Spectral Efficiency of Multi-Antenna Index Modulation

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2021 IEEE International Symposium on Information Theory (ISIT) | 978-1-5386-8209-8/21/$31.00 ©2021 IEEE | DOI: 10.1109/ISIT45174.2021.9517864

***Abstract*—Index modulation emits information through the index of the activated component of a vector signal as well as the value of the activated component. Indexing could occur across antennas, subcarriers, or other degrees of freedom. When index modulation is applied to antennas, it is known as spatial modulation. Earlier bounds or estimates for the spectral efficiency of spatial modulation have been too loose for determining the parameters of coding and modulation, which are important in practice. Furthermore, the best bounds formerly available did not effectively elucidate the relationship of spatial modulation capacity with SIMO and MIMO capacity at low- and high-SNR. The present work develops novel, tighter bounds on the spectral efficiency of spatial modulation. Specifically, for a 4×2 antenna configuration at 8 bits/s/Hz, our results are 2dB tighter than the best bounds available in the literature.**

I. Introduction

Index modulation conveys information by selecting a subset of elements of a transmit vector such as antennas, subcarriers, etc., in addition to the information conveyed through the values of the selected elements [1], [2]. Spatial modulation is the most popular index modulation technique in which one out of n_t available antennas is activated per channel use, and a modulation symbol is transmitted from the active antenna [3], [4]. Both the index of the active antenna and the transmitted symbol carry information. Spatial modulation is driven by the idea of utilizing the available multi-antenna channel while limiting complexity. Spatial modulation has low transmitter hardware complexity by requiring only one transmit RF chain. Also, spatial modulation receivers are fairly simple since multi-antenna interference is absent. Spatial modulation has been studied extensively in the literature. For a recent survey, see [5] and the references therein.

Knowledge of spectral efficiency is necessary for the design of modulation and coding. The work in [6] shows via simulations that spatial modulation with a single receive antenna can achieve higher capacity compared to SISO, but it does not provide closed-form capacity or bounds. The works in [7], [8], [9], [10] derive approximations or bounds on capacity of spatial modulation, but the results are not sufficiently tight for determining parameters of modulation and coding. The work in [11] states that the capacity of spatial modulation is equal to the number of receive antennas times the capacity of AWGN channel [11, Eq. (30)], which on the face of it violates the MIMO bound on the degrees of freedom of the $2 \times n_r$ channels, for large n_r.

This work was made possible in part by the grants 1711689 and 1718551 from the National Science Foundation.

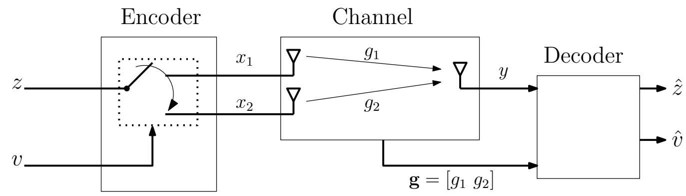

Fig. 1. Schematic diagram of 2×1 spatial modulation.

The difficulty in calculating the spectral efficiency for spatial modulation is due to the character of its signaling: the signal transmitted from each antenna is effectively the product of two information-carrying variables, one choosing the antenna index and the other representing the signal emitted from the chosen antenna. Thus far, lower bounds or approximations have either directly calculated the mutual information by approximating the statistics of the spatial modulation transmissions [9], or have used the chain rule [7].

Our lower bound also utilizes a chain rule. The distinction of our lower bound is that its calculation does not employ further conditioning or approximation of the chain rule terms; its only departure from optimality is due to choosing a codebook distribution which has not been proven optimal.

In contrast, [7] further bounds the information term for the antenna index, which weakens their inner bound. The work in [9] uses a Gaussian mixture model for the transmit signals and bounds its entropy. The resulting lower bound does not provide insight into the relationship of spatial modulation with MIMO and SIMO capacity, and under-performs SIMO capacity at low-SNR. At high-SNR, the lower bound of [9] has been the best available, but is significantly improved by the present work. For example, for a 4×2 spatial modulation, at 8 bits/s/Hz, our bound is 2 dB tighter than [9].

Our outer bounds are a combination of the MIMO bound and a genie-aided SIMO bound.

