

## Enhancing Entangled Two-Photon Absorption for Picosecond Quantum Spectroscopy

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Supporting Information

**ABSTRACT:** Entangled two-photon absorption (ETPA) is known to create photoinduced transitions with extremely low light intensity, reducing the risk of phototoxicity compared to classical two-photon absorption. Previous works have predicted the ETPA cross-section,  $\sigma_e$ , to vary inversely with the product of entanglement time ( $T_e$ ) and entanglement area ( $A_e$ ), i.e.,  $\sigma_e \sim 1/A_e T_e$ . The decreasing  $\sigma_e$  with increasing  $T_e$  has limited ETPA to fs-scale  $T_e$ , while ETPA applications for ps-scale spectroscopy have been unexplored. However, we show that spectral–spatial coupling, which reduces  $A_e$  as the SPDC bandwidth ( $\sigma_f$ ) decreases, plays a significant role in determining  $\sigma_e$  when  $T_e > \sim 100$  fs. We experimentally measured  $\sigma_e$  for zinc tetraphenylporphyrin at several  $\sigma_f$  values. For type-I ETPA,  $\sigma_e$  increases as  $\sigma_f$  decreases down to  $0.1 \text{ ps}^{-1}$ . For type-II SPDC,  $\sigma_e$  is constant for a wide range of  $\sigma_f$ . With a theoretical analysis of the data, the maximum type-I  $\sigma_e$  would occur at  $\sigma_f = 0.1 \text{ ps}^{-1}$  ( $T_e = 10 \text{ ps}$ ). At this maximum,  $\sigma_e$  is 1 order of magnitude larger than fs-scale  $\sigma_e$  and 3 orders of magnitude larger than previous predictions of ps-scale  $\sigma_e$ . By utilizing this spectral–spatial coupling, narrowband type-I ETPA provides a new opportunity to increase the efficiency of measuring nonlinear optical signals and to control photochemical reactions requiring ps temporal precision.

Entangled two-photon absorption (ETPA) was shown to be measurable in atoms<sup>1,2</sup> and organic chromophores<sup>3</sup> using extremely low light intensity more than a decade ago. The well-known linear absorption rate for ETPA vs photon flux significantly enhances the ETPA rate compared to the classical two-photon absorption (TPA) rate at these low light intensities.<sup>4,5</sup> To date, ETPA has only been performed in the fs scale of the fourth-order temporal correlation, or entanglement time,  $T_e$ . ETPA with ps  $T_e$  would be beneficial for probing chemical processes such as isomerization,<sup>6,7</sup> proton transfer,<sup>8,9</sup> water solvation,<sup>10,11</sup> and ligand binding/unbinding.<sup>12,13</sup> However, previous reports on ETPA suggest that ETPA would not be measurable practically with ps-scale  $T_e$  because the ETPA cross-section,  $\sigma_e$ , was predicted to decrease with increasing  $T_e$ .<sup>14–18</sup> Other parameters of SPDC, such as the fourth-order transverse spatial correlation, or entanglement area,  $A_e$ , may be used to counteract the effect of increasing  $T_e$ .

With spontaneous parametric down-conversion (SPDC) as the entangled photon source, the ETPA cross-section with a Gaussian frequency filter has been derived previously as<sup>19,20</sup>

$$\sigma_e = \frac{\sqrt{2} \omega_s^0 \omega_i^0}{\hbar^2 \epsilon_0^2 c^2 A_e T_e} \frac{k_f}{(\epsilon_f - \epsilon_g - \omega_s^0 - \omega_i^0)^2 + \left(\frac{k_f}{2}\right)^2} \times \left| \sum_j \mu_{fg} \mu_{ig} T_e F[(\Delta_j - ik_j/2) T_e] \right|^2 \quad (1)$$

Here,  $\epsilon_g$ ,  $\epsilon_j$ , and  $\epsilon_f$  are the energies of the molecule's ground, intermediate, and final states, respectively;  $\mu_{ig}$  and  $\mu_{fg}$  are the transition dipole moments (TDMs) from the ground-to-intermediate and intermediate-to-final states, respectively;  $\omega_{s/i}^0$  are the central frequencies of the signal/idler entangled

