

BOUNDED SEMIGROUP WELLPOSEDNESS FOR A LINEARIZED COMPRESSIBLE FLOW STRUCTURE PDE INTERACTION WITH MATERIAL DERIVATIVE*

PELIN G. GEREDELI†

Abstract. We consider a compressible flow-structure interaction (FSI) PDE system which is linearized about some reference rest state. The deformable interface is under the effect of an ambient field generated by the underlying and unbounded material derivative term which further contributes to the nondissipativity of the FSI system with respect to the standard energy inner product. In this work we show that, on an appropriate subspace, only one dimension less than the entire finite energy space, the FSI system is wellposed and is moreover associated with a continuous semigroup which is *uniformly bounded* in time. Our approach involves establishing maximal dissipativity with respect to a special inner product which is equivalent to the standard inner product for the given finite energy space. Among other technical features, the necessary PDE estimates require the invocation of a multiplier which is intrinsic to the given compressible FSI system.

Key words. flow-structure interaction, compressible flows, wellposedness, uniformly bounded semigroup, material derivative

AMS subject classifications. 34A12, 74F10, 35Q35, 76N10

DOI. 10.1137/20M1345840

1. Introduction. Compressible flow phenomena arise in fluid mechanics, particularly in the modeling of gas dynamics. The motion of such flows is typically described via the Navier–Stokes equations by way of providing qualitative information on the three basic physical variables: the pressure of the fluid $p = p(x, t)$, the mass density $\rho = \rho(x, t)$, and the fluid velocity field $u = u(x, t)$. Unlike the case of incompressible flows wherein density ρ is a constant, the pressure associated with compressible flow has a nonlocal character and is an unknown function determined (implicitly) by the fluid motion. Moreover, in compressible flow dynamics the density of the fluid is considered to be an additional variable component, the resolution of which represents substantial difficulties in the associated mathematical analysis.

In this work, we consider the linearization of a coupled flow-structure interaction (FSI) PDE system, with compressible fluid flow PDE component. In the context of real-world applications, this FSI finds its key application in aeroelasticity: this PDE system involves the strong coupling between a dynamically deforming structure (e.g., the wing) and the air flow which streams past it. In short, this system describes the interaction between plate and flow dynamics through a deformable interface.

The description of our FSI PDE model is given as follows: Let the flow domain $\mathcal{O} \subset \mathbb{R}^3$ with Lipschitz boundary $\partial\mathcal{O}$. We assume that $\partial\mathcal{O} = \overline{S} \cup \overline{\Omega}$, with $S \cap \Omega = \emptyset$, and the (structure) domain $\Omega \subset \mathbb{R}^3$ is a *flat* portion of $\partial\mathcal{O}$ with C^2 -boundary. In particular, $\partial\mathcal{O}$ has the following specific configuration:

$$(1) \quad \Omega \subset \{x = (x_1, x_2, 0)\} \text{ and surface } S \subset \{x = (x_1, x_2, x_3) : x_3 \leq 0\}.$$

Let $\mathbf{n}(\mathbf{x})$ be the unit outward normal vector to $\partial\mathcal{O}$, and $\mathbf{n}|_{\Omega} = [0, 0, 1]$. Also,

*Received by the editors June 16, 2020; accepted for publication (in revised form) February 1, 2021; published electronically March 24, 2021.

<https://doi.org/10.1137/20M1345840>

Funding: The work of the author was partially supported by the National Science Foundation grant DMS-1907823.

†Department of Mathematics, Iowa State University, Ames, IA 50011 USA (peling@iastate.edu).



FIG. 1. Polyhedral flow-structure geometries.

we denote the unit outward normal vector to $\partial\Omega$ by $\nu(\mathbf{x})$. In addition to the above properties given for the fluid domain $\mathcal{O}$, we impose the following conditions which will be necessary for the application of some elliptic regularity results for solutions of second order boundary value problems (BVPs) on corner domains [22, 24].

CONDITION 1. *Flow domain $\mathcal{O}$ should be a curvilinear polyhedral domain which satisfies the following condition:*

- Each corner of the boundary $\partial\mathcal{O}$ —if any—is diffeomorphic to a convex cone,
- Each point on an edge of the boundary $\partial\mathcal{O}$ is diffeomorphic to a wedge with opening $< \pi$.

Some examples of geometries can be seen in Figure 1.

We assume that the pressure is a linear function of the density; $p(x, t) = C\rho(x, t)$ as mostly done in the compressible fluid literature and it is chosen as a primary variable to solve. This assumption can be arrived at in two ways, which we now briefly describe (see [9] for further detailed explanation): For *barotropic* flows, the relationship between pressure and density is (see, e.g., [21, 35])

$$(2) \quad \tilde{p} = C[\tilde{\rho}]^\gamma,$$

where C is a constant evaluated for the pressure and density in the far field and $\gamma > 0$ —for air, $\gamma = 1.4$. (We note that so-called *isentropic flows* [25, pp. 169–200] are barotropic.) This equation can be linearized by the above perturbation convention, taking p_* and ρ_* to be the far field pressure and density. This gives the linear relation

$$(3) \quad p = C(\gamma, \rho_*, p_*)\rho.$$

On the other hand, if one considers *isothermal* flow, the ideal gas law reads $\tilde{p} = \tilde{\rho}RT$, where T is the temperature and R is a fluid-dependent constant. This equation also presents a linear relation between pressure and density if T is a constant. Isothermal flows are used in low speed situations, i.e., with velocities much less than the speed of sound. Isentropic flow is used for compressible flows with small viscosity, and the ideal gas law is used for compressible viscous flows. We do note that T is typically taken as an unknown for compressible viscous flows; if this consideration is made, an energy balance equation is required.

With respect to some equilibrium point of the form $\{p_*, \mathbf{U}, \varrho_*\}$ where the pressure and density components p_*, ϱ_* are assumed to be scalars (for simplicity assuming

$p_* = \varrho_* = 1$), and the arbitrary ambient field $\mathbf{U} : \mathcal{O} \rightarrow \mathbb{R}^3$,

$$\mathbf{U}(x_1, x_2, x_3) = [U_1(x_1, x_2, x_3), U_2(x_1, x_2, x_3), U_3(x_1, x_2, x_3)],$$

is given, the small perturbations give the following physical equations by generalizing the forcing functions:

$$(\partial_t + \mathbf{U} \cdot \nabla)p + \operatorname{div}(u) + (\operatorname{div} \mathbf{U})p = f(\mathbf{x}) \quad \text{in } \mathcal{O} \times \mathbb{R}_+,$$

$$(\partial_t + \mathbf{U} \cdot \nabla)u - \nu \Delta u - (\nu + \lambda) \nabla \operatorname{div} u + \nabla p + \nabla \mathbf{U} \cdot u + (\mathbf{U} \cdot \nabla \mathbf{U})p = \mathbf{F}(\mathbf{x}) \quad \text{in } \mathcal{O} \times \mathbb{R}_+.$$

(For further discussion, see also [9, 19].) When we delete some of the noncritical lower order and the benign inhomogeneous terms in the above equations, this linearization produces the following system of equations, in solution variables $u(x_1, x_2, x_3, t)$ (flow velocity), $p(x_1, x_2, x_3, t)$ (pressure), $w_1(x_1, x_2, t)$ (elastic plate displacement), and $w_2(x_1, x_2, t)$ (elastic plate velocity):

$$(4) \quad \begin{cases} p_t + \mathbf{U} \cdot \nabla p + \operatorname{div} u + \operatorname{div}(\mathbf{U})p = 0 & \text{in } \mathcal{O} \times (0, \infty), \\ u_t + \mathbf{U} \cdot \nabla u - \operatorname{div} \sigma(u) + \eta u + \nabla p = 0 & \text{in } \mathcal{O} \times (0, \infty), \\ (\sigma(u)\mathbf{n} - p\mathbf{n}) \cdot \boldsymbol{\tau} = 0 & \text{on } \partial\mathcal{O} \times (0, \infty), \\ u \cdot \mathbf{n} = 0 & \text{on } S \times (0, \infty), \\ u \cdot \mathbf{n} = w_2 + \mathbf{U} \cdot \nabla w_1 & \text{on } \Omega \times (0, \infty), \end{cases}$$

$$(5) \quad \begin{cases} w_{1t} - w_2 - \mathbf{U} \cdot \nabla w_1 = 0 & \text{on } \Omega \times (0, \infty), \\ w_{2t} + \Delta^2 w_1 + [2\nu \partial_{x_3}(u)_3 + \lambda \operatorname{div}(u) - p]_\Omega = 0 & \text{on } \Omega \times (0, \infty), \\ w_1 = \frac{\partial w_1}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0, \infty), \end{cases}$$

$$(6) \quad [p(0), u(0), w_1(0), w_2(0)] = [\bar{p}, \bar{u}, \bar{w}_1, \bar{w}_2] \in H_N^\perp,$$

where H_N is the null space of the generator of the above problem (see (18)) and the space $H_N^\perp$ is characterized (see (1)) as follows:

$$H_N^\perp = \left\{ [p_0, u_0, w_1, w_2] \in \mathcal{H} : \int_{\mathcal{O}} p_0 d\mathcal{O} + \int_{\Omega} w_1 d\Omega = 0 \right\},$$

where

$$(7) \quad \mathcal{H} \equiv L^2(\mathcal{O}) \times \mathbf{L}^2(\mathcal{O}) \times H_0^2(\Omega) \times L^2(\Omega)$$

is the associated finite energy (Hilbert) space, topologized by the standard inner product

$$(8) \quad (\mathbf{y}_1, \mathbf{y}_2)_{\mathcal{H}} = (p_1, p_2)_{L^2(\mathcal{O})} + (u_1, u_2)_{\mathbf{L}^2(\mathcal{O})} + (\Delta w_1, \Delta w_2)_{L^2(\Omega)} + (v_1, v_2)_{L^2(\Omega)}$$

for any $\mathbf{y}_i = (p_i, u_i, w_i, v_i) \in \mathcal{H}$, $i = 1, 2$.

The quantity $\eta > 0$ represents a drag force of the domain on the viscous flow. In addition, the quantity τ in (4) is in the space $TH^{1/2}(\partial\mathcal{O})$ of tangential vector fields of Sobolev index $1/2$; that is,

$$(9) \quad \tau \in TH^{1/2}(\partial\mathcal{O}) = \{\mathbf{v} \in \mathbf{H}^{1/2}(\partial\mathcal{O}) : \mathbf{v}|_{\partial\mathcal{O}} \cdot \mathbf{n} = 0 \text{ on } \partial\mathcal{O}\}.$$

(See e.g., page 846 of [15].) In addition, we take ambient field $\mathbf{U} \in \mathbf{V}_0 \cap W$ where

$$(10) \quad \mathbf{V}_0 = \{\mathbf{v} \in \mathbf{H}^1(\mathcal{O}) : \mathbf{v}|_{\partial\mathcal{O}} \cdot \mathbf{n} = 0 \text{ on } \partial\mathcal{O}\},$$

$$(11) \quad W = \{v \in \mathbf{H}^1(\mathcal{O}) : v \in L^\infty(\mathcal{O}), \quad \operatorname{div}(v) \in L^\infty(\mathcal{O}), \quad \text{and} \quad v|_\Omega \in C^2(\overline{\Omega})\}$$

and define

$$(12) \quad \|\mathbf{U}\|_* = \|\mathbf{U}\|_{L^\infty(\mathcal{O})} + \|\operatorname{div}(\mathbf{U})\|_{L^\infty(\mathcal{O})} + \|\mathbf{U}|_\Omega\|_{C^2(\overline{\Omega})}.$$

(The vanishing of the boundary for ambient fields is a standard assumption in compressible flow literature; see [1, 21, 28, 35].) Moreover, the *stress and strain tensors* in the flow PDE component of (4)–(6) are defined, respectively, as

$$\sigma(\mu) = 2\nu\epsilon(\mu) + \lambda[I_3 \cdot \epsilon(\mu)]I_3; \quad \epsilon_{ij}(\mu) = \frac{1}{2} \left(\frac{\partial \mu_j}{\partial x_i} + \frac{\partial \mu_i}{\partial x_j} \right), \quad 1 \leq i, j \leq 3,$$

where *Lamé coefficients* $\lambda \geq 0$ and $\nu > 0$.

REMARK 2. As will be seen below, the appearance of the term $-w_2 - \mathbf{U} \cdot \nabla w_1$, in the mechanical displacement equation (5), will induce an invariance with respect to the space $H_N^\perp$ defined in (1). We will ultimately establish that solutions of (4)–(6), with initial data in $H_N^\perp$, are associated with a bounded semigroup for $\mathbf{U}$ sufficiently small with respect to an appropriate measurement (see (12)). In addition, if we set $w(t) = w_1(t)$, $w_t = w_2 + \mathbf{U} \cdot \nabla w_1$, then we have that $[p, u, w, w_t]$ solves

$$\begin{cases} p_t + \mathbf{U} \cdot \nabla p + \operatorname{div} u + \operatorname{div}(\mathbf{U})p = 0 & \text{in } \mathcal{O} \times (0, \infty), \\ u_t + \mathbf{U} \cdot \nabla u - \operatorname{div} \sigma(u) + \eta u + \nabla p = 0 & \text{in } \mathcal{O} \times (0, \infty), \\ (\sigma(u)\mathbf{n} - p\mathbf{n}) \cdot \boldsymbol{\tau} = 0 & \text{on } \partial\mathcal{O} \times (0, \infty), \\ u \cdot \mathbf{n} = 0 & \text{on } S \times (0, \infty), \\ u \cdot \mathbf{n} = w_t & \text{on } \Omega \times (0, \infty), \\ \begin{cases} w_{tt} + \Delta^2 w - \mathbf{U} \cdot \nabla w_t + [2\nu\partial_{x_3}(u)_3 + \lambda\operatorname{div}(u) - p]_\Omega = 0 & \text{on } \Omega \times (0, \infty), \\ w = \frac{\partial w}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0, \infty), \end{cases} \\ [p(0), u(0), w(0), w_t(0)] = [\bar{p}, \bar{u}, \bar{w}_1, \bar{w}_2 + \mathbf{U} \cdot \nabla \bar{w}_1] \in H_N^\perp, \end{cases}$$

where $w(0) = w_1(0) = \bar{w}_1$ and $w_t(0) = w_2(0) + \mathbf{U} \cdot \nabla w_1(0) = \bar{w}_2 + \mathbf{U} \cdot \nabla \bar{w}_1$.

Here, as is usually done for viscous fluids, we impose the so-called *impermeability condition* on Ω ; namely, we assume that no fluid passes through the elastic portion of the boundary during deflection [14, 25]. At this point, we emphasize that the FSI problem under consideration has a *material derivative* term on the deflected interaction surface. This material derivative computes the time rate of change of any quantity such as temperature or velocity (and hence also acceleration) for a portion of a material in motion. Since our material is a fluid, the movement is simply the flow field, and any particle of fluid speeds up and slows down as it flows along the specified spatial domain. With respect to the change of the speed of the said fluid, the material derivative effectively gives a true rate of change of the velocity. Hence, we describe the interface Ω in Lagrangian coordinates in $\mathbb{R}^3$ with $S(a_1, a_2, a_3) = 0$; also let $\mathbf{x} = \langle x_1, x_2, x_3 \rangle$ be the Eulerian position inside $\mathcal{O}$. Then, letting $w(x_1, x_2, t)$ represent the transverse (x_3) displacement of the plate on Ω , we have that

$$S(x_1, x_2, x_3 - w(x_1, x_2, t)) \equiv S(x_1, x_2, x_3, t) = 0$$

describes the time-evolution of the boundary. The impermeability condition requires that the material derivative $(\partial_t + \tilde{u} \cdot \nabla_{\mathbf{x}})$ vanish on the deflected surface [14, 16, 25]:

$$(\partial_t + \tilde{u} \cdot \nabla_{\mathbf{x}})S = 0, \quad \tilde{u} = u + \mathbf{U}.$$

Applying the chain rule and rearranging, we obtain

$$(13) \quad \nabla_{\mathbf{x}} S \cdot \langle 0, 0, -w_t \rangle + \mathbf{U} \cdot [\nabla_{\mathbf{x}} S + \langle -S_{x_3} w_{x_1}, -S_{x_3} w_{x_2}, 0 \rangle] = -u \cdot [\nabla_{\mathbf{x}} S + \langle -S_{x_3} w_{x_1}, -S_{x_3} w_{x_2}, 0 \rangle].$$

We identify $\nabla_{\mathbf{x}} S$ as the normal to the deflected surface; *assuming small deflections* and restricting to $(x_1, x_2) \in \Omega$, we can identify $\nabla_{\mathbf{x}} S|_{\Omega}$ with $\mathbf{n}|_{\Omega} = \langle 0, 0, 1 \rangle$. Making use of (13), imposing that $\mathbf{U} \cdot \mathbf{n} = 0$ on $\partial\mathcal{O}$ (see (10) and discussion), and discarding quadratic terms, this relation allows us to write for $(x_1, x_2) \in \Omega$

$$\mathbf{n} \cdot \langle 0, 0, w_t \rangle + \mathbf{U} \cdot \langle w_{x_1}, w_{x_2}, 0 \rangle = u \cdot \mathbf{n}.$$

This yields the desired flow boundary condition

$$(14) \quad u \cdot \mathbf{n}|_{\Omega} = w_t + \mathbf{U} \cdot \nabla w$$

in (4)₅ via the material derivative of the deflected elastic interaction surface.