This paper is organized as follows. In Section II, bounds on the capacity of 2×1 spatial modulation are derived. In Section III, the capacity bounds are extended to $n_t \times n_r$ spatial modulation. Simulation results and comparisons with previous works are presented in Section IV. Section V concludes the paper.

II. Capacity of 2×1 Spatial Modulation

Consider a 2×1 spatial modulation system shown in Fig. 1, in which information is conveyed through the index of the active antenna (captured by the variable v) and the symbol transmitted from the active antenna (denoted by z). The received signal is given by

$$y = g_1 z v + g_2 z(1-v) + w \tag{1}$$

$$= \begin{cases} g_1 z + w & \text{if } v = 1 \\ g_2 z + w & \text{if } v = 0 \end{cases} \tag{2}$$

where w obeys $\mathcal{CN}(0, \sigma^2)$, g_1, g_2 obey $\mathcal{CN}(0,1)$, and g_1, g_2, w are independent of each other and of the inputs. An average transmit power constraint of σ_z^2 is assumed on z. We do not assume z and v are independent. v is a Bernoulli random variable characterized with $\mathbb{P}(v=1) = p$, and we define the signal-to-noise ratio parameter $\rho \triangleq \frac{\sigma_z^2}{\sigma^2}$. We denote by $h(\cdot)$ the entropy of a (continuous or discrete) random variable. We denote by C_{SISO} the capacity of SISO fading channel. We begin with the following elementary, but useful result.

Proposition 1. *(2×1 spatial modulation - SISO lower bound) The capacity of 2×1 spatial modulation is lower bounded by the capacity of SISO fading channel.*

$$C_{SM} \geq C_{SISO}. \tag{3}$$

Proof. This is a simple outcome of SISO being a special case of 2×1 spatial modulation (when spatial modulation elects to send no information via the index). Therefore, the capacity of spatial modulation is no less than SISO capacity. □

Remark 1. It can be shown that the SISO lower bound is tight at low SNR, i.e., $\frac{C_{SM}}{C_{SISO}} \underset{\rho\to 0}{\longrightarrow} 1$, using a simple sandwich argument: MISO capacity upper bounds the spatial modulation capacity, and MISO capacity tends to SISO capacity at low SNR (assuming CSIR but no CSIT) [12].[1]

We now derive an upper bound on the capacity of spatial modulation.

Proposition 2. *(2×1 spatial modulation - Genie-aided upper bound) The capacity of 2×1 spatial modulation satisfies the upper bound*

$$C_{SM} \leq C_{SISO} + 1 \tag{4}$$

Proof. We have

$$I(v,z;y|g_1,g_2) = I(z;y|v,g_1,g_2) + I(v;y|g_1,g_2). \tag{5}$$

The first term is equivalent to the mutual information across a SISO channel with CSIR.

$$\begin{aligned} I(z;y|v,g_1,g_2) =& pI(z;y|v=1,g_1,g_2) + \\ & (1-p)I(z;y|v=0,g_1,g_2) \\ =& p\, I(x_1;y|g_1) + (1-p)\, I(x_2;y|g_2) \\ =& I(x_1;y|g_1) \qquad \text{(by symmetry)} \end{aligned} \tag{6}$$

[1]The tightness of the SISO lower bound can also be shown from first principles, but the sandwich argument is more compact and is presented here for brevity.

Now consider the second term in (5)

$$\begin{aligned} I(v;y|g_1,g_2) &= h(v|g_1,g_2) - h(v|y,g_1,g_2) \\ &= h(v) - h(v|y,g_1,g_2), \end{aligned} \tag{7}$$

where the last equality is due to the independence of channel gains from input values. Combining Eqs. (5), (6), and (7)

$$I(v,z;y|g_1,g_2) = I(x_1;y|g_1) + h(v) - h(v|y,g_1,g_2) \tag{8}$$

It then follows that

$$\begin{aligned} C_{SM} &= \max_{p(v,z)} I(v,z;y|g_1,g_2) \\ &= \max_{p(v,z)} \big[I(x_1;y|g_1) + h(v) - h(v|y,g_1,g_2)\big] \\ &\leq \max_{p(v,z)} \big[I(x_1;y|g_1) + h(v)\big] \\ &\leq \max_{p(v,z)} I(x_1;y|g_1) + \max_{p(v,z)} h(v) \\ &= C_{SISO} + 1 \end{aligned}$$

□

We refer to the above upper bound as the Genie-aided upper bound since the bound can be achieved when the index information is revealed for free at the receiver.