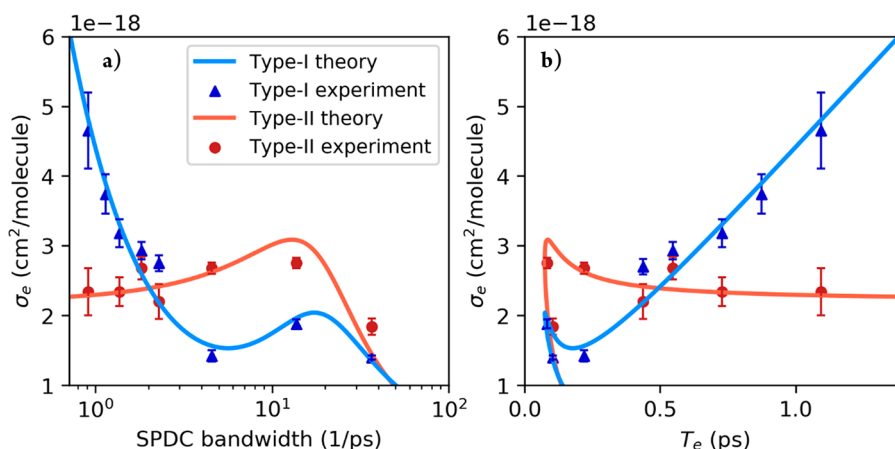
photons, respectively;  $\Delta_j = \epsilon_j - \epsilon_g - \omega_s^0$  is the detuning energy; and  $F$  is the plasma dispersion function. We assume a monochromatic pump. The molecule's excited states are Lorentzian broadened with line widths  $k_j$  and  $k_f$  for the intermediate and final states, respectively. Previous work showed that the lifetime of the final ETPA excited state can be calculated using the spontaneous radiative lifetime for two-photon emission.<sup>17</sup>

While eq 1 provides the exact  $\sigma_e$  dependence on  $T_e$ , a simpler probabilistic model is used to describe  $\sigma_e$  when the  $T_e$  dependence of the modulus squared term is negligible ( $T_e > \sim 500$  fs):<sup>14</sup>

$$\sigma_e = \frac{\delta_r}{2A_e T_e} \quad (2)$$

Here  $\delta_r$  is the classical TPA cross-section. In previous theoretical<sup>14–17</sup> and experimental<sup>3,21,22</sup> reports, it was assumed that  $T_e$  can be changed while  $A_e$  remains constant. From eq 2, if  $A_e$  is constant,  $\sigma_e$  decreases as  $T_e$  increases. However, there exists a coupling between the spectral and spatial properties of the entangled photon wave function (see the SI).<sup>14</sup> To change  $T_e$ , the bandwidth of the photons from SPDC must change, which should also change  $A_e$ . While this spectral–spatial coupling has been discussed in the literature before,<sup>23,24</sup> how

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**Figure 1.** (a) Experimental and theoretical type-I and type-II ETPA cross-sections,  $\sigma_e$ , vs  $\sigma_f$  for a  $\sim 20 \mu\text{M}$  solution of zinc tetraphenylporphyrin (ZnTPP). (b) Same as (a) with the  $x$ -axis converted to  $T_e$ .

the coupling affects the ETPA cross-section has not been considered nor explored experimentally.

