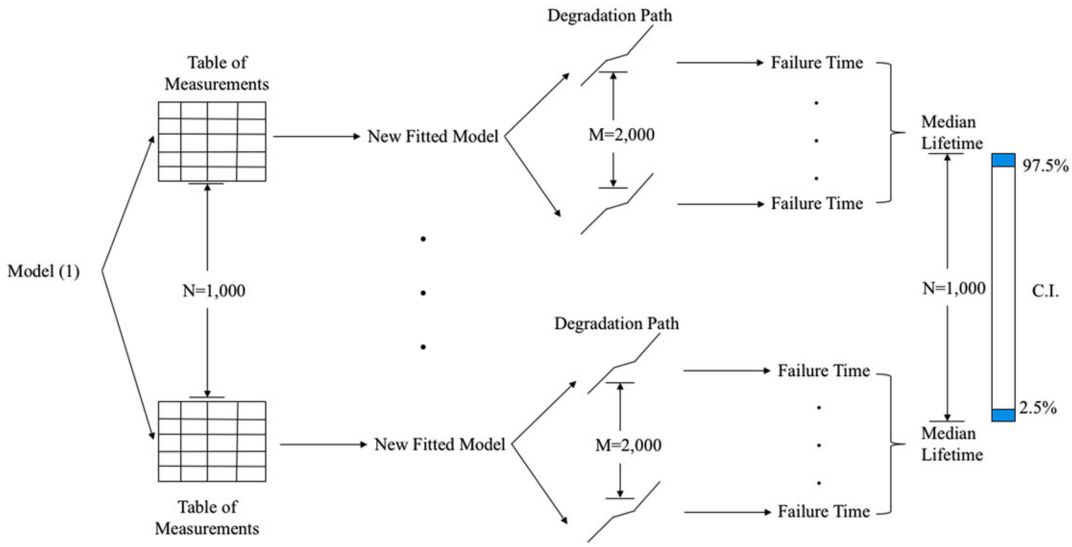

FIGURE 3 Graphical explanation of simulation algorithm [Colour figure can be viewed at wileyonlinelibrary.com]

nonlinear degradation path.This makes the CI provided by the standard skewed from that given by the hierarchical method.

5 | MODEL COMPARISON

To evaluate the performance of our hierarchical model over the method provided by ISO 10995 standard, we conduct a simulation study with the performance criterion *coverage probability* (CP), which is defined as the probability that a CI will include the true value of the parameter. In this case, we take the parameter to be the median lifetime at various conditions. Ideally, the CP should equal the nominal level for the CI, that is, a 95% CI should have a CP of about 0.95.

To approximate the CP, we ran simulations and computed the lower and upper bounds (L_k, U_k) of the CI for simulation k, $k = 1, 2, ..., K$. We then counted how often the CI covered the true value, that is,

$$CP = \frac{1}{K}\sum_{k=1}^{K} I_{\{L_k \le t_{50} \le U_k\}}. \quad (12)$$

Here, K is number of simulation runs, t_{50} is true median lifetime (approximated by the Monte Carlo algorithm described in the previous section), and $I_{\{L_k \le t_{50} \le U_k\}}$ is an indicator variable equal to 1 if the k^{th} CI contains t_{50} and 0 otherwise. The study is summarized as follows:

1. 100 000 samples of lifetime data are generated by using Model (1) and the parameter values in Table 2. Find the sample median, which is viewed as the true median life.
2. The Monte Carlo simulation mentioned in Section 4 is conducted $K = 100$ times to produce K CIs. Meanwhile, the standard method is also performed to generate CIs. For each time, the resulted CIs are checked to cover the true median life or not.
3. The CP is calculated using Equation 12.

Table 3 shows the coverage rates from the Monte Carlo study for each stress condition and for the use condition. The CP of the hierarchical method is much larger and closer to the nominal level of 95% than that provided by the standard method. As described in Section 1, by the standard method, measurements of optical media error rates are taken periodically on each disk under different stress conditions. Then, by fitting a simple linear regression model, the projected lifetime of each unit is obtained. Under the test condition 1, the highest environmental stress level results into the optical media error rates that can exceed the failure threshold in the testing period, so the estimation of lifetime is within the range of the original data. This CP is low but is a reasonably satisfactory CP of the median lifetime in the highest stress

TABLE 3 Coverage probability comparison

Condition	Hierarchical Method	Standard Method
1	86%	75%
2	90%	37%
3	92%	26%
4	100%	35%
Use	96%	0%

condition. However, as the stress level goes down, this estimation must be extrapolated to a future time, which causes poor accuracy. In other words, when these projected lifetimes are treated as real failure times, the uncertainty in the failure time prediction is ignored. Moreover, when extrapolating the median lifetime to the use condition, the standard again performs a simple regression to obtain the normal use median lifetime and ignores the uncertainties in the ADT median lifetime estimations. Combining these two issues, the large deviation of the predicted normal-use median lifetime from its true value is not surprising for the standard method. By contrast, our method accounts for both the measurement variability and unit-to-unit variability completely in a hierarchical form. This feature allows one to break a complex task into a series of manageable pieces without losing accuracy on lifetime prediction.