We note that the flow linearization is taken with respect to a general inhomogeneous compressible Navier–Stokes system. However, unlike the papers [7, 9], where some forcing and energy level terms in the pressure and flow equations have been neglected due to their relative unimportance therein, in the present study the particular energy level term $\operatorname{div}(\mathbf{U})p$ in (4)₁ cannot be neglected, inasmuch as it plays a part in establishing that the associated FSI semigroup is uniformly bounded (and invariant) with respect to the subspace $H_N^{\perp}$. Accordingly, the term $\operatorname{div}(\mathbf{U})p$ is one of the ingredients in the “feedback” operator B defined in (17).

1.1. Notation. Throughout, for a given domain D , the norm of corresponding space $L^2(D)$ will be denoted as $\|\cdot\|_D$ (or simply $\|\cdot\|$ when the context is clear). Inner products in $L^2(\mathcal{O})$ or $\mathbf{L}^2(\mathcal{O})$ will be denoted by $(\cdot, \cdot)_{\mathcal{O}}$, whereas inner products $L^2(\partial\mathcal{O})$ will be written as $\langle \cdot, \cdot \rangle_{\partial\mathcal{O}}$. We will also denote pertinent duality pairings as $\langle \cdot, \cdot \rangle_{X \times X'}$ for a given Hilbert space X . The space $H^s(D)$ will denote the Sobolev space of order s , defined on a domain D ; $H_0^s(D)$ will denote the closure of $C_0^\infty(D)$ in the $H^s(D)$ -norm $\|\cdot\|_{H^s(D)}$. We make use of the standard notation for the boundary trace of functions defined on $\mathcal{O}$, which are sufficiently smooth: i.e., for a scalar function $\phi \in H^s(\mathcal{O})$, $\frac{1}{2} < s < \frac{3}{2}$, $\gamma(\phi) = \phi|_{\partial\mathcal{O}}$, which is a well-defined and surjective mapping on this range of s , owing to the Sobolev trace theorem on Lipschitz domains (see, e.g., [33], or Theorem 3.38 of [31]).

1.2. Functional setting. With respect to the above setting, the PDE system given in (4)–(6) can be written as an ODE in Hilbert space $\mathcal{H}$. That is, if $\Phi(t) = [p, u, w_1, w_2] \in C([0, T]; \mathcal{H})$ solves the problem (4)–(6), then there is a modeling operator $\mathcal{A} + B : D(\mathcal{A} + B) \subset \mathcal{H} \rightarrow \mathcal{H}$ such that $\Phi(\cdot)$ satisfies

$$(15) \quad \begin{aligned} \frac{d}{dt} \Phi(t) &= (\mathcal{A} + B) \Phi(t), \\ \Phi(0) &= \Phi_0 \end{aligned}$$

as follows:

$$(16) \quad \mathcal{A} = \begin{bmatrix} -\mathbf{U} \cdot \nabla(\cdot) & -\operatorname{div}(\cdot) & 0 & 0 \\ -\nabla(\cdot) & \operatorname{div} \sigma(\cdot) - \eta I - \mathbf{U} \cdot \nabla(\cdot) & 0 & 0 \\ 0 & 0 & 0 & I \\ [\cdot]|_{\Omega} & -[2\nu \partial_{x_3}(\cdot)_3 + \lambda \operatorname{div}(\cdot)]_{\Omega} & -\Delta^2 & 0 \end{bmatrix};$$

and

$$(17) \quad B = \begin{bmatrix} -\operatorname{div}(\mathbf{U})(\cdot) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{U} \cdot \nabla(\cdot) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then, $D(\mathcal{A} + B) \subset \mathcal{H}$ is given by

$$D(\mathcal{A} + B) = \{(p_0, u_0, w_1, w_2) \in L^2(\mathcal{O}) \times \mathbf{H}^1(\mathcal{O}) \times H_0^2(\Omega) \times L^2(\Omega) : \text{properties (A.i)–(A.vi) hold}\},$$

where the following hold:

- (A.i) $\mathbf{U} \cdot \nabla p_0 \in L^2(\mathcal{O})$.
- (A.ii) $\operatorname{div} \sigma(u_0) - \nabla p_0 \in \mathbf{L}^2(\mathcal{O})$. (So, $[\sigma(u_0)\mathbf{n} - p_0\mathbf{n}]_{\partial\mathcal{O}} \in \mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O})$.)
- (A.iii) $-\Delta^2 w_1 - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]_{\Omega} + p_0|_{\Omega} \in L^2(\Omega)$ (by elliptic regularity theory $w_1 \in H^3(\Omega)$).
- (A.iv) $(\sigma(u_0)\mathbf{n} - p_0\mathbf{n}) \perp TH^{1/2}(\partial\mathcal{O})$. That is,

$$\langle \sigma(u_0)\mathbf{n} - p_0\mathbf{n}, \tau \rangle_{\mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O}) \times \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O})} = 0 \quad \text{in } \mathcal{D}'(\mathcal{O}) \text{ for every } \tau \in TH^{1/2}(\partial\mathcal{O}).$$

- (A.v) $w_2 + \mathbf{U} \cdot \nabla w_1 \in H_0^2(\Omega)$ (and so $w_2 \in H_0^1(\Omega)$).

- (A.vi) The flow velocity component $u_0 = \mathbf{f}_0 + \tilde{\mathbf{f}}_0$, where $\mathbf{f}_0 \in \mathbf{V}_0$ and $\tilde{\mathbf{f}}_0 \in \mathbf{H}^1(\mathcal{O})$ satisfies¹

$$\tilde{\mathbf{f}}_0 = \begin{cases} 0 & \text{on } S, \\ (w_2 + \mathbf{U} \cdot \nabla w_1)\mathbf{n} & \text{on } \Omega \end{cases}$$

(and so $\mathbf{f}_0|_{\partial\mathcal{O}} \in TH^{1/2}(\partial\mathcal{O})$).

Moreover, we denote

$$(18) \quad H_N = \operatorname{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ \mathring{A}^{-1}(1) \\ 0 \end{bmatrix} \right\},$$

where $\mathring{A} : L^2(\Omega) \rightarrow L^2(\Omega)$ is the elliptic operator

$$\mathring{A}\varpi = \Delta^2 \varpi, \text{ with } D(\mathring{A}) = \{w \in H_0^2(\Omega) : \Delta^2 w \in L^2(\Omega)\},$$

and

$$(19) \quad H_N^\perp = \left\{ [p_0, u_0, w_1, w_2] \in \mathcal{H} : \int_{\mathcal{O}} p_0 d\mathcal{O} + \int_{\Omega} w_1 d\Omega = 0 \right\}$$

(see [26, Lemma 6]).

REMARK 3. We note that having $\mathbf{U} \in \mathbf{V}_0$ and the pressure term $p_0 \in L^2(\mathcal{O})$ implies a priori that $\mathbf{U}\nabla p_0 \in [H^1(\mathcal{O})]'$. In fact, for $\phi \in H^1(\mathcal{O})$

$$\int_{\mathcal{O}} (\mathbf{U}\nabla p_0)\phi d\mathcal{O} = \int_{\partial\mathcal{O}} (\mathbf{U} \cdot \mathbf{n})p_0\phi d\partial\mathcal{O} - \int_{\mathcal{O}} p_0 \operatorname{div}(\phi\mathbf{U}) d\mathcal{O}.$$

¹The existence of an $\mathbf{H}^1(\mathcal{O})$ -function $\tilde{\mathbf{f}}_0$ with such a boundary trace on Lipschitz domain $\mathcal{O}$ is ensured; see e.g., Theorem 3.33 of [31].

So, the domain requirement $\mathbf{U}\nabla p_0 \in L^2(\mathcal{O})$ given in (A.i) means that $\mathbf{U}\nabla p_0$ is “one unit higher” in regularity, i.e., in $L^2(\mathcal{O})$.

Moreover, for the reader’s convenience, in defining the domain of the flow-structure generator, in order to justify the existence of the trace values and the “integration by parts” to be performed in what follows, we provide the following fact.

PROPOSITION 4. Let $\mathcal{O} \subset \mathbb{R}^3$ be a Lipschitz domain. Also, assume that $(f, q) \in \mathbf{H}^1(\mathcal{O}) \times L^2(\mathcal{O})$ satisfies $[-\operatorname{div} \sigma(f) + \nabla q] \in [\mathbf{H}^1(\mathcal{O})]'$. Then

$$[\sigma(f)\mathbf{n} - q\mathbf{n}] \in \mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O}),$$

and

$$\|\sigma(f)\mathbf{n} - q\mathbf{n}\|_{\mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O})} \leq C \left[\|f\|_{\mathbf{H}^1(\mathcal{O})} + \|q\|_{L^2(\mathcal{O})} + \|-\operatorname{div} \sigma(f) + \nabla q\|_{[\mathbf{H}^1(\mathcal{O})]'} \right].$$

Proof. Since the domain $\mathcal{O}$ is Lipschitz, with respect to the Sobolev trace map $\gamma_0 \in \mathcal{L}(\mathbf{H}^1(\mathcal{O}), \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O}))$, there exists $\gamma_0^+ \in \mathcal{L}(\mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O}), \mathbf{H}^1(\mathcal{O}))$ such that

$$\gamma_0 \gamma_0^+(g_0) = g_0 \quad \text{for all } g_0 \in \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O}).$$

(See, e.g., Theorem 3.38 of [31].) Therewith, let $g_0 \in \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O})$. Then

$$\begin{aligned} \langle g_0, \sigma(f)\mathbf{n} - q\mathbf{n} \rangle_{\partial\mathcal{O}} &= \langle \gamma_0 \gamma_0^+(g_0), \sigma(f)\mathbf{n} - q\mathbf{n} \rangle_{\partial\mathcal{O}} \\ &= (\epsilon(\gamma_0^+(g_0)), \sigma(f))_{\mathcal{O}} + (\gamma_0^+(g_0), \operatorname{div} \sigma(f))_{\mathcal{O}} \\ &\quad - (\operatorname{div} \gamma_0^+(g_0), g_0)_{\mathcal{O}} - (\gamma_0^+(g_0), \nabla q)_{\mathcal{O}}. \end{aligned}$$

Estimating the right-hand side (RHS), using the continuity of right inverse $\gamma_0^+(\cdot)$, we obtain

$$|\langle g_0, \sigma(f)\mathbf{n} - q\mathbf{n} \rangle_{\partial\mathcal{O}}| \leq C \|g_0\|_{\mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O})} \left[\|f\|_{\mathbf{H}^1(\mathcal{O})} + \|q\|_{L^2(\mathcal{O})} + \|\operatorname{div} \sigma(f) - \nabla q\|_{[\mathbf{H}^1(\mathcal{O})]'} \right].$$

Dividing and taking the supremum with respect to $g_0 \in \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O})$ completes the proof. $\square$

1.3. Literature. The PDEs which describe FSIs have been considered from a variety of viewpoints and with different objectives in mind [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 26, 32]. Analysis of FSI generally constitutes a broad area of research with applications in aeroelasticity, biomechanics, biomedicine, etc. In particular, the study of wellposedness of various linearized incompressible/compressible FSI models which manifest parabolic-hyperbolic coupling has a large presence in the literature; see, e.g., [2, 7, 8, 9, 10, 11, 13, 19, 32], wherein the Navier–Stokes equations are coupled with the wave/plate equation along a fixed interface. The parabolic-hyperbolic nature of the system generally results in major mathematical difficulties, principally because the coupling mechanisms between the fluid and the solid PDE components inevitably involve boundary terms which are strictly above the level of finite energy. In the case of a *compressible* flow component in the FSI system, the analysis is further complicated: whereas for incompressible flows the density of the fluid is assumed to be a constant and pressure an unknown function determined by the fluid motion, for compressible flows the main difficulty in the analysis of the density or pressure term arises from the fact that the density variable is no longer constant. Although in most of the works in the literature, the motion of an isentropic compressible fluid—i.e., the density is a linear function of pressure—is solely considered, having

to contend with this additional density (pressure) variable presents a mathematical challenge, even at the level of wellposedness.

In contrast to the growing literature on incompressible fluids, the knowledge about compressible fluids interacting with elastic solids is relatively limited. In fact, the very first contribution to this problem is the pioneering paper [19], where both wellposedness and the existence of global attractors were shown. In [19], the author addresses the simple case that the ambient vector field $\mathbf{U} = 0$; i.e., the linearization takes place about the trivial flow steady state. For this canonical situation, he used Galerkin approximations to prove the wellposedness result. However, the author duly noted that the case $\mathbf{U} \neq 0$ cannot be handled in a similar fashion due to the existence of the troublesome, i.e., unbounded, term $\mathbf{U} \cdot \nabla p$ in the pressure equation (4)₁.

Subsequently, the linearized model in [19] with $\mathbf{U} \neq 0$ was considered in [7]. The linearization in [7], about an arbitrary nonzero state, gives rise to terms which induce a nondissipativity of the resulting FSI system. For this nondissipative FSI in [7], a *pure* velocity matching condition is imposed at the interface (i.e., no material derivative is present in this boundary condition). In contrast to the Galerkin approach applied in [19], the authors in [7] invoke a certain Lumer–Phillips methodology, with a view of associating solutions of the fluid-structure dynamics with a continuous semigroup which is not uniformly bounded. Subsequently, a more convoluted FSI model was considered in [9], where, in addition to the aforementioned nondissipative and unbounded terms brought about by ambient field $\mathbf{U} \neq 0$, the associated flow-structure interface is also under the effect of this ambient field $\mathbf{U} \neq 0$. In particular, the flow and structure velocity matching boundary condition also contains the *material derivative* of the structure, which again refers to the rate of change of the velocity on the deflected interaction surface. In [9] semigroup wellposedness is established by an appropriate invocation of the Lumer–Phillips theorem; this semigroup generation is posed with respect to the *entire* phase space $\mathcal{H}$, as defined in (7) above.

However, this wellposedness result in [9] is not totally satisfactory from the standpoint of future studies into the long-time behavior of FSI solutions: while [9] does provided existence and uniqueness of solutions to the FSI system in the entire finite energy space $\mathcal{H}$, the resulting semigroup is *not uniformly bounded*. In particular, the semigroup estimate obtained in [9] is $\mathcal{O}(e^{C(\mathbf{U})t})$ for $t > 0$, where $C(\mathbf{U}) = \frac{1}{2} \|\operatorname{div}(\mathbf{U})\|_\infty + \epsilon$. This lack of FSI semigroup boundedness in [9] will therefore forestall any subsequent discussion of FSI stability. Accordingly, keeping in mind future investigations of the asymptotic behavior of FSI solutions, we are led to the following question: Is it possible to obtain a semigroup wellposedness result, with the semigroup being bounded uniformly in time, at least in some (inherently invariant) subspace of the finite energy space?

Motivated by this question, in the present work we consider the linearized compressible FSI model (4)–(6), where $\mathbf{U} \neq 0$ and the material derivative term $\mathbf{U} \cdot \nabla w_1$ is in place in the matching velocity boundary condition. Since our main objective here is to obtain a *uniformly bounded* semigroup, our departure point is to find an appropriate subspace for the analysis. In order to have semigroup generation on this sought-after subspace, the prospective generator of the PDE system (4)–(6) should be invariant with respect to it. In this connection, it was shown in [26] that if operator $\mathcal{A}_0 : \mathcal{H} \rightarrow \mathcal{H}$ is the FSI generator in [7], which models the “material derivative” free FSI PDE interaction, then zero is an eigenvalue of $\mathcal{A}_0$. (In particular, the action of $\mathcal{A}_0$ is given by $\mathcal{A}$ of (16), with the appropriate domain of definition, which includes the pure matching velocity boundary condition; see [26] and [7]). In fact, the null space of $\mathcal{A}_0$ is one dimensional, denoted here by H_N , and is given explicitly in (18) below.