Here we consider the effect of a spectral filter, with bandwidth  $\sigma_f$  that is applied to the SPDC photons after they emerge from the SPDC process. Full details of our experimental setup have been reported previously<sup>25</sup> and can be found in the SI. In Figure 1a, the experimental  $\sigma_e$  for type-I and type-II filtered SPDC are plotted as a function of  $\sigma_f$  ranging from 0.9 to 40  $\text{ps}^{-1}$ . In our passive filtering technique, where SPDC photons are produced in a crystal and filtered after their generation, the input photon rate for ETPA decreases with decreasing  $\sigma_f$ . We found that we can choose  $\sigma_f$  as small as 0.9  $\text{ps}^{-1}$  before the input rate was too small to measure an ETPA signal. Due to the limited wavelength detection range of APDs, we could not obtain  $\sigma_e$  at  $\sigma_f > 40 \text{ ps}^{-1}$ . Given that eqs 1 and 2 show that  $\sigma_e$  varies inversely with  $T_e$ , it is of interest to convert Figure 1a to show dependence on  $T_e$  (Figure 1b). Standard filter theories show that  $T_e$  varies inversely with  $\sigma_f$ <sup>26–28</sup> which can be controlled by the bandwidth of a filter or monochromator. Dispersion was considered when calculating  $T_e$  (see SI). The smooth curves in this figure are the results of an analytical theory that we describe below.

The experimental data in Figure 1 show that for type-I SPDC at small  $\sigma_f$ ,  $\sigma_e$  increases as  $\sigma_f$  decreases ( $T_e$  increases). For type-II SPDC at small  $\sigma_f$ ,  $\sigma_e$  is constant as  $\sigma_f$  decreases. Not only do the two SPDC types differ from each other in their cross-section trends, they are also incompatible with the constant  $A_e$  prediction that  $\sigma_e$  should decrease as  $\sigma_f$  decreases. We note that dispersion leads to two branches in the curves in Figure 1b: an upper branch corresponding to small  $\sigma_f$  in which  $T_e$  is determined by  $T_e = 1/\sigma_f$  and a lower branch corresponding to large  $\sigma_f$  where dispersion dominates, and  $T_e$  is proportional to  $\sigma_f$ . Figure 1 shows that both branches are involved in the data fit. However, we are only concerned with the small  $\sigma_f$  branch. Even with dispersion considered, the variation of  $\sigma_e$  with  $\sigma_f$  is not explained by the assumption of a constant  $A_e$ . We then consider the spectral–spatial coupling and how changing  $\sigma_f$  would change  $A_e$ . The solid curves in Figure 1 are the theoretical model in eq 1 including the spectral–spatial coupling explained below.

The origin of  $A_e$  is the uncertainty in the SPDC emission angles from the crystal source, as determined by the polar and azimuthal entanglement angles,  $\theta^e$  and  $\varphi^e$ , respectively.<sup>29,30</sup>

Because each frequency within the SPDC frequency superposition experiences a different refractive index inside the crystal, each frequency propagates at a different angle. Therefore, uncertainty in the frequency causes an uncertainty in the propagation angle and ultimately the spatial position. Decreasing the frequency uncertainty ( $\sigma_f$ ) is then expected to decrease  $A_e$ , which should increase  $\sigma_e$ . However, decreasing  $\sigma_f$  also increases  $T_e$ , which should decrease  $\sigma_e$ . From the experimental data in Figure 1a, for type-II SPDC, these competing effects cancel each other, leaving  $\sigma_e$  constant for small  $\sigma_f$ . For type-I SPDC, the decreasing  $A_e$  outweighs the increasing  $T_e$ , resulting in a net increase in  $\sigma_e$ . We propose the following relationships between  $A_e$  and the SPDC frequency bandwidth (for small  $\sigma_f$ ) for type-I (eq 3) and type-II (eq 4) (see SI for more details):

$$A_e = \left( \frac{\alpha}{\sigma_f^2} + \frac{\beta}{\sigma_L^2} \right)^{-1} + A_d \propto (\alpha T_{e,f}^2 + \beta T_{e,L}^2)^{-1} + A_d \quad (3)$$

$$A_e = \left( \frac{\gamma}{\sigma_f} + \frac{\zeta}{\sigma_L} \right)^{-1} + A_d \propto (\gamma T_{e,f} + \zeta T_{e,L})^{-1} + A_d \quad (4)$$

