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IMPROVED ATOMIC NORM BASED CHANNEL ESTIMATION FOR TIME-VARYING NARROWBAND LEAKED CHANNELS

Jianxiu Li and Urbashi Mitra

Ming Hsieh Department of Electrical Engineering, Viterbi School of Engineering
University of Southern California, Los Angeles, CA
E-mail: {jianxiul, ubli}@usc.edu

ABSTRACT

In this paper, improved channel gain delay estimation strategies are investigated when practical pulse shapes with finite block length and transmission bandwidth are employed. Pilot-aided channel estimation with an improved atomic norm based approach is proposed to promote the low rank structure of the channel. All the channel parameters, *i.e.*, delays, Doppler shifts and channel gains are recovered. Design choices which ensure unique estimates of channel parameters for root-raised-cosine pulse shapes are examined. Furthermore, a perturbation analysis is conducted. Finally, numerical results verify the theoretical analysis and show performance improvements over the previously proposed method.

***Index Terms*—** Leakage, atomic norm, re-sampling, channel estimation, time-varying narrowband channel.

1. INTRODUCTION

Many wireless communication applications necessitate high performance channel estimation in order to ensure reliable communications. In particular, frequency and temporal distortion [1] is a challenge in high mobility scenarios, such as high-speed railway system [2], vehicle-to-vehicle communications [3], and positioning systems [4].

To combat channel distortion, equalization with accurate channel estimation has been persistently studied (see *e.g.* [5] - [9]). Inherent channel sparsity has been exploited in [7] - [9], reducing the number of observations needed for estimation; these works ignored the impact of practical pulse shapes which lead to a loss in sparsity (channel leakage) in the Doppler-delay domain and challenge performance of these sparse methods.

Channel leakage has been addressed by enhancing classical sparse approximation [3, 10, 11]. Distinct from these approaches, [12] exploits the atomic norm heuristic [13, 14] to promote structure while explicitly considering the leakage leading to strong performance improvements over [10]. The extensions of [12] to the orthogonal frequency-division multiplexing leaked channel are presented in [15] - [17]. However, in [12], [15] - [17], only the Doppler shift and leakage vector are estimated, *i.e.*, the delays and channel gains are not separately recovered. While equalization can be achieved via the methods in [12], improved equalization is enabled via the direct estimation of channel gains and delays. In addition, based on the estimated delays and channel gains, time-of-arrival or received signal-strength information can be exploited for localization [18, 19], critical to many IoT applications. Herein, we further improve upon the atomic norm based approach, by directly estimating these quantities. Properties of the particular pulse shape employed are analyzed.

The main contributions of this paper are:

1) A simple improvement to the atomic norm based channel estimation scheme [12] is proposed, where the delay, Doppler shift, and channel gain of each path can be individually estimated, in contrast to [12].
2) Specific to the root-raised-cosine (RRC) pulse shape, the proposed approach is theoretically analyzed, where we prove the uniqueness of the delay estimate in the noiseless case.
3) A perturbation analysis for the noisy scenario is investigated to understand the impact of noise on the estimates.
4) Numerical comparisons to [12] show the proposed scheme, surprisingly, offers an average 5dB improvement with respect to the bit error rate (BER) achieved by equalizing with the estimated channel matrix of [12]. Furthermore, an improvement in the normalized-mean-square-error (NMSE) of the Doppler shift is achieved with re-estimation with the re-sampled signal.

