Coherence Diversity DoF in MIMO Relays: Generalization, Transmission Schemes, and Multi-Relay Strategies

Fan Zhang and Aria Nosratinia
Department of Electrical Engineering, University of Texas at Dallas, TX, USA.
Email: fan.zhang5@utdallas.edu;aria@utdallas.edu

***Abstract*—This paper studies the MIMO relay with non-identical link coherence times, a condition known as *coherence diversity*. This happens in practice when a node or the scatterers surrounding it have different mobility compared with the conditions at or around other nodes. In this paper, coherence diversity in relays is studied under link coherence intervals with arbitrary (unequal) length and alignment. Second, a new transmission scheme under coherence diversity is proposed in which the relay transmission is given a duty cycle based on the balance between the gain versus the channel training costs in the relay-destination link. Finally, we investigate multiple parallel relays operating under non-identical coherence intervals, propose transmission strategies, and calculate degrees of freedom.**

I. Introduction

In wireless networks, due to the mobility of nodes and the scattering environment, coherence times of difference links are often non-identical. This also happens in the context of the relay channel, for example, it is not uncommon that the destination might be a node with high mobility, which is being assisted by a relatively stationary relay.

The performance of fading relay channel has been extensively studied [1]–[6] under *identical* coherence intervals. Exploration of *unequal* coherence intervals in a network began with studies of the broadcast channels [7]–[9], showing the interesting phenomena that ensue. Inner and outer bounds for multiple access channel with unequal coherence times were calculated in [9]. Achievable rates were calculated for frequency-selective multiuser downlink channels under mismatched coherence conditions in [10]. The impact of hybrid channel state information on MISO broadcast channel with unequal coherence times was studied in [11]. In [12], it was shown that when coherence intervals are identical, a relay does not improve the degrees of freedom compared with the direct link alone. Also, For a relay with *non-identical* coherence times, [12] studied the effect of aligned coherence intervals with integer ratios, and calculated achievable degrees of freedom.

Previous analyses are insufficient for conditions often observed in practice, in which different links in a relay channel can experience different coherence intervals that are not a correct multiple of each other (integer ratio). Then, the coherence intervals cannot stay aligned as time progresses. This paper begins with the analysis of the degrees of freedom of the relay channel when the coherence intervals are arbitrary and unaligned. The transmit design principles under this condition are clarified and the corresponding degrees of freedom are calculated. Further, a new scheme combining the product superposition and relay scheduling is proposed, motivated by the following observation: Whenever a pilot-based relay is activated, the relay pilots impose a cost in degrees of freedom due to their interference with source-destination transmission. In the new scheme, this cost is matched against the relay gains and the relay is activated accordingly. We show the extent to which this new scheme improves the degrees of freedom of the relay channel. Finally, this paper studies multiple parallel relays under non-identical coherence intervals. Transmission strategies and achievable degrees of freedom are presented.

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II. Arbitrary Coherence Times

A. Unaligned Coherence Blocks

Consider a MIMO Gaussian relay in full-duplex mode. The source and destination are equipped with N_S and N_D antennas. The relay has N_R receive antennas and uses n_r antennas for transmitting. The received signals at the relay and destination are:

$$\mathbf{y}_R = \mathbf{H}_{SR}\mathbf{x}_S + \mathbf{w}_R \tag{1}$$
$$\mathbf{y}_D = \mathbf{H}_{SD}\mathbf{x}_S + \mathbf{H}_{RD}\mathbf{x}_R + \mathbf{w}_D, \tag{2}$$

where $\mathbf{x}_S$ and $\mathbf{x}_R$ are signals transmitted from the source and relay. $\mathbf{w}_R$ and $\mathbf{w}_D$ are i.i.d. Gaussian noise and $\mathbf{H}_{SR}$, $\mathbf{H}_{RD}$ and $\mathbf{H}_{SD}$ are channel gain matrices whose entries are i.i.d. Gaussian. Channel gain entries and noise components are zero-mean and have unit variance. Channel gains experience block fading, remaining constant during the coherence intervals but changing independently across intervals. The coherence intervals are denoted T_{SR}, T_{RD} and T_{SD}, satisfying $T_{SR} \geq 2\max(N_S, N_R)$, $T_{RD} \geq 2\max(N_R, N_D)$ and $T_{SD} \geq 2\max(N_S, N_D)$.[1] The source and relay obey power constraints $\mathbb{E}[\mathrm{tr}(\mathbf{x}_S\mathbf{x}_S')] \leq \rho$ and $\mathbb{E}[\mathrm{tr}(\mathbf{x}_R\mathbf{x}_R')] \leq \rho$.