$\sigma_L$  is the SPDC bandwidth emitted by the crystal with length,  $L$ , and represents the upper limit for  $\sigma_f$  in our experiment.  $T_{e,L}$  is the corresponding entanglement time. The term  $A_d$  recognizes that there is a lower bound to  $A_e$  that is determined by diffraction. The coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\zeta$  are solved for by fitting with the experimental data in Figure 1. Substituting eqs 3 and 4 into eq 2 provides new probabilistic models (to highest order in  $T_e$ ) for type-I (eq 5) and type-II (eq 6) ETPA, respectively:

$$\sigma_e \propto \delta_f T_{e,f} \quad (5)$$

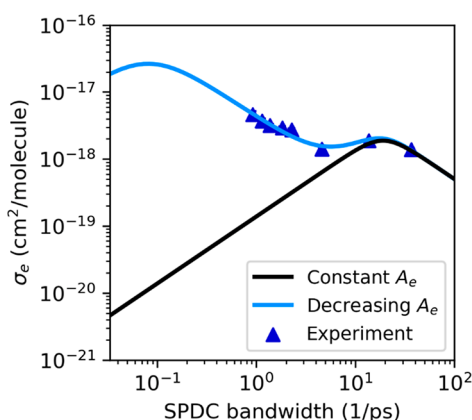
$$\sigma_e \propto \delta_f \quad (6)$$

Equations 5 and 6 predict the  $\sigma_e$  vs  $T_e$  trends (for small  $\sigma_f$ ) that we see with the experimental data in Figure 1b for both SPDC types. To confirm the validity of eqs 5 and 6, we substitute the equations into eq 1 and fit the resulting equations with our experimental data to determine the coefficient values and extract  $A_e$ . See the SI for calculation parameters. The models are accurate until  $\sigma_f$  decreases to the

point where  $A_e$  reaches  $A_d$ , as determined by eqs 3 and 4, which we calculate below to occur around  $T_e = 10$  ps.

The resulting fits in Figure 1 agree reasonably well with the experimental data. The trends for  $\sigma_i < 5$  ps<sup>-1</sup> are the same if a sinc SPDC spectrum is considered (see the SI). For  $\sigma_i > 10$  ps<sup>-1</sup>, dispersion significantly increases  $T_e$  and decreases  $\sigma_e$  (bottom branch in Figure 1b). For  $\sigma_i$  in the range 0.9–40 ps<sup>-1</sup>, the type-I  $A_e$  ranges from  $1.95 \times 10^{-7}$  to  $7.39 \times 10^{-6}$  cm<sup>2</sup>, and the type-II  $A_e$  ranges from  $4.12 \times 10^{-7}$  to  $6.91 \times 10^{-6}$  cm<sup>2</sup>. These ranges are within an order of magnitude of the assumption of  $A_e = 1 \times 10^{-6}$  cm<sup>2</sup> in previous theoretical works for fs-scale  $T_e$ .<sup>14,16,17</sup>

Figure 2 shows how the type-I ETPA cross-section changes as the SPDC bandwidth is decreased until  $A_e$  reaches the



**Figure 2.** Comparison of the type-I ETPA experimental data with the theoretical model (eq 1) for constant  $A_e$  (black) and decreasing  $A_e$  (blue).

diffraction limit ( $(\lambda/2)^2 = 1.6 \times 10^{-9}$  cm<sup>2</sup>). For comparison, the same theoretical model (eq 1) but using a constant  $A_e$  is shown.