2. SIGNAL MODEL

We adopt the signal model of [12], thus, the transmitted signal $x(t)$ is given by

$$x(t) = \sum_{n=-\infty}^{+\infty} x[n] p_t\left(t - nT_s\right), \tag{1}$$

where $p_t\left(t\right)$, T_s, and $x[n]$ represent the transmit pulse, the sampling period, and the pilot sequence, respectively. Since the signal is transmitted over a linear, time-varying, narrowband channel whose impulse response is given by

$$g(t,\tau) = \sum_{k=1}^{p_0} \eta_k \delta\left(\tau - t_k\right) e^{j2\pi\nu_k t}, \tag{2}$$

the received signal can be written as

$$y(t) = \int_{-\infty}^{+\infty} g(t,\tau) x(t-\tau) d\tau + z(t), \tag{3}$$

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where $z(t)$ is a complex, Gaussian, white noise process, p_0 denotes the number of dominant paths, η_k, v_k, and t_k represent the channel gain, Doppler shift, and delay of the kth path, respectively, with $1 \leq k \leq p_0$. We label the paths according to the delay values, thus the first path has the smallest delay value t_1. At the receiver, the received signal is converted to the discrete-time equivalent by matched filtering with $p_r(t)$, that is, $y(t) = x(t) \otimes g(t, \tau)$, and then sampled at $t = nT_s + t^\star(n)$, where the offset, $t^\star(n)$, is a design parameter and is set to 0 for each n in [12]. Hence, the corresponding discrete-time signal is given by

$$\begin{aligned} y[n] &= \left(x(t) \otimes g(t,\tau) \otimes p_r(t) + z(t)\right)|_{t=nT_s+t^\star(n)} \\ &= \sum_{m=0}^{M-1}\sum_{k=1}^{p_0} \eta_k e^{j2\pi v_k[(n-m)T_s+t_k]} p\left(mT_s + t^\star(n) - t_k\right) \\ &\qquad \times x[n-m] + z[n], \end{aligned} \tag{4}$$

where $p(t) = p_t(t) \otimes p_r(t)$, $M \leq n < N+M-1$, $0 < m < M = \left\lfloor \frac{\tau_{\max}}{T_s} \right\rfloor + 1$, and $\tau_{\max} = \max(t_1, ..., t_{p_0})$. Here, we assume $p_t(t) = p_r(t)$. Note that, the receiver knows the pilot sequence as well as the transmit and receive pulse shapes. We seek to estimate the channel parameters (η_k, v_k, and t_k for $1 \leq k \leq p_0$).

3. IMPROVED ATOMIC NORM BASED CHANNEL ESTIMATION

Prior to providing our algorithmic improvements to the methods of [12], we briefly review the estimation strategy of [12]. Defining $\bar{v}_k = v_k T_s$, $l_k(t) = p(t-t_k)e^{-j2\pi v_k t}$, $\boldsymbol{l}_k = \frac{1}{\sum_{m=0}^{M-1} l_k(mT_s)} \left[l_k(0T_s), \cdots, l_k((M-1)T_s)\right]^T$, and $\boldsymbol{\alpha}(\bar{v}) = \left[e^{j2\pi M\bar{v}}, \cdots, e^{-j2\pi(N+M-1)\bar{v}}\right]^T$, the received discrete-time signal can be rewritten as

$$\begin{aligned} y[n] &= \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{\alpha}(v_k)^H \boldsymbol{c}_{n-M+1}\boldsymbol{x}_n^T \boldsymbol{l}_k + z[n] \\ &= \operatorname{trace}\left(\boldsymbol{c}_{n-M+1}\boldsymbol{x}_n^T \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{l}_k \boldsymbol{\alpha}(v_k)^H\right) + z[n], \end{aligned} \tag{5}$$

where $\bar{\eta}_k = \eta_k e^{j2\pi v_k t_k} \sum_{m=0}^{M-1} l_k(mT_s)$, $\bar{v}_k \in [-\frac{1}{2}, \frac{1}{2}]$, $\boldsymbol{x}_n = [x[n], \cdots, x[n-(M-1)]]^T$, $t^\star(n) = 0$, and $\boldsymbol{c}_n$ with $1 \leq n \leq N$ is a canonical basis for $\mathbb{R}^{N\times 1}$. The channel matrix is a function of the unknown channel parameters:

$$\mathbf{H} = \sum_{k=1}^{p_0} \bar{\eta}_k \boldsymbol{l}_k \boldsymbol{\alpha}(v_k)^H. \tag{6}$$