The point of our mentioning $\mathcal{A}_0$ in the present problem is that, by way of obtaining a uniformly bounded semigroup, we will take our candidate space of wellposedness to be the orthogonal complement $H_N^\perp$, which is characterized by (1) below.

The necessity of finding an appropriate invariant subspace for uniformly bounded FSI semigroup analysis motivates the presence of the additional (and unbounded) term $w_2 + \mathbf{U} \cdot \nabla w_1$ in (4)–(6). Let $\mathcal{A}_1 : \mathcal{H} \rightarrow \mathcal{H}$ be the FSI generator which gives rise to the wellposedness result in [9]; the action of $\mathcal{A}_1$ is given by $\mathcal{A}$ of (16) with the appropriate domain of definition, which includes the material derivative term matching velocity boundary condition; see page 342 of [9]. As thus constituted, $H_N^\perp$ is *not* invariant with respect to $\mathcal{A}_1$. However, if we define an operator B which abstractly models the unbounded term $w_2 + \mathbf{U} \cdot \nabla w_1$ in (4)–(6), as well as the energy level term $\operatorname{div}(\mathbf{U})p$, then with the appropriate domain of definition, $H_N^\perp$ is invariant with respect to the modeling operator $(\mathcal{A} + B)$ of (4)–(6). (This is Lemma 5 below.)

Having established said invariance, we will subsequently proceed to show that, with respect to a certain inner product which is equivalent to the standard $\mathcal{H}$ -inner product, $(\mathcal{A} + B)$ generates a contraction semigroup on $H_N^\perp$ for ambient field $\mathbf{U}$ small enough in norm (and so the semigroup will be uniformly bounded with respect to the standard $\mathcal{H}$ -norm). In consequence, the PDE system (4)–(6) is wellposed for initial data $[p_0, u_0, w_a, w_b]$ taken from $H_N^\perp$.

1.4. Challenges encountered and novelty. In the present work, we establish a result of semigroup wellposedness so as to ascertain the existence and uniqueness of solutions to (4)–(6) for Cauchy data in $H_N^\perp$. Moreover, we find that this FSI semigroup is uniformly bounded in time. This boundedness will have implications in our future analysis of long-time behavior of the solutions to the PDE system (4)–(6). The main challenging points and improvements in our treatment are as follows.

(a) *Uniformly bounded semigroup in $H_N^\perp \subset \mathcal{H}$.* By way of fulfilling our objective of obtaining a uniformly bounded semigroup, we adopt a Lumer–Phillips approach in an appropriate inner product. To wit, to establish dissipativity we topologize the $(\mathcal{A} + B)$ -invariant space $H_N^\perp$ with an inner product which is equivalent to the standard $\mathcal{H}$ -inner product. In this construction, we make use of a multiplier $\nabla\psi$ introduced in [19] (defined in (22) below) and previously used in [26]; the multiplier exploits the characterization of $H_N^\perp$ in (1). In addition, inasmuch as we are after an FSI solution semigroup which is uniformly bounded in time, we give a proof for the maximality (or the range condition) of the operator $(\mathcal{A} + B)$ which is quite different from that in [9]. Unlike [9], where the theory of linear perturbations is used to yield a semigroup whose bound is of said exponential order, in the present paper we totally eschew the Lax–Milgram approach of [9] and instead invoke functional analytical and PDE methods to show that $[\lambda I - (\mathcal{A} + B)]$ is invertible for any $\lambda > 0$. This entails showing that $[\lambda I - (\mathcal{A} + B)]$ is a closed linear operator that has a dense range in $H_N^\perp$ and enjoys the inverse estimate (100) below. By these means we establish that $(\mathcal{A} + B)$ is maximal dissipative with respect to said appropriate inner product, and so then a uniformly bounded semigroup on the standard $\mathcal{H}$ -inner product. Our uniformly bounded semigroup result is valid under the assumption that ambient vector field $\mathbf{U}$ is small enough with respect to an appropriate measurement; see (12) below. However, one should bear in mind that the presence of $\mathbf{U} \neq 0$ gives rise to terms—namely, $\mathbf{U} \cdot \nabla p$ and $\mathbf{U} \cdot \nabla w_1$ (as it appears twice)—which are *unbounded* with respect to the underlying finite energy of the FSI system. Thus, our method of proof does not at all involve some bounded perturbation result which exploits the smallness of $\mathbf{U}$.

(b) *$H_N^\perp$ -invariant generator.* Subsequent to our work [9], our original immediate

objective was to analyze the stability properties of the material derivative FSI system in [9]. However, because of the presence of the zero eigenvalue, as mentioned above, it is problematic to consider the strong or exponential decay problem in the entire phase space $\mathcal{H}$. Accordingly, we are led here to consider wellposedness (and future stability) analysis on $H_N^\perp$ as given in (1) below. (Since H_N of (18) is only one dimensional—see [26, Lemma 6]—we would not lose too much.) However, as we said above, $H_N^\perp$ is not invariant with respect to the material derivative FSI generator $\mathcal{A}_1 : \mathcal{H} \rightarrow \mathcal{H}$ in [9]. (The unbounded material derivative term in particular contributes to the noninvariance.) However, the presence of the terms $-w_2 - \mathbf{U} \cdot \nabla w_1$ and $\operatorname{div}(\mathbf{U})p$ in the respective structural displacement and pressure equations in (4)–(6) gives rise to an invariance on $H_N^\perp$. (Actually, the term $\operatorname{div}(\mathbf{U})p$ was blithely discarded during the linearization process in [9], since it is a benign energy level term.) Thus, these two terms are captured abstractly by the “feedback” operator B in (17) below. We say “feedback” since B is incorporated so as to beneficently provide the prerequisite that $H_N^\perp$ is $(\mathcal{A} + B)$ -invariant. We note that the presence of B does *not* at all give rise to a fortuitous cancellation of terms so as to have dissipativity with respect to the standard $\mathcal{H}$ -inner product. The operator B allows only for said invariance property, so that our wellposedness and uniform bounded semigroup problem can be considered on the slightly smaller subspace $H_N^\perp$. As we said, our finding that the FSI semigroup is uniformly bounded in time in $H_N^\perp$ will constitute a departure point in our future work on stability properties of the FSI PDE model.

(c) *Less regularity required on the ambient vector field $\mathbf{U}$.* The presence of the nontrivial ambient flow field $\mathbf{U}$ causes substantial difficulties in the wellposedness analysis. In this case $\mathbf{U} \neq 0$, the desired result for an FSI system—with material derivative present in the matching velocities BC—on the entire phase space $\mathcal{H}$ was obtained in the earlier work [9] (with recall, the semigroup estimate $\mathcal{O}(e^{C(\mathbf{U})t})$, for $t > 0$, where $C(\mathbf{U}) = \frac{1}{2} \|\operatorname{div}(\mathbf{U})\|_\infty + \epsilon$). In the course of applying the Lax–Milgram theorem in [9], there is the need to deal with the pressure PDE component of an associated static compressible FSI system. In this regard, a methodology, based upon a treatment of (uncoupled) transport equations in [21], was applied to solve for the pressure and fluid velocity components of said static FSI system. However this approach compelled the authors in [9] to impose that $\mathbf{U} \in \mathbf{H}^3(\mathcal{O})$. In the present work, we require that the small enough ambient field $\mathbf{U} \in \mathbf{H}^1(\mathcal{O})$ obey the less stringent regularity assumptions in (11).

1.5. Plan of the paper. The paper is organized as follows: In section 2, we first provide the framework which will be required for our proof of semigroup wellposedness. In particular, we carefully describe the FSI generator $(\mathcal{A} + B)$ and its domain, as well as the equivalent inner product which will be used for our proof of wellposedness on subspace $H_N^\perp$ of (1) below. Moreover, we show that $H_N^\perp$ is $(\mathcal{A} + B)$ -invariant. In section 3, we establish the maximal dissipativity of $(\mathcal{A} + B)$ with respect to said special inner product, thereby allowing for an appeal to the Lumer–Phillips theorem. In the course of our work, we will have need of a classic lemma of functional analysis, as well as the adjoint of $(\mathcal{A} + B)$. These ingredients are given in section 4, the appendix.

2. Preliminaries. As stated before, in order to be able to obtain a uniformly bounded (contraction) semigroup, we analyze the wellposedness of problem (4)–(6) in the reduced space $H_N^\perp$ defined in (1). This will require us to retopologize the phase space $\mathcal{H}$ with a new inner product which will be used in $H_N^\perp$ and is equivalent to the natural inner product given in (7). Now, with the above notation let us take $\varphi = [p_0, u_0, w_1, w_2] \in H_N^\perp$, $\tilde{\varphi} = [\tilde{p}_0, \tilde{u}_0, \tilde{w}_1, \tilde{w}_2] \in H_N^\perp$. Then the new inner product is

given as

$$\begin{aligned} ((\varphi, \tilde{\varphi}))_{H_N^\perp} &= (p_0, \tilde{p}_0)_{\mathcal{O}} + (u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1), \tilde{u}_0 - \alpha D(g \cdot \nabla \tilde{w}_1)e_3 + \xi \nabla \psi(\tilde{p}_0, \tilde{w}_1))_{\mathcal{O}} \\ (20) \quad &+ (\Delta w_1, \Delta \tilde{w}_1)_{\Omega} + (w_2 + h_{\alpha} \cdot \nabla w_1 + \xi w_1, \tilde{w}_2 + h_{\alpha} \cdot \nabla \tilde{w}_1 + \xi \tilde{w}_1)_{\Omega}, \end{aligned}$$

and in turn the norm

$$\begin{aligned} \|\varphi\|_{H_N^\perp}^2 &= ((\varphi, \varphi))_{H_N^\perp} \\ (21) \quad &= \|p_0\|_{\mathcal{O}}^2 + \|u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 + \|w_2 + h_{\alpha} \cdot \nabla w_1 + \xi w_1\|_{\Omega}^2 \end{aligned}$$

for every $\varphi = [p_0, u_0, w_1, w_2] \in H_N^\perp$. Here, the following hold.

(i) The function $\psi = \psi(f, \chi) \in H^1(\mathcal{O})$ is considered to solve the following BVP for data $f \in L^2(\mathcal{O})$ and $\chi \in L^2(\Omega)$:

$$(22) \quad \begin{cases} -\Delta \psi = f & \text{in } \mathcal{O}, \\ \frac{\partial \psi}{\partial n} = 0 & \text{on } S, \\ \frac{\partial \psi}{\partial n} = \chi & \text{on } \Omega, \end{cases}$$

with the compatibility condition

$$(23) \quad \int_{\mathcal{O}} f d\mathcal{O} + \int_{\Omega} \chi d\Omega = 0.$$

We should note that by known elliptic regularity results for the Neumann problem on Lipschitz domains (see, e.g., [27]) we have

$$(24) \quad \|\psi(f, \chi)\|_{H^{\frac{3}{2}}(\mathcal{O})} \leq [\|f\|_{\mathcal{O}} + \|\chi\|_{\partial \mathcal{O}}].$$

(ii) The map $D(\cdot)$ is the Dirichlet map that extends boundary data φ defined on Ω to a harmonic function in $\mathcal{O}$ satisfying

$$D\varphi = f \Leftrightarrow \begin{cases} \Delta f = 0 & \text{in } \mathcal{O}, \\ f|_{\partial \mathcal{O}} = \varphi|_{ext} & \text{on } \partial \mathcal{O}, \end{cases}$$

where

$$\varphi|_{ext} = \begin{cases} 0 & \text{on } S, \\ \phi & \text{on } \Omega. \end{cases}$$

Then by, e.g., [31, Theorem 3.3.8] and the Lax–Milgram theorem, we deduce that

$$(25) \quad D \in \mathcal{L}(H_0^{1/2+\epsilon}(\Omega); H^1(\mathcal{O})).$$

(iii) The vector field $h_{\alpha}(\cdot)$ is defined as $h_{\alpha}(\cdot) = \mathbf{U}|_{\Omega} - \alpha g$, where $g(\cdot)$ is a C^2 extension of the normal vector $\nu(x)$ (recall, with respect to Ω) and we specify the parameter α to be

$$(26) \quad \alpha = 2 \|\mathbf{U}\|_*,$$

where $\|\mathbf{U}\|_*$ is defined as in (12). Also, ξ is eventually specified in (63). Since the main goal of this paper is to have the semigroup wellposedness in the subspace $H_N^\perp$, in what follows, for the sake of simplicity, we will use the notation

$$(\mathcal{A} + B)|_{H_N^\perp} = (\mathcal{A} + B).$$

Before beginning our wellposedness analysis, we first need to justify that the semigroup generator is indeed $H_N^\perp$ -invariant. This is given in the following lemma.

LEMMA 5. *The operator $(\mathcal{A} + B)$ is $H_N^\perp$ -invariant; that is, $(\mathcal{A} + B) : D(\mathcal{A} + B) \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$.*

Proof. Let $\varphi = [p_0, u_0, w_1, w_2] \in H_N^\perp$, $\tilde{\varphi} = [\tilde{p}_0, \tilde{u}_0, \tilde{w}_1, \tilde{w}_2] \in H_N$. Recalling the adjoint operator $\mathcal{A}^*$ in (109), we have

$$\begin{aligned} (\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} &= (\varphi, \mathcal{A}^*\tilde{\varphi})_{\mathcal{H}} = (\varphi, L_1\tilde{\varphi})_{\mathcal{H}} + (\varphi, L_2\tilde{\varphi})_{\mathcal{H}} = 0 + (\varphi, L_2\tilde{\varphi})_{\mathcal{H}} \\ &= \int_{\mathcal{O}} p_0 \operatorname{div}(\mathbf{U}) 1 d\mathcal{O} + \int_{\Omega} \Delta w_1 \Delta \dot{A}^{-1} \{\operatorname{div}[U_1, U_2]\} 1 d\Omega \\ &= \int_{\mathcal{O}} p_0 \operatorname{div}(\mathbf{U}) 1 d\mathcal{O} + \int_{\Omega} w_1 \operatorname{div}[U_1, U_2] 1 d\Omega \\ &= \int_{\mathcal{O}} \operatorname{div}(\mathbf{U}) p_0 1 d\mathcal{O} - \int_{\Omega} (\nabla w_1 \cdot \mathbf{U}) 1 d\Omega \\ &= \int_{\mathcal{O}} \operatorname{div}(\mathbf{U}) p_0 1 d\mathcal{O} - \int_{\Omega} \Delta(\nabla w_1 \cdot \mathbf{U}) \Delta \dot{A}^{-1}(1) d\Omega \\ &= \left(\begin{bmatrix} \operatorname{div}(\mathbf{U}) p_0 \\ 0 \\ -\nabla w_1 \cdot \mathbf{U} \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ \dot{A}^{-1}(1) \\ 0 \end{bmatrix} \right)_{\mathcal{H}} \\ &= -(B\varphi, \tilde{\varphi})_{\mathcal{H}}, \end{aligned}$$

which yields that

$$(\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} = -(B\varphi, \tilde{\varphi})_{\mathcal{H}} \quad \text{or} \quad ((\mathcal{A} + B)\varphi, \tilde{\varphi})_{\mathcal{H}} = 0$$

for every $\varphi = [p_0, u_0, w_1, w_2] \in H_N^\perp$. Hence, $(\mathcal{A} + B)$ is $H_N^\perp$ -invariant. $\square$

3. Wellposedness. This section is devoted to showing the semigroup wellposedness of the PDE system (4)–(6). The main result of this paper is given as follows.

THEOREM 6. *With reference to problem (4)–(6), let Condition 1 hold. Moreover, assume that $\mathbf{U} \in \mathbf{V}_0 \cap W$ with $\|\mathbf{U}\|_* = \|\mathbf{U}\|_{L^\infty(\mathcal{O})} + \|\operatorname{div}(\mathbf{U})\|_{L^\infty(\mathcal{O})} + \|\mathbf{U}|_{\Omega}\|_{C^2(\overline{\Omega})}$ sufficiently small (see also (12)). Then the operator $(\mathcal{A} + B) : D(\mathcal{A} + B) \cap H_N^\perp \rightarrow H_N^\perp$, as defined via (16) and (17), generates a strongly continuous semigroup $\{e^{(\mathcal{A} + B)t}\}_{t \geq 0}$ on $H_N^\perp$. Hence, for every initial datum $[\bar{p}, \bar{u}, \bar{w}_1, \bar{w}_2] \in H_N^\perp$, the solution $[p(t), u(t), w_1(t), w_2(t)]$ of problem (4)–(6) is given continuously by*

$$(27) \quad \begin{bmatrix} p(t) \\ u(t) \\ w_1(t) \\ w_2(t) \end{bmatrix} = e^{(\mathcal{A} + B)t} \begin{bmatrix} \bar{p} \\ \bar{u} \\ \bar{w}_1 \\ \bar{w}_2 \end{bmatrix} \in C([0, T]; H_N^\perp).$$

Moreover, this semigroup is uniformly bounded in time with respect to the standard $\mathcal{H}$ -inner product. (With respect to the special norm in (21), the semigroup is in fact a contraction.)

