Optimal Joint Channel Estimation and Data Detection by L_1-norm PCA for Streetscape IoT

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***Abstract*—We prove, for the first time in the literature of communication theory and machine learning, the equivalence of joint maximum-likelihood (ML) optimal channel estimation and data detection (JOCEDD) to the problem of finding the L_1-norm principal components of a real-valued data matrix. Optimal algorithms for L_1-norm principal component analysis (PCA) are therefore direct solvers to the problem of interest, thus the proposed JOCEDD approach requires a polynomial number of operations. To avoid high computational costs incurred by the exact calculation of optimal L_1 principal components, we implement an efficient bit flipping-based algorithm for L_1-norm PCA in a software-defined radio. In particular, we carry out experiments with two radios that operate at Wi-Fi frequencies in a multipath indoor radio environment and have no direct line-of-sight. We apply L_1-norm PCA for JOCEDD over short frames that are transmitted over the single-input single-output communication link. We compare the performance of supervised data-aided channel estimation techniques versus JOCEDD in terms of bit-error-rate and demonstrate the superiority of the proposed approach across a wide range of signal-to-noise ratios.**

Keywords*—*blind channel estimation, ML data detection, PCA, L_1-norm, software-defined radio testbed, IoT, streetscapes

I. Introduction

Urban and city planners currently work toward smart connected streetscapes that are equipped with high throughput data communication technologies to enable vehicular automation and last-meter logistics, assist people with various disabilities with wayfinding and other outdoor activities with the goal to improve the operational quality of streets and cities. High throughput data communication systems require high quality maximum-likelihood (ML) type channel estimation techniques to provide reliable data detection at the receiver, however joint channel estimation and data detection over time varying, fast-fading channels such as the ones often arising in streetscape IoT is a challenging task.

The time-varying nature of the urban wireless channel typically requires the use of frequent channel re-training at the receiver, which in turn increases the data overhead due to training/pilot signaling, thus reducing the system's overall spectral efficiency. Blind joint channel/data estimation techniques [1] have received significant attention in the context of spectrally efficient networking in challenging communication environments [2]–[5], as they eliminate the overhead of training/pilot symbols at the cost of slower convergence and/or lower estimation accuracy. Channel phase ambiguities are however introduced. On the other hand, semi-blind estimation methods [6] lie between training-based and blind estimation approaches, require less computational complexity than blind methods and fewer training symbols than training/pilot-based methods, making them attractive for practical implementation.

The fact that channel knowledge is not used by blind detectors renders them applicable even to degraded and fast-fading channel conditions often encountered in streetscape environments. Nevertheless, if the channel is unknown at the receiver, single-symbol detection is no longer optimal. Instead, the ML optimal blind detector takes the form of a sequence detector and has exponential (in the sequence length) complexity when implemented through a conventional exhaustive search among all possible data sequences [7], [8].

Work in [9] describes a polynomial-complexity algorithm for non-coherent physical-layer network coding of frequency-shift-keying (FSK) signals in flat fading channels, while complementary work in [10] shows that for Rayleigh fading channels, optimal blind sequence detection of minimum-shift-keying (MSK) modulated signals can be carried out with log-linear complexity. Both [9] and [10] leverage the principles of polynomial-complexity optimization [11], [12] and complement efficient optimal noncoherent detection techniques that have been developed for phase-shift-keying (PSK) [13], pulse-amplitude modulation (PAM) or quadrature amplitude modulation (QAM) [14].