Given that many channels of interest have a small number of paths relative to the number of observations $p_0 \ll N$, we can formulate a parametric low-rank matrix recovery problem. Stacking $y[n]$ for $M < n < N+M-1$ in a vector $\boldsymbol{y}$, we can write

$$\boldsymbol{y} = \Pi(\mathbf{H}) + \boldsymbol{z}, \tag{7}$$

where $\boldsymbol{z} = [z[M], \cdots, z[N+M-1]]^T$ and the linear operator $\Pi : \mathbb{C}^{M\times N} \to \mathbb{C}^{N\times 1}$ is defined as $\Pi(\mathbf{H})[n] = \operatorname{trace}\left(\boldsymbol{c}_{n-M+1}\boldsymbol{x}_n^T\mathbf{H}\right)$. Since each term in the sum in Equation (6) is a rank-one matrix, [12] proposes the use of the atomic norm [13], [14] to promote sparsity. Given a set of atoms, $\mathcal{A} = \left\{e^{j\theta}\boldsymbol{l}\boldsymbol{\alpha}(\bar{v})^H : \bar{v} \in [-\frac{1}{2}, \frac{1}{2}], \|\boldsymbol{l}\|_2 = 1, \boldsymbol{l} \in \mathbb{C}^{M\times 1}, \theta \in [0, 2\pi)\right\}$, the atomic norm is defined as

$$\begin{aligned} \|\mathbf{H}\|_{\mathcal{A}} &= \inf\left\{c > 0 : \mathbf{H} \in c\operatorname{conv}(\mathcal{A})\right\} \\ &= \inf_{\bar{\eta}_k, \bar{v}_k, \|\boldsymbol{l}_k\|_2 = 1} \left\{\sum_k |\bar{\eta}_k| : \mathbf{H} = \sum_k \bar{\eta}_k \boldsymbol{l}_k \boldsymbol{\alpha}(\bar{v}_k)^H\right\}, \end{aligned} \tag{8}$$

where $\operatorname{conv}(\mathcal{A})$ is the convex hull of $\mathcal{A}$. Using the atomic norm, we solve the following optimization problem to estimate the channel,

$$\underset{\mathbf{H}}{\operatorname{minimize}} \|\mathbf{H}\|_{\mathcal{A}} \text{ s.t. } \|\boldsymbol{y} - \Pi(\mathbf{H})\|_2^2 \leq N\sigma_z^2, \tag{9}$$

where σ_z^2 is the variance of $z[n]$. In [14], our optimization problem in Equation (9) is shown to have an equivalent semidefinite program representation, which enables efficient solution of Equation (9) via solvers such as CVX [20]. Under key conditions (C-1 and C-2 of [12]), the Doppler shifts can be estimated by exploiting the dual problem of Equation (9) since there is no duality gap [12]. We refer readers to [12] for the details of the atomic norm based Doppler shift estimation. This concludes the summary of prior work.

In Equation (5), we replace the Doppler shifts v_k by their estimated values, $\hat{v}_k$ for $1 \leq k \leq p_0$, to construct an estimate of the channel leakage vector, $\hat{\boldsymbol{h}}_{\boldsymbol{m}}^{\boldsymbol{l}}$. We assume that the estimates are perturbed from the true values as follows

$$\hat{\boldsymbol{h}}_m^l = \boldsymbol{h}_m^l + \boldsymbol{e}_m^{(h)}, \tag{10}$$

where the kth element of the true channel leakage vector $\boldsymbol{h}_m^l$ with $0 \leq m \leq M-1$, is given by

$$\boldsymbol{h}_m^l[k] = \eta_k e^{j2\pi v_k(t_k - mT_s)} p\left(mT_s - t_k\right), \tag{11}$$

for $1 \leq k \leq p_0$. The vector $\boldsymbol{e}_m^{(h)}$ represents the error induced by the errors in the estimation of the Doppler shifts. From Equations (5) and (11), we can see that the channel delays appear only in the channel leakage vector. How to obtain the estimated delays and channel gains based on $\hat{\boldsymbol{h}}_m^l$, $0 \leq m \leq M-1$, is the focus of this paper.