[1]This guarantees each interval can fit link training, based on transmit antennas.

We assume there is no free channel state information at the destination and no CSI feedback to transmitters.

To better illustrate the issues that are addressed in this paper, and to motivate our developments, we begin with a small example: $N_S = N_R = 2$ and $N_D = 3$. The coherence intervals $T_{SD} = T_{RD} = 8$ and $T_{SR} = \infty$, i.e., the source-relay channel is static, therefore the cost of training over this link is amortized over a large number of samples, so we can assume the relay knows $\mathbf{H}_{SR}$. From [12], degrees of freedom d = 1.75 can be achieved when the coherence blocks are aligned. If the coherence blocks of the relay are unaligned, the following calculation shows that when there is an offset of $\Delta T = 4$ between the transition times of $\mathbf{H}_{SD}$ and $\mathbf{H}_{RD}$, i.e. , the coherence blocks of the channel $\mathbf{H}_{RD}$ starts from time slot 5 of $\mathbf{H}_{SR}$, the same degrees of freedom can still be achieved.

The source uses product superposition, sending

$$\mathbf{X}_S = \mathbf{X}_u[\mathbf{I}_2, \mathbf{0}_{2\times 1}, \mathbf{X}_d], \tag{3}$$

where $\mathbf{X}_u \in \mathbb{C}^{2\times 2}$ and $\mathbf{X}_d \in \mathbb{C}^{2\times 5}$.

At the relay, the received signal is

$$\mathbf{Y}_R = \mathbf{H}_{SR}\mathbf{X}_u[\mathbf{I}_2, \mathbf{0}_{2\times 1}, \mathbf{X}_d] + \mathbf{W}_R. \tag{4}$$

The received signal from time slot 1 to 2 is

$$\mathbf{Y}'_R = \mathbf{H}_{SR}\mathbf{X}_u + \mathbf{W}'_R. \tag{5}$$

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u$. Denote the signal decoded by the relay in the previous block is $\mathbf{X}'_u$ and the two rows of $\mathbf{X}'_u$ are $\mathbf{x}'_1, \mathbf{x}'_2 \in \mathbb{C}^{1\times 2}$.

The relay uses one antenna for transmission and sends

$$\mathbf{X}_R = [\mathbf{0}_{1\times 2}\ 1, \mathbf{x}'_1, \mathbf{x}'_2, 0] \in \mathbb{C}^{1\times 8}. \tag{6}$$

In one coherence block of $\mathbf{H}_{SD}$, because the blocks of $\mathbf{H}_{RD}$ are unaligned with $\mathbf{H}_{SD}$, the received signal at the destination experiences two realizations of $\mathbf{H}_{RD}$ ($\mathbf{H}_{RD_1}$ and $\mathbf{H}_{RD_2}$), from time slot 1 to 4 is

$$\begin{aligned}\mathbf{Y}_D &= \mathbf{H}_{SD}\mathbf{X}_S + \mathbf{H}_{RD_1}\mathbf{X}_R + \mathbf{W}_D \\ &= [\mathbf{H}_{SD}, \mathbf{H}_{RD_1}] \begin{bmatrix} \mathbf{X}_u[\mathbf{I}_2, \mathbf{0}_{2\times 1}, \mathbf{X}_{d1}] \\ \mathbf{0}_{1\times 2},\ 1,\ \mathbf{x}'_1(1) \end{bmatrix} + \mathbf{W}_D \\ &= [\mathbf{H}_{SD}\mathbf{X}_u, \mathbf{H}_{RD_1}][\mathbf{I}_3, \mathbf{X}_D] + \mathbf{W}_D,\end{aligned} \tag{7}$$

where

$$\mathbf{X}_D = \begin{bmatrix} \mathbf{X}_{d1} \\ \mathbf{x}'_1(1) \end{bmatrix}. \tag{8}$$

The destination estimates the equivalent channel $\mathbf{H}_D = [\mathbf{H}_{SD}\mathbf{X}_u\ \mathbf{H}_{RD_1}]$ from time slot 1 to 3 and decodes $\mathbf{X}_{d1}, \mathbf{x}'_1(1)$.

