

Towards a phenomenological model on the deformation and orientation dynamics of finite-sized bubbles in both quiescent and turbulent media

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A phenomenological model is proposed to describe the deformation and orientation dynamics of finite-sized bubbles in both quiescent and turbulent aqueous media. This model extends and generalizes a previous work that is limited to only the viscous deformation of neutrally buoyant droplets, conducted by Maffettone & Minale (*J. Non-Newtonian Fluid Mech.*, vol. 78, 1998, pp. 227–241), into a high Reynolds number regime where the bubble deformation is dominated by flow inertia. By deliberately dividing flow inertia into contributions from the slip velocity and velocity gradients, a new formulation for bubble deformation is constructed and validated against two experiments designed to capture the deformation and orientation dynamics of bubbles simultaneously with two types of surrounding flows. The relative importance of each deformation mechanism is measured by its respective dimensionless coefficient, which can be isolated and evaluated independently through several experimental constraints without multi-variable fitting, and the results agree with the model predictions well. The acquired coefficients imply that bubbles reorient through body rotation as they rise in water at rest but through deformation along a different direction in turbulence. Finally, we provide suggestions on how to implement the proposed framework for characterizing the dynamics of deformable bubbles/drops in simulations.

Key words: bubble dynamics, gas/liquid flow, isotropic turbulence

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1. Introduction

The deformation and breakup of gas bubbles and oil droplets in turbulent water are ubiquitous in nature, from bubble-mediated gas transfer in the ocean (Wanninkhof & McGillis 1999) to the fragmentation and dispersion of an oil spill (Delvigne 1988; Yang *et al.* 2015). Despite advances made in the field of multiphase flows and droplet-laden turbulence (Risso 2018; Elghobashi 2019; Mathai, Lohse & Sun 2020), our understanding of these problems is still limited due to the complex nonlinear interactions between two phases across a deformable interface and the manifestation of these interactions across multiple length and time scales in turbulence. Unlike rigid objects, the geometries of deformable objects have nearly infinite degrees of freedom.

The deformation of the dispersed phase is sensitive to many parameters. Other than the size, it is primarily determined by the competition between the intensity of local inertial/viscous stresses and surface tension. In a regime where the viscous stress dominates, Taylor (1932, 1934) showed that the droplet deformation is determined by the capillary number $Ca = \mu_c G D / \sigma$, where μ_c is the viscosity of the carrier fluid, G is the local shear rate around the droplet, D is the droplet diameter, and σ is the surface tension coefficient. The larger the value of Ca , the more intense the flow shear rate; when Ca surpasses a critical value (Ca_{cr}), the drop breaks. In addition to the drop deformation, the orientation dynamics of drops is also closely associated with the local shear rate.

To capture both the deformation and orientation dynamics of drops in viscous shear flows, Maffettone & Minale (1998) developed a model (the MnM model, hereafter) to characterize the dynamics of a single neutrally buoyant drop immersed in an infinite medium with a generic flow field. In particular, this model assumes ellipsoidal drop shapes based on experimental observations (Guido & Villone 1998) and numerical simulations (Kennedy, Pozrikidis & Skalak 1994). The MnM model is well suited for characterizing drop shape and orientation in simple shear (Guido & Villone 1998), planar and uniaxial elongational flows, as well as the linear combinations of these basic flows (Bentley & Leal 1986). The agreement can be extended to Ca that is not far from Ca_{cr} , as long as the nonlinear effect remains small.

For drops deforming in viscous shear flows, there are two main assumptions: (i) the relative motion between the centre of mass of the drops with the surrounding flows is negligible compared with the drop deformation. This can be accomplished by matching the density of both phases (ii) the viscous shear stress is more important than the dynamic pressure exerted by the surrounding moving fluid. For both assumptions to hold, one needs only to make sure that the drop-based Reynolds number $Re_b = u^s D / \nu_c$ is smaller than one. In Re_b , ν_c is the kinematic viscosity of the carrier phase, and u^s is the magnitude of the drop slip velocity with respect to the surrounding flow.

Unlike drops, gas bubbles tend to have a large density difference with the surrounding liquid. One such canonical problem is the rise motion of finite-sized bubbles in water at rest. In this case, Re_b ranges from $O(10^2)$ to $O(10^3)$ (Magnaudet & Eames 2000), so the viscous stress ($\mu_c G$) becomes less important than the dynamic pressure exerted on the bubble interface due to their buoyant rising motion ($\Delta \rho g D$), and Ca is consequently much smaller than the Eötvös number $Eo = \Delta \rho g D^2 / \sigma$, where $\Delta \rho = \rho_c - \rho_d$ is the density difference between the dispersed phase (ρ_d) and the carrier phase (ρ_c), and g is the gravitational constant. Moore (1965) developed a simple model to link the bubble aspect ratio α to Eo for small Eo close to 1, and extended it to larger Eo by employing the potential flow method. It was found that a maximum α of 6 could be achieved for Eo close to 3.745, beyond which symmetric deformation could be attained.

In many papers discussing the bubble deformation due to their rise motion, the dimensionless number often used is the Weber number: $We = \rho_c u^2 D / \sigma$. One can see that $We \approx Eo$ if the bubble rise velocity $u \approx \sqrt{gD}$. However, in turbulence, when the slip velocity between bubbles and surrounding flows is not solely driven by the bubble rise velocity, these two dimensionless numbers are no longer the same. To draw a clear distinction, Eo will be used only to represent the bubble deformation by buoyancy, whereas We is reserved for characterizing the deformation driven by turbulence in this paper. In addition, the problem depends on other dimensionless numbers, such as the Galilei number: $Ga = gD^3/\nu^2$ (similar to Reynolds number) or the Morton number: $Mo \equiv Eo^3/Ga^4$. The Morton number is a constant for a given gas–liquid system. Rising bubbles exhibit a wide range of behaviours depending on the exact values in the phase diagram of Eo and Ga (Tripathi, Sahu & Govindarajan 2015). In this paper, we consider only air bubbles rising in quiescent water with Ga larger than Eo . For this regime, although the bubble shape is not axisymmetric, it is still close to an ellipsoid without exhibiting complex topological changes, such as skirted, spherical cap bubbles, that no linear model can capture.

In turbulence, the problem can be roughly categorized based on the drop/bubble size, in either the dissipative range ($D \ll \eta$) or the inertial range ($\eta \ll D \ll L$), where η and L are the Kolmogorov and integral length scales, respectively. For a small neutrally buoyant drop that responds to only the local and instantaneous flow, its centre-of-mass (CoM) motion can be integrated based on the Maxey–Riley equation. The drop shape, if assumed to be an ellipsoid, can be solved based on the MnM model (Maffettone & Minale 1998). Although the MnM model was originally proposed to describe the drop deformation in viscous shear flows, it is still valid to use in turbulence for calculating the deformation of infinitesimal drops, which are primarily driven by the local viscous shear stresses. This framework has been utilized to study the drop deformation in homogeneous and isotropic turbulence by Biferale, Meneveau & Verzicco (2014) and in a Taylor–Couette system by Spandan, Lohse & Verzicco (2016). Such an approach enables the simultaneous simulation of 10^4 to 10^5 deforming drops in turbulence. Moreover, similar frameworks coupling carrier-phase simulation with simple models for the dispersed phase have also been utilized to study the tumbling motion of non-spherical particles (Marchioli, Fantoni & Soldati 2010; Challabotla, Zhao & Andersson 2015), the stretching and buckling of flexible rods (Allende, Henry & Bec 2018), and the breakup of ductile aggregates (Marchioli & Soldati 2015).

The study of the deformation of the finite-sized bubbles in turbulence is much more complicated. As detailed by Elghobashi (2019) in a recent review, there are three main direct numerical simulation (DNS) approaches to the study of finite-sized bubbles and droplets in turbulence: (i) tracking individual points on the bubble interface, e.g. front tracking (Unverdi & Tryggvason 1992; Tryggvason *et al.* 2001); (ii) tracking a scalar function, e.g. volume of fluid (Scardovelli & Zaleski 1999; Dodd & Ferrante 2016), level set (Sussman, Smereka & Osher 1994; Osher & Fedkiw 2001), lattice-Boltzmann (Shan & Chen 1993) or the phase field method; (iii) a more recently developed hybrid method that couples the immerse boundary method with a phenomenological interaction potential. All these sophisticated models are valuable tools for investigating finite-sized bubble/droplet deformation in turbulence. However, they are expensive to perform for a large number of bubbles/droplets in a large system even with the most advanced computational methods and resources.

The objective of this paper is to develop a phenomenological model to characterize the deformation and orientation dynamics of finite-sized bubbles without considering

two-way couplings or the bubble–bubble interaction. Similar to the MnM model, the proposed model takes the known flow conditions, including the velocity gradients and the bubble slip velocity as inputs and predicts the time evolution of the bubble shape and orientation. In § 2, we introduce the experimental set-up and the measurement techniques that enable the simultaneous measurements of both phases, including the three-dimensional (3-D) shape of the bubbles and their surrounding turbulence. Then a new phenomenological model accounting for the contribution of the slip velocity to the bubble deformation and orientation dynamics is discussed and explained in § 3. Following this, the model parameters are calibrated against experimental results for bubbles deforming in quiescent and turbulence media in § 4. In the same section, we present how to extend the proposed new model to characterize the deformation of finite-sized droplets with different viscosities and densities.

2. Experimental set-up and measurements

2.1. Experimental set-up

Although the focus of this paper is on developing a phenomenological model to describe the deformation and orientation dynamics of finite-sized bubbles with any given surrounding flow fields, the experimental data for constraining such a model was not available until recently thanks to the advance of the simultaneous measurements of the bubble 3-D shape and surrounding turbulence. Here, we introduce a unique facility and the associated 3-D simultaneous two-phase measurement technique to aid in developing a phenomenological model for deformable bubbles and drops.