Remark 2. Since $C_{SISO} = \Theta(\log \rho)$, the ratio of lower and upper bounds goes to one in high-SNR limit. Also, another obvious upper bound on the capacity of 2×1 spatial modulation is the capacity of 2×1 MISO since 2×1 spatial modulation is a special case of 2×1 MISO when the transmitter decides to activate one of the antennas in each transmission interval.

We now offer a Lemma that provides insight into the behavior of index modulation decoding at high SNR.

Lemma 1.

$$h(v|z,y,g_1,g_2) \overset{\rho\to\infty}{\longrightarrow} 0 \tag{9}$$

Proof. Let $\epsilon, \epsilon' > 0$ be arbitrary positive constants. Consider g_1 and g_2 such that $|g_1 - g_2| > \epsilon$. Then, there exists $0 < \delta < 1$ such that

$$\mathbb{P}(|g_1 - g_2| > \epsilon) = 1 - \delta. \tag{10}$$

Now, consider $z = \sqrt{\rho}\cdot z'$, where $z' \sim \mathcal{CN}(0,1)$. We consider a receiver strategy where we throw away the symbols with $|z'| < \epsilon'$. There exists $0 < \delta' < 1$ such that

$$\mathbb{P}(|z'| \geq \epsilon') = 1 - \delta'. \tag{11}$$

The 2×1 spatial modulation system model can be written as

$$y = (g_1 - g_2)vz + g_2 z + w. \tag{12}$$

It can be seen that, conditioned on y, z, g_1, and g_2, the effective SNR for the detection of v is bounded below by:

$$\begin{cases} \epsilon^2 \epsilon'^2 \rho & \text{w.p. } (1-\delta)(1-\delta') \\ 0 & \text{w.p. } \delta + \delta' - \delta\delta' \triangleq \delta'' \end{cases}$$

We now argue in reverse by assigning arbitrary small values for δ, δ' and hence for δ''. This induces fixed values for ϵ, ϵ' through Eqs. (10), (11). For these fixed values, we can increase

ρ sufficiently to make $\epsilon^2\epsilon'^2\rho$ as large as desired. Under these conditions, via symbol-by-symbol detection, the symbols with a positive SNR lower bound will be correctly detected (because their SNR is made arbitrarily high as $\rho \to \infty$). Therefore the overall probability of error is limited to the symbols that did not have a positive SNR guarantee, i.e.

$$\mathbb{P}(\hat{v} \neq v) \leq \delta'' \tag{13}$$

and as mentioned earlier, we can make δ'' as small as desired. By data processing inequality:

$$h(v|y,z,g_1,g_2) \leq h(v|\hat{v}(y,z,g_1,g_2)), \tag{14}$$

where $\hat{v}(y,z,g_1,g_2)$ is the estimate of v. By Fano's inequality

$$\begin{aligned} h(v|\hat{v}(y,z,g_1,g_2)) &\leq H(\mathbb{P}(\hat{v} \neq v)) + \mathbb{P}(\hat{v} \neq v)\log_2(|\chi|-1) \\ &= H(\mathbb{P}(\hat{v} \neq v)), \end{aligned} \tag{15}$$

where the equality follows since $|\chi| = 2$. Due to the continuity of $H(\cdot)$, Eq. (13) implies $H(\mathbb{P}(\hat{v} \neq v)) < \delta'''$ for a positive δ''' that can be made arbitrarily small. This, together with (14) and (15) completes the proof. □

Remark 3. Lemma 1 implies that in spatial modulation, at high-SNR, decoding of z implies the decoding of the index v. This says that in the high-SNR regime, arbitrarily long sequences of antenna indices can be recovered at the receiver with negligible error, as long as the rate of the codebook z is sufficiently low. This exposes the tension between the recovery of antenna indices on the one hand, and the rate for the codebook z on the other hand, in the high-SNR regime.

