Dependence Modeling for Multivariate System Reliability Prediction

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SUMMARY & CONCLUSIONS

Both the reliability-wise system structure and the multivariate component lifetime distributions are required for accurately predicting a complex system's reliability. Most of existing research work either assumes these components' lifetime distributions are statistically independent or they are subject to a well-defined multivariate joint distribution, such as a multivariate Gaussian distribution. However, oftentimes the independence assumption does not match engineering practice since components usually have interactions with each other due to common manufacturing defects and shared environmental conditions, etc. On the other hand, a multivariate joint Gaussian distribution may not be adequate, because it cannot describe distribution skewness or upper/lower tail dependency among multivariate lifetime data that are often observed in real data sets. As a result, the system reliability assessment may be biased.

In this study, we present a data-centric multivariate distribution construction framework that is based on a sequence of copula functions. Under this framework, historical degradation data from different components within a system are utilized to derive the multivariate degradation model, and various types of dependency among these components are explicitly scrutinized and used for either component or system level performance prediction. Our contributions include that 1) we apply the pair copula construction (PCC) method on more than two degradation processes to explicitly model the association of these processes; 2) we connect the system structure and system failure prior information to the PCC structure to simplify the construction of multivariate distribution; and 3) we demonstrate the biasness in system reliability prediction if the dependencies existed in component failure processes are ignored. This study highlights the applicability and flexibility of the pair copula construction method for conducting multivariate reliability analysis for complex systems. A case study of degradation analysis of optical materials is used to demonstrate our proposed approach.

1 INTRODUCTION

Modern engineer system is a complex system that consists of hundreds to thousands of components, and system performance relies on the proper execution of individual component function. Therefore, a system failure could be caused by any one of numerous combinations of component malfunctions. And even worse, these system failure modes are not independent to each other due to either known or unknown interactions/interferences of component functions. As such, predicting the reliability of a complex system requires data collection from multiple sources at multiple system levels and these data are inherently correlated. At the same time, we should utilize the system knowledge extracted from system physics to support and reinforce the empirical evidence obtained from data analysis. In this paper, we explore a copula-based multivariate distribution construction technique, Pair Copula Construction (PCC), for system characterization and reliability prediction.

The copula approach to multivariate distribution modeling originates from the study of parametric bivariate distribution, which is to study how two random variables or two random processes are coupling together [1]. To extend bivariate copula (or bi-copula) to multivariate cases, the PCC method utilizes a sequence of bi-copula functions. This method was originally proposed in [2], and it has been further explored and discussed by [3-6]. More recently, it has been applied on system reliability analysis; see [7-11].