From time slot 5 to 8, the received signal is:

$$\begin{aligned}\mathbf{Y}_D &= \mathbf{H}_{SD}\mathbf{X}_S + \mathbf{H}_{RD_2}\mathbf{X}_R + \mathbf{W}_D \\ &= [\mathbf{H}_{SD}\mathbf{X}_u, \mathbf{H}_{RD_2}] \begin{bmatrix} \mathbf{X}_{d2} \\ \mathbf{x}'_1(2), \mathbf{x}'_2, 0 \end{bmatrix} + \mathbf{W}_D.\end{aligned} \tag{9}$$

First part of the equivalent channel, $\mathbf{H}_{SD}\mathbf{X}_u$, is already estimated. Second part $\mathbf{H}_{RD_2}$ will be estimated in the next transmission block. The destination decodes $\mathbf{X}_{d2}, \mathbf{x}'_1(2)$ and $\mathbf{x}'_2$. Therefore, when the coherence blocks from the source and relay to the destination are not aligned, The destination achieves the same DoF $d = (2 \times 5 + 2 \times 1 \times 2)/8 = 1.75$.

B. Arbitrary Coherence Times

The following can be achieved using a transmit strategy for arbitrary coherence intervals via product superposition.

Theorem 1. *In a three-node relay with coherence diversity, $T_{SR} > T_{SD}$, $T_{RD} > T_{SD}$. and $N_S, N_R < N_D$, denote $N_S^* = \min\{N_S, N_R\}$, the following degree of freedom is achievable:*

$$\begin{aligned} d = \frac{1}{T_{SR}T_{SD}T_{RD}} \max_{n_r} \{ & N_S(T_{SR}T_{SD}T_{RD} - N_S T_{SR}T_{RD} \\ & - n_r T_{SR}T_{SD}) + \min\{N_S^* N_S(T_{SR}T_{RD} - T_{SD}T_{RD}), \\ & n_r(T_{SR}T_{SD}T_{RD} - N_S T_{SR}T_{RD} - n_r T_{SR}T_{SD})\}\}. \end{aligned} \tag{10}$$

Proof. Design the pilot-based achievable scheme in the following manner:

- On the multiple-access side, pilots sent from the relay and the source will be allocated in different time slots, such that they will not interfere with each other. In addition, during these time slots no data is sent, avoiding pilot contamination.
- On the broadcast side, the source-relay link needs fewer pilots than the source-destination. Thus, product superposition enables transmission of additional data to the relay.

In the following, we consider a super-interval of length $T_{SR}T_{RD}T_{SD}$, after which the coherence intervals will come back to their original alignment. The achievable degrees of freedom are calculated as follows: In each source-destination coherence interval T_{SD}, N_S pilot symbols are transmitted. We call the pilot symbols in each coherence block a *pilot sequence*.

Therefore, for source-destination link, we repeat the length-N_S pilot sequence $T_{SR}T_{RD}$ times over the length-$T_{SR}T_{RD}T_{SD}$ super-interval. Having coherence time $T_{SR}T_{RD}$, the relay needs $T_{SD}T_{RD}$ pilot sequences. Hence, product superposition can be applied during $(T_{SR}T_{RD} - T_{SD}T_{RD})$ pilot sequences of length $N)S$to send data to the relay. Data with N_S^* degrees of freedom per symbol can be sent.

Over each super-interval, the relay-destination link needs $T_{SR}T_{SD}$ pilot sequences of length n_r. The pilots slots will be non-overlapping with pilots transmitted from the source terminal.

In each super-interval, the source and the relay each have $(T_{SR}T_{SD}T_{RD} - N_S T_{SR}T_{RD} - n_r T_{SR}T_{SD})$ time slots available for sending data. The source has N_S degrees of freedom available per transmission, and the relay n_r degrees of freedom per transmission.

The relay can decode at most $N_S^* N_S(T_{SR}T_{RD} - T_{SD}T_{RD})$ degrees of freedom, therefore, it provides $\min\{N_S^* N_S(T_{SR}T_{RD} - T_{SD}T_{RD}), n_r(T_{SR}T_{SD}T_{RD} - N_S T_{SR}T_{RD} - n_r T_{SR}T_{SD})\}$ degrees of freedom, the minimum of the degrees of freedom the relay can receive and can transmit.

We can now sum the degrees of freedom by the source transmission (subject to relay constraints) and the degrees of freedom provided by the relay transmission, and optimize the number of relay antennas to be activated. This concludes the proof.