In this work, we show that ML sequence detection of PSK-modulated signals over multipath fading channels can be performed with polynomial (in the sequence length) complexity by proving that the problem of joint channel estimation and data detection is equivalent to the problem of finding the L_1-norm principal component of a real-valued data matrix. In particular, we focus on the special case of binary-PSK (i.e. binary antipodal symbols) and prove that the problem of ML-optimal joint channel estimation and data detection can be rewritten as a real-valued maximum-projection-L_1-PCA problem [15]–[18]. In [15], we showed that the K L_1-norm principal components of a real-valued data matrix $\mathbf{X} \in \mathbb{R}^{D \times N}$ (N data samples of D dimensions) can be exactly calculated with cost $\mathcal{O}(2^{NK})$. To avoid high computational costs incurred by the exact calculation of optimal L_1 principal components over transmitted data frames with "big" data size (i.e. large N), we implement an efficient bit-flipping-based algorithm for L_1-norm PCA [18] and carry out experiments with two software-defined radios in a multipath radio environment. In [18], we showed that the cost of the bit flipping algorithm for the calculation of the $K < \operatorname{rank}(\mathrm{X})$ L_1 principal components of $\mathbf{X}$ is $\mathcal{O}(ND\min\{N, D\} + N^2K^2(K^2 + \operatorname{rank}(\mathrm{X})))$. We evaluate the bit-error-rate (BER) of the link by applying L_1-norm PCA for JOCEDD over short frames transmitted over the single-input single-output (SISO) channel. Both radios operate at Wi-Fi frequencies and have no direct line-of-sight. The proposed method outperforms supervised data-aided channel estimation techniques and demonstrates superior BER performance in low signal-to-noise ratio (SNR).

II. System Model

We consider information symbol transmissions over a SISO channel with M resolvable propagation paths. Each symbol $b\left[n\right], n = 1, 2, \ldots$ is drawn from a unit energy binary constellation $\mathcal{A}$ and is modulated with an all-spectrum digital waveform $s(t)$ of duration T. The transmitted signal is written as

$$x(t) \triangleq \sum_{n} \sqrt{E} b\left[n\right] s\left(t - nT\right) e^{j2\pi f_c t + \phi} \tag{1}$$

where $E > 0$ denotes the transmitted energy per symbol, ϕ is the carrier phase offset, and f_c denotes the carrier frequency. The n-the transmitted symbol $b[n] \in \mathcal{A}$ is modulated by a digitally coded waveform $s(t)$ of duration T that is given by

$$s(t) \triangleq \sum_{l=1}^{L} s\left[l\right] g_{T_c}\left(t - lT_c\right) \tag{2}$$

where $g_{T_c}(\cdot)$ is an square-root-raised cosine (SRRC) pulse of duration T_c, so that $T = LT_c$, and $\mathbf{s} \in \left\{\pm\frac{1}{\sqrt{L}}\right\}^L$ is a unit-norm binary antipodal code sequence of length L. Without loss of generality, after carrier frequency demodulation, pulse-matched filtering and sampling at the code rate T_c over the multipath-extended bit period of $L_M = L + M - 1$ chips, the received signal vector $\mathbf{y}\left[n\right] \in \mathbb{C}^{L_M}$ at the receiver can be written as

$$\mathbf{y}\left[n\right] = \sqrt{E} b\left[n\right] \mathbf{H}\mathbf{s} + \mathbf{n}\left[n\right], \quad n = 1, 2, \ldots \tag{3}$$

where $\mathbf{H} \in \mathbb{C}^{L_M \times L}$ is the multipath channel matrix between the transmitter and receiver

$$\mathbf{H} \triangleq \begin{bmatrix} h_1 & 0 & \cdots & 0 & 0 \\ h_2 & h_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_M & h_{M-1} & & 0 & 0 \\ 0 & h_M & & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & h_M & h_{M-1} \\ 0 & 0 & \cdots & 0 & h_M \end{bmatrix} \tag{4}$$

with $h_m, m = 1, 2, \ldots, M$, denoting an independent zero-mean complex Gaussian random variable that models the complex baseband channel coefficient of the m-th resolvable path and $\mathbf{n}\left[n\right]$ models additive noise with a zero-mean white Gaussian noise (AWGN) vector with autocorrelation matrix $\mathbb{E}\left[\mathbf{n}\mathbf{n}^H\right] = \sigma^2 \mathbf{I}_{L_M}$.