2 A MOTIVATING EXAMPLE

Optical fibers are widely used for transmitting analog and

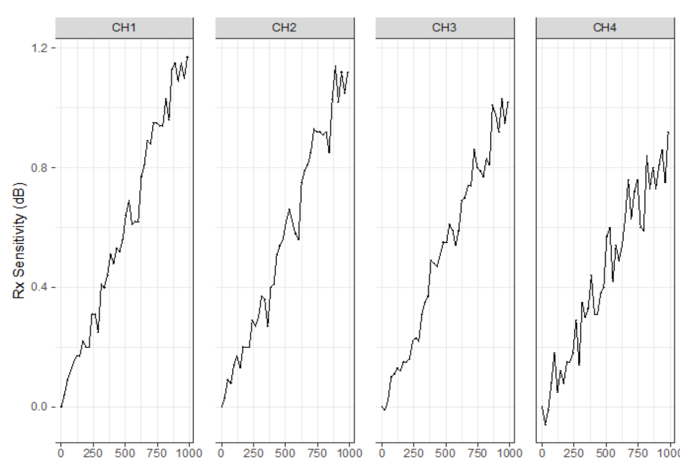


Figure 1: The Degradation Process Chart for 4 Channels

digital signals, along with transceivers being the devices of sending and receiving signals. The performance of optical fiber is usually measured by Receiver Sensitivity or so-called Rx Sensitivity, which is defined as the minimum signal optical power level required at the receiver to achieve a certain level of Bit Error Ratio (BER). Figure 1 below shows a test result of a sample transceiver. This specimen is used to convert between light signals and electric signals and is able to capture 4 types of light with different wavelength (i.e. 4 different channels). As time elapsed, each channel's Rx Sensitivity gradually deteriorates. It can be seen that the degradation processes for the 4 channels indicate a similar pattern, which implies possible underlying dependence. Thus, to evaluate the reliability of the optical system, a multivariate dependence modeling framework is needed.

3 *MUTIVARIATE DISTRIBUTION*

Bi-copula function

A copula function is defined as a multivariate distribution function with standard uniform univariate margins:

$$C(\boldsymbol{u}) = C(u_1, u_2, \dots, u_d) = P(U_1 \le u_1, U_2 \le u_2, \dots, U_d \le u_d),$$

where $\boldsymbol{U} = (U_1, U_2, \dots, U_d)^T$ is a d-diomensional random vector with $U_i \sim Unif(0,1), \forall\, i = 1,2,\dots,d$.

According to Sklar's theorem, any multivariate joint (continuous) distribution can be expressed by a copula function as follows:

Sklar's Theorem: Let $X = (X_1, X_2, \dots, X_d)^T$ be a random vector with marginal cdfs, $F_1(x_1), F_2(x_2), \dots, F_d(x_d)$, and let $F(x_1, x_2, \dots, x_d)$ be their joint cdf. Define $u_i = F_i(x_i) = P(X_i \le x_i), \forall i = 1,2,\dots,d$. Then, there exists a copula C such that

$$\begin{aligned} C(u_1, u_2, \dots, u_d) &= C\big(F_1(x_1), F_2(x_2), \dots, F_d(x_d)\big) \\ &= P(X_1 \le x_1, X_2 \le x_2, \dots, X_d \le x_d) \\ &= F(x_1, x_2, \dots, x_d)\,. \end{aligned} \tag{1}$$

In other words, any continuous univariate random variable can be transformed to a continuous uniform random variable via its marginal distribution function, and then a copula function can assemble a group of random variables to form a multivariate distribution. Therefore, the copula function characterizes the intrinsic dependency between random variables, which is separated from the marginal distribution of individual variable. Defining a copula function is equivalent to defining the intrinsic dependency between random variables, while the marginals of these variables could be any continuous distribution. In fact, marginal distribution can be directly determined by fitting the data collected for the variable of interest.

Consider a bivariate distribution. Eq. (1) becomes a bi-copula such that $C: [0,1]^2 \to [0,1]$

$$C(u_1, u_2) = F(x_1, x_2). \tag{2}$$

Subsequently, copula density function can be defined as

$$c(u_1, u_2) = \frac{\partial^2 C(u_1, u_2)}{\partial u_1 \partial u_2}, \tag{3}$$

which is the same as the joint probability density function (pdf) of u_1 and u_2. A further derivation of Eq. (2) shows that multiplying this copula density function with marginal densities of individual variables yields the joint density function of these variables. That is,

$$\begin{aligned} f(x_1, x_2) &= \frac{\partial^2 F(x_1, x_2)}{\partial x_1 \partial x_2} \\ &= \frac{\partial^2 C\big(F_1(x_1), F_2(x_2)\big)}{\partial F_1(x_1) \partial F_2(x_2)} \frac{\partial F_1(x_1)}{\partial x_1} \frac{\partial F_2(x_2)}{\partial x_2} \\ &= c\big(F_1(x_1), F_2(x_2)\big) f_1(x_1) f_2(x_2), \end{aligned} \tag{4}$$

where $f_1(x_1)$ and $f_2(x_2)$ are the marginal pdf's of X_1 and X_2, respectively.