To ensure that the proposed model works for all cases from the buoyancy-dominated to the turbulence-dominated regimes, it is important to develop an experimental set-up that can produce a strong turbulent environment with the turbulence-induced deformation stronger than or at least similar to the buoyancy-induced bubble deformation. This requires that $We \geq Eo$ and $We > 1$, where the Weber number is defined as $We \approx \rho_c (\langle \epsilon \rangle D)^{2/3} D / \sigma$ based on the Kolmogorov theory. To satisfy this requirement, it is important for $\langle \epsilon \rangle$ to reach approximately $O(0.1) \text{ m}^2 \text{ s}^{-3}$.

A vertical water tunnel (V-ONSET) was constructed to produce both quiescent and turbulent media. Turbulence in the tunnel was generated by shooting 88 high-speed momentum jets ($\sim 12 \text{ m s}^{-1}$), through a jet array, into the test section co-axially with the mean flow. All jet nozzles (the nozzle diameter d is 5 mm) were connected to the same high pressure water tank and controlled by a dedicated solenoid valve. The measurement volume of $6 \text{ cm} \times 6 \text{ cm} \times 5 \text{ cm}$ was located about $40d$ below the jet array. Such a large separation is designed to ensure that the jets are well mixed and that turbulence generated is close to homogeneous and isotropic. A randomized firing pattern of the jets was set similar to that of a previous study by Variano, Bodenschatz & Cowen (2004), which was shown to generate homogeneous and isotropic turbulence without inducing a persistent mean flow or any secondary flow structures without considering two-way couplings or the bubble–bubble interaction. Detailed discussions of these statistics and flow distributions have been discussed by Masuk *et al.* (2019b).

In addition to the randomly fired jets, the vertical tunnel features an independently controlled downward mean flow of $\sim 0.25 \text{ m s}^{-1}$ to maintain bubbles inside the test section for an extended residence time, which helps to obtain longer bubble trajectories. Bubbles were injected through a capillary island consisting of four arrays of vertically oriented hypodermic needles of two different sizes. After these bubbles reached the

Bubble deformation model

	f'_1	f'_2	K_s	K_o	We_{slip}	We_{vg}	Re_{slip}	Re_λ	Eo	Mo
Quiescent	1	0	0.14	15	[2.03, 3.84]	0	[540, 895]	0	[0.54, 1.23]	2.6×10^{-11}
Turbulent	1	0.5	0.14	0	[2.28, 9.56]	[1.28, 4.3]	[697, 1945]	435 ± 20	[1.65, 6.2]	2.6×10^{-11}

Table 1. Values of the dimensionless parameters used in the model and experiments.

interrogation volume, the typical size range of the bubbles turned out to be $D = 2\text{--}7$ mm. The dimensionless numbers of these bubbles are listed in [table 1](#).

The test section of the tunnel features an octagonal cross-section with eight vertical walls to allow optical access by six high-speed cameras that are uniformly distributed all around the test section. Each of these cameras can operate at 4000 frames per second at a full resolution of 1 megapixel. One dedicated LED panel for each high-speed camera was used to provide diffused back lighting. By using such an optical arrangement, both bubbles and a high concentration of tracer particles (density-matched Polyamide particles with $50\text{ }\mu\text{m}$ in diameter) appeared as dark shadows in front of a bright background. For further description of the tunnel and its diagnostics, the reader may refer to a recent work published by Masuk *et al.* ([2019b](#)).

2.2. Bubble and flow measurements

From raw images, bubbles and tracers were segmented based on their size and contrast difference. Each phase was processed separately to quantify its dynamics. For the fluid phase, an in-house particle tracking code OpenLPT (Tan *et al.* [2020](#)) that utilizes the Shake-The-Box (Schanz, Gesemann & Schröder [2016](#)) method was used to obtain the tracer trajectories. For each frame, the centre of each particle was triangulated to obtain its 3-D location, which was then connected through time to acquire the particle trajectory. For the gas phase, a recently developed virtual-camera visual hull method (Masuk, Salibindla & Ni [2019a](#)) was employed to construct the bubble shape. The 3-D volume of the bubble was first reconstructed by calculating the intersection of the cone-like volume extruding from the bubble shadow on each camera. Then, the shape was refined iteratively by projecting the initial reconstructed volume onto virtual cameras to remove any artefacts generated. In each iteration, the refined geometry was also projected back onto the real cameras to ensure that no real material was removed. Based on the shape reconstruction and refinement, the bubble geometry consisting of many surface vertices can be acquired. These surface vertices can be averaged to determine the bubble centre, which is connected over time, as with how the tracer trajectories were calculated. Both the bubble and tracer trajectories are smoothed by convoluting with Gaussian kernels (Mordant, Crawford & Bodenschatz [2004](#)), from which the tracer velocity $\mathbf{u}^p$ and acceleration $\mathbf{a}^p$ and the bubble velocity $\mathbf{u}_b$ and acceleration $\mathbf{a}_b$ can be obtained along their trajectories.

Along with each bubble trajectory, two important flow parameters of interest in this study are the fluid velocity at the CoM of a bubble $\mathbf{u}_f$ if the bubble is not present and the coarse-grained velocity gradient tensor $\tilde{A}_{ij}$. For every bubble in our experiments, a spherical search volume with diameter $D_s = 2D$ centred at the bubble CoM was used to select the surrounding tracer particles. Note that for each of these n ($p = 1, 2, \dots, n$) tracer particles, their velocities and locations have already been determined. The value of $\mathbf{u}_f$ is estimated by taking an average of the particle velocity of all n tracer particles,

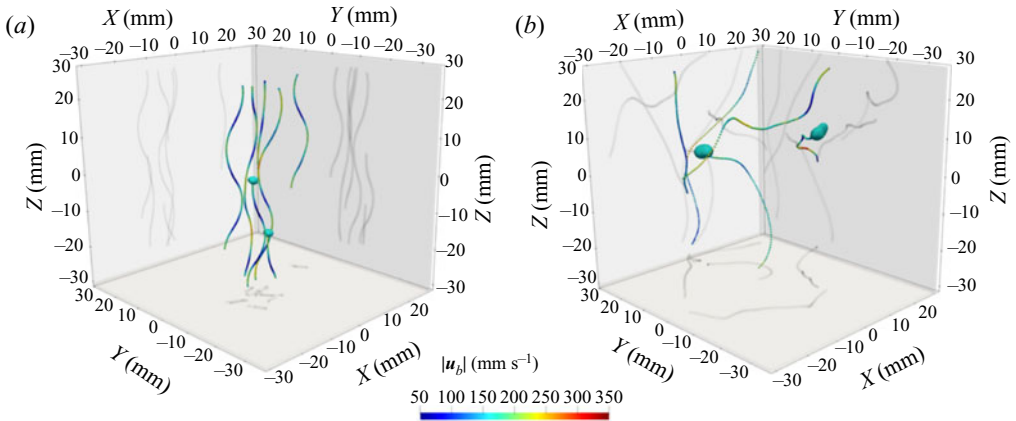


Figure 1. Example 3-D bubble trajectories colour coded with their instantaneous velocity magnitude and the 3-D reconstructed shapes at one time instant in (a) an otherwise quiescent medium, and (b) intense turbulence.

$\mathbf{u}_f = \sum_{p=1}^n \mathbf{u}^p / n$. This allows us to calculate the instantaneous slip velocity $\mathbf{u}^s$ between the two phases: $\mathbf{u}^s = \mathbf{u}_b - \mathbf{u}_f$. To obtain the coarse-grained velocity gradient $\tilde{A}_{ij} = \partial u_i^p / \partial x_j^p$, a least-square fit was performed to minimize the residuals of $\sum_p (u_i^p - \tilde{A}_{ij} x_j^p)^2$ using the velocity and location of all n nearby tracer particles. In particular, $\mathbf{x}^p$ is the separation vector of the p th tracer particle away from the CoM of a bubble. For additional details about the calculation and reliability of such coarse-grained velocity gradients, please refer to previous works by Masuk, Salibindla & Ni (2021a) and Ni *et al.* (2015).

Figure 1 shows examples of the trajectories of deforming bubbles in both (a) quiescent and (b) turbulent media. The trajectories are colour coded with their velocity magnitudes. The zigzag path oscillation is clearly visible in the quiescent rising case, whereas more chaotic trajectories become evident for bubbles travelling in turbulence. In addition to their 3-D trajectories, the shadows they cast on the three main planes are presented to illustrate their projected tracks. Although a high concentration of tracer particles are present, their trajectories are not shown to highlight the difference of bubble tracks in different flows.

2.3. Flow characterization

Before attempting to develop a model for bubble deformation and orientation, the characteristics of the single-phase turbulence generated in our system are briefly discussed here for completeness. Additional details regarding the turbulence characteristics can be found in a recent work by Masuk *et al.* (2019b). In figure 2, the second-order (D_{LL}) and third-order longitudinal structure functions (D_{LLL}) are shown, both of which follow the scaling laws for a range of scales that the Kolmogorov theory predicts. Specifically, between 2η and 200η , D_{LL} appears to exhibit a $2/3$ power law, indicated by the dashed line, whereas the power-law exponent seems to be close to 1 for D_{LLL} , as indicated by the solid line. Furthermore, the green region in figure 2 indicates the range of the bubble size. It is evident that the bubble size lies within the inertial range, as expected. Moreover, based on the structure function in figure 2, a reasonable estimation of the energy dissipation rate can be obtained, namely $\langle \epsilon \rangle \approx 0.16 \pm 0.02 \text{ m}^2 \text{ s}^{-3}$, which satisfies the required turbulence intensity $\langle \epsilon \rangle \approx O(0.1) \text{ m}^2 \text{ s}^{-3}$ for $We > Eo$. All relevant dimensionless parameters of the generated turbulence can be found in table 1.