Since the channel gains are unknown, complex values, we can not use the phase of $\boldsymbol{h}_m^l$ defined in Equation (11) to directly obtain the delay estimate. However, knowledge of the pulse shape $p(t)$ can be exploited to infer the delays. We define the *ratio function*, which will be used in our estimation strategy, as

$$r_k(m_1, m_2, t_k) = \frac{p\left(m_2T_s - t_k\right)}{p\left(m_1T_s - t_k\right)} + e^{(r_k)}, \tag{12}$$

with $0 \leq m_1, m_2 \leq M-1$ and $m_1, m_2 \in \mathbb{Z}$. Here, the error in the ratio function, $e^{(r_k)}$ is

$$e^{(r_k)} = \operatorname{sign}\left(\frac{p\left(m_2T_s - t_k\right)}{p\left(m_1T_s - t_k\right)}\right)\left|\frac{\hat{\boldsymbol{h}}_{m_2}^l[k]}{\hat{\boldsymbol{h}}_{m_1}^l[k]}\right| - \frac{p\left(m_2T_s - t_k\right)}{p\left(m_1T_s - t_k\right)}. \tag{13}$$

Algorithm 1 Ratio Functions for Delay Estimation.

Input: $r_k^o(m_1, m_2, t_k) = \frac{\hat{\boldsymbol{h}}_{m_2}^l[k]}{\hat{\boldsymbol{h}}_{m_1,k}^l[k]} = |r_k^o(m_1, m_2, t_k)| \times e^{j\phi_{r_k^o(m_1,m_2,t_k)}}$ and $\hat{v}_k$;
Output: $r_k(m_1, m_2, t_k)$, with $1 \leq k \leq p_0$;
for $k = 1$ to p_0 **do**
  $z_k \leftarrow \left\langle \frac{2\pi(m_2-m_1)\hat{v}_k T_s + \phi_{r_k^o(m_1,m_2,t_k)}}{\pi} \right\rangle$;
  if $p(t)$ is positive for all t **then**
    $r_k(m_1, m_2, t_k) \leftarrow |r_k^o(m_1, m_2, t_k)|$;
  else
    $r_k(m_1, m_2, t_k) \leftarrow (-1)^{z_k} |r_k^o(m_1, m_2, t_k)|$;
  end if
  $k \leftarrow k + 1$;
end for
return $r_k(m_1, m_2, t_k)$, with $1 \leq k \leq p_0$.

Note that, if $p(t)$ is positive for all t, we can directly set the ratio function $r_k(m_1, m_2, t_k)$ to $\left|\frac{\hat{\boldsymbol{h}}_{\boldsymbol{m_2}}^{\boldsymbol{l}}[k]}{\hat{\boldsymbol{h}}_{\boldsymbol{m_1}}^{\boldsymbol{l}}[k]}\right|$. If $p(t)$ is not positive everywhere, we need to first obtain the sign of $\frac{p(m_2T_s-t_k)}{p(m_1T_s-t_k)}$ and then set $r_k(m_1, m_2, t_k)$ properly. Algorithm 1 illustrates the generation of these ratios for delay estimation, where $< d >$ represents the integer that is closest to d. Perhaps surprisingly, the choices of the design parameters m_1 and m_2 strongly influence the uniqueness and quality of the estimates; furthermore, they are pulse-shape dependent. Thus in Section 4, we explore good choices of these parameters for RRC pulse shapes. Once these parameters are selected, the ratio function is simply a function of delays. If we set m_1 and m_2 to a and b (integers in $[0, M-1]$), the delay estimate of the kth path can be obtained by $\hat{t}_k = (f(t_k))^{-1}$, where $f(t_k) = r_k(m_1, m_2, t_k)\mid_{m_1=a, m_2=b}$, $(f(\cdot))^{-1}$ represents the inverse function of function $f(\cdot)$, and $\hat{t}_k$ is the estimate of t_k. After delays are estimated, we can substitute their values into Equation (5) to achieve a linear system of equations to compute the channel gains.