□

III. Product Superposition with Relay Scheduling

In this section, a new scheme combining product superposition and relay scheduling is introduced. The following theorem highlights the main result of this section. We begin by defining:

$$d_1 \triangleq N_S(T - n_r - N_S),$$
$$d_2 \triangleq n_r(T - n_r - N_S),$$
$$d_3 \triangleq N_S^2,$$

Theorem 2. *In a relay, if $T_{SR} = \infty$, $T_{SD} = T_{SR} = T$ and $N_S = N_R < N_D$,*

- *If $d_2 \le d_3$, the DoF $d = \frac{1}{T}\max_{n_r}(d_1+d_2)$ is achievable.*
- *If $d_2 > d_3$, the following DoF is achievable.*

$$d = \frac{1}{T}\max_{n_r}\Big(\frac{d_2 - d_3}{d_2}N_S(T - N_S) + \frac{d_3}{d_2}(d_1 + d_2)\Big), \quad (11)$$

Proof. If $d_2 \le d_3$, the achievable DoF follows [12]. Consider the case $d_2 > d_3$. The transmit scheme with relay scheduling has two phases, each of them lasting an integer multiple of the coherence interval T. In both phases, product superposition is used at the source, but the relay action is different in the two phases, as described in the sequel. We propose to transmit for $d_2 - d_3$ coherence intervals in Phase 1, followed by transmitting d_3 coherence intervals in Phase 2.

During Phase 1, the relay transmission is deactivated but the source continues to transmit via product superposition. In this phase, in each coherence interval of length T, the source delivers to the destination data rates corresponding to its point-to-point DoF bound, which is $N_S(T - N_s)$, while delivering *additional* data to the relay with DoF d_3. We transmit in Phase 1 for $d_2 - d_3$ coherence intervals, therefore, the normalized (per-symbol) average DoF contribution of this phase is $\frac{d_2-d_3}{d_2}\frac{1}{T}N_S(T - N_S)$.

During Phase 2, the relay is activated and the source sends the product superposition signal:

$$\mathbf{X}_S = \mathbf{X}_u[\mathbf{I}_{N_S}, \mathbf{0}_{N_S\times n_r}, \mathbf{X}_d], \quad (12)$$

where $n_r \le \min\{N_S, N_D - N_S\}$, $\mathbf{X}_u \in \mathbb{C}^{N_S\times N_S}$ and $\mathbf{X}_d \in \mathbb{C}^{N_S\times(T-n_r-N_S)}$.

The relay knows $\mathbf{H}_{SR}$ and decodes $\mathbf{X}_u$. Denote by $\mathbf{X}'_u$ the message decoded by the relay in the previous block. The relay uses n_r antennas for transmission, sending

$$\mathbf{X}_R = [\mathbf{0}_{n_r\times N_S}, \mathbf{I}_{n_r}, \mathbf{X}_{r,d}] \in \mathbb{C}^{n_r\times T}, \quad (13)$$

where $\mathbf{X}_{r,d} \in \mathbb{C}^{n_r\times(T-n_r-N_S)}$.

The destination estimates the equivalent channel $\mathbf{H}_D = [\mathbf{H}_{SD}\mathbf{X}_u, \mathbf{H}_{RD}]$ during the first $(N_S + n_r)$ time slots and then decodes its messages. Destination receives: $\mathbf{X}_d$ from the source and $\mathbf{X}_{r,d}$ from the relay, providing degrees of freedom d_1 and d_2, respectively. Phase 2 consists of d_3 coherence intervals, further, recall that the relay has stored data available from Phase 1 in addition to the data it is receiving in Phase 2. Therefore, the relay can send data with DoF d_2 to the destination. Hence during phase 2, the normalized per-symbol DoF are $\frac{1}{T}\frac{d_3}{d_2}(d_1 + d_2)$.

Adding the degrees of freedom achieved in Phase 1 and Phase 2 and optimizing the number of relay transmit antennas to be activated produces (11). This completes the proof. □

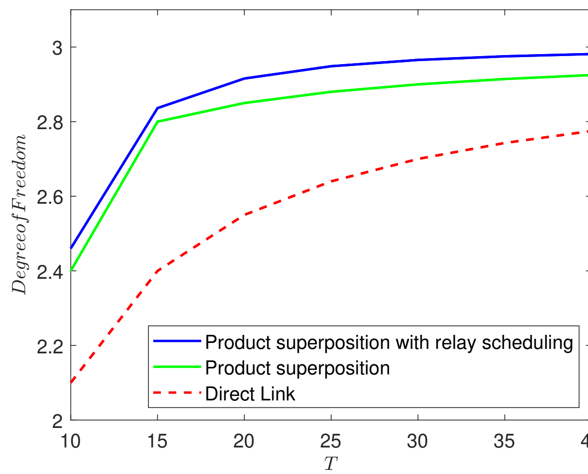

Fig. 1. Achievable DoF in Theorem (2)

Remark 1. *For comparison, we also mention the degrees of freedom* without *relay scheduling. For a relay with the following setup $T_{SR} = \infty$, $T_{SD} = T_{SR} = T$ and $N_S = N_R < N_D$. From Theorem 2 in [12], the following degrees of freedom are achievable:*

$$d = \frac{1}{T}\max_{n_r}\min\{d_1 + d_2, d_1 + d_3\}, \quad (14)$$

Figure 1 shows the comparison between the achievable degrees of freedom of product superposition alone and with relay scheduling when $N_S = 3, N_D = 5$ for different T.