REMARK 7. In point of fact, for ambient field $\mathbf{U}$ smooth enough, the operator $(\mathcal{A} + B)$ generates a continuous semigroup in the entire phase space $\mathcal{H}$. This conclusion can be straightforwardly obtained by invoking the machinery of [9]. However, this wellposedness on all of $\mathcal{H}$ has its downsides: (i) The ambient field requires the stronger regularity $\mathbf{H}^3(\mathcal{O})$. (ii) The argumentation in [7, 9], which partly involves linear perturbation theory, will culminate in the semigroup of $(\mathcal{A} + B)$ not having a uniform bound; in fact, the semigroup estimate on all of $\mathcal{H}$ will be of exponential order.

To prove Theorem 6, we will appeal to the Lumer–Phillips theorem, which requires the analysis of the dissipativity and maximality properties of the semigroup generator $(\mathcal{A} + B)$. We start with the dissipativity, for which our main tool will be the use of the inner product defined in (20).

3.1. Dissipativity of the generator $(\mathcal{A} + B)$. We show the dissipativity property of the generator operator $(\mathcal{A} + B)$ in the following lemma.

LEMMA 8. With reference to problem (4)–(6), the semigroup generator $(\mathcal{A} + B) : D(\mathcal{A} + B) \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$ is dissipative with respect to inner product $((\cdot, \cdot))_{H_N^\perp}$ for $\|\mathbf{U}\|_*$ (defined in (12)) small enough. In particular, for $\varphi = [p_0, u_0, w_1, w_2] \in D(\mathcal{A} + B) \cap H_N^\perp$,

$$(28) \quad \operatorname{Re}(([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} \leq -\frac{(\sigma(u_0), \epsilon(u_0))_{\mathcal{O}}}{4} - \frac{\eta \|u_0\|_{\mathcal{O}}^2}{4} - \frac{\xi \|p_0\|_{\mathcal{O}}^2}{2} - \frac{\xi \|\Delta w_1\|_{\Omega}^2}{2},$$

where ξ is as specified in (63).

Proof. Given $\varphi = [p_0, u_0, w_1, w_2] \in D(\mathcal{A} + B) \cap H_N^\perp$, we have

$$\begin{aligned} & (([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} = (-\mathbf{U}\nabla p_0 - \operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0, p_0)_{\mathcal{O}} \\ & + (-\nabla p_0 + \operatorname{div}\sigma(u_0) - \eta u_0 - \mathbf{U}\nabla u_0, u_0 - \alpha D(g \cdot \nabla w_1)e_3)_{\mathcal{O}} \\ & + (-\nabla p_0 + \operatorname{div}\sigma(u_0) - \eta u_0 - \mathbf{U}\nabla u_0, \xi \nabla \psi(p_0, w_1))_{\mathcal{O}} \\ & - \alpha(D(g \cdot \nabla[w_2 + \mathbf{U}\nabla w_1])e_3, u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1))_{\mathcal{O}} \\ & + \xi(\nabla \psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0, w_2 + \mathbf{U}\nabla w_1), u_0 - \alpha D(g \cdot \nabla w_1)e_3)_{\mathcal{O}} \\ & + \xi^2(\nabla \psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0, w_2 + \mathbf{U}\nabla w_1), \nabla \psi(p_0, w_1))_{\mathcal{O}} \\ & + (\Delta w_2, \Delta w_1)_{\Omega} + (\Delta(\mathbf{U}\nabla w_1), \Delta w_1)_{\Omega} \\ & + (p_0|_{\Omega} - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_{\Omega}, w_2 + h_{\alpha} \cdot \nabla w_1 + \xi w_1)_{\Omega} \end{aligned}$$

$$+(h_\alpha \cdot \nabla[w_2 + \mathbf{U}\nabla w_1], w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega$$

$$-(\Delta^2 w_1, w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega$$

$$+\xi(w_2 + \mathbf{U}\nabla w_1, w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega.$$

After integration by parts we then arrive at

$$\begin{aligned} (([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} &= -(\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} - \eta \|u_0\|_{\mathcal{O}}^2 + \frac{1}{2} \int_{\mathcal{O}} \operatorname{div}(\mathbf{U})[|u_0|^2 - |p_0|^2] d\mathcal{O} \\ &+ 2i\operatorname{Im}[(p_0, \operatorname{div}(u_0))_{\mathcal{O}} + (\Delta w_2, \Delta w_1)_\Omega] - i\operatorname{Im}[(\mathbf{U}\nabla p_0, p_0)_{\mathcal{O}} + (\mathbf{U}\nabla u_0, u_0)_{\mathcal{O}}] \\ &+ \sum_{j=1}^8 I_j, \end{aligned} \quad (29)$$

where the I_j are given by

$$\begin{aligned} I_1 &= (\nabla p_0 - \operatorname{div}\sigma(u_0) + \eta u_0 + \mathbf{U}\nabla u_0, \alpha D(g \cdot \nabla w_1) e_3)_{\mathcal{O}} \\ &- \alpha(p_0|_\Omega - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_\Omega, g \cdot \nabla w_1)_\Omega, \end{aligned} \quad (30)$$

$$\begin{aligned} I_2 &= (-\nabla p_0 + \operatorname{div}\sigma(u_0) - \eta u_0 - \mathbf{U}\nabla u_0, \xi \nabla \psi(p_0, w_1))_{\mathcal{O}} - \xi(\Delta^2 w_1, w_1)_\Omega \\ &+ (p_0|_\Omega - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_\Omega, \xi w_1)_\Omega, \end{aligned} \quad (31)$$

$$I_3 = -\alpha(D(g \cdot \nabla[w_2 + \mathbf{U}\nabla w_1])e_3, u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1))_{\mathcal{O}}, \quad (32)$$

$$I_4 = \xi(\nabla \psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0, w_2 + \mathbf{U}\nabla w_1), u_0 - \alpha D(g \cdot \nabla w_1)e_3)_{\mathcal{O}}, \quad (33)$$

$$I_5 = \xi^2(\nabla \psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0, w_2 + \mathbf{U}\nabla w_1), \nabla \psi(p_0, w_1))_{\mathcal{O}}, \quad (34)$$

$$I_6 = (\Delta(\mathbf{U}\nabla w_1), \Delta w_1)_\Omega - (\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega, \quad (35)$$

$$I_7 = (h_\alpha \cdot \nabla[w_2 + \mathbf{U}\nabla w_1], w_2)_\Omega, \quad (36)$$

$$\begin{aligned} I_8 &= (h_\alpha \cdot \nabla[w_2 + \mathbf{U}\nabla w_1], h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega \\ &+ \xi(w_2 + \mathbf{U}\nabla w_1, w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega, \end{aligned} \quad (37)$$

where we also recall the definition $h_\alpha = \mathbf{U}|_\Omega - \alpha g$. In the course of estimating the terms (30)–(37) above, we will invoke the polynomial

$$(38) \quad r(a) = a + a^2 + a^3,$$

and for simplicity we set

$$(39) \quad r_{\mathbf{U}} = r(\|\mathbf{U}\|_*).$$

We start with I_1 ; integrating by parts, we have

$$(40) \quad \begin{aligned} I_1 = & -\alpha(p_0, \operatorname{div}[D(g \cdot \nabla w_1)e_3])_{\mathcal{O}} + \alpha\sigma(u_0), \epsilon(D(g \cdot \nabla w_1)e_3)_{\mathcal{O}} \\ & + \alpha\eta(u_0, D(g \cdot \nabla w_1)e_3)_{\mathcal{O}} + \alpha(\mathbf{U}\nabla u_0, D(g \cdot \nabla w_1)e_3)_{\mathcal{O}}. \end{aligned}$$

Using the fact that Dirichlet map $D \in L(H_0^{\frac{1}{2}+\epsilon}(\Omega), H^1(\mathcal{O}))$, we have

$$(41) \quad I_1 \leq r_{\mathbf{U}}C \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|p_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 \right\}.$$

We continue with I_2 ; using the definition of the map $\psi(\cdot, \cdot)$ in (22) and integrating by parts, we get

$$\begin{aligned} I_2 = & -\xi \int_{\mathcal{O}} |p_0|^2 d\mathcal{O} - \xi(\sigma(u_0), \epsilon(\nabla\psi(p_0, w_1)))_{\mathcal{O}} \\ & + \xi \langle \sigma(u_0)n - p_0n, (\nabla\psi(p_0, w_1), n)n \rangle_{\partial\mathcal{O}} - \eta(u_0, \xi\nabla\psi(p_0, w_1))_{\mathcal{O}} \\ & (-\mathbf{U}\nabla u_0, \xi\nabla\psi(p_0, w_1))_{\mathcal{O}} - (\Delta^2 w_1, \xi w_1)_{\Omega} \\ & + (p_0|_{\Omega} - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_{\Omega}, \xi w_1)_{\Omega}, \end{aligned}$$

whence we obtain

$$(42) \quad \begin{aligned} I_2 \leq & -\xi \|p_0\|_{\mathcal{O}}^2 - \xi \|\Delta w_1\|_{\Omega}^2 + \xi r_{\mathbf{U}}C \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|p_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 \right\} \\ & + \xi C \left\{ \|u_0\|_{H^1(\mathcal{O})} [\|p_0\|_{\mathcal{O}} + \|\Delta w_1\|_{\Omega}] \right\}. \end{aligned}$$

For I_3 , recalling the boundary condition

$$(u_0)_3|_{\Omega} = w_2 + \mathbf{U}\nabla w_1,$$

making use of Lemma 6.1 of [9], and considering the assumptions made on the geometry in Condition 1, we have

$$(43) \quad \begin{aligned} I_3 \leq & \alpha C \|g \cdot \nabla(u_0)_3\|_{H^{-\frac{1}{2}}(\Omega)} \|u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi\nabla\psi(p_0, w_1)\|_{\mathcal{O}} \\ \leq & C \left[r_{\mathbf{U}} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|\Delta w_1\|_{\Omega}^2 \right\} + \xi^2 \left\{ \|p_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 \right\} \right], \end{aligned}$$

where we have also implicitly used the Sobolev embedding theorem. To continue with I_4 , we have

$$\begin{aligned} I_4 &= \xi(\nabla\psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(\mathbf{U})p_0, 0), u_0 - \alpha D(g \cdot \nabla w_1)e_3)_\mathcal{O} \\ &\quad + \xi(\nabla\psi(-\operatorname{div}(u_0), u_0 \cdot \mathbf{n}), u_0 - \alpha D(g \cdot \nabla w_1)e_3)_\mathcal{O} \\ (44) \qquad &= I_{4a} + I_{4b}. \end{aligned}$$

Since $\mathbf{U} \cdot \mathbf{n}|_{\partial\mathcal{O}} = \mathbf{0}$, we have that $(\mathbf{U}\nabla p_0 + \operatorname{div}(\mathbf{U})p_0) \in [H^1(\mathcal{O})]'$ with

$$(45) \qquad \|\mathbf{U}\nabla p_0 + \operatorname{div}(\mathbf{U})p_0\|_{[H^1(\mathcal{O})]'} \leq C \|\mathbf{U}\|_* \|p_0\|_\mathcal{O}.$$

By the Lax–Milgram theorem, we then have

$$\begin{aligned} I_{4a} &\leq C\xi \|\nabla\psi(-\mathbf{U}\nabla p_0 - \operatorname{div}(\mathbf{U})p_0, 0)\|_\mathcal{O} \|u_0 - \alpha D(g \cdot \nabla w_1)e_3\|_\mathcal{O} \\ (46) \qquad &\leq C\xi r_\mathbf{U} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|p_0\|_\mathcal{O}^2 + \|\Delta w_1\|_\Omega^2 \right\} \end{aligned}$$

and similarly

$$(47) \qquad I_{4b} \leq C\xi r_\mathbf{U} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|\Delta w_1\|_\Omega^2 \right\}.$$

Now, applying (46)–(47) to (44) gives

$$(48) \qquad I_4 \leq C\xi r_\mathbf{U} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|p_0\|_\mathcal{O}^2 + \|\Delta w_1\|_\Omega^2 \right\}.$$

Estimating I_5 , we proceed as we did for I_4 and invoke (45), the Lax–Milgram theorem, and the estimate (24) to obtain

$$(49) \qquad I_5 \leq C\xi^2 \left[\|\mathbf{U}\|_* \left\{ \|p_0\|_\mathcal{O}^2 + \|\Delta w_1\|_\Omega^2 \right\} + \|u_0\|_{H^1(\mathcal{O})}^2 \right].$$

For I_6 , in order to estimate the second term in (35), we follow the standard calculations used for the flux multipliers and the commutator symbol given by

$$(50) \qquad [P, Q]f = P(Qf) - Q(Pf)$$

for the differential operators P and Q . Hence,

$$(51) \qquad -(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega = (\nabla \Delta w_1, \nabla(h_\alpha \cdot \nabla w_1))_\Omega$$

$$(52) \qquad = -(\Delta w_1, \Delta(h_\alpha \cdot \nabla w_1))_\Omega + \int_{\partial\Omega} (h_\alpha \cdot \nu) |\Delta w_1|^2 d\partial\Omega,$$

where, in the first identity we have directly invoked the clamped plate boundary conditions, and in the second we have used the fact that $w_1 = \partial_\nu w_1 = 0$ on $\partial\Omega$, which yields that

$$\frac{\partial}{\partial\nu}(h_\alpha \cdot \nabla w_1) = (h_\alpha \cdot \nu) \frac{\partial^2 w_1}{\partial\nu} = (h_\alpha \cdot \nu)(\Delta w_1|_{\partial\Omega}).$$

(See [29] or [30, p. 305]). Using the commutator bracket $[\cdot, \cdot]$, we can rewrite the latter relation as

$$-(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega = -(\Delta w_1, [\Delta, h_\alpha \cdot \nabla] w_1)_\Omega - (\Delta w_1, h_\alpha \cdot \nabla (\Delta w_1))_\Omega + \int_{\partial\Omega} (h_\alpha \cdot \nu) |\Delta w_1|^2 d\partial\Omega.$$

With Green's relations, once more

$$\begin{aligned} -(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega &= -(\Delta w_1, [\Delta, h_\alpha \cdot \nabla] w_1)_\Omega - \frac{1}{2} \int_{\partial\Omega} (h_\alpha \cdot \nu) |\Delta w_1|^2 d\partial\Omega \\ &\quad + \frac{1}{2} \int_{\Omega} [\operatorname{div}(h_\alpha)] |\Delta w_1|^2 d\Omega - i \operatorname{Im}(\Delta w_1, h_\alpha \cdot \nabla (\Delta w_1))_\Omega \\ (53) \quad &\quad + \int_{\partial\Omega} (h_\alpha \cdot \nu) |\Delta w_1|^2 d\partial\Omega. \end{aligned}$$

Thus,

$$\begin{aligned} -(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega &= -(\Delta w_1, [\Delta, h_\alpha \cdot \nabla] w_1)_\Omega + \frac{1}{2} \int_{\partial\Omega} (h_\alpha \cdot \nu) |\Delta w_1|^2 d\partial\Omega \\ (54) \quad &\quad + \frac{1}{2} \int_{\Omega} [\operatorname{div}(h_\alpha)] |\Delta w_1|^2 d\Omega - i \operatorname{Im}(\Delta w_1, h_\alpha \cdot \nabla (\Delta w_1)). \end{aligned}$$

Since $h_\alpha = \mathbf{U}|_\Omega - \alpha g$, where g is an extension of $\nu(\mathbf{x})$, we will have then

$$\begin{aligned} -\operatorname{Re}(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega &= \frac{1}{2} \int_{\partial\Omega} (\mathbf{U} \cdot \nu - \alpha) |\Delta w_1|^2 d\partial\Omega + \frac{1}{2} \int_{\Omega} \operatorname{div}(h_\alpha) |\Delta w_1|^2 d\Omega \\ (55) \quad &\quad - \operatorname{Re}(\Delta w_1, [\Delta, h_\alpha \cdot \nabla] w_1)_\Omega. \end{aligned}$$