After collecting N data vectors $\mathbf{y}\left[n\right],\ n = 1, 2, \ldots, N$, the received signal can be written in matrix form as

$$\mathbf{Y} \triangleq \sqrt{E}\mathbf{H}\mathbf{s}\mathbf{b}^T + \mathbf{N} = \mathbf{S}\mathbf{h}\mathbf{b}^T + \mathbf{N} \in \mathbb{C}^{L_M \times N} \tag{5}$$

where $\mathbf{Y} \in \mathbb{C}^{L_M \times N}$ is the complex-valued received signal matrix and $\mathbf{N}$ denotes an $L_M \times N$ Gaussian noise matrix. $\mathbf{Y}_{\Re}$ and $\mathbf{Y}_{\Im}$ denote the real and imaginary parts of the received signal matrix, respectively. We consider that $\mathbf{S}$ denotes an $L_M \times M$ (energy-including) code matrix that has the same structure as $\mathbf{H}$ in (4) and is written as

$$\mathbf{S} \triangleq \sqrt{E} \begin{bmatrix} \mathbf{s}\left[1\right] & & \mathbf{0} \\ \vdots & \ddots & \\ \mathbf{s}\left[L\right] & & \mathbf{s}\left[1\right] \\ & \ddots & \vdots \\ \mathbf{0} & & \mathbf{s}\left[L\right] \end{bmatrix}.$$

In this work, our goal is to jointly estimate the channel vector $\mathbf{h}$ and transmitted symbols $\mathbf{b}$ by solving the following least-squares (LS) problem

$$(\mathbf{h}^{\text{opt}}, \mathbf{b}^{\text{opt}}) = \underset{\substack{\mathbf{h} \in \mathbb{C}^M \\ \mathbf{b} \in \{\pm 1\}^N}}{\operatorname{argmin}} \ \|\mathbf{Y} - \mathbf{S}\mathbf{h}\mathbf{b}^T\|_F^2 \tag{6}$$

where $\|\cdot\|_F^2$ denotes the Frobenius norm. The solution $(\mathbf{h}^{\text{opt}}, \mathbf{b}^{\text{opt}})$ to the above LS problem is maximum-likelihood (ML) optimal as long as $\mathbf{N}$ is a white Gaussian noise matrix. We will show that the problem of interest is equivalent to maximum-projection L_1-norm principal-component analysis (PCA) [15]–[18]. An immediate corollary that can be derived due to the equivalence of both problems is the direct applicability of methods that have been originally developed for L_1-norm PCA, to solve joint channel estimation and data detection problems.

III. ML Joint Channel Estimation and Data Detection by L_1-norm PCA

In this section, we present how the problem in (6) can be transformed to an instance of the L_1-norm PCA maximum-projection problem and provide a summary of the developed algorithm for polynomial time joint ML-optimal channel estimation and data detection.

For a given symbol vector $\mathbf{b}$, the ML-optimal estimation of channel vector $\mathbf{h} \in \mathbb{C}^M$ is given by

$$\begin{aligned} \mathbf{h}^{\text{opt}}(\mathbf{b}) &= \underset{\mathbf{h} \in \mathbb{C}^M}{\operatorname{argmin}} \ \|\mathbf{Y} - \mathbf{S}\mathbf{h}\mathbf{b}^T\|_F^2 \\ &= \underset{\mathbf{h} \in \mathbb{C}^M}{\operatorname{argmin}} \ \|\mathbf{Y}\|_F^2 + \|\mathbf{S}\mathbf{h}\mathbf{b}^T\|_F^2 - 2\Re\left\{\operatorname{Tr}\left\{\mathbf{Y}^H\mathbf{S}\mathbf{h}\mathbf{b}^T\right\}\right\} \\ &= \underset{\mathbf{h} \in \mathbb{C}^M}{\operatorname{argmin}} \ N\mathbf{h}^H\mathbf{S}^T\mathbf{S}\mathbf{h} - 2\Re\left\{\mathbf{h}^H\mathbf{S}^T\mathbf{Y}\mathbf{b}\right\} \\ &= \underset{\mathbf{h} \in \mathbb{C}^M}{\operatorname{argmin}} \ \|\sqrt{N}\mathbf{S}\mathbf{h} - \frac{1}{\sqrt{N}}\mathbf{Y}\mathbf{b}\|_2^2 \\ &= \frac{1}{N}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T\mathbf{Y}\mathbf{b}. \end{aligned} \tag{7}$$