Some common bi-copula functions (in short, copula functions) are given in Table 1 below.

Table 1: Commonly-used Bi-Copula Functions

Copula	$C(\boldsymbol{u})$
Gaussian	$\Phi_{\Sigma}\big(\Phi^{-1}(u_1), \Phi^{-1}(u_2)\big)$
Student's *t*	$T_{\Sigma,v}\big(T_v^{-1}(u_1), T_v^{-1}(u_2)\big)$
Frank	$-\frac{1}{\delta}\ln\left\{1 + \frac{[\exp(-\delta u_1) - 1][\exp(-\delta u_2) - 1]}{(\exp(-\delta) - 1)}\right\}$
Clayton	$\left(u_1^{-\delta} + u_2^{-\delta} - 3\right)^{-\frac{1}{\delta}}$
Gumbel	$\exp\left\{-\left[(-\ln u_1)^{\delta} + (-\ln u_2)^{\delta}\right]^{\frac{1}{\delta}}\right\}$

Note that for bivariate Gaussian or Student's t distribution, if the marginal distributions of both individual variables are univariate Gaussian or Student's t distribution, then the bi-copula function is indeed parametric bivariate Gaussian or bivariate Student's t distribution function. The density functions of these two distributions are bell-shaped and symmetric and there is no tail dependency in bivariate Gaussian distribution.

Although Frank copula does not have tail dependence either, it does have a more squared, wider-spread distribution shape. Clayton copula has lower-tail dependency, thus the two random variables (marginally with uniform distribution) are more correlated with each other when they are having smaller values (close to 0). In contrast, Gumbel copula has upper-tail dependency, so the two random variables are more correlated at

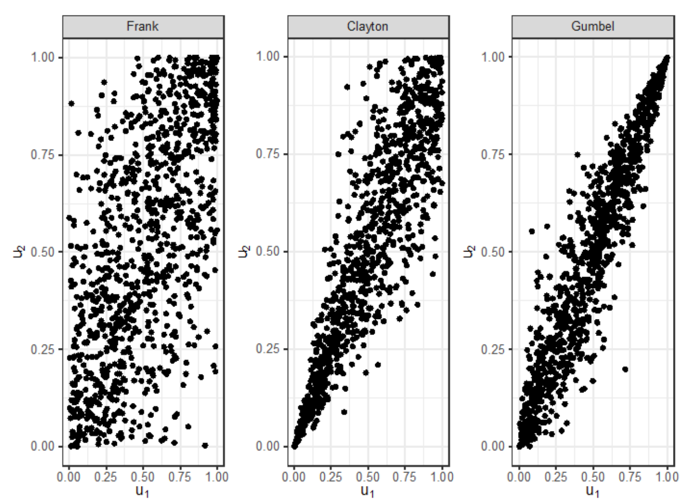


Figure 2: Scatter Plots for Frank, Clayton, and Gumbel Copula Densities

larger values (close to 1). See Figure 2 for these copulas. The data plotted in these graphs are the simulated data from each corresponding copula function. Note that the lower-tail or upper-tail association patterns of two random variables cannot be captured by any bivariate Gaussian or Student-t distribution.

Pair copula construction

To model the dependency among more than two random variables, a flexible multivariate distribution construction method is introduced in this section. First, bi-copulae, as described in the last section, are used to define the relationship between any two random variables. Next, a systematic approach to be discussed below constructs any general multivariate distribution.

The pair copula construction (PCC) method utilizes the natural data structure in an application and existing copula functions to embed one cupula function within another one, so as to construct a larger model for more than two random variables. It involves a sequence of copula functions, with most of them applied to pairs of univariate conditional distributions.

As Sklar's theorem can be applied on a set of univariate conditional distributions, all conditioning on variables in an index set *S*, a sequential mixture of conditional distributions leads to the pair copula construction.