Since we can explicitly compute the commutator

$$\begin{aligned} [\Delta, h_\alpha \cdot \nabla] w_1 &= (\Delta h_1)(\partial_{x_1} w_1) + 2(\partial_{x_1} h_1)(\partial_{x_1}^2 w_1) + 2(\partial_{x_2} h_2)(\partial_{x_2}^2 w_1) + (\Delta h_2)(\partial_{x_2} w_1) \\ &\quad + 2\operatorname{div}(h_\alpha)(\partial_{x_1} \partial_{x_2} w_1) \end{aligned}$$

and

$$(56) \quad \|[\Delta, h_\alpha \cdot \nabla] w_1\|_{L^2(\Omega)} \leq r_{\mathbf{U}} \|\Delta w_1\|_{L^2(\Omega)},$$

combining (55)–(56) we eventually get

$$(57) \quad -\operatorname{Re}(\Delta^2 w_1, h_\alpha \cdot \nabla w_1)_\Omega \leq \frac{1}{2} \int_{\partial\Omega} [\mathbf{U} \cdot \nu - \alpha] |\Delta w_1|^2 d\partial\Omega + C r_{\mathbf{U}} \|\Delta w_1\|_\Omega^2.$$

Moreover, for the first term of (35), we have

$$\begin{aligned} (\Delta(\mathbf{U} \nabla w_1), \Delta w_1)_\Omega &= ((\mathbf{U} \nabla w_1), \Delta w_1)_\Omega - ([\mathbf{U} \cdot \nabla, \Delta] w_1, \Delta w_1)_\Omega \\ &= \int_{\partial\Omega} (\mathbf{U} \cdot \nu) |\Delta w_1|^2 d\partial\Omega - \int_{\partial\Omega} \operatorname{div}(\mathbf{U}) |\Delta w_1|^2 d\partial\Omega \\ &\quad - ([\mathbf{U} \cdot \nabla, \Delta] w_1, \Delta w_1)_\Omega - \int_{\Omega} \Delta w_1 \mathbf{U} \cdot \nabla (\Delta w_1) d\Omega, \end{aligned}$$

where we also use the commutator expression in (50). This gives us

$$(58) \quad \operatorname{Re}(\Delta(\mathbf{U}\nabla w_1), \Delta w_1)_\Omega \leq \frac{1}{2} \int_{\partial\Omega} (\mathbf{U} \cdot \nu) |\Delta w_1|^2 d\partial\Omega + Cr_{\mathbf{U}} \|\Delta w_1\|_\Omega^2.$$

Now applying (57)–(58) to (35), we obtain

$$(F) \quad \operatorname{Re} I_6 \leq \int_{\partial\Omega} \left[\mathbf{U} \cdot \nu - \frac{\alpha}{2} \right] |\Delta w_1|^2 d\partial\Omega + Cr_{\mathbf{U}} \|\Delta w_1\|_\Omega^2.$$

To estimate I_7 , since $w_2 \in H_0^1(\Omega)$, we have

$$\begin{aligned} \operatorname{Re}(h_\alpha \cdot \nabla w_2, w_2)_\Omega &= -\frac{1}{2} \int_\Omega \operatorname{div}(h_\alpha) |w_2|^2 d\Omega \\ &= -\frac{1}{2} \int_\Omega \operatorname{div}(h_\alpha) |(u_0)_3 - \mathbf{U}\nabla w_1|^2 d\Omega \end{aligned}$$

after using the boundary condition in $(\mathbf{A} \cdot \mathbf{v})$. Applying the last relation to the RHS of (36) and recalling that $h_\alpha = \mathbf{U}|_\Omega - \alpha g$, we get

$$\begin{aligned} \operatorname{Re} I_7 &= \operatorname{Re}(h_\alpha \cdot \nabla w_2, w_2)_\Omega + \operatorname{Re}(h_\alpha \cdot \nabla(\mathbf{U}\nabla w_1), (u_0)_3 - \mathbf{U}\nabla w_1)_\mathcal{O} \\ (59) \quad &\leq Cr_{\mathbf{U}} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|\Delta w_1\|_\Omega^2 \right\}, \end{aligned}$$

where we also implicitly use the Sobolev trace theorem. Finally, for the term I_8 , we proceed in a manner similar to that adopted for I_7 , and we have

$$\begin{aligned} I_8 &= (h_\alpha \cdot \nabla(u_0)_3, h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega \\ &\quad + \xi((u_0)_3, (u_0)_3 - \mathbf{U} \cdot \nabla w_1 + h_\alpha \cdot \nabla w_1 + \xi w_1)_\Omega \\ &\leq C[r_{\mathbf{U}} + \xi^2] \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|\Delta w_1\|_\Omega^2 \right\} \\ (60) \quad &\quad + C\xi \left[\|u_0\|_{H^1(\mathcal{O})}^2 + r_{\mathbf{U}} \left\{ \|u_0\|_{H^1(\mathcal{O})}^2 + \|\Delta w_1\|_\Omega^2 \right\} \right]. \end{aligned}$$

Now, if we apply (41)–(60) to the RHS of (29), we obtain

$$\begin{aligned} \operatorname{Re}(([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} &\leq -(\sigma(u_0), \epsilon(u_0))_\mathcal{O} - \eta \|u_0\|_\mathcal{O}^2 - \xi \|p_0\|_\mathcal{O}^2 - \xi \|\Delta w_1\|_\Omega^2 \\ &\quad + \int_{\partial\Omega} \left[\mathbf{U} \cdot \nu - \frac{\alpha}{2} \right] |\Delta w_1|^2 d\partial\Omega \\ &\quad + C[r_{\mathbf{U}} + \xi r_{\mathbf{U}} + \xi^2 + \xi] \|u_0\|_{H^1(\mathcal{O})}^2 \\ &\quad + C[r_{\mathbf{U}} + \xi r_{\mathbf{U}} + \xi^2 + \xi^2 r_{\mathbf{U}}] \left\{ \|p_0\|_\mathcal{O}^2 + \|\Delta w_1\|_\Omega^2 \right\} \end{aligned}$$

$$(61) \quad + C\xi \|u_0\|_{H^1(\mathcal{O})}^2 \{ \|p_0\|_{\mathcal{O}} + \|\Delta w_1\|_{\Omega} \}.$$

We recall now the value of $\alpha = 2\|\mathbf{U}\|_*$ (see (26)) to get

$$(62) \quad \begin{aligned} \operatorname{Re}(([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} &\leq -(\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} - \eta \|u_0\|_{\mathcal{O}}^2 - \xi \|p_0\|_{\mathcal{O}}^2 - \xi \|\Delta w_1\|_{\Omega}^2 \\ &\quad + [(C_1 + C_2 r_{\mathbf{U}})\xi^2 + C_2 r_{\mathbf{U}}\xi + C_2 r_{\mathbf{U}}] \left\{ \|p_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 \right\} \\ &\quad + \frac{1}{2} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 \right\} + C_3 [r_{\mathbf{U}} + \xi r_{\mathbf{U}} + \xi^2 + \xi] \|u_0\|_{H^1(\mathcal{O})}^2, \end{aligned}$$

where the positive constants C_1 , C_2 , and C_3 are obtained with the application of the Holder–Young and Korn inequalities and C_2 depends on the constant in Korn’s inequality. We now specify ξ be a zero of the equation

$$(C_1 + C_2 r_{\mathbf{U}})\xi^2 + \left(C_2 r_{\mathbf{U}} - \frac{1}{2}\right)\xi + C_2 r_{\mathbf{U}} = 0.$$

Namely,

$$(63) \quad \xi = \frac{\frac{1}{2} - C_2 r_{\mathbf{U}}}{2(C_1 + C_2 r_{\mathbf{U}})} - \frac{\sqrt{(\frac{1}{2} - C_2 r_{\mathbf{U}})^2 - 4C_2(C_1 + C_2 r_{\mathbf{U}})r_{\mathbf{U}}}}{2(C_1 + C_2 r_{\mathbf{U}})},$$

where the radicand is nonnegative for $\|\mathbf{U}\|_*$ sufficiently small. Then (62) becomes

$$\begin{aligned} \operatorname{Re}(([\mathcal{A} + B]\varphi, \varphi))_{H_N^\perp} &\leq -\frac{(\sigma(u_0), \epsilon(u_0))_{\mathcal{O}}}{4} - \eta \frac{\|u_0\|_{\mathcal{O}}^2}{4} - \frac{\xi}{2} \|p_0\|_{\mathcal{O}}^2 - \frac{\xi}{2} \|\Delta w_1\|_{\Omega}^2 \\ &\quad - \frac{(\sigma(u_0), \epsilon(u_0))_{\mathcal{O}}}{4} - \eta \frac{\|u_0\|_{\mathcal{O}}^2}{4} \\ &\quad + C_K [r_{\mathbf{U}} + \xi r_{\mathbf{U}} + \xi^2 + \xi] \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 \right\}. \end{aligned}$$

With ξ as prescribed in (63), we now have the dissipativity estimate (28) for $\|\mathbf{U}\|_*$ small enough. (Here we also implicitly reuse Korn’s inequality and C_K is the constant there.) This concludes the proof of Lemma 8. $\square$

3.2. Maximality of the generator $(\mathcal{A} + B)$. In order to complete the proof of Theorem 6, we also need to show that the semigroup generator $(\mathcal{A} + B) : D(\mathcal{A} + B) \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$ is maximal dissipative. This is given in the following lemma.

LEMMA 9. *With reference to problem (4)–(6), the semigroup generator $(\mathcal{A} + B) : D(\mathcal{A} + B) \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$ is maximal dissipative. In other words, the following range condition holds:*

$$(64) \quad \operatorname{Range}[\lambda I - (\mathcal{A} + B)] = H_N^\perp$$

for some $\lambda > 0$.

Proof of Lemma 9. Proof of relation (64) is based on showing that $[\lambda I - (\mathcal{A} + B)]^{-1} \in \mathcal{L}(H_N^\perp)$. For this, we appeal to linear operator theory and exploit Lemma 14 in the appendix as our main tool. So, with respect to Lemma 14, the requirements to be shown are as follows:

(M – I) $\text{Range}[\lambda I - (\mathcal{A} + B)]$ is dense in $H_N^\perp$.

(M – II) $[\lambda I - (\mathcal{A} + B)]$ is a closed operator.

(M – III) There is an $m > 0$ such that

$$\|[\lambda I - (\mathcal{A} + B)]\varphi\|_{H_N^\perp} \geq m \|\varphi\|_{H_N^\perp}$$

for all $\varphi \in D([\lambda I - (\mathcal{A} + B)]) \cap H_N^\perp = D(\mathcal{A} + B) \cap H_N^\perp$.

STEP (M-I). First, to prove that $\text{Range}[\lambda I - (\mathcal{A} + B)]$ is dense in $H_N^\perp$, we use the fact that

$$\text{Range}[\lambda I - (\mathcal{A} + B)] = \text{Null}([\lambda I - (\mathcal{A} + B)]^*)^\perp,$$

which is given in the following lemma.

LEMMA 10. Let parameter $\lambda > 0$ be given. Then for $\|\mathbf{U}\|_*$ sufficiently small,

$$\text{Null}[\lambda I - (\mathcal{A} + B)]^* = \{0\}.$$

Proof. Suppose that $\varphi = [p_0, u_0, w_1, w_2] \in D((\mathcal{A} + B)^*) \cap H_N^\perp$ satisfies

$$(65) \quad [\lambda I - (\mathcal{A} + B)]^* \varphi = 0.$$

In PDE terms, this is

$$(66) \quad \left\{ \begin{array}{l} \lambda p_0 - \mathbf{U} \nabla p_0 - \text{div}(u_0) = 0 \quad \text{in } \mathcal{O}, \\ \lambda u_0 - \nabla p_0 - \text{div} \sigma(u_0) + \eta u_0 - \mathbf{U} \nabla u_0 + \text{div}(\mathbf{U}) u_0 = 0 \quad \text{in } \mathcal{O}, \\ u_0 \cdot n = 0 \quad \text{on } S, \\ u_0 \cdot n = w_2 \quad \text{on } \Omega, \\ \lambda w_1 + w_2 - \dot{A}^{-1} \{ \text{div}[U_1, U_2] + \mathbf{U} \cdot \nabla \} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \text{div}(u_0) - \Delta^2 w_1]_\Omega \\ \quad - \mathbf{U} \cdot \nabla w_1 - \Delta \dot{A}^{-1} \nabla^* (\nabla \cdot (\mathbf{U} \cdot \nabla w_1)) = 0 \quad \text{in } \Omega, \\ \lambda w_2 + [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \text{div}(u_0)]|_\Omega - \Delta^2 w_1 = 0 \quad \text{in } \Omega, \\ w_1|_{\partial\Omega} = \frac{\partial w_1}{\partial \nu}|_{\partial\Omega} = 0. \end{array} \right.$$

Since we have from (65)

$$(67) \quad 0 = \lambda \|\varphi\|_{\mathcal{H}}^2 - ((\mathcal{A} + B)^* \varphi, \varphi)_{\mathcal{H}},$$

integrating by parts as usual, we get

$$(68) \quad \begin{aligned} & \lambda \|\varphi\|_{\mathcal{H}}^2 + (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 = -\frac{1}{2} \int_{\mathcal{O}} \text{div}(\mathbf{U}) [|p_0|^2 + 3|u_0|^2] d\mathcal{O} \\ & + (\{ \text{div}[U_1, U_2] + \mathbf{U} \cdot \nabla \} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \text{div}(u_0) - \Delta^2 w_1]_\Omega, w_1)_\Omega \\ & + (\Delta[\mathbf{U} \cdot \nabla w_1], \Delta w_1)_\Omega + (\nabla^* (\nabla \cdot (\mathbf{U} \cdot \nabla w_1)), \Delta w_1)_\Omega. \end{aligned}$$

To handle the terms on the RHS of (68), we first invoke the map given in (22) and apply the multiplier $\nabla \psi(p_0, w_1)$ to the fluid equation (66)₂. This gives

$$\lambda (u_0, \nabla \psi(p_0, w_1))_{\mathcal{O}} - (\nabla p_0, \nabla \psi(p_0, w_1))_{\mathcal{O}} - (\text{div} \sigma(u_0), \nabla \psi(p_0, w_1))_{\mathcal{O}}$$

$$(69) \quad +\eta(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} - (\mathbf{U}\nabla u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} + (\operatorname{div}(\mathbf{U})u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} = 0.$$

Let us look at the terms of (69):

$$(70) \quad \begin{aligned} -(\nabla p_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} &= -\int_{\partial\mathcal{O}} p_0(\nabla\psi(p_0, w_1) \cdot \mathbf{n})d\partial\mathcal{O} + \int_{\mathcal{O}} p_0\operatorname{div}(\nabla\psi(p_0, w_1))d\mathcal{O} \\ &= -\int_{\Omega} p_0 w_1 d\Omega - \int_{\mathcal{O}} |p_0|^2 d\mathcal{O}. \end{aligned}$$

Also,

$$(71) \quad \begin{aligned} &-(\operatorname{div}\sigma(u_0), \nabla\psi(p_0, w_1))_{\mathcal{O}} + \eta(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} \\ &= (\sigma(u_0), \epsilon(\nabla\psi(p_0, w_1)))_{\mathcal{O}} - \langle \sigma(u_0) \cdot n, \nabla\psi(p_0, w_1) \rangle_{\partial\mathcal{O}} \\ &\quad + \eta(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}}. \end{aligned}$$

Applying (70)–(71) to (69), we then have

$$(72) \quad \begin{aligned} \int_{\mathcal{O}} |p_0|^2 d\mathcal{O} &= \lambda(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} - (\mathbf{U}\nabla u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} \\ &\quad + (\operatorname{div}(\mathbf{U})u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} - ([p_0 + 2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]_{\Omega}, w_1)_{\Omega} \\ &\quad + (\sigma(u_0), \epsilon(\nabla\psi(p_0, w_1)))_{\mathcal{O}} + \eta(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}}. \end{aligned}$$

Subsequently, we apply the multiplier w_1 to the structural equation in (66)₇ and use (72) to get

$$(73) \quad \begin{aligned} \int_{\mathcal{O}} |p_0|^2 d\mathcal{O} + (\Delta^2 w_1, w_1)_{\Omega} &= \lambda(w_2, w_1)_{\Omega} + \lambda(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} \\ &\quad + (\sigma(u_0), \epsilon(\nabla\psi(p_0, w_1)))_{\mathcal{O}} + \eta(u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} \\ &\quad - (\mathbf{U}\nabla u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}} + (\operatorname{div}(\mathbf{U})u_0, \nabla\psi(p_0, w_1))_{\mathcal{O}}. \end{aligned}$$