By substituting (7) to the objective function in (6), we can rewrite the optimization problem as

$$\underset{\substack{\mathbf{h}\in\mathbb{C}^M\\ \mathbf{b}\in\{\pm1\}^N}}{\text{minimize}} \|\mathbf{Y}-\mathbf{S}\mathbf{h}\mathbf{b}^T\|_F^2 \Leftrightarrow \underset{\mathbf{b}\in\{\pm1\}^N}{\text{minimize}} \|\mathbf{Y}-\mathbf{X}\mathbf{b}\mathbf{b}^T\|_F^2 \quad (8)$$

where $\mathbf{X}_{L_M\times N} \triangleq \frac{1}{N}\mathbf{P}\mathbf{Y}$ and $\mathbf{P}_{L_M\times L_M} \triangleq \mathbf{S}(\mathbf{S}^T\mathbf{S})^{-1}\mathbf{S}^T$. We notice that $\mathbf{P}$ is the projection matrix of the span of the matrix $\mathbf{S}$. As any projection matrix, $\mathbf{P}$ satisfies the following property

$$\mathbf{P}\mathbf{P}^H = \mathbf{P}. \quad (9)$$

By expanding the objective function of the problem in (8), and using property (9) we get

$$\begin{aligned}\|\mathbf{Y}-\mathbf{X}\mathbf{b}\mathbf{b}^T\|_F^2 &= \|\mathbf{Y}\|_F^2 + N\|\mathbf{X}\mathbf{b}\|_2^2 - 2\mathbf{b}^T\Re\{\mathbf{Y}^H\mathbf{X}\}\mathbf{b}\\ &= \|\mathbf{Y}\|_F^2 - \frac{1}{N}\mathbf{b}^T\mathbf{Y}^H\mathbf{P}\mathbf{Y}\mathbf{b}\\ &= \|\mathbf{Y}\|_F^2 - \frac{1}{N}\mathbf{b}^T\Re\left\{\mathbf{Y}^H\mathbf{P}\mathbf{Y}\right\}\mathbf{b}.\end{aligned}$$

Interestingly, the problem of interest in (8) is now rewritten as an antipodal binary quadratic *maximization*[1]. By finding $\mathbf{b}^{\text{opt}}$, we will get the ML-optimally detected bits at the receiver. The matrix $\Re\left\{\mathbf{Y}^H\mathbf{P}\mathbf{Y}\right\}$ is real positive semi-definite and can be written as $\Re\left\{\mathbf{Y}^H\mathbf{P}\mathbf{Y}\right\} = \widetilde{\mathbf{Y}}^T\widetilde{\mathbf{Y}}$ where

$$\widetilde{\mathbf{Y}}_{2L_M\times N} \triangleq \begin{bmatrix}\mathbf{P}_\Re\mathbf{Y}_\Re - \mathbf{P}_\Im\mathbf{Y}_\Im\\ \mathbf{P}_\Re\mathbf{Y}_\Im + \mathbf{P}_\Im\mathbf{Y}_\Re\end{bmatrix} \quad (10)$$

where $P_\Re$, $Y_\Re$ and $P_\Im$, $Y_\Im$ denote the real and imaginary parts of matrices $\mathbf{P}$ and $\mathbf{Y}$, respectively. In conclusion, the problem in (8) can be written as

$$\underset{\mathbf{b}\in\{\pm1\}^N}{\text{maximize}} \|\widetilde{\mathbf{Y}}\mathbf{b}\|_2. \quad (11)$$