Bubble deformation model

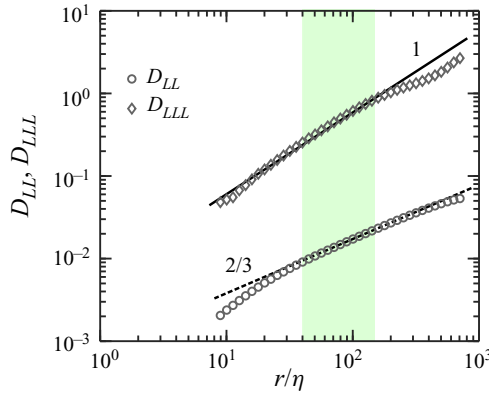


Figure 2. The second-order and third-order longitudinal structure functions (D_{LL} and D_{LLL}) of turbulence. The green region indicates the range of the bubble size.

3. Finite-sized bubble deformation model (FBD model)

This paper has three main objectives. The first objective is to develop a model to capture the deformation dynamics of finite-sized bubbles with diameter D ($\eta < D < L$) both in turbulence and in water at rest. Second, the developed model should correctly account for two deformation mechanisms driven by the local velocity gradients and the slip velocity between the two phases. For one extreme limit, when bubbles rise in an otherwise quiescent medium, the bubble deformation is dominated by the slip velocity. In this case, Eu is much greater than 1, and We driven by the velocity gradients is close to zero. For the other limit, the turbulent energy dissipation rate becomes so large that the bubble deformation driven by the dynamic pressure gradient ($We \geq 1$) becomes important. Finally, the third objective is to use our experimental results to validate the model and constrain different dimensionless coefficients in the model.

Before introducing our model, we begin with the MnM model originally proposed by Maffettone & Minale (1998) to describe the shape evolution of neutrally buoyant droplets in a linear velocity gradient, with the droplet shape represented by a symmetric positive-definite second-order tensor P_{ij} , which can be expressed as

$$\frac{dP_{ij}}{dt} = \Omega_{ik}P_{kj} - P_{ik}\Omega_{kj} + f_2(\mu)(S_{ik}P_{kj} + P_{ik}S_{kj}) - \frac{f_1(\mu)}{\tau}(P_{ij} - g(P)\delta_{ij}), \quad (3.1)$$

where the three eigenvalues of P_{ij} represent the squared lengths of three semi-axes of an ellipsoid. Here, S_{ij} and Ω_{ij} are the symmetric and anti-symmetric parts of the velocity gradient tensor (A_{ij}) that the droplet is subject to. In particular, $S_{ij} = (A_{ij} + A_{ji})/2$ and $\Omega_{ij} = (A_{ij} - A_{ji})/2$. For a simple shear flow with a small Reynolds number, f_1 and f_2 are functions of the viscosity ratio $\mu = \mu_d/\mu_c$, where μ_d and μ_c represent the dynamic viscosity of the inner fluid of bubbles/drops and their surrounding carrier fluid, respectively. The last term on the right-hand side of (3.1) is the restoring term, in which $\tau = \mu_d D/2\sigma$ is the relaxation time scale of the droplet determined by μ_d and the coefficient of surface tension σ ; D is the equivalent sphere diameter of the droplet. Volume conservation is ensured in the model with $g(P) = 3III_P/II_P$, where III_P and II_P are the invariants of P_{ij}

$$I_P = P_{ii}, \quad II_P = -\frac{1}{2}(P_{ij}P_{ji} - I_P^2), \quad III_P = \frac{1}{3}(P_{ik}P_{kj}P_{ji} - I_P^3 + 3I_P II_P). \quad (3.2a-c)$$

This model is suitable for describing the deformation of bubbles in simple flows, and it has been validated against several experimental results (Torza, Cox & Mason 1972; Bentley & Leal 1986; Guido & Villone 1998; Guido, Minale & Maffettone 2000). Moreover, the MnM model has been used to characterize the shape evolution of small neutrally buoyant droplets in turbulence (Biferale *et al.* 2014; Spandan *et al.* 2016). This works under two assumptions: (i) D is in the dissipative range ($D \ll \eta$), where the viscous stress still dominates; and (ii) drops are neutrally buoyant with no significant slip velocity.

For finite-sized bubbles with D in the inertial range ($\eta \ll D \ll L$), neither of these two assumptions holds any longer. In particular, the bubble slip velocity is so large such that the bubble-scale Reynolds number Re_b becomes much larger than one, implying that the flow inertia plays a more important role than viscous stresses. The two basic sources of flow inertia come from the surrounding straining flows and the velocity mismatch between the two phases due to the density mismatch and finite bubble size.

Although an analytic model of the bubble deformation and orientation dynamics in turbulence is not available, the potential flow calculations for bubble deformation in either uniform flows or pure straining flows have been conducted, which can lend us some insights into this problem. First, for a nearly spherical bubble with small deformation, the bubble shape can be taken to be the ellipsoid of revolution: $r = D[1 + \zeta(We)P_2(\cos \theta)]/2$, where the deviation of the bubble radius at a different location θ away from the equivalent spherical radius $D/2$ is a function of the Weber number $\zeta(We)$ and the Legendre polynomial $P_2(\cos \theta)$. By calculating the irrotational flow passing a sphere, the local radius can be determined by the difference of pressure across the interface. From this relationship, the bubble radius and its aspect ratio can be calculated. For both uniform flows (Moore 1959, 1965) and straining flows (Kang & Leal 1987) around a nearly spherical particle, $\zeta(We)$ appears to be a linear function of We

$$\left. \begin{aligned} r &= \frac{D}{2} \left[1 - \frac{3}{32} We_{slip} P_2(\cos \theta) \right] & (\text{uniform flows}), \\ r &= \frac{D}{2} \left[1 + \frac{We_{strain}}{4} \frac{25}{168} P_2(\cos \theta) \right] & (\text{straining flows}), \end{aligned} \right\} \quad (3.3)$$

where $We_{slip} = \rho(u^s)^2 D / \sigma$ and $We_{strain} = \rho(\lambda_3 D / 2)^2 D / \sigma$ are the Weber numbers defined by the slip velocity (u^s) and the smallest eigenvalue of the rate-of-strain tensor λ_3 , respectively.

In this paper, the contribution of the uniform flows (driven by the slip velocity between the two phases) and straining flows are considered separately in the model. However, rigorously, when one applies the potential flow solution that accounts for both, the nonlinearity of the Bernoulli equation results in not only these terms that depend on We_{slip} and We_{strain} but also some coupled terms as a function of $\sqrt{We_{slip} We_{strain}}$. Nevertheless, since these coupled terms are associated with the higher-order Legendre polynomials that do not contribute to the ellipsoidal deformation (which can be proved with a simple expansion), they are ignored in our framework and the deformation driven by the slip velocity and straining flows can therefore be cleanly separated.

In the MnM model, the term that characterizes the bubble deformation driven by the straining flow is $f_2(\mu)(S_{ik}P_{kj} + P_{ik}S_{kj})$. This term cannot be directly applied to finite-sized bubbles for two reasons: (i) S_{ij} is the local strain rate tensor at a length scale close to or smaller than η . For bubbles with $D \gg \eta$, it should be a coarse-grained $\tilde{S}_{ij}$ that dominates the bubble deformation, i.e. $\tilde{S}_{ik}P_{kj} + P_{ik}\tilde{S}_{kj}$. This argument is supported by our recent work, which showed that the bubble deformation can be estimated using a Weber number

calculated based on $\tilde{S}_{ij}$ (Masuk *et al.* 2021a). In addition, the bubble orientation appears to be preferentially aligned with the eigenvectors of $\tilde{S}_{ij}$, further confirming that $\tilde{S}_{ij}$ is the relevant flow strain rate that contributes to bubble deformation (Masuk, Salibindla & Ni 2021b). (ii) For sub-Kolmogorov bubbles or drops, $f_2(\mu)(S_{ik}P_{kj} + P_{ik}S_{kj})$, scales linearly with the Capillary number. However, if we follow the same expression but replace S_{ij} with $\tilde{S}_{ij}$ for finite-sized bubbles, the deformation scales with $We^{1/2}$, which differs from the potential flow predictions in (3.3).

Although we cannot adopt the deformation term induced by the coarse-grained strain rate as it is, i.e. $\tilde{S}_{ik}P_{kj} + P_{ik}\tilde{S}_{kj}$, this formulation does capture one important feature of the process: the resulting principal axes of the deformed shape P_{ij} will align with the eigenvectors of $\tilde{S}_{ij}$, which is consistent with the experimental observation (Masuk *et al.* 2021b). One might attempt to improve this formulation by assuming a more general form from potential flow theory and retaining terms of a higher order in the departures from the small deformation limit. However, as Moore (1959) indicated, not only would such a calculation be very lengthy, but it is also unlikely for one to obtain a valid result for large deformation without using an excessively large number of terms. In this paper, we intend to retain a simple formulation, i.e. $\tilde{S}_{ik}P_{kj} + P_{ik}\tilde{S}_{kj}$, but correct its scaling with We . Here, we introduce a stress tensor

$$\gamma_{ij} = \rho V_{ik} \Lambda_{kp} |\Lambda_{pq}| V_{qj} \left(\frac{D}{2}\right)^2, \quad (3.4)$$

where $\Lambda_{ij} = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$ is a diagonal matrix with λ_i representing the eigenvalues of $\tilde{S}_{ij}$, and V_{ij} is a matrix with each column corresponding to one of the eigenvectors $\hat{e}_i$ of $\tilde{S}_{ij}$. This stress term is formulated such that it scales with the dynamic pressure and the Weber number linearly and the directions of maximum and minimum pressure align with the maximum compression and stretching directions of the strain-rate tensor, respectively.