To estimate the terms on the RHS of (73), we appeal to the elliptic regularity results for solutions of second order BVPs on corner domains [23]. At this point, using the geometrical assumptions in Condition 1 and the higher regularity estimate

$$(74) \quad \begin{aligned} \|\psi(p, w)\|_{H^2(\mathcal{O})} &\leq C \left[\|p\|_{\mathcal{O}} + \|w_{ext}\|_{H^{\frac{1}{2}+\varepsilon}(\partial\mathcal{O})} \right] \\ &\leq C [\|p\|_{\mathcal{O}} + \|w\|_{H_0^2(\Omega)}], \end{aligned}$$

where

$$w_{ext}(x) = \begin{cases} 0, & x \in S, \\ w(x), & x \in \Omega, \end{cases}$$

we obtain

$$(75) \quad \int_{\mathcal{O}} |p_0|^2 d\mathcal{O} + \int_{\Omega} |\Delta w_1|^2 d\Omega \leq C_{\epsilon} r_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}.$$

Here, we also used the Holder–Young inequalities, and $r_{\mathbf{U}}$ and $\|\mathbf{U}\|_*$ are given as in (39) and (12), respectively. Now, to proceed with the second term on the RHS of (68),

$$\begin{aligned} & (\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0) - \Delta^2 w_1]_{\Omega}, w_1)_{\Omega} \\ &= (\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega}, w_1)_{\Omega} \\ &\quad - (\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} \Delta^2 w_1, w_1)_{\Omega} \\ (76) \quad &= K_1 + K_2. \end{aligned}$$

For K_1 ,

$$\begin{aligned} K_1 &= (\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega}, w_1)_{\Omega} \\ (77) \quad &= -([p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega}, \mathbf{U} \cdot \nabla w_1)_{\Omega}. \end{aligned}$$

To handle the term on the RHS of (77), let $D_{\Omega} : H_0^{\frac{1}{2}+\epsilon}(\Omega) \rightarrow H^1(\mathcal{O})$ be defined by

$$(78) \quad D_{\Omega} g = f \Leftrightarrow \begin{cases} -\Delta f = 0 & \text{in } \mathcal{O}, \\ f|_S = 0 & \text{on } S, \\ f|_{\Omega} = g & \text{on } \Omega. \end{cases}$$

Therewith,

$$\begin{aligned} & ([p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega}, \mathbf{U} \cdot \nabla w_1)_{\Omega} = (\sigma(u_0), \epsilon(D_{\Omega}(\mathbf{U} \cdot \nabla w_1)))_{\mathcal{O}} \\ &+ (\nabla p_0, D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}} + (p_0, \operatorname{div}(D_{\Omega}(\mathbf{U} \cdot \nabla w_1)))_{\mathcal{O}} + (\operatorname{div} \sigma(u_0), D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}} \\ &= (\sigma(u_0), \epsilon(D_{\Omega}(\mathbf{U} \cdot \nabla w_1)))_{\mathcal{O}} + \eta (u_0, D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}} + (p_0, \operatorname{div}(D_{\Omega}(\mathbf{U} \cdot \nabla w_1)))_{\mathcal{O}} \\ (79) \quad &+ \lambda (u_0, D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}} - (\mathbf{U} \cdot \nabla u_0, D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}} + (\operatorname{div}(\mathbf{U}) u_0, D_{\Omega}(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}}. \end{aligned}$$

Now, applying (79) to the RHS of (77) and invoking (75), we then have

$$\begin{aligned} & |K_1| = |(\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega}, w_1)_{\Omega}| \\ (80) \quad &\leq C r_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}, \end{aligned}$$

where again $r_{\mathbf{U}}$ and $\|\mathbf{U}\|_*$ are given as in (39) and (12), respectively. Let us now continue with K_2 :

$$(81) \quad K_2 = -(\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} \Delta^2 w_1, w_1)_\Omega = (\Delta^2 w_1, \mathbf{U} \cdot \nabla w_1)_\Omega.$$

If we argue as in the estimates (53)–(54) by replacing h_α with $\mathbf{U}$, we then have

$$(82) \quad (\Delta^2 w_1, \mathbf{U} \cdot \nabla w_1)_\Omega = (\Delta w_1, [\Delta, \mathbf{U} \cdot \nabla] w_1)_\Omega - \frac{1}{2} \int_{\partial\Omega} (\mathbf{U} \cdot \nu) |\Delta w_1|^2 d\partial\Omega - \frac{1}{2} \int_{\Omega} \operatorname{div}(\mathbf{U}) |\Delta w_1|^2 d\Omega.$$

For the second term on the RHS of (82), let $\gamma(x)$ be a C^2 -extension of the normal vector $\nu(\mathbf{x})$ to the boundary of Ω . Applying the multiplier $\gamma \cdot \nabla w_1$ to the structural equation (66)₇, we get

$$(83) \quad (\Delta^2 w_1, \gamma \cdot \nabla w_1)_\Omega = ([p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]|_\Omega, \gamma \cdot \nabla w_1)_\Omega + \lambda(w_2, \gamma \cdot \nabla w_1)_\Omega.$$

Revolving the elliptic map (78), we have

$$(84) \quad \begin{aligned} & ([p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]|_\Omega, \gamma \cdot \nabla w_1)_\Omega \\ &= (\sigma(u_0), \epsilon(D_\Omega(\gamma \cdot \nabla w_1)))_{\mathcal{O}} + \eta(u_0, D_\Omega(\gamma \cdot \nabla w_1))_{\mathcal{O}} + (p_0, \operatorname{div}(D_\Omega(\gamma \cdot \nabla w_1)))_{\mathcal{O}} \\ &+ \lambda(u_0, D_\Omega(\gamma \cdot \nabla w_1))_{\mathcal{O}} - (\mathbf{U} \cdot \nabla u_0, D_\Omega(\gamma \cdot \nabla w_1))_{\mathcal{O}} + (\operatorname{div}(\mathbf{U})u_0, D_\Omega(\mathbf{U} \cdot \nabla w_1))_{\mathcal{O}}. \end{aligned}$$

Moreover, proceeding as in (82), we get

$$(85) \quad (\Delta^2 w_1, \gamma \cdot \nabla w_1)_\Omega = (\Delta w_1, [\Delta, \gamma \cdot \nabla] w_1)_\Omega - \frac{1}{2} \int_{\partial\Omega} |\Delta w_1|^2 d\partial\Omega - \frac{1}{2} \int_{\Omega} \operatorname{div}(\gamma) |\Delta w_1|^2 d\Omega.$$

Now, applying (84), (85) to (83), using (56) (replacing h_α with γ), and subsequently reinvoking (75), we obtain

$$(86) \quad \int_{\partial\Omega} |\Delta w_1|^2 d\partial\Omega \leq Cr_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}.$$

Combining now (81), (82), (86), and (75), we have

$$(87) \quad \begin{aligned} |K_2| &= |(\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} \Delta^2 w_1, w_1)_\Omega| \\ &\leq Cr_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}. \end{aligned}$$

Hence, the second term of (68) can be handled by

$$\left| (\{\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla\} [p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0) - \Delta^2 w_1]_\Omega, w_1)_\Omega \right|$$

$$\begin{aligned}
&\leq |K_1| + |K_2| \\
(88) \quad &\leq Cr_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}.
\end{aligned}$$

Also, for the third and fourth terms of (68),

$$\begin{aligned}
&(\Delta[\mathbf{U} \cdot \nabla w_1], \Delta w_1)_{\Omega} + (\nabla^*(\nabla \cdot (\mathbf{U} \cdot \nabla w_1)), \Delta w_1)_{\Omega} \\
&= (\mathbf{U} \cdot \nabla(\Delta w_1), \Delta w_1)_{\Omega} + ([\Delta, \mathbf{U} \cdot \nabla]w_1, \Delta w_1)_{\Omega} + (\nabla[\mathbf{U} \cdot \nabla w_1], \nabla(\Delta w_1))_{\Omega} \\
&= \frac{1}{2} \int_{\partial\Omega} (\mathbf{U} \cdot \nu) |\Delta w_1|^2 d\Omega - \frac{1}{2} \int_{\Omega} \operatorname{div}(\mathbf{U}) |\Delta w_1|^2 d\Omega \\
&\quad + ([\Delta, \mathbf{U} \cdot \nabla]w_1, \Delta w_1)_{\Omega} - (\mathbf{U} \cdot \nabla w_1, \Delta^2 w_1)_{\Omega}.
\end{aligned}$$

Proceeding as above, we then have

$$\begin{aligned}
&|(\Delta[\mathbf{U} \cdot \nabla w_1], \Delta w_1)_{\Omega} + (\nabla^*(\nabla \cdot (\mathbf{U} \cdot \nabla w_1)), \Delta w_1)_{\Omega}| \\
(89) \quad &\leq Cr_{\mathbf{U}} \left\{ (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 + \lambda \|\varphi\|_{\mathcal{H}}^2 \right\}.
\end{aligned}$$

Finally, if we apply the estimates (75), (88), and (89) to the RHS of (68), we arrive at

$$\begin{aligned}
&\lambda \|\varphi\|_{\mathcal{H}}^2 + (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 \\
&\leq C \|\mathbf{U}\|_* \left\{ \lambda \|\varphi\|_{\mathcal{H}}^2 + (\sigma(u_0), \epsilon(u_0))_{\mathcal{O}} + \eta \|u_0\|_{\mathcal{O}}^2 \right\}.
\end{aligned}$$

For $\|\mathbf{U}\|_*$ (see (12)) small enough, independent of $\lambda > 0$, we infer that the solution φ of (65) is zero, which concludes the proof of Lemma 10. $\square$

STEP (M-II). We continue with showing that $[\lambda I - (\mathcal{A} + B)]$ is a closed operator. For this, it will be enough to prove the following lemma.

LEMMA 11. *The operator $\mathcal{A} + B : D(\mathcal{A} + B) \cap H_N^{\perp} \rightarrow H_N^{\perp}$ is closed.*

Proof. Let $\{\varphi_n\} = \{[p_{0n}, u_{0n}, w_{1n}, w_{2n}]\} \subseteq D(\mathcal{A} + B) \cap H_N^{\perp}$ satisfy

$$\begin{aligned}
\varphi_n &\rightarrow \varphi \quad \text{in } H_N^{\perp}, \\
(\mathcal{A} + B)\varphi_n &\rightarrow \varphi^* \quad \text{in } H_N^{\perp}.
\end{aligned}$$

We must show that $\varphi \in D(\mathcal{A} + B) \cap H_N^{\perp}$ and $(\mathcal{A} + B)\varphi = \varphi^*$. To start, via the relation (28) in Lemma 8, we have

$$\frac{(\sigma(u_{0m} - u_{0n}), \epsilon(u_{0m} - u_{0n}))_{\mathcal{O}}}{4} \leq -\operatorname{Re}([\mathcal{A} + B](\varphi_m - \varphi_n, \varphi_m - \varphi_n))_{H_N^{\perp}},$$

from which we infer that

$$(90) \quad u_{0n} \rightarrow u \quad \text{in } H^1(\mathcal{O}).$$

Assume that for $\varphi_n^* = \{[p_{0n}^*, u_{0n}^*, w_{1n}^*, w_{2n}^*]\} \subseteq H_N^\perp$

$$(91) \quad (\mathcal{A} + B)\varphi_n = \varphi_n^*.$$

In PDE terms this gives

$$(92) \quad \begin{cases} -\mathbf{U}\nabla p_{0n} - \operatorname{div}(u_{0n}) - \operatorname{div}(\mathbf{U})p_{0n} = p_{0n}^* & \text{in } \mathcal{O}, \\ -\nabla p_{0n} + \operatorname{div}\sigma(u_{0n}) - \eta u_{0n} - \mathbf{U}\nabla u_{0n} = u_{0n}^* & \text{in } \mathcal{O}, \\ w_{2n} + \mathbf{U} \cdot \nabla w_{1n} = w_{1n}^* & \text{in } \Omega, \\ p_{0n} - [2\nu\partial_{x_3}(u_{0n})_3 + \lambda\operatorname{div}(u_{0n})]_{|\Omega} - \Delta^2 w_{1n} = w_{2n}^* & \text{in } \Omega. \end{cases}$$

If we read off the first equation in (92) to have

$$\mathbf{U}\nabla p_{0n} = -\operatorname{div}(u_{0n}) - \operatorname{div}(\mathbf{U})p_{0n} - p_{0n}^*$$

and take upon the limit when $n \rightarrow \infty$, we get

$$(93) \quad \mathbf{U}\nabla p_0 = [-\operatorname{div}(u_0) - \operatorname{div}(\mathbf{U})p_0 - p_0^*] \in L^2(\mathcal{O}).$$

Moreover, using the third equation in (92), we have

$$(94) \quad w_2 = \lim_{n \rightarrow \infty} w_{2n} = \lim_{n \rightarrow \infty} [w_{1n}^* - \mathbf{U} \cdot \nabla w_{1n}] = [w_1^* - \mathbf{U} \cdot \nabla w_1] \in H_0^1(\Omega).$$

In addition, from the domain criteria for $(\mathcal{A} + B)$, we have $u_{0n} = \mu_{0n} + \tilde{\mu}_{0n}$, where $\mu_{0n} \in \mathbf{V}_0$ and $\tilde{\mu}_{0n} \in H^1(\mathcal{O})$ satisfies

$$\tilde{\mu}_{0n} = \begin{cases} 0 & \text{on } S, \\ (w_{2n} + \mathbf{U} \cdot \nabla w_{1n})\mathbf{n} & \text{on } \Omega. \end{cases}$$

Since $\mathbf{V}_0$ is closed, then by (90), (94), and the Sobolev trace theorem, we have

$$(95) \quad u_0 = \mu_0 + \tilde{\mu}_0,$$

where $\mu_0 \in \mathbf{V}_0$ and $\tilde{\mu}_0 \in H^1(\mathcal{O})$ satisfies

$$\tilde{\mu}_0 = \begin{cases} 0 & \text{on } S, \\ (w_2 + \mathbf{U} \cdot \nabla w_1)\mathbf{n} & \text{on } \Omega. \end{cases}$$

Furthermore, we recall the form of the adjoint $(\mathcal{A} + B)^* : D(\mathcal{A} + B)^* \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$ in (109) and given arbitrary $\Phi \in \mathcal{D}(\mathcal{O})$ we will have then $[0, \Phi, 0, 0] \in D(\mathcal{A} + B)^* \cap H_N^\perp$. Therewith, we have

$$\begin{aligned} \left(\varphi, (\mathcal{A} + B)^* \begin{bmatrix} 0 \\ \Phi \\ 0 \\ 0 \end{bmatrix} \right)_{\mathcal{H}} &= \lim_{n \rightarrow \infty} \left(\varphi_n, (\mathcal{A} + B)^* \begin{bmatrix} 0 \\ \Phi \\ 0 \\ 0 \end{bmatrix} \right)_{\mathcal{H}} \\ &= \lim_{n \rightarrow \infty} \left((\mathcal{A} + B)\varphi_n, \begin{bmatrix} 0 \\ \Phi \\ 0 \\ 0 \end{bmatrix} \right)_{\mathcal{H}} = \left(\varphi^*, \begin{bmatrix} 0 \\ \Phi \\ 0 \\ 0 \end{bmatrix} \right)_{\mathcal{H}}, \end{aligned}$$

or

$$(p_0, \operatorname{div}(\Phi))_{\mathcal{O}} + (u_0, \operatorname{div}\sigma(\Phi) - \eta\Phi + \mathbf{U} \cdot \nabla\Phi + \operatorname{div}(\mathbf{U})\Phi)_{\mathcal{O}} = (u_0^*, \Phi)_{\mathcal{O}}.$$

Upon an integration by parts this relation now becomes

$$-(\nabla p_0, \Phi)_{\mathcal{O}} + (\operatorname{div}\sigma(u_0), \Phi)_{\mathcal{O}} - \eta(u_0, \Phi)_{\mathcal{O}} - (\mathbf{U} \cdot \nabla u_0, \Phi)_{\mathcal{O}} = (u_0^*, \Phi)_{\mathcal{O}} \quad \forall \Phi \in \mathcal{D}(\mathcal{O}).$$

Applying a density argument to the above relation gives

$$(96) \quad -\nabla p_0 + \operatorname{div}\sigma(u_0) - \eta u_0 - \mathbf{U} \cdot \nabla u_0 = u_0^* \in L^2(\mathcal{O}).$$