III. Capacity of $n_t \times n_r$ Spatial Modulation

Consider a multi-antenna system with n_t transmit and n_r receive antennas. The $n_t \times n_r$ spatial modulation activates a single transmit antenna in a channel use and transmits a symbol from the activated antenna. The system model for $n_t \times n_r$ spatial modulation can be written as

$$\mathbf{y} = \left(\sum_{i=1}^{n_t} \mathbf{g}_i v_i\right) z + \mathbf{w}, \tag{16}$$

where $\mathbf{g}_i$ is the channel gain vector from ith transmit antenna to n_r receive antennas, v_i is the antenna activation variable for ith antenna such that only one of the v_is is one and remaining (n_t-1) v_is are zeros, and $\mathbf{w}$ is $n_r \times 1$ noise vector with its entries being i.i.d $\mathcal{CN}(0,\sigma^2)$. The system model in (16) can be written as

$$\mathbf{y} = \mathbf{G}\mathbf{v}z + \mathbf{w}, \tag{17}$$

where $\mathbf{G} = [\mathbf{g}_1 \ \mathbf{g}_2 \ \cdots \ \mathbf{g}_{n_t}]$ and $\mathbf{v} = [v_1 \ v_2 \ \cdots \ v_{n_t}]^T$ such that $\mathbf{v} \in \{\mathbf{e}_i, \ i = 1, \ldots, n_t\}$ with $\mathbf{e}_i$ being $n_t \times 1$ vector with its ith entry being 1 and other entries being 0s. We define $p_i \triangleq \mathbb{P}(\mathbf{v} = \mathbf{e}_i)$ as the probability of activating ith antenna. Let $\mathbf{x} = \mathbf{v}z = [x_1 x_2 \ldots x_{n_t}]^T$ denote the $n_t \times 1$ transmit vector. We denote by C_{SIMO} the capacity of SIMO fading channel. With this, we have the following propositions on the capacity of $n_t \times n_r$ spatial modulation.

Proposition 3. *(Spatial modulation - SIMO lower bound) The capacity of $n_t \times n_r$ spatial modulation is lower bounded by the capacity of $1 \times n_r$ SIMO fading channel.*

$$C_{SM} \geq C_{SIMO} \tag{18}$$

Proof. The proof follows from the fact that SIMO is a special case of spatial modulation when it emits no information through the index. Therefore, the capacity of $n_t \times n_r$ spatial modulation is no less than the capacity of the SIMO fading channel with the same number of receive antennas. □

Proposition 4. *(Spatial modulation - Genie-aided upper bound) The capacity of $n_t \times n_r$ spatial modulation satisfies the upper bound*

$$C_{SM} \leq C_{SIMO} + \log_2 n_t. \tag{19}$$

Proof. We have

$$I(\mathbf{v},z;\mathbf{y}|\mathbf{G}) = I(z;\mathbf{y}|\mathbf{v},\mathbf{G}) + I(\mathbf{v};\mathbf{y}|\mathbf{G}). \tag{20}$$

The first term is equivalent to the mutual information of a SIMO channel with CSIR.

$$\begin{aligned} I(z;\mathbf{y}|\mathbf{v},\mathbf{G}) &= \sum_{i=1}^{n_t} p_i I(z;\mathbf{y}|\mathbf{v}=\mathbf{e}_i,\mathbf{G}) \\ &= \sum_{i=1}^{n_t} p_i I(x_i;\mathbf{y}|\mathbf{G}) \\ &= I(x_1;\mathbf{y}|\mathbf{G}) \quad \text{(by symmetry)} \end{aligned} \tag{21}$$

Now consider the second term in (20)

$$\begin{aligned} I(\mathbf{v};\mathbf{y}|\mathbf{G}) &= h(\mathbf{v}|\mathbf{G}) - h(\mathbf{v}|\mathbf{y},\mathbf{G}). \\ &= h(\mathbf{v}) - h(\mathbf{v}|\mathbf{y},\mathbf{G}), \end{aligned} \tag{22}$$

where the last equality follows from the independence of channel and inputs. Combining Eqs. (20), (21), (22)

$$I(\mathbf{v},z;\mathbf{y}|\mathbf{G}) = I(x_1;\mathbf{y}|\mathbf{G}) + h(\mathbf{v}) - h(\mathbf{v}|\mathbf{y},\mathbf{G}) \tag{23}$$