IV. Multiple Parallel Relays

This section studies the MIMO relay channel with K full-duplex relays, under coherence diversity. The source and destination are equipped with N_S and N_D antennas. Relay k has $N_R(k)$ receive antennas and uses $n_R(k) \le N_R(k)$ antennas for transmission. The received signals at the relays and the destination are:

$$\mathbf{y}_R(k) = \mathbf{H}_{SR}(k)\mathbf{x}_S + \mathbf{w}_R(k), \qquad k = 1, \dots, K \quad (15)$$
$$\mathbf{y}_D = \mathbf{H}_{SD}\mathbf{x}_S + \sum_{k=1,\dots,K}\mathbf{H}_{RD}(k)\mathbf{x}_R(k) + \mathbf{w}_D, \quad (16)$$

where $\mathbf{x}_S$ and $\mathbf{x}_R(k)$ are signals transmitted from the source and Relay k. $\mathbf{w}_R$ and $\mathbf{w}_D$ are i.i.d. zero-mean Gaussian noise and $\mathbf{H}_{SR}(k)$, $\mathbf{H}_{RD}(k)$ and $\mathbf{H}_{SD}$ are channel gain matrices, whose entries are i.i.d. Gaussian. We assume there is no free channel state information at the destination and no CSIT at the source or relay. In the parallel relay geometry, there are no inter-relay links. Denote the coherence time of the link between the source and Relay k as $T_{SR}(k)$ and the coherence time of the link between Relay k and the destination as $T_{RD}(k)$.

A. Achievable DoF for Two Parallel Relays

Consider the following channel with two parallel relays. $T_{SR}(2) = K_2 T_{SR}(1) = K_2 K_1 T_{SD} = K_2 K_1 T$ and the destination knows the channel state of $\mathbf{H}_{RD}(1)$ and $\mathbf{H}_{RD}(2)$, i.e., $T_{RD}(1) = T_{RD}(2) = \infty$. Denote $N_S^*(i) = \min\{N_S, N_R(i)\}$. If Relay 1 or Relay 2 is activated alone, the achievable degrees of freedom are:

$$d_i = \max_{n_R(i)} \Big\{ N_S(1 - \frac{N_S}{T}) + \min\big\{(1 - \frac{1}{K_i})\frac{N_S^*(i) N_S}{T}, n_R(i)(1 - \frac{N_S}{T})\big\}\Big\}. \tag{17}$$

When Relay 1 and Relay 2 are both activated. Consider a transmission interval of length $K_2 K_1 T$. During each coherence interval, Relay 1 and Relay 2 send the message they decoded in the previous interval of length $K_2 K_1 T$. The transmitted signal from Relay 1 and 2 over each sub-interval of length T has the following structure and is repeated $K_2 K_1$ times:

$$\mathbf{X}_R(i) = [\mathbf{0}_{n_R(i)\times N_S}, \mathbf{X}_{di}], \quad i = 1, 2. \tag{18}$$

During the first coherence interval of length $K_1 T$, in the first sub-interval of length T, the source sends $\mathbf{X}_S = [\mathbf{I}_{N_S}, \mathbf{X}_D]$. Relay 1 and Relay 2 estimate their channel. The signal at the destination is:

$$\begin{aligned}\mathbf{Y}_D &= [\mathbf{H}_{SD}, \mathbf{H}_{RD}(1), \mathbf{H}_{RD}(2)] \begin{bmatrix} \mathbf{I}_{N_S}, \mathbf{X}_D \\ \mathbf{0}_{n_R(1)\times N_S}, \mathbf{X}_{d1} \\ \mathbf{0}_{n_R(2)\times N_S}, \mathbf{X}_{d2} \end{bmatrix} + \mathbf{W}_D, \\ &= \big[\mathbf{H}_{SD}, [\mathbf{H}_{SD}, \mathbf{H}_{RD}(1), \mathbf{H}_{RD}(2)]\bar{\mathbf{X}}_D\big] + \mathbf{W}_D \end{aligned} \tag{19}$$

where

$$\bar{\mathbf{X}}_D = \begin{bmatrix} \mathbf{X}_D \\ \mathbf{X}_{d1} \\ \mathbf{X}_{d2} \end{bmatrix}. \tag{20}$$

The destination estimates $\mathbf{H}_{SD}$ and decodes the messages in $\mathbf{X}_D, \mathbf{X}_{d1}$ and $\mathbf{X}_{d2}$, which provide $N_S, n_R(1), n_R(2)$ degrees of freedom per symbol over this interval of length $(T - N_S)$.