We can view the problem in (11) as the following equivalent problem

$$\underset{\mathbf{b}\in\{\pm1\}^N,\ \mathbf{v}\in\mathbb{R}^{2L_M}}{\text{maximize}} \mathbf{v}^T\widetilde{\mathbf{Y}}\mathbf{b}. \quad (12)$$

which can be derived by leveraging the Cauchy-Schwarz inequality, since, for any $\mathbf{a},\mathbf{v} \in \mathbb{R}^N$ with $\|\mathbf{v}\|_2 = 1$, $\mathbf{a}^T\mathbf{v} \leq \|\mathbf{a}\|_2$ with equality if and only if $\mathbf{v} = \mathbf{a}\|\mathbf{a}\|_2^{-1}$. The necessary and sufficient condition of equality couples the two optimal vectors $\mathbf{v}^{\text{opt}}$ and $\mathbf{b}^{\text{opt}}$ by the following identity

$$\mathbf{v}^{\text{opt}} = \widetilde{\mathbf{Y}}^T\mathbf{b}^{\text{opt}}\cdot\|\widetilde{\mathbf{Y}}^T\mathbf{b}^{\text{opt}}\|_2^{-1}.$$

Lastly, we can rewrite (12) as

$$\underset{\mathbf{v}\in\mathbb{R}^{2L_M}}{\text{maximize}} \|\widetilde{\mathbf{Y}}^T\mathbf{v}\|_1.$$

This is attributed to a basic property of the L_1-norm, which is that for any $\mathbf{a}\in\mathbb{R}^N$ and $\mathbf{u}\in\{\pm1\}^N$, $\mathbf{a}^T\mathbf{u}\leq\|\mathbf{a}\|_1$ with equality if and only if $\mathbf{u} = \text{sgn}(\mathbf{a})$. For a binary antipodal symbol alphabet $b[n]\in\{\pm1\}, n = 1,2,\ldots$ the optimal bit vector $\mathbf{b}^{\text{opt}}$ can be estimated by

$$\mathbf{b}^{\text{opt}} = \text{sgn}\left(\widetilde{\mathbf{Y}}^T\mathbf{v}^{\text{opt}}\right). \quad (13)$$

[1]Minimization with the negative sign becomes maximization.

Algorithm: ML-optimal joint channel estimation and data detection via L_1-norm PCA.

Input: Observation data matrix $\mathbf{Y}_{L_M\times N}$ and code $\mathbf{s}\in\{\pm1/\sqrt{L}\}^L$.
1: Use the known code $\mathbf{s}$ to build $\mathbf{S}$ and $\mathbf{P}$.
2: Calculate the matrix $\widetilde{\mathbf{Y}}$ as per (10).
3: Calculate $\mathbf{v}^{\text{opt}}$, the L_1-norm PC of matrix $\widetilde{\mathbf{Y}}$ by [18].
4: Calculate $\mathbf{b}^{\text{opt}}$, the ML-optimal bits as per (13).
5: Calculate $\mathbf{h}^{\text{opt}}$, the ML-optimal channel estimates as per (7).
Output: $(\mathbf{h}^{\text{opt}}, \mathbf{b}^{\text{opt}})$ the solution pair to (6)

Fig. 1. The proposed algorithm for joint ML-optimal channel estimation and data detection from the complex-valued received signal matrix $\mathbf{Y}\in\mathbb{C}^{L_M\times N}$.

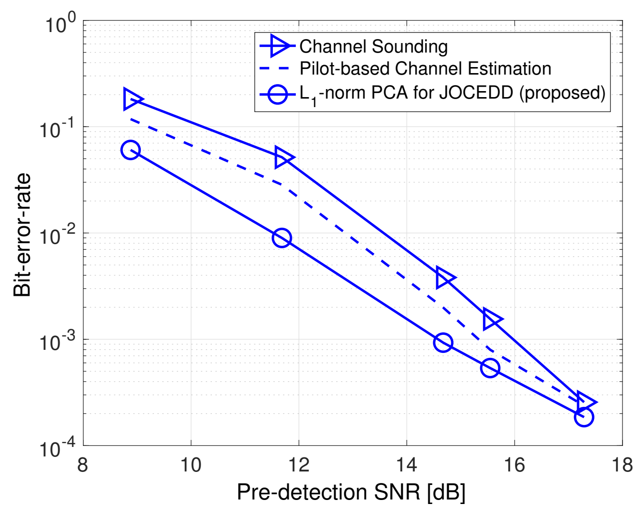

Fig. 2. BER vs. estimated pre-detection SNR at the receiver.