Consider d random variables $X_1, X_2, \dots, X_d$ with multivariate distribution *F*. If we separate these variables to two groups, say g_1 and g_2, then Sklar's theorem implies that there is a copula $C_{g_1;g_2}(.\,; \boldsymbol{x}_{g_2})$ such that

$$F_{g_1|g_2}(\boldsymbol{x}_{g_1}|\boldsymbol{x}_{g_2}) = C_{g_1;g_2}(F_{j|g_2}(x_j|\boldsymbol{x}_{g_2}): j \in g_1;\ \boldsymbol{x}_{g_2}),$$

where $F_{j|g_2}(x_j|\boldsymbol{x}_{g_2})$ is the univariate conditional distribution of x_j, for $x_j \in g_1$, conditioning on the group g_2. Jointly, the multivariate distribution *F* can be expressed as

$$F_{\boldsymbol{X}}(\boldsymbol{x}) = \int C_{g_1;g_2} dF_{g_2}(\boldsymbol{x}_{g_2}).$$

For variables in g_1 and g_2, we can continue to separate them into two distinct groups and construct the copula functions for them, until each group consists of only two variables. Therefore, there are many possible ways to permutate these variables to be grouped and nested together, to construct a multivariate distribution.

As an example, consider four random variables X_1, X_2, X_3 and X_4. One possible multivariate distribution construction is as follows:

(1) $F_{12} = C_{12}(F_1, F_2)$
(2) $F_{23} = C_{23}(F_2, F_3)$
(3) $F_{34} = C_{34}(F_3, F_4)$
(4) $F_{13|2} = C_{13;2}(F_{1|2}, F_{3|2})$
(5) $F_{24|3} = C_{24;3}(F_{2|3}. F_{4|3})$
(6) $F_{14|23} = C_{14;23}(F_{1|23}, F_{4|23})$

Finally, we integrate $F_{13|2}$ with respect to f_2 (marginal density of X_2) to obtain F_{123} (joint distribution function of X_1, X_2, X_3), integrate $F_{24|3}$ with respect to f_3 to obtain F_{234}, and integrate $F_{14|23}$ with respect to f_{23} to obtain F_{1234}. This construction method can be illustrated by Figure 3 and it is called D-vine (D for drawable).

Similarly, the pairing of these variables can be arranged in a different manner such as:

(1) $F_{12} = C_{12}(F_1, F_2)$
(2) $F_{13} = C_{13}(F_1, F_3)$
(3) $F_{14} = C_{14}(F_1, F_4)$
(4) $F_{23|1} = C_{23;1}(F_{2|1}, F_{3|1})$
(5) $F_{24|1} = C_{24;1}(F_{2|1}, F_{4|1})$
(6) $F_{34|12} = C_{34;12}(F_{3|12}, F_{4|12})$

And the joint distribution function F_{1234} can be obtained by integration $F_{34|12}$ with respect to the density function f_{12}. Graphically, this construction method is shown in Figure 4 and it is called C-vine (C for "Canonical").