One can construct an equalizer from the estimated Doppler shifts and channel leakage vectors versus estimating the individual path parameters. However, with the estimated delays and channel gains, we can properly re-sample the received signals for sequence detection yielding lower BERs. Specifically, we re-sample the received signal at $nT_s + \hat{t}_{k'}$ with $1 \leq k' \leq p_0$, and thus have

$$
\begin{aligned}
y[n] &= (x(t) \otimes g(t,\tau) \otimes p_r(t) + z(t))\mid_{t=nT_s+\hat{t}_{k'}} \\
&= \sum_{m=0}^{m_0-1} \sum_{k=1}^{p_0} \eta_i e^{j2\pi v_i[(n-m)T_s+t_i]} p\left(mT_s + \hat{t}_{k'} - t_k\right) \\
&\qquad \times x[n-m] + z[n],
\end{aligned} \tag{14}
$$

Using these re-sampled discrete-time signals that are matched to the channel for detection, we can decrease BER of the data sequence and establish a more reliable transmission link. Also, re-estimation with the re-sampled signals can further reduce the mean-squared estimation error, which will be seen in Section 5. The improved estimation strategy is denoted as the Atomic Norm based Delay-Doppler Estimation (ANDE).

4. PULSE-SHAPE BASED DESIGN CHOICES

Given that typical pulse-shapes are non-linear with respect to the time argument, the inversion function $f(\cdot)$ needed to compute the delay estimates will also be non-linear. As such, we need further constraints to ensure unique delay estimates, even in the noiseless case. To this end, we show that proper choice of m_1 and m_2 can ensure uniqueness for RCC pulse shape functions[1], using the noiseless ratio function, *i.e.*, $e^{(r_k)} = 0$, and present the perturbation analysis, where we assume $m_2 > m_1$. Since the RCC pulse shapes are adopted for $p_t(t)$ and $p_r(t)$, $p(t)$ is defined as

$$
p(t) = \begin{cases} \frac{\pi}{4T} \operatorname{sinc}\left(\frac{1}{2\beta}\right), & t = \pm\frac{T}{2\beta}; \\ \frac{1}{T} \operatorname{sinc}\left(\frac{t}{T}\right) \frac{\cos\left(\frac{\pi\beta t}{T}\right)}{1-\left(\frac{2\beta t}{T}\right)^2}, & \text{otherwise}, \end{cases} \tag{15}
$$

where $\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$, β and T are the roll-off factor and a design parameter of $p(t)$, respectively. First, viewing the design parameters m_1 and m_2 as constants, we consider the following noiseless ratio function for delay estimation,

$$
\begin{aligned}
&f_{RRC}(t_k) = \frac{p\left(m_2T_s - t_k\right)}{p\left(m_1T_s - t_k\right)} \\
&= \frac{\operatorname{sinc}\left(\frac{m_2T_s-t_k}{T}\right)\cos\left(\frac{\pi\beta(m_2T_s-t_k)}{T}\right)\left(1-\left(\frac{2\beta(m_1T_s-t_k)}{T}\right)^2\right)}{\operatorname{sinc}\left(\frac{m_1T_s-t_k}{T}\right)\cos\left(\frac{\pi\beta(m_1T_s-t_k)}{T}\right)\left(1-\left(\frac{2\beta(m_2T_s-t_k)}{T}\right)^2\right)},
\end{aligned} \tag{16}
$$

where we assume $p\left(m_1T_s - t_k\right) \neq 0$[2]. In general, $f_{RRC}(t_k)$ is not a monotonic function with respect to t_k and thus the uniqueness of the delay estimate can not be guaranteed. To make the delay estimate unique, we need to properly select the design parameters m_i, which is discussed as follows,