It then follows that

$$\begin{aligned} C_{SM} &= \max_{p(\mathbf{v},z)} I(\mathbf{v},z;\mathbf{y}|\mathbf{G}) \\ &= \max_{p(\mathbf{v},z)} \left[I(x_1;\mathbf{y}|\mathbf{G}) + h(\mathbf{v}) - h(\mathbf{v}|\mathbf{y},\mathbf{G})\right] \\ &\leq \max_{p(\mathbf{v},z)} \left[I(x_1;\mathbf{y}|\mathbf{G}) + h(\mathbf{v})\right] \\ &\leq \max_{p(\mathbf{v},z)} I(x_1;\mathbf{y}|\mathbf{G}) + \max_{p(\mathbf{v},z)} h(\mathbf{v}) \\ &= C_{SIMO} + \log_2 n_t. \end{aligned}$$

□

Remark 4. Using similar arguments as before, it can be shown that the lower bound in Proposition 3 is tight at low-SNR. Also, the ratio of lower and upper bounds goes to one at high-SNR. Also, another obvious upper bound on the capacity of spatial modulation is the capacity of the MIMO system using the same number of transmit and receive antennas, i.e., $C_{SM} \leq C_{MIMO}$.

We now present a tighter lower bound on the capacity of spatial modulation.

Proposition 5. *(Spatial modulation - Independence lower bound) The capacity of $n_t \times n_r$ spatial modulation is lower bounded as follows:*

$$C_{SM} \geq C_{SIMO} + \log_2 n_t - \mathbb{E}_{\mathbf{y},\mathbf{G}}\left[\sum_{i=1}^{n_t} \frac{\log_2 \sum_{j=1}^{n_t} \sqrt{\frac{|\Sigma_i|}{|\Sigma_j|}} \exp\left(\frac{1}{2}\mathbf{y}^H(\Sigma_i^{-1} - \Sigma_j^{-1})\mathbf{y}\right)}{\sum_{j=1}^{n_t} \sqrt{\frac{|\Sigma_i|}{|\Sigma_j|}} \exp\left(\frac{1}{2}\mathbf{y}^H(\Sigma_i^{-1} - \Sigma_j^{-1})\mathbf{y}\right)}\right], \tag{24}$$

where $\Sigma_i = \mathbf{g}_i\mathbf{g}_i^H\sigma_z^2 + \sigma^2\mathbf{I}_{n_r}$.

Proof. We have

$$I(\mathbf{v}, z; \mathbf{y}|\mathbf{G}) = I(z; \mathbf{y}|\mathbf{v}, \mathbf{G}) + I(\mathbf{v}; \mathbf{y}|\mathbf{G})$$

Therefore,

$$C_{SM} = \max_{p(\mathbf{v},z)} I(\mathbf{v}, z; \mathbf{y}|\mathbf{G}) \geq I(z; \mathbf{y}|\mathbf{v}, \mathbf{G}) + I(\mathbf{v}; \mathbf{y}|\mathbf{G}).$$

To evaluate the right-hand side,[2] consider the signalling where $\mathbf{v}$ and z are independent, $z \sim \mathcal{CN}(0, \sigma_z^2)$, and $\mathbf{v}$ is selected uniformly. As seen before, when $z \sim \mathcal{CN}(0, \sigma_z^2)$, the mutual information $I(z; \mathbf{y}|\mathbf{v}, \mathbf{G}) = C_{SIMO}$ and the above inequality becomes

$$\begin{aligned} C_{SM} &\geq C_{SIMO} + I(\mathbf{v}; \mathbf{y}|\mathbf{G}) \\ &= C_{SIMO} + h(\mathbf{v}|\mathbf{G}) - h(\mathbf{v}|\mathbf{y}, \mathbf{G}) \\ &= C_{SIMO} + \log_2 n_t - h(\mathbf{v}|\mathbf{y}, \mathbf{G}). \end{aligned} \tag{25}$$