This stress tensor must be converted back to a strain-rate tensor to (i) maintain the same formulation as the one used in the MnM model, and (ii) ensure that the volume is conserved during deformation. In particular, a new deformation-rate tensor $\tilde{S}_{ij}^s$ (the superscript s indicates that the eigenvalues of this tensor are the square of those of $\tilde{S}_{ij}$, i.e. $\lambda_i|\lambda_i|$) can be expressed by following the generalized Hooke's law

$$S_{ij}^s = \frac{1}{E\tau_n} [(1 + \nu)\gamma_{ij} - \nu\gamma_{kk}\delta_{ij}], \quad (3.5)$$

to relate stress with strain for isotropic materials (Ugural & Fenster 2003). Here, E is a constant related to the restoring stress of bubble, which can be linked to the surface tension coefficient, i.e. $E = \sigma/D$. Another constant in (3.5) is ν , often referred to as the Poisson ratio in solid mechanics, which can be set as 0.5 for incompressible fluid. Since only gas bubbles rather than vapour bubbles are considered in our case, this assumption should be valid.

The new deformation-rate tensor obtained from (3.5) scales linearly with We and can be used directly with P_{ij} to represent the deformation of finite-sized bubbles under a straining flow, i.e. $f_2'(\tilde{S}_{ik}^s P_{kj} + P_{ik} \tilde{S}_{kj}^s)$. Note that the coefficient f_2' in this case does not depend on the carrier-phase viscosity as the deformation of finite-sized bubbles is driven by inertia rather than viscous stresses.

In addition to the coarse-grained velocity gradients, another effect caused by finite-sized bubbles is the stresses on the bubbles induced by the slip velocity between the two phases.

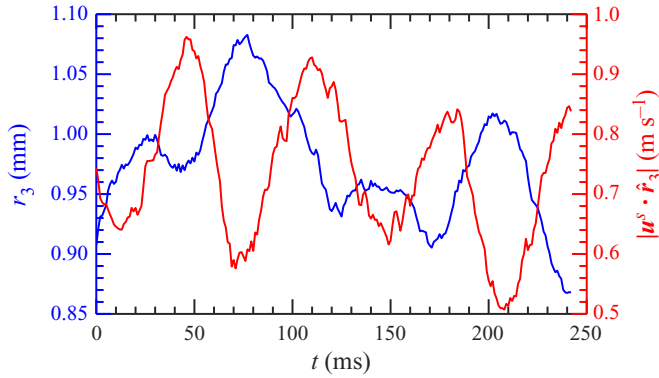


Figure 3. An example time trace of the semi-minor axis (r_3) and the bubble slip velocity projected onto the direction of r_3 for an air bubble rising in an otherwise quiescent water medium.

The slip velocity results from both the density mismatch and finite-size effect (Bellani & Variano 2012; Cisse, Homann & Bec 2013). The importance of the slip velocity was clearly illustrated in a recent work by Masuk *et al.* (2021b). In particular, the bubble semi-minor axis appears to preferentially align not only with the eigenvector ($\hat{e}_3$) associated with the smallest eigenvalue (λ_3 , strongest compression) of $\tilde{\mathcal{S}}_{ij}$ but also with the slip velocity $\mathbf{u}^s$. This indicates that the role played by the slip velocity cannot be ignored for finite-sized bubbles.

To include the contribution of the slip velocity, we decide to seek a simpler case in which the slip velocity is the only dominant mechanism for bubble deformation with nearly zero velocity gradients coarse grained at the bubble scale. Such an example is readily available for a bubble rising in an otherwise quiescent aqueous medium at a high Reynolds number. In this case, the vorticity production induced by the bubble motion is assumed to be strictly confined to the interface and zero everywhere else in the fluid. At high Reynolds numbers, the boundary layer is so thin that this assumption should hold.

We performed such an experiment in the same V-ONSET facility by simply turning all jets and the mean flow off to allow for individual bubbles to rise in an undisturbed environment. The same diagnostic system was used to extract the bubble rise motion and their shapes in three dimensions. One such example is shown in figure 3. Here, the blue line indicates the time trace of the bubble dimension along the semi-minor axis (r_3), while the red line represents the slip velocity projected onto the direction of $\hat{r}_3$, i.e. ($|\mathbf{u}^s \cdot \hat{r}_3|$). Both signals exhibit some apparent oscillations in time because of the well-known path instability developed due to the wake–bubble interaction (Mougin & Magnaudet 2001, 2006; Ern *et al.* 2012; Tayler *et al.* 2012). It appears that the time traces of r_3 and $|\mathbf{u}^s \cdot \hat{r}_3|$ are out of phase with each other, indicating that an increase of the slip velocity results in a decrease in the bubble minor axis. This is consistent with our expectation that a stronger dynamic pressure from a larger slip velocity tends to compress the bubble along that direction. When the slip velocity weakens, the bubble can relax back toward a spherical geometry resulting in an increase of r_3 .

In this case, the background flow is almost stagnant, and the velocity gradient around the bubble is negligible if we do not consider the bubble-induced flows. Following this argument, the terms associated with velocity gradients in the MnM model become close to zero. It is evident that, without these two terms, there is no other driving force that can deform a bubble. Thus, the roles played by the slip velocity must be added. In figure 3, it is clear that the slip velocity primarily compresses bubbles along its direction. To add its

contribution, a new stress tensor, i.e. γ'_{ij} , is defined in a way that it must be aligned with the slip velocity $\mathbf{u}^s$ following: $\gamma'_{ij} = -\rho_c u_i^s u_j^s$. This allows us to define a pseudo-strain-rate tensor $\tilde{S}'_{ij}$ following (3.5). Adding this term into the revised MnM model yields

$$\begin{aligned} \frac{dP_{ij}}{dt} = & f'_2(\tilde{S}'_{ik}P_{kj} + P_{ik}\tilde{S}'_{kj}) + K_s(\tilde{S}'_{ik}P_{kj} + P_{ik}\tilde{S}'_{kj}) - \frac{f'_1}{\tau_n}(P_{ij} - g(\Pi_P, \text{III}_P)\delta_{ij}) \\ & + \tilde{\Omega}_{ik}P_{kj} - P_{ik}\tilde{\Omega}_{kj}, \end{aligned} \quad (3.6)$$

where $\tilde{\Omega}_{ij}$ in the MnM model is replaced with the coarse-grained version ($\tilde{\Omega}_{ij} = (\tilde{A}_{ij} - \tilde{A}_{ji})/2$). Here, f'_1 and f'_2 are two dimensionless coefficients, and τ_n is the typical relaxation time scale of the bubble, which will be discussed in § 4.1. The pseudo-strain-rate term, i.e. $K_s(\tilde{S}'_{ik}P_{kj} + P_{ik}\tilde{S}'_{kj})$, has a coefficient K_s that measures its relative importance. To validate the new model, we numerically integrate equation (3.6) and set $\tilde{S}'_{ij} = 0$ and $\tilde{\Omega}_{ij} = 0$ for bubbles rising in water at rest. The time series of $\mathbf{u}^s$ were used to calculate the time series of $\tilde{S}'_{ij}$. The integration was performed using the fourth-order Runge–Kutta method. The initial condition for P_{ij} was set such that the semi-minor axis of P_{ij} is identical to that of the measured bubble. The dimensions along the other two axes were assumed to be the same, and the total volumes of the modelled and measured geometries were also kept the same. This initial condition essentially fits the measured bubble geometry with an oblate spheroidal shape. The same initial conditions and integration methods will be employed throughout the remainder of this paper.

Both the semi-major and semi-minor axes integrated from the new model with the pseudo-strain-rate term are shown in figure 4(b) along with the experimental result presented in figure 4(a) for comparison. In this case, $f'_1 = 1$ and $K_s = 0.14$ were used. The reasons for selecting these values will be discussed in § 4.1. It can be seen that the model successfully reproduces the oscillation of both r_3 and r_1 simply based on the oscillation of the slip velocity. In addition, the oscillation amplitude differs slightly between the experimental results and the model prediction as the modelled oblate spheroidal geometry is an approximation and the actual bubble will always have some deviation from this idealized case.

In addition to the shape oscillation, decades of works have revealed the following essential picture of the dynamics of finite-sized air bubbles rising in purified water with no contaminants or surfactants. The bubble first rises along a straight line, followed by a zigzag motion and subsequent spiral circular motion. During this process, its orientation also oscillates along with its slip velocity. It has been shown by many previous works that the semi-minor axis of a bubble oscillates between 0° and 30° (Mougin & Magnaudet 2001) with respect to the vertical direction. Such an oscillation arises from the wake dynamics (Mougin & Magnaudet 2001, 2006; Ern *et al.* 2012; Tayler *et al.* 2012). The wake vortices break and reform with reversed rotation near the inflection point of the zigzag motion (Mougin & Magnaudet 2001), whereas the wake is continuously generated during the spiral motion.

Figure 5 displays an example time trace of the relative orientation between $\hat{\mathbf{r}}_3$ and the vertical direction $\hat{\mathbf{z}}$. The orientation oscillation can be clearly captured by the 3-D shape reconstruction. The black line represents the prediction based on the modified MnM model with the addition of the pseudo-strain-rate term. It is clear that the modelled results appear to capture the orientation oscillation. However, for each oscillation period, as the relative orientation reaches the peak and begins to drop, the predicted time trace consistently lags behind the experimental one. We found that the lag is linked to the fact that the predicted

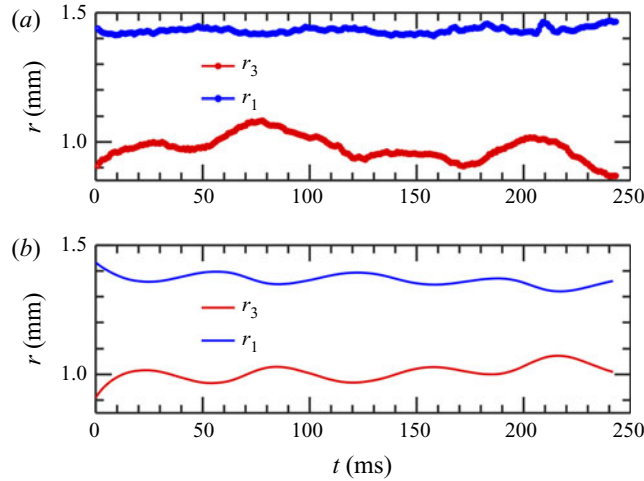


Figure 4. Example time traces of the semi-major (blue) and semi-minor (red) axes of an air bubble rising in water at rest from (a) direct experimental measurements and (b) the model calculation by using the pseudo-strain-rate term based on the slip velocity.