A further integration by parts assigns a meaning to the trace $[\sigma(u_0)\mathbf{n} - p_0\mathbf{n}]_{\partial\mathcal{O}}$ in the $H^{-\frac{1}{2}}$ -sense. What is more, if $\gamma_0^+(\cdot) \in L(H^{\frac{1}{2}}(\partial\mathcal{O}), H^1(\mathcal{O}))$ is the right inverse of the Sobolev trace map $\gamma_0(\cdot) = (\cdot)|_{\partial\mathcal{O}}$, then for every $g \in H^{\frac{1}{2}}(\partial\mathcal{O})$ we have

$$\begin{aligned} \langle [\sigma(u_0)\mathbf{n} - p_0\mathbf{n}]_{\partial\mathcal{O}}, g \rangle_{\partial\mathcal{O}} &= (\sigma(u_0), \epsilon(\gamma_0^+(g)))_{\mathcal{O}} + (\operatorname{div}\sigma(u_0), \gamma_0^+(g))_{\mathcal{O}} \\ &\quad - (p_0, \operatorname{div}\gamma_0^+(g))_{\mathcal{O}} - (\nabla p_0, \gamma_0^+(g))_{\mathcal{O}} \\ &= (\sigma(u_0), \epsilon(\gamma_0^+(g)))_{\mathcal{O}} + \eta(u_0, \gamma_0^+(g))_{\mathcal{O}} + (\mathbf{U} \cdot \nabla u_0, \gamma_0^+(g))_{\mathcal{O}} \\ &\quad + (u_0^*, \gamma_0^+(g))_{\mathcal{O}} - (p_0, \operatorname{div}\gamma_0^+(g))_{\mathcal{O}} \\ &= \lim_{n \rightarrow \infty} [(\sigma(u_{0n}), \epsilon(\gamma_0^+(g)))_{\mathcal{O}} + \eta(u_{0n}, \gamma_0^+(g))_{\mathcal{O}} + (\mathbf{U} \cdot \nabla u_{0n}, \gamma_0^+(g))_{\mathcal{O}} \\ &\quad + (u_{0n}^*, \gamma_0^+(g))_{\mathcal{O}} - (p_{0n}, \operatorname{div}\gamma_0^+(g))_{\mathcal{O}}] \\ &= \lim_{n \rightarrow \infty} \langle [\sigma(u_{0n})\mathbf{n} - p_{0n}\mathbf{n}]_{\partial\mathcal{O}}, g \rangle_{\partial\mathcal{O}}. \end{aligned}$$

That is,

$$(97) \quad [\sigma(u_{0n})\mathbf{n} - p_{0n}\mathbf{n}]_{\partial\mathcal{O}} \rightarrow [\sigma(u_0)\mathbf{n} - p_0\mathbf{n}]_{\partial\mathcal{O}} \quad \text{in } H^{\frac{1}{2}}(\partial\mathcal{O}).$$

The last relation in turn allows us to pass to limit in (92)₄, and we get

$$(98) \quad [p_0 - (2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0))]_{\Omega} - \Delta^2 w_1 = w_2^* \in L^2(\Omega).$$

Finally, from (95) and (96) and the Lax–Milgram theorem, the flow component $u_0 = \mu_0 + \tilde{\mu}_0$ can be characterized via the solution $\mu_0 \in \mathbf{V}_0$ of the following variational problem for all $\chi \in \mathbf{V}_0$:

$$\begin{aligned} (\sigma(\mu_0), \epsilon(\chi))_{\mathcal{O}} + \eta(\mu_0, \chi)_{\mathcal{O}} &= -(\sigma(\tilde{\mu}_0), \epsilon(\chi))_{\mathcal{O}} - \eta(\tilde{\mu}_0, \chi)_{\mathcal{O}} \\ &\quad + (p_0, \operatorname{div}(\chi))_{\mathcal{O}} - (\mathbf{U} \cdot \nabla u_0, \chi)_{\mathcal{O}} - (u_0^*, \chi)_{\mathcal{O}} \end{aligned}$$

An integration by parts with respect to this relation now gives, for all $\chi \in V_0$,

$$-(\operatorname{div} \sigma(u_0), \chi)_{\mathcal{O}} + \eta(u_0, \chi)_{\mathcal{O}} + \langle \sigma(u_0) \mathbf{n}, \chi \rangle_{\partial \mathcal{O}}$$

$$= -(\nabla p_0, \chi)_{\mathcal{O}} + \langle p_0 \mathbf{n}, \chi \rangle_{\partial \mathcal{O}} - (\mathbf{U} \cdot \nabla u_0, \chi)_{\mathcal{O}} - (u_0^*, \chi)_{\mathcal{O}}$$

or, after using (96),

$$\langle \sigma(u_0) \mathbf{n} - p_0 \mathbf{n}, \chi \rangle_{\partial \mathcal{O}} = 0 \quad \text{for every } \chi \in V_0,$$

which gives in the sense of distributions

$$(99) \quad [\sigma(u_0) \mathbf{n} - p_0 \mathbf{n}] \cdot \tau = 0 \quad \forall \tau \in TH^{\frac{1}{2}}(\partial \mathcal{O}).$$

Hence, the estimates (90)–(99) now give the desired conclusion and complete the proof of Lemma 11. $\square$

STEP (M-III). Finally, we prove the following fact.

LEMMA 12. *For given $\lambda > 0$, we have the existence of a constant $\varrho > 0$ such that for all $\varphi \in D(\mathcal{A} + B) \cap H_N^{\perp}$*

$$(100) \quad \|[\lambda I - (\mathcal{A} + B)]\varphi\|_{H_N^{\perp}} \geq \varrho \|\varphi\|_{H_N^{\perp}},$$

where the norm $\|\cdot\|_{H_N^{\perp}}$ is as defined in (21).

Proof. Using the estimate (28) in Lemma 8, we have, for given $\lambda > 0$,

$$\begin{aligned} & (([\lambda I - (\mathcal{A} + B)]\varphi, \varphi))_{H_N^{\perp}} \\ & \geq \lambda \|\varphi\|_{H_N^{\perp}}^2 + C_1 \|u_0\|_{H^1(\mathcal{O})}^2 + \frac{\epsilon}{2} [\|p_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2] \\ (101) \quad & \geq \lambda \|\varphi\|_{H_N^{\perp}}^2 + \left(C_1 - \frac{\epsilon}{2}\right) \|u_0\|_{H^1(\mathcal{O})}^2 + \frac{\epsilon}{2} [\|p_0\|_{\mathcal{O}}^2 + \|u_0\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2]. \end{aligned}$$

With respect to the RHS, we first add and subtract so as to have

$$\begin{aligned} \|u_0\|_{\mathcal{O}}^2 &= \|[u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1)] + \alpha D(g \cdot \nabla w_1)e_3 - \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 \\ &= \|[u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1)]\|_{\mathcal{O}}^2 \\ &\quad + 2\operatorname{Re}(u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1), \alpha D(g \cdot \nabla w_1)e_3 - \xi \nabla \psi(p_0, w_1))_{\mathcal{O}} \\ (102) \quad &\quad + \|\alpha D(g \cdot \nabla w_1)e_3 - \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2. \end{aligned}$$

By using the Holder–Young inequalities we get

$$\|u_0\|_{\mathcal{O}}^2 \geq (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2$$

$$(103) \quad + (1 - C_\delta) \|\alpha D(g \cdot \nabla w_1) e_3 - \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2.$$

Using the boundedness of the maps $D(\cdot)$ and $\psi(\cdot, \cdot)$ defined in (25) and (24), respectively, we then have

$$(104) \quad \begin{aligned} & \|u_0\|_{\mathcal{O}}^2 \geq (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1) e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 \\ & + C_2(1 - C_\delta) \left[\|\mathbf{U}\|_*^2 + \xi^2 \right] \|\Delta w_1\|_{\Omega}^2, \end{aligned}$$

where $\|\mathbf{U}\|_*$ is defined as in (12). Now, applying (104) to the RHS of (101), we get

$$(105) \quad \begin{aligned} & (([\lambda I - (\mathcal{A} + B)]\varphi, \varphi))_{H_N^\perp} \geq \lambda \|\varphi\|_{H_N^\perp}^2 + \left(C_1 - \frac{\epsilon}{2}\right) \|u_0\|_{H^1(\mathcal{O})}^2 \\ & + \frac{\epsilon}{2} \left\{ \|p_0\|_{\mathcal{O}}^2 + (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1) e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 \right. \\ & \left. + \left[1 + C_2(1 - C_\delta) \left[\|\mathbf{U}\|_*^2 + \xi^2\right]\right] \|\Delta w_1\|_{\Omega}^2 \right\}. \end{aligned}$$

If we take now $\|\mathbf{U}\|_*$ so small such that

$$\|\mathbf{U}\|_*^2 + \xi^2 < \frac{1}{2C_2(C_\delta - 1)},$$

we then have

$$(106) \quad \begin{aligned} & (([\lambda I - (\mathcal{A} + B)]\varphi, \varphi))_{H_N^\perp} \geq \lambda \|\varphi\|_{H_N^\perp}^2 + \left(C_1 - \frac{\epsilon}{2}\right) \|u_0\|_{H^1(\mathcal{O})}^2 \\ & + \frac{\epsilon}{2} \left\{ \|p_0\|_{\mathcal{O}}^2 + (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1) e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 + \frac{1}{2} \|\Delta w_1\|_{\Omega}^2 \right\} \\ & \geq \frac{\epsilon}{2} \left\{ \|p_0\|_{\mathcal{O}}^2 + (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1) e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 + \frac{1}{2} \|\Delta w_1\|_{\Omega}^2 \right\} \\ & + \lambda \|w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1\|_{\mathcal{O}}^2. \end{aligned}$$

Using the Cauchy-Schwarz inequality, now we obtain

$$(107) \quad \begin{aligned} & \|[\lambda I - (\mathcal{A} + B)]\varphi\|_{H_N^\perp} \|\varphi\|_{H_N^\perp} \\ & \geq \frac{\epsilon}{2} \left\{ \|p_0\|_{\mathcal{O}}^2 + (1 - \delta) \|u_0 - \alpha D(g \cdot \nabla w_1) e_3 + \xi \nabla \psi(p_0, w_1)\|_{\mathcal{O}}^2 + \frac{1}{2} \|\Delta w_1\|_{\Omega}^2 \right\} \\ & + \lambda \|w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1\|_{\mathcal{O}}^2, \end{aligned}$$

which gives the desired estimate (100), with therein

$$\varrho = \min \left\{ \frac{\epsilon}{4}, \lambda \right\},$$

and finishes the proof of Lemma 12. $\square$

Now, combining Lemmas 10, 11, and 12 gives that the map $[\lambda I - (\mathcal{A} + B)]$ satisfies the requirements of Lemma 14 in the appendix, which, in turn, yields that

$$[\lambda I - (\mathcal{A} + B)]^{-1} \in \mathcal{L}(H_N^\perp),$$

and the range condition (64) holds. This finishes the proof of Lemma 9.

By Lemmas 8 and 9, we have the desired contraction semigroup generation with respect to the special inner product $((\cdot, \cdot))_{H_N^\perp}$. Hence we have the asserted wellposedness statement of Theorem 6. Moreover, from the values of the parameters α and ξ in (26) and (63), respectively, as well as the definition of $((\cdot, \cdot))_{H_N^\perp}$ in (20), we infer that $e^{(\mathcal{A}+B)t}$ is uniformly bounded in time, in the standard $\mathcal{H}$ -norm. In fact, given $\phi^* = [p^*, u^*, w_1^*, w_2^*] \in H_N^\perp$, set

$$(108) \quad \phi(t) = \begin{bmatrix} p(t) \\ u(t) \\ w_1(t) \\ w_2(t) \end{bmatrix} = e^{(\mathcal{A}+B)t} \begin{bmatrix} p^* \\ u^* \\ w_1^* \\ w_2^* \end{bmatrix}.$$

Then,

$$\begin{aligned} \|\phi(t)\|_{\mathcal{H}}^2 &= \|p\|_{\mathcal{O}}^2 + \|u\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 + \|w_2\|_{\Omega}^2 \\ &\leq C \left[\|p\|_{\mathcal{O}}^2 + \|u - \alpha D(g \cdot \nabla w_1)e_3 + \xi \nabla \psi(p, w_1)\|_{\mathcal{O}}^2 + \alpha^2 \|D(g \cdot \nabla w_1)e_3\|_{\mathcal{O}}^2 \right. \\ &\quad \left. + \xi^2 \|\nabla \psi(p, w_1)\|_{\mathcal{O}}^2 + \|\Delta w_1\|_{\Omega}^2 + \|w_2 + h_\alpha \cdot \nabla w_1 + \xi w_1\|_{\Omega}^2 + \|h_\alpha \cdot \nabla w_1 + \xi w_1\|_{\Omega}^2 \right] \\ &\leq C \left[\|e^{(\mathcal{A}+B)t} \phi^*\|_{H_N^\perp}^2 + \alpha^2 \|D(g \cdot \nabla w_1)e_3\|_{\mathcal{O}}^2 + \xi^2 \|\nabla \psi(p, w_1)\|_{\mathcal{O}}^2 + \|h_\alpha \cdot \nabla w_1 + \xi w_1\|_{\Omega}^2 \right]. \end{aligned}$$

Using the fact that $e^{(\mathcal{A}+B)t}$ is a contraction semigroup on $H_N^\perp$ with respect to the norm $\|\cdot\|_{H_N^\perp}$, and then combining this fact with (21), we have

$$\|\phi(t)\|_{\mathcal{H}}^2 \leq C[\|\mathbf{U}\|_*^2 + \xi^2] \|\phi(t)\|_{\mathcal{H}}^2 + C_1 \|\phi^*\|_{\mathcal{H}}^2.$$

For $\|\mathbf{U}\|_*$ small enough, we then have

$$\|\phi(t)\|_{\mathcal{H}} \leq C^* \|\phi^*\|_{\mathcal{H}} \quad \text{for all } t > 0.$$

This concludes the proof of Theorem 6.

4. Appendix. In this section we will provide some useful lemmas that are critical in this manuscript. In reference to problem (4)–(6), we start with defining the adjoint operator $(\mathcal{A} + B)^* : D((\mathcal{A} + B)^*) \cap H_N^\perp \subset H_N^\perp \rightarrow H_N^\perp$ of the semigroup generator $\mathcal{A} + B$ in the following lemma.

LEMMA 13. *The adjoint operator of the generator $(\mathcal{A} + B)$ (given via (16)–(17)) is defined as*

$$(\mathcal{A} + B)^* = \mathcal{A}^* + B^*$$

$$\begin{aligned}
&= \begin{bmatrix} \mathbf{U} \cdot \nabla(\cdot) & \operatorname{div}(\cdot) & 0 & 0 \\ \nabla(\cdot) & \operatorname{div} \sigma(\cdot) - \eta I + \mathbf{U} \cdot \nabla(\cdot) & 0 & 0 \\ 0 & 0 & 0 & -I \\ -[\cdot]_{\Omega} & -[2\nu \partial_{x_3}(\cdot)_3 + \lambda \operatorname{div}(\cdot)]_{\Omega} & \Delta^2 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} \operatorname{div}(\mathbf{U})(\cdot) & 0 & 0 & 0 \\ 0 & \operatorname{div}(\mathbf{U})(\cdot) & 0 & 0 \\ \dot{A}^{-1} \{(\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla)(\cdot)|_{\Omega}\} & \dot{A}^{-1} \{ \operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla \} [2\nu \partial_{x_3}(\cdot)_3 + \lambda \operatorname{div}(\cdot)]_{\Omega} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} -\operatorname{div}(\mathbf{U})(\cdot) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\dot{A}^{-1} \{(\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla) \Delta^2(\cdot)\} + \mathbf{U} \cdot \nabla(\cdot) + \Delta \dot{A}^{-1} \nabla^*(\nabla \cdot (\mathbf{U} \cdot \nabla(\cdot))) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
&= L_1 + L_2 + B^*.
\end{aligned}
\tag{109}$$

Here, $\nabla^* \in \mathcal{L}(L^2(\Omega), [H^1(\Omega)]')$ is the adjoint of the gradient operator $\nabla \in \mathcal{L}(H^1(\Omega), L^2(\Omega))$ and the domain of $(\mathcal{A} + B)^*|_{H_N^\perp}$ is given as

$$\begin{aligned}
D((\mathcal{A} + B)^*) \cap H_N^\perp &= \{(p_0, u_0, w_1, w_2) \in L^2(\mathcal{O}) \times \mathbf{H}^1(\mathcal{O}) \times H_0^2(\Omega) \times L^2(\Omega) : \\
&\quad \text{properties } (\mathbf{A}^*.i) - (\mathbf{A}^*.vii) \text{ hold}\},
\end{aligned}$$

where the following hold:

1. **(A*.i)** $\mathbf{U} \cdot \nabla p_0 \in L^2(\mathcal{O})$.
2. **(A*.ii)** $\operatorname{div} \sigma(u_0) + \nabla p_0 \in \mathbf{L}^2(\mathcal{O})$ (So, $[\sigma(u_0)\mathbf{n} + p_0\mathbf{n}]_{\partial\mathcal{O}} \in \mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O})$.)
3. **(A*.iii)** $\Delta^2 w_1 - [2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega} - p_0|_{\Omega} \in L^2(\Omega)$.
4. **(A*.iv)** $(\sigma(u_0)\mathbf{n} + p_0\mathbf{n}) \perp TH^{1/2}(\partial\mathcal{O})$. That is,

$$\langle \sigma(u_0)\mathbf{n} + p_0\mathbf{n}, \tau \rangle_{\mathbf{H}^{-\frac{1}{2}}(\partial\mathcal{O}) \times \mathbf{H}^{\frac{1}{2}}(\partial\mathcal{O})} = 0 \quad \text{in } \mathcal{D}'(\mathcal{O}) \text{ for every } \tau \in TH^{1/2}(\partial\mathcal{O}).$$

5. **(A*.v)** The flow velocity component $u_0 = \mathbf{f}_0 + \tilde{\mathbf{f}}_0$, where $\mathbf{f}_0 \in \mathbf{V}_0$ and $\tilde{\mathbf{f}}_0 \in \mathbf{H}^1(\mathcal{O})$ satisfies

$$\tilde{\mathbf{f}}_0 = \begin{cases} 0 & \text{on } S, \\ w_2 \mathbf{n} & \text{on } \Omega \end{cases}$$

(and so $\mathbf{f}_0|_{\partial\mathcal{O}} \in TH^{1/2}(\partial\mathcal{O})$).