In the remaining $K_1 - 1$ intervals of length T, the source sends the signal:

$$\mathbf{X}_S = \mathbf{X}_R^i(1)[\mathbf{I}_{N_S}, \mathbf{X}_D^i], \quad i = 1, 2, \ldots, K_1 - 1, \tag{21}$$

where $\mathbf{X}_R^i(1) \in \mathbb{C}^{N_S \times N_S}$. Relay 1 has already estimated its channel in the first interval of length T. It can decode $\mathbf{X}_R^i(1)$, providing $N_S^*(1) N_S$ degrees of freedom. The total degrees of freedom Relay 1 can decode are $(K_1 - 1) N_S^*(1) N_S$. The received signal at the destination is:

$$\begin{aligned}\mathbf{Y}_D = \big[&\mathbf{H}_{SD}\mathbf{X}_R^i(1), [\mathbf{H}_{SD}\mathbf{X}_R^i(1), \\ &\mathbf{H}_{RD}(1), \mathbf{H}_{RD}(2)]\bar{\mathbf{X}}_D\big] + \mathbf{W}_D \end{aligned} \tag{22}$$

The destination estimates $\mathbf{H}_{SD}\mathbf{X}_R^i(1)$ and decodes $\bar{\mathbf{X}}_D$.

During the remaining $K_2 - 1$ coherence intervals of length $K_1 T$, the transmitter sends, every $K_1 T$-length interval, the signal with the same structure as the first sub-interval of length T multiplying it from the left by $\mathbf{X}_R^j(2)$, which contains the message for Relay 2. During each interval of length $K_1 T$, the transmitted signal from the source has the following structure:

$$\begin{aligned}\mathbf{X}_S = \mathbf{X}_R^j(2)\big[&[\mathbf{I}_{N_S}\ \mathbf{X}_D^1], \mathbf{X}_R^1(1)[\mathbf{I}_{N_S}\ \mathbf{X}_D^2], \\ &\mathbf{X}_R^2(1)[\mathbf{I}_{N_S}\mathbf{X}_D^3], \ldots, \mathbf{X}_R^{(K_1-1)}(1)[\mathbf{I}_{N_S}\ \mathbf{X}_D^{K_1}]\big]. \end{aligned} \tag{23}$$

During these $K_2 - 1$ coherence intervals with length $K_1 T$, the channel $\mathbf{H}_{SR}(2)$ remains the same as in the first sub-interval of length $K_1 T$. Therefore, in each interval of length $K_1 T$, Relay 2 can decode $\mathbf{X}_R^j(2)$ of $N_S^*(2) N_S$ degrees of freedom. The total degrees of freedom Relay 2 can decode are $(K_2 - 1) N_S^*(2) N_S$ over coherence interval of length $K_2 K_1 T$.

The first N_S symbols during the first sub-interval of length $K_1 T$ received at Relay 1 are:

$$\mathbf{Y}_R(1) = \mathbf{H}_{SR}^j \mathbf{X}_R^j(2) + \mathbf{W}_R(1). \tag{24}$$

The first N_S symbols during the remaining sub-interval of length $K_1 T$ received at Relay 1 are:

$$\mathbf{Y}_R(1) = \mathbf{H}_{SR}^j \mathbf{X}_R^j(2)\mathbf{X}_R^i(1) + \mathbf{W}_R(1), i = 1, \ldots, K_1 - 1. \tag{25}$$

Relay 1 first estimates its equivalent channel

$$\tilde{\mathbf{H}}_{SR}^j(1) = \mathbf{H}_{SR}^j(1)\mathbf{X}_R^j(2), \tag{26}$$

and decodes $\mathbf{X}_R^i(1)$, which provides $N_S^*(1) N_S$ degrees of freedom. The total degrees of freedom Relay 1 can decode are $(K_2 - 1)(K_1 - 1) N_S^*(1) N_S$.