Fig. 1 presents the pseudo-algorithm for the proposed ML-optimal joint channel estimation and data detection method via L_1-norm PCA.

IV. Experimental Setup and Results

We implement and evaluate the proposed algorithm for joint ML-optimal channel estimation and data detection in terms of BER performance in a software-defined radio testbed with two USRP N210s. Both USRPs are interfaced to SBX 400-4400 daughtercards, which allow operation at a carrier frequency of $f_c = 2.485$ GHz. Experiments are carried out indoors in a multipath radio environment in a laboratory. Software-defined radios are positioned such that there is no direct line-of-sight between the two radios. We consider an M-tap channel to model multipath propagation with $M = 2$ paths. We use GNU Radio software to control and collect data from the two USRPs. More specifically, we develop GNU Radio out-of-tree signal processing software blocks that implement a spread-spectrum transmitter and receiver and use spreading codes of length $L = 8$ and use SRRC pulses of duration $T_c = 4\ \mu s$ to shape each chip. We consider frame-based transmissions of $N = 1024$ Binary Phase Shift Keying (BPSK)-modulated symbols per frame. More information regarding the transmitter and receiver architecture can be found in [19], [20]. At the software-defined radio receiver of this paper we implement and test for the first time algorithms [16], [17] for L_1-norm principal component analysis to evaluate the proposed algorithm for joint ML-optimal channel estimation and data detection in terms of BER performance.

We consider a total of 10000 frame transmissions by changing the transmission energy of each frame in every 2000 frame transmissions at the software-defined radio transmitter. We test 5 different transmit energy settings and calculate the BER of each transmitted frame by applying the proposed algorithm for joint ML-optimal channel estimation and data detection (as described in Fig. 1). We implement the efficient L_1-norm PCA bit flipping algorithm (as described in [18]) and calculate the L_1-norm principal component of matrix $\widetilde{\mathbf{Y}}$ (step 3 of the algorithm in Fig. 1) by considering only the information-bearing part of each frame (i.e. $N = 576$ symbols). Subsequently, we acquire ML-optimal estimates of the channel and payload bits by following the steps 4 and 5 of the algorithm in Fig. 1.

We estimate the pre-detection SNR at the software-defined radio receiver as

$$\mathrm{SNR} = \frac{\|\mathbf{S}\widehat{\mathbf{h}}(\widehat{\mathbf{b}})^T\|_F^2}{\|\mathbf{Y} - \mathbf{S}\widehat{\mathbf{h}}(\widehat{\mathbf{b}})^T\|_F^2} \tag{14}$$

where $\widehat{\mathbf{h}}$ is the acquired channel estimate and $\widehat{\mathbf{b}}$ the estimated payload bits. Fig. 2 depicts the BER versus the estimated pre-detection SNR at the receiver (calculated from (14)) and compares the performance of the proposed algorithm for joint channel estimation and data detection to supervised data-aided channel estimation algorithms based on channel-sounding and on pilot/training data [19], [20] for a wide range of SNRs (8-17 dB). We observe that the proposed approach achieves superior BER performance, particularly in low SNR.

V. Conclusions

In this paper, we consider the fundamental problem of joint ML-optimal channel estimation and data detection. We prove, for the first time in the communication theory and machine learning literature, the connection of this problem to real-valued L_1-norm principal component analysis. In future work, we plan to evaluate the performance of the proposed approach in realistic streetscape IoT and congested spectrum environments by leveraging NSF PAWR platforms.

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