Now, we have

$$h(\mathbf{v}|\mathbf{y}, \mathbf{G}) = \mathbb{E}_{\mathbf{y},\mathbf{G}}\left[-\sum_{i=1}^{n_t} p(\mathbf{v} = \mathbf{e}_i|\mathbf{y}, \mathbf{G}) \log_2 p(\mathbf{v} = \mathbf{e}_i|\mathbf{y}, \mathbf{G})\right], \tag{26}$$

where

$$p(\mathbf{v} = \mathbf{e}_i|\mathbf{y}, \mathbf{G}) = \frac{p(\mathbf{y}|\mathbf{v} = \mathbf{e}_i, \mathbf{G})p(\mathbf{v} = \mathbf{e}_i|\mathbf{G})}{\sum_{j=1}^{n_t} p(\mathbf{y}|\mathbf{v} = \mathbf{e}_j, \mathbf{G})p(\mathbf{v} = \mathbf{e}_j|\mathbf{G})}.$$

From the system model of $n_t \times n_r$ spatial modulation, we have

$$\mathbb{E}(\mathbf{y}|\mathbf{v} = \mathbf{e}_i, \mathbf{G}) = \mathbf{g}_i\mathbb{E}(z) + \mathbb{E}(w) = 0$$

and

$$\begin{aligned} \text{Cov}(\mathbf{y}|\mathbf{v} = \mathbf{e}_i, \mathbf{G}) &= \mathbf{g}_i\mathbf{g}_i^H\mathbb{E}(|z|^2) + \sigma^2\mathbf{I}_{n_r} \\ &= \mathbf{g}_i\mathbf{g}_i^H\sigma_z^2 + \sigma^2\mathbf{I}_{n_r} \triangleq \Sigma_i. \end{aligned}$$

Therefore,

$$p(\mathbf{y}|\mathbf{v} = \mathbf{e}_i, \mathbf{G}) = \frac{1}{\sqrt{(2\pi)^{n_r}|\Sigma_i|}} \exp\left(-\frac{1}{2}\mathbf{y}^H\Sigma_i^{-1}\mathbf{y}\right),$$

[2]So that any maximization of individual terms on the right hand side does not compromise the inequality, all mutual information terms on the right are calculated according to one and the same distribution.

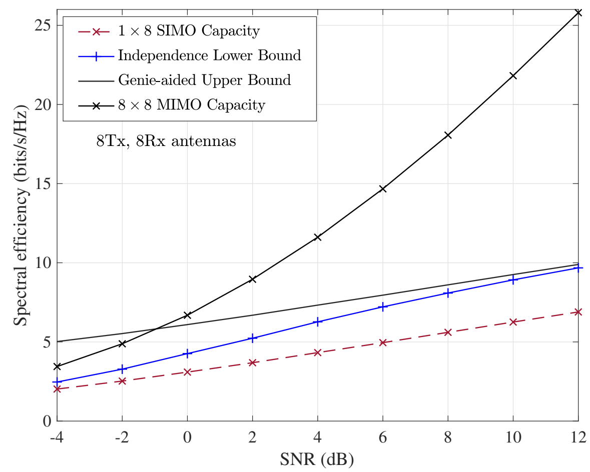

Fig. 2. Capacity bounds for 8×8 spatial modulation.

and hence

$$\begin{aligned} p(\mathbf{v} = \mathbf{e}_i|\mathbf{y}, \mathbf{G}) &= \frac{\frac{1}{\sqrt{|\Sigma_i|}} \exp\left(-\frac{1}{2}\mathbf{y}^H\Sigma_i^{-1}\mathbf{y}\right)}{\sum_{j=1}^{n_t} \frac{1}{\sqrt{|\Sigma_j|}} \exp\left(-\frac{1}{2}\mathbf{y}^H\Sigma_j^{-1}\mathbf{y}\right)} \\ &= \frac{1}{\sum_{j=1}^{n_t} \sqrt{\frac{|\Sigma_i|}{|\Sigma_j|}} \exp\left(\frac{1}{2}\mathbf{y}^H(\Sigma_i^{-1} - \Sigma_j^{-1})\mathbf{y}\right)}. \end{aligned}$$

Using this expression in (26) and substituting the resulting expression in (25) proves the proposition. □

IV. Simulations

Figure 2 shows the upper and lower bounds on the capacity of 8×8 spatial modulation derived in the previous section. The capacity of 8×8 MIMO is also shown in the figure. Firstly, it can be seen that the independence lower bound meets the genie-aided upper bound at high-SNR, which illustrates the tightness of the derived bounds. Next, it can be seen that although 8×8 spatial modulation may achieve a capacity greater than 1×8 SIMO, its capacity falls significantly below the 8×8 MIMO capacity at high-SNR. This observation suggests that spatial modulation should be considered when there is a strict hardware constraint that allows using only one RF chain. Using spatial modulation when there is no such hardware constraint is spectrally inefficient.