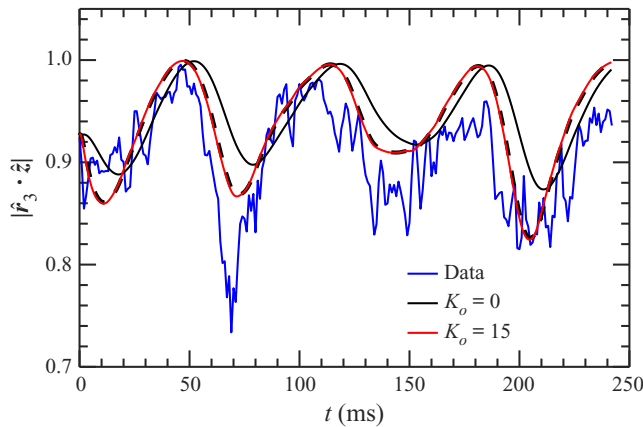


Figure 5. Time evolution of the cosine of the angle between the semi-minor axis of a bubble and the vertical direction $\hat{z}$, including the direct experimental measurements (blue) and the model predictions with $K_o = 0$ (black solid line), $K_o = 15$ (red solid line) and $f_2' = 0$ and $K_s = 0$ (rigid-particle limit, black dashed line).

bubble semi-minor axis does not rotate away from the vertical axis as quickly as the slip velocity does.

To illustrate the underlying mechanism, figure 6 displays the possible processes of a bubble following the re-orientation of the slip velocity. In this case, the bubble is shown as an oblate spheroid geometry at t_0 as if it was compressed due to a slip velocity aligned with the vertical z axis prior to t_0 . At t_0 , $\mathbf{u}^s$ switches to a new direction aligning with the y axis. There are two possible means by which a bubble can adjust its orientation to the new $\mathbf{u}^s$. The first way is through deformation. The pseudo-strain rate in this case acts to first assist the restoring force to help the bubble to return to a sphere and then continues compressing the bubble along the new $\mathbf{u}^s$ direction. In this case, the bubble axes do change their orientations, but rather their lengths. However, as a result, the semi-minor axis appears to switch from the z axis to the y axis.

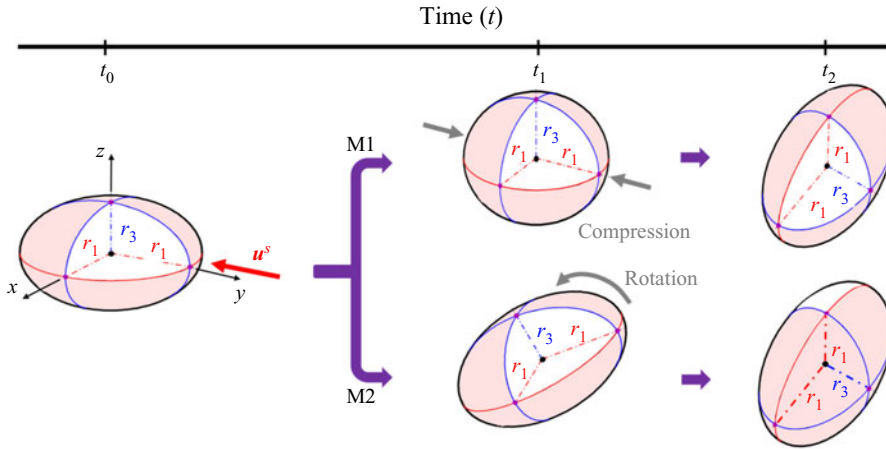


Figure 6. Schematic of two possible bubble-reorientation mechanisms as the slip velocity changes its direction, including (M1) deformation along a different direction, or (M2) simple rotation while maintaining the original geometry.

Alternatively, the bubble could simply rotate toward the new direction of $\mathbf{u}^s$ while maintaining its original oblate spheroidal geometry. The evidence to support such a mechanism can be drawn from decades of research on the path instability observed both for bubbles (Magneaud & Eames 2000; Mougin & Magnaudet 2001, 2006) and rigid non-spherical particles rising/settling in an otherwise quiescent medium (Ern *et al.* 2012). It has been shown that rigid oblate spheroidal particles can exhibit similar orientation oscillation as deformable bubbles (Mougin & Magnaudet 2006; Fernandes *et al.* 2008; Cano-Lozano *et al.* 2016). This observation has led to a conclusion that the deformability effect is not important for path instability, and it also implies that the oscillation of the bubble orientation is likely connected to rotation rather than deformation. Following this argument, the pseudo-strain-rate term might not be sufficient to account for all effects introduced by the slip velocity, as it does not contain an anti-symmetric component to describe the bubble rotation due to wake–bubble interaction.

The rigorous method of modelling this bubble rotation due to wake–bubble interaction is through the Kevin–Kirchhoff equation (Kirchhoff 1870; Mougin & Magnaudet 2002)

$$(m\mathbf{I} + \mathbf{A}) \cdot \frac{d\mathbf{u}^s}{dt} + m\boldsymbol{\omega} \times \mathbf{u}^s = \mathbf{F} + (m - \rho_c V)\mathbf{g}, \quad (3.7)$$

$$(\mathbf{J} + \mathbf{D}) \cdot \frac{d\boldsymbol{\omega}}{dt} + \boldsymbol{\omega} \times (\mathbf{J} \cdot \boldsymbol{\omega}) = \boldsymbol{\Gamma}, \quad (3.8)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{u}^s$ and $\boldsymbol{\omega}$ indicate body translational and rotational velocities respectively with their main axes aligning with the principal axes of the body. To be consistent with the rest of the paper, the translational velocity is denoted the same as the slip velocity since the background flow velocity is close to zero. Additionally $\mathbf{A}$ and $\mathbf{D}$ are the translational and rotational added-mass tensors, respectively. For bubbles with negligible inertia, both mass m and moment of inertia tensor $\mathbf{J}$ should be close to zero; $\mathbf{F}$ and $\boldsymbol{\Gamma}$ are the instantaneous hydrodynamic force and torque, respectively, obtained by integrating local stresses and moments over the bubble interface. Many attempts have been made to solve the Kevin–Kirchhoff equations by coupling them with the Navier–Stokes equation (Mougin & Magnaudet 2002), potential flow approximation (Fernandes *et al.* 2008) and subcritical bifurcation model of the lateral lift force (Shew & Pinton 2006).

Through these models and simplifications, the importance of the wake dynamics and the added-mass effects have been established. Note that, in this framework, bubbles are considered as rigid spheroids without the deformation oscillation, which suggests that the deformation and orientation oscillation can be separately modelled.

To model the orientation oscillation, a new pseudo-rotation tensor Ω'_{ij} is introduced to characterize the bubble rotation induced by the wake dynamics

$$\Omega'_{ij} = -\frac{1}{2}\epsilon_{ijk}\omega'_k, \quad (3.9)$$

where ω' is the pseudo-vorticity vector. This equation connects with the Kelvin–Kirchhoff equation (3.8) based on the relationship between vorticity and the object angular velocity: $\omega' = 2\omega$. Although it might appear that ω' is readily available after solving (3.7) and (3.8) together, the inputs to these equations, i.e. $\mathbf{F}$ and $\mathbf{\Gamma}$, can only be acquired by integrating hydrodynamic forces over the entire bubble interface, which requires access to the entire flow field nearby a bubble. Since the goal of the proposed framework is to model the bubble dynamics based on simplified flow information, we cannot rely on the Kelvin–Kirchhoff equation directly, at least not in its complete form without a simple but realistic model for $\mathbf{F}$ and $\mathbf{\Gamma}$. In the current framework, we turn to a simple experimental observation that $\hat{\mathbf{r}}_3$ always attempts to align with the direction of $\mathbf{u}^s_i$, which is consistent with what has also been reported in previous works (Mougin & Magnaudet 2001). Based on this observation, we propose that $\omega' = \phi(\hat{\mathbf{r}}_3 \times \hat{\mathbf{u}}^s)/|\hat{\mathbf{r}}_3 \times \hat{\mathbf{u}}^s|$. The variable ϕ is the angle between two unit vectors: $\hat{\mathbf{r}}_3$ and $\hat{\mathbf{u}}^s$. This pseudo-vorticity vector ω' points to a direction that is perpendicular to both $\hat{\mathbf{r}}_3$ and $\hat{\mathbf{u}}^s$. In this way, ω' is designed such that the pseudo-rotation tensor Ω'_{ij} rotates the semi-minor axis of the bubble towards the direction of the slip velocity at a rate linearly proportional to ϕ .