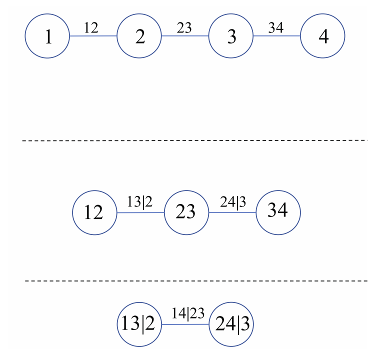


Figure 3: D-vine of 4 variables

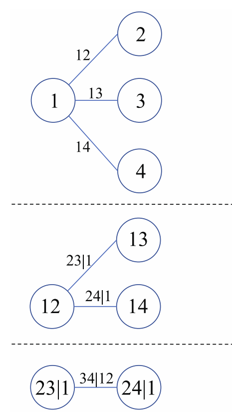


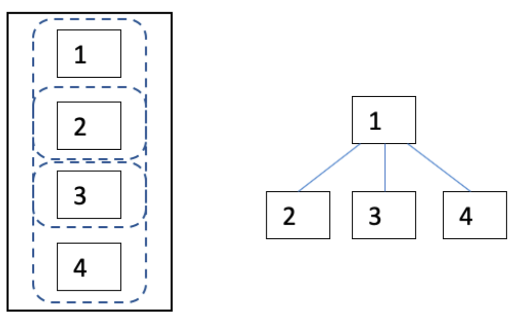
Figure 4: C-vine of 4 variables

Both D-vine and C-vine utilize six copula functions and they are flexible enough to characterize various association patterns among different variables. Therefore, they are examples of PCC. Obviously, there could be many other ways to construct this 4-variate distribution. However, for studying system reliability, we can make use of the physical configuration of the system and/or the data proximity among these variables to select a sounded multivariate distribution construction procedure, and, consequently, connect the system lifetime behavior to its physical or data models more closely.

PCC pairing structure

Choosing a specific PCC pairing structure seems to be a daunting task. However, with enough understanding of the context of the system under study, we may conveniently integrate some prior information into the PCC structure to make it theoretically sounded and computationally simpler at the same time. For instance, consider a safety-critical system with hot standbys as illustrated in Figure 5(a), where there are always two components – primary and secondary components – sharing loads at the same time, and when the primary is failed, the secondary becomes the primary one and the next standby component moves to take the secondary role, and so on. Therefore, the lifetimes of the first and second components should be closely correlated since they are sharing the same load in the same time period. With the same argument, the second and third components or the third and fourth components are closely related. Therefore, D-vine is a proper choice here.

As another example, suppose a system has multiple components and the failure of each component may induce a complete or partial failure of the whole system. Figure 5(b) represents the relationship between system-level failures (Block 1) and component-level failures (Blocks 2, 3 and 4). To



(a) (b)

Figure 3: (a) safe-critical system with hot standbys; (b) system-component failure relationship.

investigate the relationship between system-level performance and component failures, C-vine becomes a handy tool for this case, because the dependencies between 1 and 2, 1 and 3, and 1 and 4 become more important.

In fact, D-vine and C-vine are well defined structures that can be represented by matrices with regular patterns. Consider the 4-variate D-vine example aforementioned. Using an upper triangle matrix to express the pairs of variables at each level, we have

$$\begin{bmatrix} - & 12 & 23 & 34 \\ & - & 13|2 & 24|3 \\ & & - & 14|23 \\ & & & - \end{bmatrix}, \tag{5}$$

and it can be more succinctly written as

$$\begin{bmatrix} 1 & 1 & 2 & 3 \\ & 2 & 1 & 2 \\ & & 3 & 1 \\ & & & 4 \end{bmatrix}, \tag{6}$$

so that on the top line of Matrix (5) the pairs can be constructed by the variables appearing on the diagonal of Matrix (6) and the variables on the top line of Matrix (6), and for subsequent lines, the entries of Matrix (5) are again formed by the variables on this line on Matrix (6) and its diagonal variable while conditioning on the variables above this line.

For C-vine, these matrices become

$$\begin{bmatrix} - & 12 & 13 & 14 \\ & - & 23|1 & 24|1 \\ & & - & 34|12 \\ & & & - \end{bmatrix} \tag{7}$$

or

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ & 2 & 2 & 2 \\ & & 3 & 3 \\ & & & 4 \end{bmatrix} \tag{8}$$

The structure patterns of matrices (6) and (8) are clear to see. The variable indices appear in order on diagonals of both matrices. In D-vine matrix, off-diagonal elements are filled with variables with a same-number-on-slant pattern, while in C-vine matrix, it is a same-number-on-horizonal pattern. Thus, the construction process of these regular PCCs follows certain rules and can be automated.