In Figure 2, at the smallest experimental bandwidth (0.9 ps<sup>-1</sup>), the model with a constant  $A_e$  is more than an order of magnitude smaller than the experimental value. For the decreasing  $A_e$ , the maximum occurs at  $\sigma_i = 0.1$  ps<sup>-1</sup> ( $T_e = 10$  ps), and here  $\sigma_e$  is  $\sim 3 \times 10^{-17}$  cm<sup>2</sup>, which is over 3 orders of magnitude larger than what the constant  $A_e$  predicts. Comparison of ETPA with one-photon absorption (OPA) at the intermediate state has previously been explored.<sup>20,31,32</sup> In this report, we considered a molecular system where the intermediate states are far off-resonance with the single photon energies, such that OPA at the intermediate states is negligible. At the maximum,  $\sigma_e$  for ZnTPP is within an order of magnitude of its final-state OPA cross-section ( $\sim 2 \times 10^{-16}$  cm<sup>2</sup>/molecule at 405 nm),<sup>33</sup> rather than 2 orders of magnitude smaller in the tens of ps<sup>-1</sup> range where ETPA experiments are performed today.<sup>43–45</sup> To obtain enough narrowband photons to measure an ETPA signal at  $\sigma_i < 0.9$  ps<sup>-1</sup>, an active filtering technique using a cavity-enhanced SPDC source would be useful.<sup>34</sup> The SPDC photons can have a  $\sigma_i$  of MHz or smaller,<sup>35–40</sup> making  $T_e$  as large as 1  $\mu$ s for visible wavelengths.<sup>41</sup> Alternatively, passive filtering of a periodically poled SPDC source could be used since they can provide a 3 orders of magnitude larger SPDC rate while using the same pump power as our thin crystal source.<sup>42</sup>

In the SI, experimental ETPA cross-sections for flavin dinucleotide (FAD) are provided, showing the same cross-

section trends as ZnTPP for type-I and type-II SPDC. For these chromophores, which are much smaller than  $A_e$  in the experiment and the diffraction-limited  $A_e$ , the  $T_e$  with the maximum ETPA cross-section in the ps regime is determined by the diffraction limit of the SPDC, not the molecule. This means that any molecule of this size can receive this enhancement to its ETPA cross-section, which is useful for label-free sensing and imaging by decreasing  $A_e$  to enhance the signal from the target molecule. For analytes that are much larger in size, such as a large protein or organelle within a cell, the  $T_e$  with maximum ETPA cross-section may depend on the analyte itself. In that case, ETPA could be a new means of measuring the size of a molecule in a cell. This relationship between the size of the absorbing molecule and its ETPA cross-section will be explored in future work.

$A_e$  at the maximum  $\sigma_e$  is more than 3 orders of magnitude smaller than  $A_e$  at the fs-scale  $T_e$  (see SI). This much smaller  $A_e$  would improve the spatial resolution of ETPA microscopy by more than 3 orders of magnitude, while also yielding brighter images due to the enhanced  $\sigma_e$ .<sup>22</sup> This enhancement would also provide chemists the ability to measure nonlinear optical signals, which are notoriously much weaker than linear signals, with the same absorption efficiency as linear OPA. Chemists could then more easily identify, for example, molecules with efficient charge transfer for solar cells or OLEDs.

Most interestingly, ETPA at ps-scale  $T_e$  provides a new opportunity to control chemical reactions that require intermediate conversion processes with fs–ps times, such as an isomerization, proton transfer, or intersystem crossing to a triplet state. A primary example is the photochemical reactivity of  $T_n$  triplet states in green fluorescence proteins (GFPs).<sup>43,44</sup> With ps  $T_e$  ETPA, the first photon would excite the GFP to the  $S_1$  state and allow the population to cross to the  $T_1$  state, and then the second photon would excite the GFP to the  $T_n$  state. Entangled photons are unique in that they can provide high-frequency resolution simultaneously with the required temporal resolution.<sup>45</sup> Classical ps laser pulses are Fourier limited, so they would lack frequency resolution in this photochemical reaction. Since excited states become closer in energy as the energy increases, maintaining high-frequency resolution with temporal resolution is crucial for control over the photochemical reaction.

## ■ ASSOCIATED CONTENT

### Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/jacs.1c09728>.

Calculation of entanglement time with dispersion effects; origin and derivation of entanglement area as a function of entanglement time; experimental details; ETPA cross-section measurements in flavin adenine dinucleotide; effect of vibrational relaxation and of molecular motion (PDF)

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## Notes

The authors declare no competing financial interest.

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