Finally, adding both the pseudo-strain-rate (S'_{ij}) and pseudo-rotation (Ω'_{ij}) terms to equation (3.6) leads to a new model (the finite-sized bubble deformation model, or the FBD model hereafter) for describing the affine deformation of finite-sized bubbles in both linear flows and turbulence

$$\begin{aligned} \frac{dP_{ij}}{dt} = & \tilde{\Omega}_{ik}P_{kj} - P_{ik}\tilde{\Omega}_{kj} + f'_2(\tilde{S}_{ik}^sP_{kj} + P_{ik}\tilde{S}_{kj}^s) - \frac{f'_1}{\tau_n}(P_{ij} - g(\Pi_P, \text{III}_P)\delta_{ij}) \\ & + K_o(\Omega'_{ik}P_{kj} - P_{ik}\Omega'_{kj}) + K_s(S'_{ik}P_{kj} + P_{ik}S'_{kj}). \end{aligned} \quad (3.10)$$

The coefficients K_o and K_s are constants that set the relative roles played by these two new terms, respectively. These two new coefficients must be constrained through comprehensive experimental results that are introduced in § 4.1. Here, to demonstrate that they can capture the observed oscillations for bubbles rising in an otherwise quiescent medium, we fix them at $K_s = 0.14$ and $K_o = 15$.

The red line in figure 5 shows the predicted relative orientation of the semi-minor axis of the bubble, which appears to better agree with the measured results than does the black line obtained using the pseudo-strain-rate term alone, capturing not only the overall trend but also the shape of each peak. Moreover, the lag between the blue line and black line at the trailing edge of each peak is reduced by the addition of the pseudo-rotation term.

Furthermore, to ensure that the model can reproduce the key observation that the bubble deformation does not affect the orientation oscillation, we performed another test of the model by setting $f'_2 = 0$ and $K_s = 0$, which returns to the solid-particle limit that forces the bubble to retain its initial geometry throughout the entire time trace without any deformation. The results are shown as the black dashed line in figure 5. This line falls

directly on top of the red line, suggesting that the orientation oscillation can be reproduced even for rigid particles without deformation.

Similar tests were performed to repeat the calculation of the shape oscillation in figure 4(b) for cases with or without the pseudo-rotation term. The results are almost identical, suggesting that the deformation dynamics is primarily dominated by the pseudo-strain-rate term. Combining this test with the previous one on the deformation oscillation, we conclude that K_o and K_s , can be separately evaluated based on the deformation and orientation oscillation. In addition, the FBD model, in essence, is a first-order linear model that cannot capture the free oscillation of a bubble. The emergence of the oscillation in both the deformation and orientation dynamics arises from the oscillation in the inputted slip velocity.

4. Results and discussion

In (3.10), there are four dimensionless coefficients: f'_1 , f'_2 , K_o and K_s . Other than f'_1 for the relaxation term, each driving mechanism has a coefficient that needs to be determined from experimental results. The advantage of our experimental results is that we have a rather unique dataset with simultaneous measurements of the two phases. This independent measurement of these two phases provides a way to constrain different coefficients and validate the proposed FBD model. In this section, we intend to link different coefficients to different statistics so that we can constrain them one by one without multi-variable fitting.

4.1. Quiescent rising

The time traces of both the bubble geometry and orientation have already been used in the previous section to introduce the additional terms. It is clear that the new model works well. In this section, the focus is shifted to the discussion of different coefficients. Here, special attention must be paid to two new coefficients f'_1 and f'_2 that replace $f_1(\mu)$ and $f_2(\mu)$ in the MnM model; $f_1(\mu)$ and $f_2(\mu)$ were introduced as non-dimensional and non-negative terms that quantify the relative roles played by the relaxation and shear stresses in linear shear flows, both of which are related to the viscosity ratio μ . For finite-sized bubbles in turbulence, the capillary number is negligible and the turbulence Weber number becomes large so the bubbles are deformed by inertial forces. In this case, the gradient of the dynamic pressure is more important than the shear stresses. Therefore, the flow viscosity becomes secondary, and $f_1(\mu)$ and $f_2(\mu)$ are replaced with f'_1 and f'_2 , both of which are independent of the viscosity ratio.

Note that $f_1(\mu)$ enters the MnM model along with the relaxation time scale τ , which is also a function of μ . For the same aforementioned reason, τ should not depend on μ for finite-sized bubbles. Therefore, the relaxation frequency $f_1(\mu)/\tau$ is replaced with the natural frequency of a bubble under small deformation, namely $1/\tau_n = \sqrt{(96\sigma)/(\rho_c D^3)}/2\pi$ based on Lamb's mode 2 frequency. Following this argument, f'_1 should be of order unity, and τ_n should follow $2\pi/\sqrt{(96\sigma)/(\rho_c D^3)}$ for all cases discussed in this paper. For convenience, f'_1 is fixed to 1.

Once f'_1 is fixed (and $f'_2 = 0$ for quiescent rising), the two remaining coefficients are K_s and K_o , which are related to the pseudo-strain-rate and pseudo-rotation terms, respectively. As we have shown previously in figures 4 and 5, not all statistics depend on both coefficients at the same time: the deformation oscillation is not sensitive to K_o , whereas the orientation oscillation does not rely on K_s .

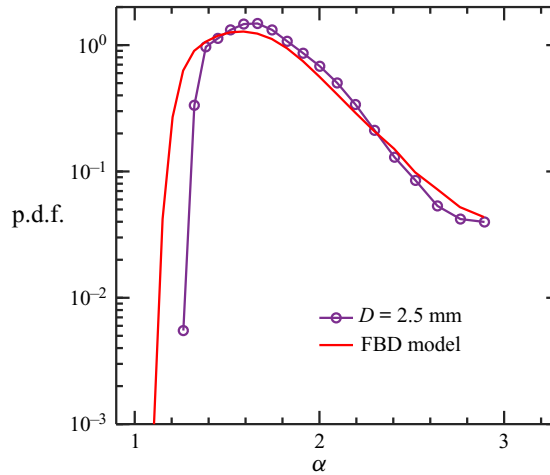


Figure 7. The probability density distribution of aspect ratio α for bubbles with diameter of $D = 2\text{--}3$ mm rising in an otherwise quiescent medium, including direct measurements (open circles) and model predictions (red solid line).

Symbols in figure 7 show the probability distribution function (p.d.f.) of the bubble aspect ratio $\alpha = r_1/r_3$ from our experiments, i.e. $p(\alpha_e)$. In this particular case, the bubble size D is around 2.5 mm. Most bubbles have small aspect ratios, but the tail of $p(\alpha_e)$ extends to a large α close to 3. In addition to the measured $p(\alpha_e)$, the distribution of α can also be determined by integrating the FBD model ($p(\alpha_m)$) using (3.10) with a selected K_s . For each bubble trajectory, the initial condition of P_{ij} is set the same as the oblate fit of the bubble 3-D reconstructed geometry; P_{ij} for the remainder of the trajectory is numerically integrated using the time traces of the slip velocity as inputs.

A nonlinear search for K_s was performed to minimize the difference between the two p.d.f.s: $p(\alpha_m)$ and $p(\alpha_e)$. Based on this search, $K_s = 0.14$ was obtained, which is similar to the result if we minimize the difference between α_m and α_e for all bubble trajectories directly. The final p.d.f. of α calculated using the FBD model with $K_s = 0.14$ is shown as the red line in figure 7. With only one fitting parameter, $p(\alpha_m)$ agrees well with the directly measured p.d.f. In particular, the model captures both the long right tail for large α and a steep drop for small α close to one.

The mean α calculated from both p.d.f.s differ by only 3.5 %. This small difference comes from the lower left tail from the model prediction. Such a difference could potentially be attributed to the experimental uncertainty of the 3-D shape reconstruction, which has been discussed systematically in another paper (Masuk *et al.* 2019b). The key point is that this uncertainty is larger when the shape is close to a sphere as any reconstruction artefacts could result in an overestimation of α . In other words, we have greater confidence in large α than those close to 1. Thus, when we fit K_s , more weight is put on the right tail of α than the left.

Once K_s and f_1' are fixed, the only remaining coefficient for the quiescent rising case is K_o , which can be constrained by the bubble orientation. Note that the distribution of the alignment between $\hat{r}_3$ and $\hat{z}$ is very close to 1. As shown in figure 5, the oscillation of $|\hat{r}_3 \cdot \hat{z}|$ is from 0.8 to 1. Thus, rather than fitting K_o based on the p.d.f., we directly fit K_o based on all the instantaneous trajectories, which are all quite similar to what has been shown in figure 5 because bubbles rising in a quiescent medium are very reproducible. This fit yields $K_o \approx 15$.

Note that the experiments for both the quiescent and turbulent carrier phases were conducted in purified water. However, a high concentration of tracer particles was added to the turbulent case to resolve the flows close to bubbles but no tracer particles were used for the quiescent case. As a result, the bubble rise velocity in an otherwise quiescent medium matches with the results obtained in prior experiments for clean bubbles. However, in turbulence, the lift and drag coefficients are closer to the reported values for contaminated ones (Salibindla *et al.* 2020). Nevertheless, the interface contamination will not affect the model prediction as the slip velocity between the two phases enters into our model as an input. As shown by recent experiments (Aoyama *et al.* 2018), the relationship between the bubble aspect ratio and the Weber number appears to be the same for both clean and contaminated bubbles as long as the respective rise velocity and surface tension are used in the Weber number.

4.2. Turbulence

We assume that the three coefficients: f'_1 , K_o and K_s , determined from the quiescent case, can be directly applied to describe the bubble deformation in turbulence as the key underlying physics that bubbles are deformed by the competition between the gradient of the dynamic pressure and the restoring surface tension does not change whether bubbles move in a quiescent or a turbulent medium.

The key differences between the quiescent and the turbulent cases include: (i) the slip velocity comes not only from buoyancy but also from the random fluctuations of the surrounding turbulent flows; (ii) the local velocity gradients are not zero and must be measured along with the bubble trajectory. Their contributions to the bubble deformation introduce the additional coefficient f'_2 . In our experiments, both the slip velocity and the local coarse-grained velocity gradient can be measured accurately. Additional details can be found in a recent paper by Masuk *et al.* (2021a). Here, the same dataset is used to evaluate the new model.