6 | SUMMARY

Degradation processes can be modeled by hierarchical models that involve a random intercept for each unit, and a slope that depends on the level of the environmental stress variables. Care must be taken to select a model that fits the observed data. For the case of data in the ISO10995 standard, a linear degradation path was insufficient. Instead, we had to select the power model to fit the data.

The ISO10995 standard uses a projection method to predict the time that the degradation path will exceed the threshold for the error rate. It then treats these projections as if they were the known true failure times. Such an approach is bound to produce confidence intervals that are too narrow; in other words, the parameter estimates have a smaller standard error than they should. A hierarchical model fit with the R package `nlme` can be used to find point and interval estimates of the model parameters, which can then be used to project the median lifetime. A simulation study shows that the coverage probabilities for the ISO10995 standard are much lower than the nominal 95%. The hierarchical method we suggest has coverage probabilities that are around the nominal and performs well even for use condition.

Two types of model-based extrapolation are involved in the algorithm for predicting the use-stress-level product reliability by ADTs—predicting the degradation behavior of the product under its use stress level and estimating its mean or median lifetime by extending the degradation path to failure threshold. As shown by the simulation study, using a regression method to project lifetimes at test stress levels would cause poor lifetime prediction, particularly at the lower test stress level, and consequently, it causes the use-stress-level reliability prediction to be unreliable. In contrast, the hierarchical model that includes both types of extrapolation into the model can mitigate this effect. A higher test stress level may be able to produce failure times directly, but one should be aware that the product failure mode at a very high stress level may deviate from the failure mode observed at the use stress level. Thus, lower testing stress levels are recommended. In such case, the hierarchical model will perform much better than the simple failure time extrapolation model suggested by the ISO standard.

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Guanqi Fang is a PhD student of Industrial Engineering and a Master student of Statistics at Arizona State University, concurrently. He obtained his bachelor and master degree from Hunan University of China and North Carolina State University, respectively. His interest includes reliability data analysis, stochastic simulation, and data mining.

Steven E. Rigdon is a Professor in the Department of Epidemiology and Biostatistics in the College for Public Health and Social Justice at Saint Louis University. His is also Distinguished Research Professor Emeritus at Southern Illinois University where he served on the faculty from 1986 to 2012. He is the author of *Statistical Methods for the Reliability of Repairable Systems*, published by Wiley, and Calculus, Ninth Ed. published by Pearson. He is currently the editor in chief of *Journal of Quantitative Analysis in Sports* and is a Fellow of the American Statistical Association.

Rong Pan is an Associate Professor in the School of Computing, Informatics, and Decision Systems Engineering, Arizona State University, Tempe, AZ, USA. His research interests include failure time data analysis, design of experiments, multivariate statistical quality control, time series analysis, and control. He is a senior member of IEEE, IIE, and ASQ and a member of SRE and INFORMS.

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APPENDIX A

The data from ISO 10995 is shown in Table A1.