4 SYSTEM RELAIBILITY

Define a generic system state function to be an indicator such that it equals to 1 when the system is working at a given time *t*, and 0 otherwise. That is,

$$\phi_s(t) = \begin{cases} 1 & if\ system\ is\ working \\ 0 & if\ system\ is\ failed \end{cases}$$

The same type of state function can be defined for each component in the system.

$$\phi_i(t) = \begin{cases} 1 & if\ component\ i\ is\ working \\ 0 & if\ component\ i\ is\ failed \end{cases}$$

For a coherent system, the system's state is determined by its components' states in the sense that the system is working only if the combination of component states satisfies certain requirements. That is,

$$\phi_s(t) = \begin{cases} 1 & if\ (\phi_1(t), \dots, \phi_d(t))\ meet\ survival\ condition \\ 0 & otherwise \end{cases}$$

For example, for a series system this requirement requires all components must be working, while for a parallel system this requirement requires only one working component. Obviously, other complicated requirements may be imposed on a complex system based on the system design.

Without loss of generality, a component's survival state can be defined by requiring the component's performance characteristic to be less than a threshold value. Thus, the reliability of a working component at time *t* is given by

$$R_i(t) = P(\phi_i(t) = 1) = P(X_i(t) < x_i^T)$$

where x_i^T is the threshold value for the *i*-th component's performance characteristic. Consequently, system reliability becomes

$$R_s(t) = P(\phi_s(t) = 1) = \\ P\left((\phi_1(t), \dots, \phi_d(t))\ meet\ survival\ condition\right) = \\ P\big((X_1(t), \dots, X_d(t))\ satisfy\ some\ requirements \\ in\ relations\ to\ their\ threshold\ values\big)$$

For a series system the system reliability is given by

$$R_s(t) = P(X_1(t) < x_1^T, \dots, X_d(t) < x_d^T)$$

And for a parallel system, it is

$$R_s(t) = 1 - P(X_1(t) > x_1^T, \dots, X_d(t) > x_d^T)$$

Therefore, we can see that when the multivariate distribution of $X_1, \dots, X_d$ has been fully specified by a multivariate copula function, system reliability is ready to be evaluated.

5 DATA ANALYSIS

In this section, we revisit the motivating example and carry out its data analysis. The R package, VineCopula, has been used to assist with the analysis.

First of all, observing that the Rx Sensitivity of the four channels indicates nonmonotone trend, a well-known statistical tool – the Wiener process – is suitable for modeling the degradation processes. The model we assume is $\Delta Y_i(t_j) \sim N\left(\mu_i \Delta\Lambda(t_j, \gamma_i), \sigma_i^2 \Delta\Lambda(t_j, \gamma_i)\right)$, where $\Delta Y_i(t_j)$ is the degradation increment for channel i, $i = 1,2,\dots,4$, at time t_j, $j = 1,2,\dots,41$. $\Delta\Lambda(t_j, \gamma_i) = t_j^{\gamma_i} - t_{j-1}^{\gamma_i}$ is a time-scale transformation function to linearize the degradation processes. $\mu_i \in \mathbb{R}$ is the location parameter and $\sigma_i > 0$ is the scale parameter. Table 2 provides the results of Wiener process parameter estimation for the four channels' marginal degradation processes.

Table 2: Results of Parameters Estimation for Marginal Degradation Processes.

Channel	μ	σ	γ
1	6.35×10^{-4}	8.50×10^{-3}	1.09
2	2.14×10^{-4}	5.80×10^{-3}	1.21
3	1.00×10^{-4}	4.13×10^{-3}	1.29
4	1.98×10^{-4}	9.92×10^{-3}	1.22

Next, based on the marginal information, we calculate the corresponding univariate distribution function values and draw a scatter plot of the pairs of pseudo observations; see Figure 6.