To constrain f'_2 , the statistics of the measured bubble 3-D shape are used. In particular, figures 8(a) and 8(b) present the p.d.f.s of the semi-major and semi-minor axes for a range of bubble sizes ($D = 3\text{--}7$ mm), respectively. For both cases, the symbols represent the measured results, whereas the solid lines of the same colour show the model predictions by integrating equation (3.10) using the slip velocity and velocity gradients along the bubble trajectories.

As D grows, the p.d.f.s of both r_1 and r_3 shift monotonically rightward. At the same time, the p.d.f. becomes wider because the distribution of the Weber numbers based on the slip velocity and velocity gradients expand (this result has been shown elsewhere by Masuk *et al.* 2021a), indicating that large bubbles are more susceptible to stronger deformation. It can be seen that both features can be successfully captured almost perfectly by the new FBD model.

Note that the overall shape of the p.d.f. is controlled both by the slip velocity and by velocity gradients. However, the role played by the slip velocity, controlled by K_s , has already been fixed at 0.14 based on the discussions in § 4.1 so the only unknown here is f'_2 , which controls the contribution of the turbulent strain rate. When f'_2 increases, the p.d.f. becomes wider and the peak shifts rightward. To some degree, f'_2 is overconstrained because only one parameter is needed to capture two features (peak location and width) of the distribution of r_1 and r_3 for a range of bubble sizes. In practice, the optimization was performed for r_1 of one size at $D = 3$ mm, from which $f'_2 = 0.5$ was obtained. In figure 8, it is evident that, although f'_2 is fitted based on one size, it helps to match the p.d.f.s of both

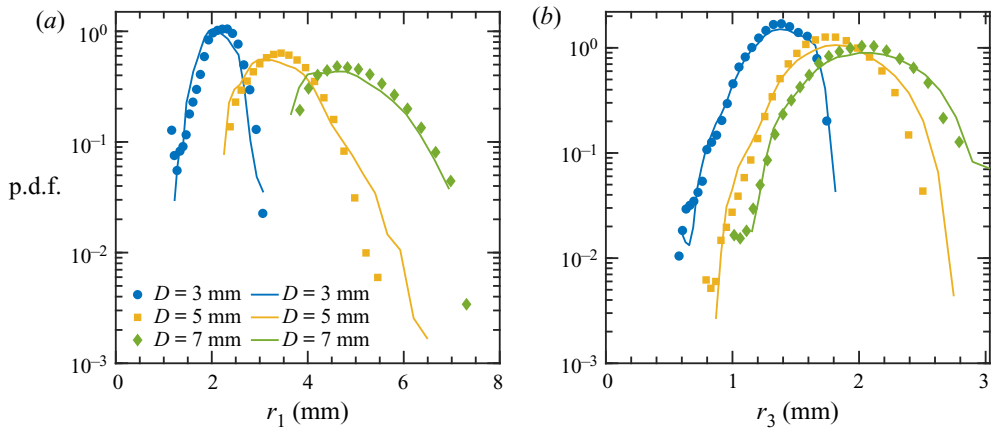


Figure 8. The probability density distributions of (a) the semi-major axis (r_1) and (b) the semi-minor axis (r_3) of bubbles with three different sizes ranging from $D = 2\text{--}4$ mm to $D = 6\text{--}8$ mm in turbulence. Symbols show experimental results while solid lines of the same colour indicate the corresponding model predictions.

r_1 and r_3 for all three sizes, which confirms that the model provides an excellent prediction of the bubble geometry with a range of sizes.

Further tests are performed to reproduce the aspect ratio $\alpha = r_1/r_3$. The distribution of α for three different sizes of bubbles are shown in figure 9. The shape and overall changes of p.d.f. as a function of D are similar to the discussions for r_1 and r_3 . For the smallest size of bubble ($D = 3$ mm), the model prediction appears to agree well with the measured p.d.f. of α as this is the size that we used to fit f'_2 . For $D = 5$ mm and $D = 7$ mm, the agreement is still very good, capturing the overall trend, including the steep drop at small α and long tail at large α . However, a small deviation can be seen: the peak location shifts towards smaller α and the long right tail seems to drop at a slower rate as α increases. This small difference can be attributed to two reasons: (i) the definition of α ; the reconstructed 3-D surface consists of many vertices. The values of r_1 and r_3 , from the experiments, are defined as the longest and shortest centre-to-vertex distances, respectively. For bubbles deforming in turbulence, any concave areas on the bubble interface could result in a smaller semi-minor axis and thus a large aspect ratio. Such a process can never be captured by a linear model which consequently results in an underprediction of α . (ii) Nonlinear deformation contributed by eddies of a size smaller than D ; this effect was not accounted for in the linear FBD model (3.10).

In addition to distributions, in figure 10, two example time traces of both experimental measurements and model predictions for semi-axes of two different bubbles deforming in turbulence are shown. Panels (a,b) show the experimental measurements of r_1 and r_3 , whereas (c,d) show the corresponding model prediction. It is evident that the model manages to predict the overall trend of the temporal fluctuations of the bubble deformation for both cases. It is important to admit that not all measured time traces of r_1 and r_3 agree with their calculated counterparts. Even for the two cases shown in figure 10, although the model captures the overall trend, it clearly misses the small-scale fluctuations, which are likely driven by eddies smaller than the bubble size. In addition, the measured peaks of r_1 appear to lag behind the calculated results. This phase lag is expected. In the FBD model, the bubble response frequency is fixed at its natural frequency obtained by assuming small-amplitude oscillation. Although it provides a good overall estimation, it is not necessarily well suited for large-amplitude deformation that is likely to be nonlinear.

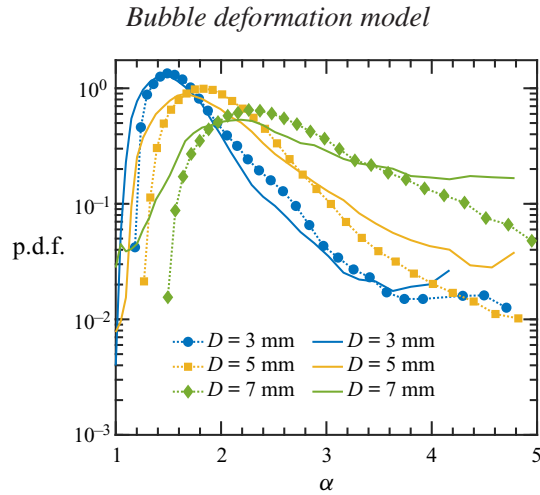


Figure 9. The probability density distribution of aspect ratio α of bubbles with different sizes in turbulence. Symbols show experimental results, while solid lines of the same colour indicate the model predictions.

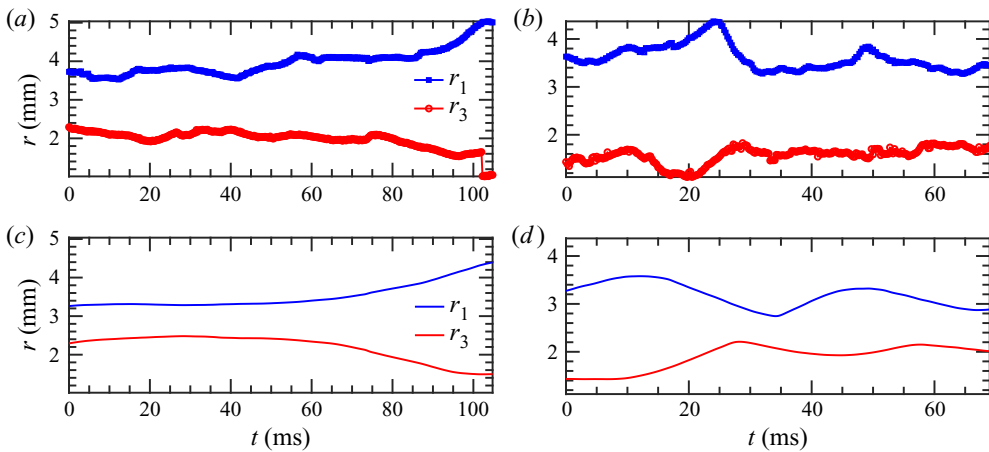


Figure 10. Two example time traces of the semi-major (r_1) and semi-minor (r_3) axes of bubbles deforming in intense turbulence, with both directly measured results (*a,b*) and model predictions (*c,d*).

As a result, one can observe phase lags between the measured and calculated r_1 . Ideally, τ_n in the FBD model should also be a function of $\alpha(t)$. For simplicity, this more complicated correction is not modelled in the current framework.

Once all four coefficients f'_1 , f'_2 , K_o and K_s are fixed, we can evaluate the performance of the FBD model in predicting the bubble orientational dynamics. Given that the bubble deformation is controlled by both the slip velocity and the velocity gradients, the bubble orientation is shown as the cosine of the angle (W) between the bubble semi-minor axis ($\hat{r}_3$) with either $\hat{u}^s$ (figure 11*a*) or one of the eigenvectors of the coarse-grained velocity gradients, i.e. $\hat{e}_3$ (figure 11*b*). If the bubble orientation is completely random, the p.d.f. of W should follow a uniform distribution at 1 for the entire range of W . If the distribution peaks at 1, this indicates that $\hat{r}_3$ preferentially aligns with that vector. In figure 11(*a*), it is evident that $\hat{r}_3$ shows the strongest alignment with the slip velocity. In figure 11(*b*), the preferential orientation of $\hat{r}_3$ with $\hat{e}_3$ confirms that bubbles are compressed along $\hat{e}_3$ and therefore align their semi-minor axes $\hat{r}_3$ with it.