Figure 3 compares the capacity results of the present paper with those of [7] and [8]. The bounds in [7] are shown as Rajashekar-Hari-Hanzo lower and upper bounds. The capacity approximations in [8] using the Taylor series are shown as Henarejos-Neira order 2 and 4 approximations. It can be seen from Fig. 3 that, while the upper bound in [7] is identical with our genie-aided upper bound, the lower bound is weaker compared to our independence lower bound at all SNR values. For example, at 8 bits/s/Hz, our lower bound is 5 dB tighter than the lower bound in [7]. It can also be observed that the

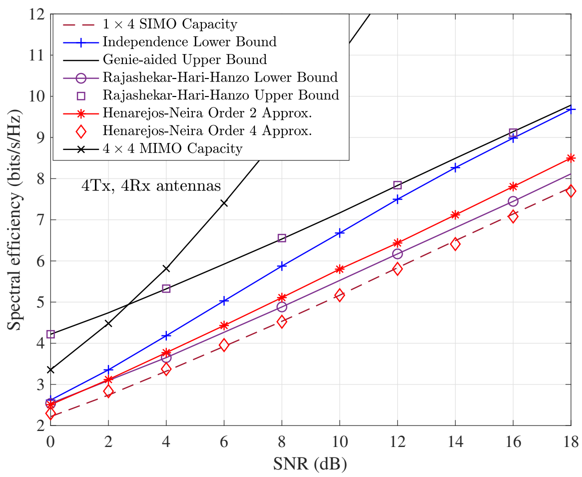

Fig. 3. Comparing bounds of the present paper with the bounds in [7] and approximations in [8] for 4×4 spatial modulation.

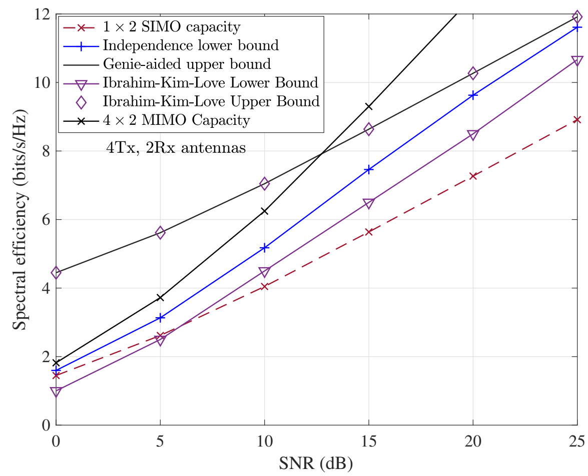

Fig. 4. Comparing bounds of the present paper with the bounds in [9] for 4×2 spatial modulation.

approximations of [8] are weaker compared to the bounds of the present paper.

Figure 4 compares the capacity bounds of the present paper with the bounds in [9] for 4×2 spatial modulation. The bounds in [9] are shown as Ibrahim-Kim-Love upper and lower bounds. It can be seen that the upper bound of [9] is identical with our genie-aided upper bound. However, the lower bound of [9] is weaker than our independence lower bound. For example, at 8 bits/s/Hz, our lower bound is 2 dB tighter than the lower bound bound in [9].

V. Conclusions

We analyzed the capacity of spatial modulation from first principles and derived upper and lower bounds on the capacity. We showed that the single-transmit-antenna lower bound is tight at low-SNR. We also showed that the bounds derived in this paper are tighter than the previous bounds or approximations in the literature. Our results revealed that at high-SNR, the information carried by the index is bounded by a constant $\log n_t$, while the SISO/SIMO capacity grows as $\log \text{SNR}$, therefore asymptotically, the ratio of spatial modulation capacity to SISO/SIMO capacity is one. Our results also revealed that the capacity loss in spatial modulation compared to MIMO can be significant at high-SNR.

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