TABLE A1 Original Data Table of Optical Medial Error Rates Provided by ISO 10995

TEMP = 85C RH = 85%							TEMP = 85C RH = 70%						
	Hours					Proj.		Hours					Proj.
Disk	0	250	500	750	1000	Failure	Disk	0	250	500	750	1000	Failure
A1	16	78	116	278	445	788	B1	10	20	67	112	156	1117
A2	25	64	134	342	532	743	B2	8	20	47	84	188	1118
A3	26	94	190	335	642	685	B3	12	26	72	185	421	880
A4	26	111	247	343	718	647	B4	20	43	120	166	219	999
A5	27	89	185	246	466	762	B5	32	45	76	103	267	1126
A6	21	111	207	567	896	607	B6	21	37	104	222	368	870
A7	26	121	274	589	781	588	B7	21	30	89	155	221	1035
A8	31	108	223	315	745	654	B8	22	26	72	125	267	1043
A9	24	118	285	723	754	578	B9	25	46	124	182	224	994
A10	12	85	178	312	988	669	B10	17	38	67	179	378	911
A11	28	111	167	312	771	671	B11	28	58	88	120	268	1065
A12	24	136	267	444	719	614	B12	8	15	36	144	189	1059
A13	35	76	265	567	610	626	B13	10	27	89	175	385	880
A14	19	53	112	278	534	778	B14	23	54	111	148	221	1037
A15	28	88	158	308	654	704	B15	28	39	125	172	278	959
A16	27	68	120	263	432	807	B16	25	53	88	130	188	1149
A17	18	87	176	302	558	723	B17	20	43	75	166	256	999
A18	26	109	238	421	641	645	B18	22	26	50	172	229	1058
A19	26	111	253	378	638	649	B19	13	38	78	124	189	1078
A20	31	91	206	367	728	656	B20	10	19	28	121	268	1046
TEMP = 65C RH = 85%							TEMP = 70C RH = 75%						
	Hours					Proj.		Hours					Proj.
Disk	0	500	1000	1500	2000	Failure		0	625	1250	1875	2500	Failure
C1	14	23	58	112	278	2057	D1	25	34	64	92	167	3240
C2	10	17	55	165	263	1948	D2	25	93	134	154	211	2596
C3	11	56	88	138	189	2078	D3	7	23	97	103	178	2615
C4	18	28	78	117	243	2106	D4	10	20	56	89	155	2920
C5	17	45	78	143	189	2167	D5	5	20	78	132	187	2496
C6	10	14	45	154	231	2031	D6	5	15	52	112	167	2644
C7	31	53	111	156	211	2151	D7	22	34	67	132	188	2851
C8	29	54	106	154	218	2128	D8	12	17	56	78	108	3318
C9	22	32	65	89	126	2799	D9	22	34	67	132	189	2847
C10	29	36	78	145	188	2297	D10	23	27	54	121	152	3129
C11	21	38	89	148	227	2075	D11	11	20	41	87	115	3249
C12	24	45	68	134	211	2236	D12	15	18	43	88	118	3343
C13	28	57	78	132	190	2352	D13	19	21	38	82	135	3435

(Continues)

TABLE A1 (Continued)

TEMP = 65C RH = 85%							TEMP = 70C RH = 75%						
	Hours					Proj.		Hours					Proj.
Disk	0	500	1000	1500	2000	Failure		0	625	1250	1875	2500	Failure
C14	19	47	61	117	150	2486	D14	18	22	86	178	245	2456
C15	25	65	89	184	256	1972	D15	22	26	73	145	252	2582
C16	10	18	57	113	178	2189	D16	18	18	29	66	127	3649
C17	21	34	45	98	121	2845	D17	22	26	93	145	178	2761
C18	12	20	34	112	176	2308	D18	18	27	56	88	134	3316
C19	28	56	108	176	243	2001	D19	11	32	44	97	143	3051
C20	29	36	57	143	238	2207	D20	12	56	66	124	249	2550
							D21	14	34	54	77	112	3500
							D22	20	23	25	50	181	3593
							D23	11	16	27	54	160	3275
							D24	17	24	25	58	108	4034
							D25	11	25	22	62	130	3488
							D26	17	24	25	70	123	3707
							D27	21	39	63	78	163	3304
							D28	20	28	45	111	243	2787
							D29	15	21	38	65	134	3453
							D30	10	34	54	96	176	2841

The R code to infer parameters is given below.

```
#Fixed effects only to get initial idea of parameters
library(nlme)
library(minpack.lm)
fm = Measurement ~ beta0 + exp(A+B*RH+H*11605/Temp)*Time^gam
#Starting value from the standard
#Use nlsLM function in minpack.lm package, which is an improved package of nlme
r1 = nlsLM(fm,data=datat,start=list(beta0=2.8,A=10,B=0.2,H=-0.726,gam=1))
# beta0 and gam as random effect
r2 = nlme(fm,data=datat,fixed = beta0+A+B+H+gam ~ 1,
         random = (beta0+gam ~ 1),
         groups = ~ Disk,
         start = coef(r1))
summary(r2)
```

The R Function to conduct a Monte Carlo experiment to generate degradation path is given below.

```
drerr = function(t,beta0,RH,Temp,A,B,H,gam,sigma){
  err = rnorm(1,0,sigma)
  beta1 = exp(A+B*log(RH)+H*11605/(Temp+273.15))
  return(exp(beta0+beta1*t^gam+err))
}
cov_var = matrix(c(fitting_value[i,6]^2,fitting_value[i,9],fitting_value[i,9],
fitting_value[i,7]^2),2,2)
beta0_gam = mvrnorm(n=1,c(fitting_value[i,1],fitting_value[i,5]),cov_var)
drerr(t_interval[k],beta0_gam[1],50,25,fitting_value[i,2],fitting_value[i,3],
                                        fitting_value[i,4],beta0_gam[2],fitting_value[i,8])
```