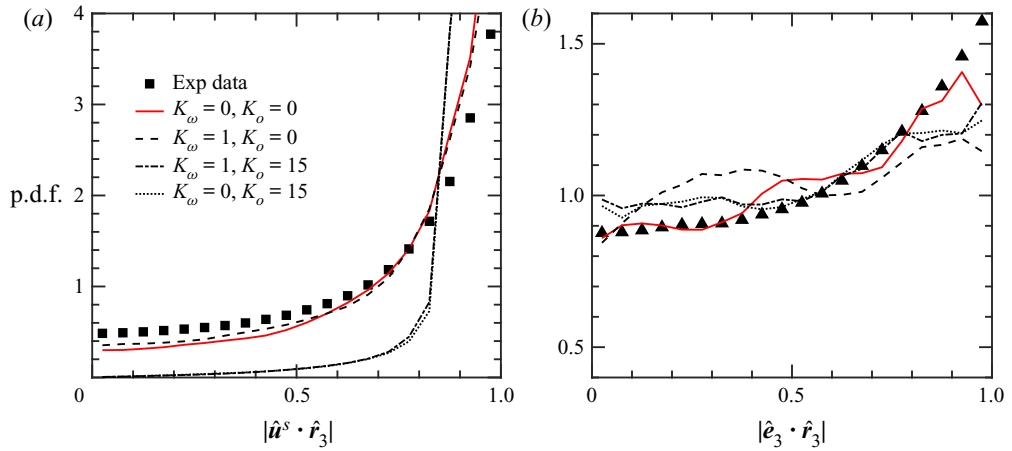


Figure 11. The probability density distribution of the alignment between (a) the bubble semi-minor axis ($\hat{r}_3$) and the slip velocity ($\hat{u}^s$), (b) the bubble semi-minor axis ($\hat{r}_3$) and the compression ($\hat{e}_3$) directions of their surrounding coarse-grained strain-rate tensor. Symbols show directly measured results, while lines indicate the model prediction using different combinations of coefficients.

In addition to the measured distribution, we also integrate the FBD model (3.10) to obtain the p.d.f.s of the alignment using the slip velocity and velocity gradients along the bubble trajectories as inputs. Since all coefficients have already been determined from other tests, we begin by calculating the bubble orientation by using the same set of coefficients ($f'_1 = 1$, $f'_2 = 0.5$, $K_s = 0.14$, $K_o = 15$) similar to the deformation dynamics. The resulting orientation is shown as dash-dotted line in figure 11(a,b). Although the relative orientation between $\hat{r}_3$ with $\hat{e}_3$ in (b) appears to reproduce a trend similar to the measured results, the model predicted $\hat{r}_3$ shows a much stronger alignment with $\hat{u}^s$.

We realize that the strong alignment between $\hat{r}_3$ and $\hat{u}^s$ is contributed by the coefficient for the pseudo-rotation term (K_o) being too large, which essentially forces $\hat{r}_3$ to immediately adjust to the direction of the new $\hat{u}^s$ at every time steps. The fact that K_o might be smaller in turbulence than in a quiescent medium is not surprising as the pseudo-rotation term is linked to the wake-induced bubble rotation. For a bubble rising in water at rest, a persistent wake forms behind the bubble. In this case, K_o is large and the pseudo-rotation term is important. In turbulence, particularly in intense turbulence with a large energy dissipation rate, the wake, even it forms, might not be sustained long enough behind the bubble before it is perturbed by the pre-existing background turbulence. Therefore, it is possible that the importance of the pseudo-rotation term becomes smaller in intense turbulence. To test this conjecture, we calculate the bubble orientation by setting $K_o = 0$ (dashed line), and the results become very close to the measured alignment between $\hat{r}_3$ and $\hat{u}^s$. But even if the wake effect becomes smaller, it should still exist for finite-sized bubbles with a large enough slip velocity. The fact that setting $K_o = 0$ does not affect the bubble orientation suggests that the reorientation of bubbles in turbulence might not be dominated by rotation, if present at all, and it is the deformation that controls the bubble orientation (M1 rather than M2 in figure 6). Continuing this logic leads to the next argument: if the bubble rotation is not important, we hypothesize that even the coarse-grained vorticity from the ambient turbulence is no longer important for the bubble orientation dynamics. Note that the coarse-grained rotation term ($\tilde{\Omega}_{ij}$) does not have a coefficient based on the original MnM model. Here, to quantify its importance,

we introduce a new coefficient K_ω , which was essentially set to be 1 for the MnM model and the new FBD model. To test the hypothesis of bubble rotation, K_ω is set to be zero here, as well. The result is shown as the red solid line in both [figure 11\(a,b\)](#). Consistent with our expectations, removing the contributions of both rotational terms provides the best prediction of bubble orientation, which implies that the bubble reorientation in turbulence occurs primarily due to deformation along another direction rather than rotation to a new direction. This is nearly opposite to the bubble reorientation in a quiescent medium, in which the wake-induced rotation could be equally as important, if not more.

4.3. Possible implementation of the model in numerical simulations

Before discussing the implementation of the FBD model in numerical simulations, we would like to briefly review how the dynamics of point spherical particles (Maxey & Riley 1983), point non-spherical particles (Voth & Soldati 2017), point bubbles (Magnaudet & Eames 2000; Lohse 2018) and point neutrally buoyant deformable droplets (Elghobashi 2019) are simulated along with the carrier-phase turbulence. In all these cases, whether the particle is spherical or deformable, the translational motion is computed by integrating the Maxey–Riley equation based on the information of the carrier phase, including the local fluid velocity to calculate the drag force. The local flow velocity gradient and acceleration must be included if the lift force and added-mass force are considered, respectively. Beyond the translational motion, the orientation dynamics of a non-spherical rigid particle can be captured by Jeffery’s equation (Jeffery 1922; Voth & Soldati 2017). If such a particle is deformable, one must include the MnM model (Elghobashi 2019). For both Jeffery’s equation and the MnM model, the local flow velocity gradient is the key input variable.

For finite-sized bubbles, although the Maxey–Riley equation cannot be applied directly due to the violation of the point-bubble assumption, one might still attempt finite-size corrections (Homann 2010) to calculate the bubbles’ translational motions. Once their trajectories are available, the deformation and orientation dynamics can be calculated from the FBD model by inputting the fluid velocity and velocity gradients coarse grained at the bubble size. Although the FBD model in (3.10) appears to be complicated, it can be implemented in simulation just like the Jeffery’s equation for non-spherical particles and the MnM model for small deformable droplets with an identical number of input variables.

Furthermore, the FBD model can be used along with simulations that consider two-way couplings and bubble-induced turbulence. The particle-induced turbulence will be solved during the first step, in which the bubble motion and its feedback to the surrounding flows are solved at the same time through the Euler–Lagrange framework (Lain *et al.* 2002). The FBD model takes inputs from the solved flows that already contain the contributions from bubble-induced turbulence to determine the bubble deformation and orientation, which in turn can be used to determine the drag tensor and added-mass tensor for the next time step.

5. Conclusion

This work focuses on developing a model capable of capturing the key deformation and orientation dynamics of finite-sized bubbles in both quiescent and turbulent media. The model uses simplified surrounding flow information as inputs and outputs the bubble geometry and orientation. Such a model can only be developed from and evaluated by experiments that have access to both the flow information and the bubble geometry simultaneously in three dimensions.

Such a simultaneous measurement was made possible through our experimental efforts. Six high-speed cameras were used to reconstruct the 3-D shape of bubbles with their surrounding 3-D tracer particle trajectories in a vertical water tunnel, allowing for the creation of both a quiescent medium for bubbles to rise up or a turbulent environment with a large energy dissipation rate to deform finite-sized bubbles. For the turbulent case, it has been found that the deformation of finite-sized bubbles is primarily driven by the dynamic pressure gradients from the local velocity gradients and the slip velocity.

Based on our observation, a new FBD model is proposed in this work. The FBD model is a linear model that comes from the potential flow solution to capture the affine ellipsoidal deformation of bubbles in both quiescent and turbulent media. The FBD model extends the work by Maffettone & Minale (1998), which was limited to small sub-Kolmogorov-scale drops, by adding a few new components: (i) replacing the local velocity gradients with the velocity gradients coarse grained at the bubble size; (ii) accounting for the bubble deformation responding to the local slip velocity by using the a pseudo-strain-rate tensor; (iii) modelling the wake-induced bubble rotation by adding a pseudo-rotation tensor. Furthermore, the two main driving terms that account for the deformation induced by the coarse-grained local velocity gradients and the slip velocity scale linearly with the Weber number rather than the Capillary number due to the finite bubble Reynolds number effect.

To test the performance of the FBD model, the time series of the coarse-grained velocity gradient and the slip velocity from the direct experimental measurements, following each bubble trajectory, in both a quiescent and a turbulent medium are input into the model. The output from the model is the time evolution of the bubble geometry and orientation, which can also be directly and independently measured from the 3-D bubble shape reconstruction. The difference between the calculated and measured geometries and orientations provides a unique way of calibrating and validating the proposed FBD model.

In the FBD model, there are four new coefficients. The coefficient for bubble relaxation can be fixed based on the bubble natural frequency, while the remaining three coefficients can be isolated and calibrated by connecting each of them to individual statistics. In particular, the coefficients associated with the deformation and rotation by the slip velocity and coarse-grained strain rate can be constrained based on the bubble deformation and reorientation in quiescent and turbulent media, respectively. Finally, based on the statistics of the bubble orientation, we determine that both the flow rotation and the pseudo-rotation terms are negligible in controlling the bubble orientation in turbulence because, in strong turbulence, the rotation of a bubble is driven by the deformation along a different direction due to the re-orientation of the strain rate and slip velocity rather than from rotation. Finally, we propose how to implement the FBD model in simulations to describe the deformation and orientation dynamics of finite-sized bubbles/drops in turbulence.

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