Proposition 1. *For the implementation of ANDE with RCC pulse shapes, the uniqueness of the delay estimates in the noiseless case ($e^{(r_k)} = 0$) can be guaranteed under the conditions: A) The design parameter m_1 is set to 0 and m_2 is restricted to $[\frac{2M}{1+\sqrt{2}}, M-1]$[3]; and B) The parameters T and β in Equation (15) are such that $T = \beta m_2 T_s$ and $\frac{1}{\beta} \in \mathbb{Z}$.*

Proof. With assumptions A and B, it can be proved that $f_{RRC}(t_k)$ is a monotonic function with respect to t_k. Hence,

[1]For Gaussian and rectangular pulse shapes with practical constraints, we also can ensure the uniqueness of the delay estimates by properly setting m_i and a larger value of $m_2 - m_1$ is preferred to reduce the mean-squared-error of delay estimates, which is not presented in this paper due to lack of space.

[2]If we have $p\left(m_1T_s - t_k\right) = 0$ and $p\left(m_2T_s - t_k\right) \neq 0$, the delay estimate of the kth path can be directly obtained.

[3]Note that, the constraint $m_2 \in [\frac{2M}{1+\sqrt{2}}, M-1]$ enforces $M \geq 6$ because $m_2 \in \mathbb{Z}$ and $m_2 > m_1 = 0$ holds, which indicates the large discrete delay spreads are taken into consideration. The condition $M \geq 6$ can be met by decreasing the sampling period T_s.

$$\hat{t}_k = \begin{cases} \dfrac{\left(5f_{RRC}(t_k) - (-1)^{\frac{1}{\beta}}\sqrt{f_{RRC}^2(t_k) - (-1)^{\frac{1}{\beta}}34f_{RRC}(t_k) + 1} - (-1)^{\frac{1}{\beta}}\right)m_2T_s}{4(f_{RRC}(t_k) + (-1)^{\frac{1}{\beta}})}, & \text{if } f_{RRC}(t_k) < 0; \\ \dfrac{m_2T_s}{2} & \text{if } f_{RRC}(t_k) = 1; \\ \dfrac{\left(5f_{RRC}(t_k) + (-1)^{\frac{1}{\beta}}\sqrt{f_{RRC}^2(t_k) - (-1)^{\frac{1}{\beta}}34f_{RRC}(t_k) + 1} - (-1)^{\frac{1}{\beta}}\right)m_2T_s}{4(f_{RRC}(t_k) + (-1)^{\frac{1}{\beta}})}, & \text{otherwise.} \end{cases} \quad (17)$$

its inverse function exists and the delay estimate is unique.[4] □

Remark 1. *With the proper selection of the design parameters based on Proposition 1, it can be verified* $f_{RRC}^2(t_k) - (-1)^{\frac{1}{\beta}}34f_{RRC}(t_k) + 1 \geq 0$ *holds for all* $t_k \in (0, MT_s)$ *and the delay estimate is given by Equation (17).*

Remark 2. *In contrast to the noiseless case, the conditions of Proposition 1 do not guarantee a unique estimate for all possible noise values, i.e., the perturbations to the ratio function* $e^{(r_k)}$. *Each perturbation could lead to a different optimal* m_2. *However, one can consider multiple values of* m_2 *and average the corresponding delay estimates.*

5. NUMERICAL RESULTS

In this section, we evaluate the performance of our proposed scheme. The channel model is as in Equation (2). Specifically, the delays are uniform random variables, normalized to $(0, 1]$ and the normalized Doppler shifts (also uniform random variables) have a support $[-\frac{1}{2}, \frac{1}{2}]$. The channel gains and noise are complex, independent, Gaussian random variables. The pilot sequence is a Binary Phase Shift Keying (BPSK) modulated random sequence. As previously noted, the pulse shapes, $p_t(t) = p_r(t)$, are chosen as RRC pulse shapes (see Equation (15)) truncated by a window with length of $2MT_s$. In all experiments, the scaling law [12] is satisfied to ensure proper behavior of the atomic norm based estimator and the transmit SNR is considered, *i.e.*, $\mathrm{E}\{\|\boldsymbol{x}\|_2^2/\|\boldsymbol{z}\|_2^2\}$. We set N, p_0 and β to 100, 3 and $\frac{1}{m_2}$, respectively.