The received signal during the first sub-interval of length $K_1 T$ at the destination is:

$$\begin{aligned}\mathbf{Y}_D = \big[&\mathbf{H}_{SD}\mathbf{X}_R^j(2), [\mathbf{H}_{SD}\mathbf{X}_R^j(2), \\ &\mathbf{H}_{RD}(1), \mathbf{H}_{RD}(2)]\bar{\mathbf{X}}_D\big] + \mathbf{W}_D \end{aligned} \tag{27}$$

and the received signals during each remaining sub-intervals of length $K_1 T$ are

$$\begin{aligned}\mathbf{Y}_D = \big[&\mathbf{H}_{SD}\mathbf{X}_R^j(2)\mathbf{X}_R^i(1), [\mathbf{H}_{SD}\mathbf{X}_R^j(2)\mathbf{X}_R^i(1), \\ &\mathbf{H}_{RD}(1), \mathbf{H}_{RD}(2)]\bar{\mathbf{X}}_D\big] + \mathbf{W}_D, \end{aligned} \tag{28}$$

where $i = 1, \ldots, K_1 - 1$, $j = 1, \ldots, K_2 - 1$. The destination estimates the equivalent channel $\mathbf{H}_{SD}\mathbf{X}_R^j(2)\mathbf{X}_R^i(1)$, $\mathbf{H}_{SD}\mathbf{X}_R^j(2)\mathbf{X}_R^i(1)$, and decodes $\mathbf{X}_D, \mathbf{X}_{d1}$ and $\mathbf{X}_{d2}$ inside $\bar{\mathbf{X}}_D$, which provide $N_S, n_R(1), n_R(2)$ degrees of freedom per symbol over these time slots.

During each interval of length $K_2 K_1 T$, the source-destination link can always provide $N_S(1 - \frac{N_S}{T})$ degrees of freedom per symbol. The maximum degrees of freedom decoded at Relay 1 are $(K_1 - 1) N_S^*(1) N_S + (K_2 - 1)(K_1 - 1) N_S^*(1) N_S = K_2(K_1 - 1) N_S^*(1) N_S$. The degrees of freedom decoded at Relay 2 are $(K_2 - 1) N_S^*(2) N_S$. The number of time slots for sending data is $K_2 K_1 (T - N_S)$. The degrees of freedom the relays can provide via the relay-destination links are $n_R(i) K_2 K_1 (T - N_S), i = 1, 2$. Noting that the emitted data by the relays is limited by what they can decode, we sum the degrees of freedom by the two relays, normalize it per symbol, and optimize the number of transmit antennas activated at the relays. The following degrees of freedom are achievable:

$$d = \max_{n_R(i)} \{N_S(1 - \frac{N_S}{T}) + \min \{(1 - \frac{1}{K_1})\frac{N_S^*(1)N_S}{T}, n_R(1)\frac{T - N_S}{T}\} + \min \{\frac{K_2 - 1}{K_1K_2}\frac{N_S^*(2)N_S}{T}, n_R(2)\frac{T - N_S}{T}\}\}. \quad (29)$$

B. Achievable DoF for K Parallel Relays

We now extend the ideas and techniques that were developed in the two-relay framework to the K-relay case. In the interest of economy of expression, the parts that are similar to the earlier discussions are condensed or omitted.

Denote with $\mathbf{T}_{SR}, \mathbf{T}_{RD}$, the size-$K$ vectors containing, respectively, source-relay and relay-destination coherence times, and $\mathbf{N}_R, \mathbf{n}_R$ the number of receive and activated transmit antennas at the relays. Also, we allow a subset k of relays to be used. We denote the coherence times of selected relays with size-k vectors $\mathbf{T}', \mathbf{T}''$ and the number of receive and activated transmit antennas in selected relays with size-k vector $\mathbf{N}', \mathbf{n}'$. The following result shows the achievable degrees of freedom, which is maximized over selected relays and their activated transmit antennas. Selection matrix $\mathbf{P}_{k\times K}$ selects the relays.

Theorem 3. *For the multi-relay system* (15) *and* (16)*, the following degrees of freedom are achievable:*

$$d = \max_{\mathbf{P},\mathbf{n}',k} \Big\{N_S(1 - \frac{N_S}{T_{SD}} - \sum_{i=1}^{k}\frac{n_i'}{T_i''}) + \sum_{i=1}^{k}\min\{N_i^*N_S(\frac{1}{T_{i-1}'} - \frac{1}{T_i'}), n_i'(1 - \frac{N_S}{T_{SD}} - \sum_{j=1}^{k}\frac{n_j'}{T_j''})\}\Big\},$$

$$\textit{subject to:} \quad [\mathbf{T}'\ \mathbf{T}''\ \mathbf{N}'\ \mathbf{n}'] = \mathbf{P}[\mathbf{T}_{SR}\ \mathbf{T}_{RD}\ \mathbf{N}_R\ \mathbf{n}_R], \quad (30)$$

where $T_0' \triangleq T_{SD}$*,* $\mathbf{P}$ *is a selection matrix consisting of* k *rows of the identity matrix of size* K*, and* $N_i^* = \min\{N_S, N_i'\}$*.*