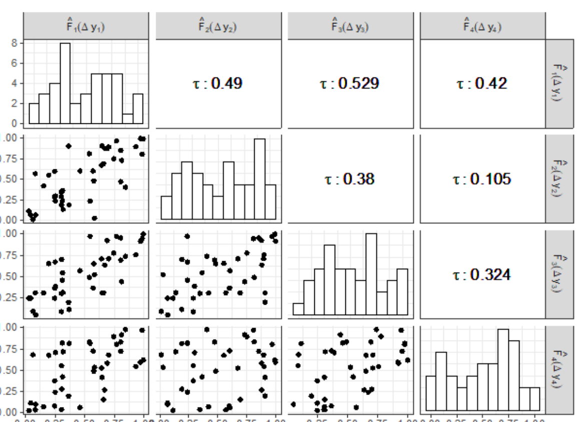


Figure 4: Scatter Plot of Pseudo Observations.

Notice that Channel 1 has strong correlations with all other channels. A C-vine model is appropriate to build such a dependency relationship. Meanwhile, we also check the tail dependence for each pair of variables. It is found that the pair of Channels 1 and 4 have similar upper-tail and lower-tail dependences. For other pairs, stronger upper-tail dependences are present. Thus, we chose the Frank copula for the pair of Channels 1 and 4 and the Gumbel copula for other pairs. The parameter estimation for the C-vine model gives $\hat{\delta}_{12} = 1.83, \hat{\delta}_{13} = 1.85, \hat{\delta}_{14} = 3.99, \hat{\delta}_{23|1} = 1.1, \hat{\delta}_{34|1} = 1.0,$ and $\hat{\delta}_{24|13} = 1.0$. Here, $\hat{\delta}$ is called the association parameter and a large value indicates a stronger association between two variables.

Finally, given the marginal and joint model, we carry out reliability analysis for the system, assuming that the system requires all four working channels (i.e., a series system). Thus, if any channel's Rx Sensitivity passes its failure threshold (in this case, 1 dB is the thresholds for all channels), the system is failed. Figure 7 has the reliability curves of system reliability functions under the dependent multivariate distribution and the independent distribution assumption, respectively. Notice that if the dependencies among four channels (which have been clearly shown through our data analysis) are ignored, the system reliability would be significantly underestimated. Underestimating a system's reliability may severely affect its maintenance plan and increase its maintenance cost. For example, one may wrongly replace the cable more frequently than actually needed.

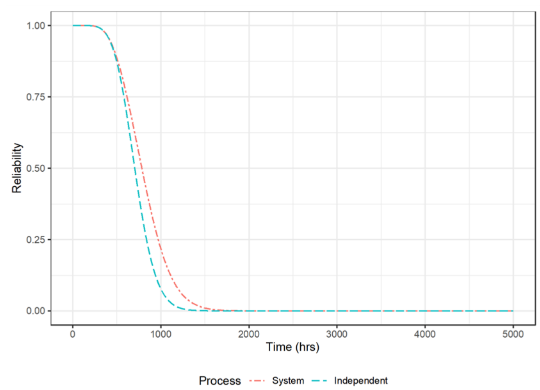


Figure 5: System Reliability Function.

6 CONCLUSION

In this paper we introduce the PCC method for constructing any multivariate distribution and apply this multivariate distribution on system reliability assessment. We argue that given the knowledge of system structure and failure causes, it is often possible to choose a specific order of copula functions that is theoretically sounded and computationally manageable. This method is also data-centric, thus the data collected from the system may lead to a natural, and oftentimes simpler, variable pairing structure.

The contribution of this paper to the reliability literature is three-folded. First, we use PCC method explicitly model the association among more than two degradation processes, which has not been done before. Second, we show that the PCC structure selection can be greatly simplified by cleverly utilizing our prior knowledge of system structure and system failure. Lastly, we demonstrate the biasness in system reliability prediction if the dependencies existed in component failure processes are ignored. In the demonstrative example of optical fiber system degradation data analysis, it has been shown that the proposed method can adequately capture the tail dependencies among data from different channels that cannot be done by traditional means. It is important to include these dependencies in system reliability assessment to avoid any potential biases.

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