In Fig. 1(a), the NMSE of delay estimation versus SNRs is shown, where we set m_1 and M to 0 and 12, respectively. Based on Proposition 1, there are two choices of m_2 to ensure unique delay estimates, *i.e.*, $m_2 = 11$ or 10. From Fig. 1(a), a slightly lower NMSE of delay estimation is achieved with a larger value of m_2 if $M = 12$ and the SNR is greater than 6 dB. To show the impact of violating the constraints of Proposition 1, we set m_1 and m_2 to 0 and 9, respectively. In this case, the delay estimates for each path are determined by minimizing $\|f_{RRC}(\hat{t}_k) - f_{RRC}(t_k)\|_2$; the associated NMSE for this case is also shown in Fig. 1(a), and as uniqueness is not guaranteed, a strong performance loss is seen, as suggested by Proposition 1.

To show the performance gains achieved by our scheme, the BER of the data sequence for ANDE and PLAN [12] are shown in Figs. 1(b), where m_1, m_2, and M are set to 0, 5 and 6 and maximum likelihood sequence equalization [21] is implemented for detection. ANDE outperforms PLAN, offering 5 dB improvement on average. This strong gain is achieved as the individual delays, Doppler values, and channel gains are estimated via ANDE enabling the construction of receive filtering based on the pulse shape and the channel; whereas PLAN only estimates the channel matrix described in Equations (6). The NMSE for the two schemes for Doppler estimation is provided in Fig. 1(c) wherein for ANDE, the channel parameter estimates are used to re-sample the signal at $nT_s + \hat{t}_1$ with re-estimation, yielding further improvements.

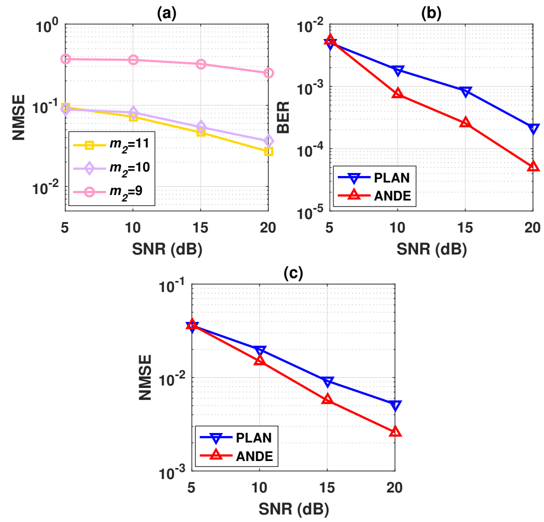

Fig. 1: (a) NMSE of delay estimation with different choices of m_2, (b) BER of data sequence versus SNRs, and (c) NMSE of Doppler shift estimation in comparison with PLAN under different SNRs.

6. CONCLUSIONS

In this paper, an improvement to atomic norm based estimation scheme is proposed for time-varying narrowband leaked channels, where all the channel parameters can be well estimated. In particular, a new strategy to estimate delays and channel gains after atomic norm based channel matrix estimation [12] is provided. An analysis regarding the uniqueness of delay estimates in the presence of root-raise cosine pulses and an accompanying perturbation study for the noisy case is provided. The new method offers strong improvement with respect to BER over the prior art (5dB) [12]. Direct estimation of the delays is an essential element of high performance localization strategies, as well as channel equalization. The proposed method can also be used to further improve Doppler estimation.

[4] See https://github.com/LJIANXIU/Improved-Atomic-Norm-Based-Channel-Estimation-for-Time-varying-Narrowband-Leaked-Channels.git for the complete proof.

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