Proof. The transmit scheme is designed in the same spirit as Theorem 1: On the multiple-access side, pilots sent from the relays and the source are allocated in different time slots; On the broadcast side, product superposition enables transmission of additional data to the relays. Throughout this proof, we index only the activated relays, e.g., Relay i refers to i-th activated relay. Without loss of generality, $T_1' \le T_2' \le \cdots \le T_k'$. Define $T_1 \triangleq \prod_{i=1}^{k} T_i'$ and $T_2 \triangleq \prod_{i=1}^{k} T_i''$. In the following, we consider a super-interval of length $T_1T_2T_{SD}$,

During each coherence interval of length T_i', Relay i needs $T_{SD}T_2T_1/T_i'$ pilot sequences each of length N_S for channel estimation. Relay $(i-1)$ needs $T_{SD}T_2T_1/T_{i-1}'$ pilot sequences each of length N_S. Therefore, product superposition can be applied during $(T_{SD}T_2T_1/T_{i-1}' - T_{SD}T_2T_1/T_i')$ pilot sequences each of length N_S to send data to Relay i, providing N_i^* degrees of freedom per symbol.

During each coherence interval of length T_{SD} in the source-destination link, N_S pilot symbols are transmitted. In each super-interval (see above) T_1T_2 pilot sequences of length N_S are transmitted.

For channel estimation between Relay i and the destination, during the super-interval of length $T_1T_2T_{SD}$, the destination needs $T_1T_2T_{SD}/T_i''$ pilot sequences of length n_i'.

Therefore, In each super-interval, the source and relays can use $(T_{SD}T_1T_2 - N_ST_1T_2 - \sum_{i=1}^{k}\frac{n_i'}{T_i''}T_1T_2T_{SD})$ time slots to send data. The source has N_S degrees of freedom available per transmission, and Relay i has n_i' degrees of freedom per transmission.

The degrees of freedom that Relay i can decode are at most $N_i'N_S(T_{SD}T_2T_1/T_{i-1}' - T_{SD}T_2T_1/T_i')$. Therefore, the degrees of freedom Relay i can provide are:

$$\min\{N_i'N_S(\frac{T_{SD}T_2T_1}{T_{i-1}'} - \frac{T_{SD}T_2T_1}{T_i'}), n_i'(T_{SD}T_1T_2 - N_ST_1T_2 - \sum_{i=1}^{k}\frac{n_i'}{T_i''}T_1T_2T_{SD})\},$$

the minimum of the degrees of freedom Relay i can receive and can transmit.

We can now sum the degrees of freedom by the source and the relays and normalize it per symbol. Optimize the relays to be activated, i.e, over k and $\mathbf{P}$, and the number transmit antennas at the relays $\mathbf{n}'$. The degrees of freedom in (30) are achieved. This concludes the proof. □

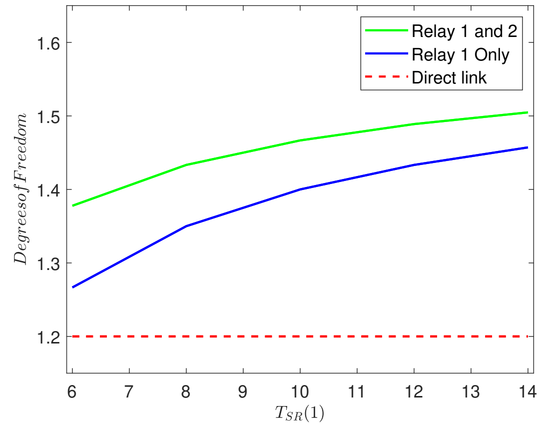

Fig. 2. Achievable degrees of freedom with two parallel relays

Figure 2 shows the achievable degrees of freedom for the system with $N_S = 3, N_R(1) = N_R(2) = 2, N_D = 6$, and $T_{SR}(1)/T_{SR}(2) = \frac{2}{3}$, over different $T_{SR}(1)$.

V. Conclusion

This paper studies the relay channel under coherence diversity. First, we extended the coherence diversity analysis to general coherence intervals that are not guaranteed to be aligned or integer multiples of each other. Second, we proposed and analyzed a more efficient relay transmission strategy under coherence diversity in which the relay, while always listening, sometimes declines to transmit. Finally, we proposed and analyzed multi-relay transmission strategies under coherence diversity.

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