6. **(A*.vi)** $[-w_2 + \mathbf{U} \cdot \nabla w_1 + \Delta \dot{A}^{-1} \nabla^*(\nabla \cdot (\mathbf{U} \cdot \nabla w_1))] \in H_0^2(\Omega)$ (and so $w_2 \in H_0^1(\Omega)$).
7. **(A*.vii)** $\int_{\mathcal{O}} [\mathbf{U} \cdot \nabla p_0 + \operatorname{div}(u_0)] d\mathcal{O} \\ + \int_{\Omega} \dot{A}^{-1} \{(\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla)([p_0 + 2\nu \partial_{x_3}(u_0)_3 + \lambda \operatorname{div}(u_0)]_{\Omega})\} d\Omega \\ - \int_{\Omega} \dot{A}^{-1} \{(\operatorname{div}[U_1, U_2] + \mathbf{U} \cdot \nabla) \Delta^2 w_1\} d\Omega \\ + \int_{\Omega} [\mathbf{U} \cdot \nabla w_1 + \Delta \dot{A}^{-1} \nabla^*(\nabla \cdot (\mathbf{U} \cdot \nabla w_1))] d\Omega \\ = 0.$

Proof. Let $\varphi = [p_0, u_0, w_1, w_2] \in D(\mathcal{A} + B) \cap H_N^\perp$, $\tilde{\varphi} = [\tilde{p}_0, \tilde{u}_0, \tilde{w}_1, \tilde{w}_2] \in D(\mathcal{A} + B)^* \cap H_N^\perp$. Then, we have

$$\begin{aligned}
(\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} &= -(\mathbf{U}\nabla p_0, \tilde{p}_0)_{\mathcal{O}} - (\operatorname{div}(u_0), \tilde{p}_0)_{\mathcal{O}} - (\nabla p_0, \tilde{u}_0)_{\mathcal{O}} \\
&\quad + (\operatorname{div}\sigma(u_0), \tilde{u}_0)_{\mathcal{O}} - \eta(u_0, \tilde{u}_0)_{\mathcal{O}} - (\mathbf{U}\nabla u_0, \tilde{u}_0)_{\mathcal{O}} \\
&\quad + (\Delta w_2, \Delta \tilde{w}_1)_{\Omega} + (p_0|_{\Omega} - [2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_{\Omega}, \tilde{w}_2)_{\Omega} - (\Delta^2 w_1, \tilde{w}_2)_{\Omega} \\
&= (p_0, \operatorname{div}(\mathbf{U})\tilde{p}_0)_{\mathcal{O}} + (p_0, \mathbf{U}\nabla\tilde{p}_0)_{\mathcal{O}} - \langle u_0 \cdot \mathbf{n}, \tilde{p}_0 \rangle_{\partial\mathcal{O}} + (u_0, \nabla\tilde{p}_0)_{\mathcal{O}} \\
&\quad + (p_0, \operatorname{div}(\tilde{u}_0))_{\mathcal{O}} - \langle p_0, \tilde{u}_0 \cdot \mathbf{n} \rangle_{\partial\mathcal{O}} - (\sigma(u_0), \epsilon(\tilde{u}_0))_{\mathcal{O}} \\
&\quad + \langle \sigma(u_0) \cdot \mathbf{n}, \tilde{u}_0 \rangle_{\partial\mathcal{O}} - \eta(u_0, \tilde{u}_0)_{\mathcal{O}} \\
&\quad + (u_0, \operatorname{div}(\mathbf{U})\tilde{u}_0)_{\mathcal{O}} + (u_0, \mathbf{U}\nabla\tilde{u}_0)_{\mathcal{O}} + (\Delta w_2, \Delta \tilde{w}_1)_{\Omega} \\
&\quad - ([2\nu\partial_{x_3}(u_0)_3 + \lambda\operatorname{div}(u_0)]|_{\Omega} - p_0|_{\Omega}, \tilde{w}_2)_{\Omega} - (\Delta w_1, \Delta \tilde{w}_2)_{\Omega}.
\end{aligned}$$

Using the domain criterion **(A.vi)**, we then have from the above equality

$$\begin{aligned}
(\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} &= (p_0, \operatorname{div}(\mathbf{U})\tilde{p}_0)_{\mathcal{O}} + (p_0, \mathbf{U}\nabla\tilde{p}_0)_{\mathcal{O}} \\
&\quad - (w_2 + \mathbf{U}\nabla w_1, \tilde{p}_0)_{\Omega} + (u_0, \nabla\tilde{p}_0)_{\mathcal{O}} + (p_0, \operatorname{div}(\tilde{u}_0))_{\mathcal{O}} \\
&\quad - (\sigma(u_0), \epsilon(\tilde{u}_0))_{\mathcal{O}} - \eta(u_0, \tilde{u}_0)_{\mathcal{O}} + (u_0, \operatorname{div}(\mathbf{U})\tilde{u}_0)_{\mathcal{O}} + (u_0, \mathbf{U}\nabla\tilde{u}_0)_{\mathcal{O}} \\
&\quad + (w_2, \Delta^2 \tilde{w}_1)_{\Omega} - (\Delta w_1, \Delta \tilde{w}_2)_{\Omega}.
\end{aligned}$$

Subsequently, integrating by parts in the third line of the last relation, we get

$$\begin{aligned}
(\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} &= (p_0, \operatorname{div}(\mathbf{U})\tilde{p}_0)_{\mathcal{O}} + (p_0, \mathbf{U}\nabla\tilde{p}_0)_{\mathcal{O}} \\
&\quad - (w_2 + \mathbf{U}\nabla w_1, \tilde{p}_0)_{\Omega} + (u_0, \nabla\tilde{p}_0)_{\mathcal{O}} + (p_0, \operatorname{div}(\tilde{u}_0))_{\mathcal{O}} \\
&\quad + (u_0, \operatorname{div}\sigma(\tilde{u}_0))_{\mathcal{O}} - \langle u_0, \sigma(\tilde{u}_0) \cdot \mathbf{n} \rangle_{\partial\mathcal{O}} - \eta(u_0, \tilde{u}_0)_{\mathcal{O}} \\
&\quad + (u_0, \operatorname{div}(\mathbf{U})\tilde{u}_0)_{\mathcal{O}} + (u_0, \mathbf{U}\nabla\tilde{u}_0)_{\mathcal{O}} \\
&\quad + (w_2, \Delta^2 \tilde{w}_1)_{\Omega} - (\Delta w_1, \Delta \tilde{w}_2)_{\Omega}.
\end{aligned}$$

Now, integrating by parts in the second line, and using again domain criterion **(A.vi)**, we have

$$\begin{aligned}
 (\mathcal{A}\varphi, \tilde{\varphi})_{\mathcal{H}} &= (p_0, \operatorname{div}(\mathbf{U})\tilde{p}_0)_{\mathcal{O}} + (p_0, \mathbf{U}\nabla\tilde{p}_0)_{\mathcal{O}} \\
 &\quad - (w_2, [\tilde{p}_0 + 2\nu\partial_{x_3}(\tilde{u}_0)_3 + \lambda\operatorname{div}(\tilde{u}_0)]|_{\Omega})_{\Omega} \\
 &\quad + (w_1, (\operatorname{div}[U_1, U_2] + \mathbf{U}\nabla)[\tilde{p}_0 + 2\nu\partial_{x_3}(\tilde{u}_0)_3 + \lambda\operatorname{div}(\tilde{u}_0)]|_{\Omega})_{\Omega} \\
 &\quad + (u_0, \nabla\tilde{p}_0)_{\mathcal{O}} + (p_0, \operatorname{div}(\tilde{u}_0))_{\mathcal{O}} + (u_0, \operatorname{div}\sigma(\tilde{u}_0))_{\mathcal{O}} \\
 &\quad - \eta(u_0, \tilde{u}_0)_{\mathcal{O}} + (u_0, \operatorname{div}(\mathbf{U})\tilde{u}_0)_{\mathcal{O}} + (u_0, \mathbf{U}\nabla\tilde{u}_0)_{\mathcal{O}} \\
 (110) \quad &\quad + (w_2, \Delta^2\tilde{w}_1)_{\Omega} - (\Delta w_1, \Delta\tilde{w}_2)_{\Omega}.
 \end{aligned}$$

Also we have

$$(111) \quad (B\varphi, \tilde{\varphi})_{\mathcal{H}} = -(\operatorname{div}(\mathbf{U})p_0, \tilde{p}_0)_{\mathcal{O}} + (\Delta(\mathbf{U}\nabla w_1), \Delta\tilde{w}_1)_{\Omega}.$$

For the second term of the RHS of the above equality, for any $w_1, \tilde{w}_1 \in H^3(\Omega)$

$$\begin{aligned}
 (\Delta(\mathbf{U}\nabla w_1), \Delta\tilde{w}_1)_{\Omega} &= \left\langle \frac{\partial}{\partial\nu}(\mathbf{U}\nabla w_1), \Delta\tilde{w}_1 \right\rangle_{\partial\Omega} \\
 &\quad - (\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega} \\
 &= \langle (\mathbf{U} \cdot \nu)\Delta w_1, \Delta\tilde{w}_1 \rangle_{\partial\Omega} - (\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega},
 \end{aligned}$$

where we have used the fact that $w_1 = \frac{\partial w_1}{\partial\nu} = 0$ and this yields

$$\frac{\partial}{\partial\nu}(\mathbf{U}\nabla w_1) = (\mathbf{U} \cdot \nu)\frac{\partial^2 w_1}{\partial\nu} = (\mathbf{U} \cdot \nu)(\Delta w_1|_{\partial\Omega}).$$

Then

$$\begin{aligned}
 (\Delta(\mathbf{U}\nabla w_1), \Delta\tilde{w}_1)_{\Omega} &= \left\langle \Delta w_1, \frac{\partial}{\partial\nu}(\mathbf{U}\nabla\tilde{w}_1) \right\rangle_{\partial\Omega} - (\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega} \\
 &= (\Delta w_1, \Delta(\mathbf{U}\nabla\tilde{w}_1))_{\Omega} + (\nabla\Delta w_1, \nabla(\mathbf{U}\nabla\tilde{w}_1))_{\Omega} - (\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega} \\
 (112) \quad &= (\Delta w_1, \Delta(\mathbf{U}\nabla\tilde{w}_1))_{\Omega} + (\Delta w_1, \nabla^*[\nabla(\mathbf{U}\nabla\tilde{w}_1)])_{\Omega} - (\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega},
 \end{aligned}$$

where $\nabla^* \in \mathcal{L}(L^2(\Omega), [H^1(\Omega)]')$ is the adjoint of the gradient operator $\nabla \in \mathcal{L}(H^1(\Omega), [L^2(\Omega)])$. To continue with the third term on the RHS of (112),

$$-(\nabla(\mathbf{U}\nabla w_1), \nabla\Delta\tilde{w}_1)_{\Omega} = (\mathbf{U}\nabla w_1, \Delta^2\tilde{w}_1)_{\Omega}$$

$$\begin{aligned}
&= -(w_1, \{\operatorname{div}[U_1, U_2] + \mathbf{U}\nabla\} \Delta^2 \tilde{w}_1)_\Omega \\
(113) \quad &= -(\Delta w_1, \Delta \mathring{A}^{-1} \{\operatorname{div}[U_1, U_2] + \mathbf{U}\nabla\} \Delta^2 \tilde{w}_1)_\Omega.
\end{aligned}$$

If we take into account (113) in (112) and invoke the biharmonic operator with clamped homogeneous boundary conditions, we take

$$\begin{aligned}
(114) \quad &(\Delta(\mathbf{U}\nabla w_1), \Delta \tilde{w}_1)_\Omega = -(\Delta w_1, \Delta \mathring{A}^{-1} \{\operatorname{div}[U_1, U_2] + \mathbf{U}\nabla\} \Delta^2 \tilde{w}_1)_\Omega \\
&+ (\Delta w_1, \Delta(\mathbf{U}\nabla \tilde{w}_1))_\Omega + (\Delta w_1, \Delta[\Delta \mathring{A}^{-1} \nabla^* [\nabla(\mathbf{U}\nabla \tilde{w}_1)]])_\Omega.
\end{aligned}$$

Now, considering (114) in (111) and combining the result with (110) gives the adjoint operator given in (109) and completes the proof of Lemma 13. $\square$

In order to establish the wellposedness result, one of the key tools that we use in our proof is the invertibility criterion of a linear, closed operator, which we recall in the following lemma [34, p. 102, Lemma 3.8.18].

LEMMA 14. *Let L be a linear and closed operator from the Hilbert space H into H . Then $L^{-1} \in \mathcal{L}(H)$ if and only if $R(L)$ is dense in H and there is an $m > 0$ such that*

$$\|Lf\| \geq m \|f\| \quad \text{for all } f \in D(L).$$

Acknowledgment. The author would like to thank the anonymous referees for their careful reading of the paper and thoughtful feedback, which improved the quality of the paper.

REFERENCES

- [1] R. AOYAMA AND Y. KAGEI, *Spectral properties of the semigroup for the linearized compressible Navier-Stokes equation around a parallel in a cylindrical domain*, Adv. Differential Equations, 21 (2016), pp. 265–300.
- [2] G. AVALOS AND T. CLARK, *A mixed variational formulation for the wellposedness and numerical approximation of a PDE model arising in a 3-D fluid-structure interaction*, Evol. Equ. Control Theory, 3 (2014), pp. 557–578.
- [3] G. AVALOS AND M. DVORAK, *A new maximality argument for a coupled fluid-structure interaction, with implications for a divergence free finite element method*, Appl. Math. (Warsaw), 35 (2008), pp. 259–280.
- [4] G. AVALOS AND F. BUCCI, *Exponential decay properties of a mathematical model for a certain flow-structure interaction*, in New Prospects in Direct, Inverse and Control Problems for Evolution Equations, Springer, New York, 2014, pp. 49–78.
- [5] G. AVALOS AND F. BUCCI, *Rational rates of uniform decay for strong solutions to a flow-structure PDE system*, J. Differential Equations, 258 (2015), pp. 4398–4423.
- [6] G. AVALOS AND P. G. GERDELI, *Exponential stability of a nondissipative, compressible flow-structure PDE model*, J. Evol. Equ., 20 (2020), pp. 1–38.
- [7] G. AVALOS, P. G. GERDELI, AND J. T. WEBSTER, *Semigroup well-posedness of a linearized, compressible flow with an elastic boundary*, Discrete Contin. Dyn. Syst. Ser. B, 23 (2018), pp. 1267–1295.
- [8] G. AVALOS, P. G. GERDELI, AND B. MUHA, *Wellposedness, spectral analysis and asymptotic stability of a multilayered heat-wave-wave system*, J. Differential Equations, 269 (2020), pp. 7129–7156.
- [9] G. AVALOS, P. G. GERDELI, AND J. T. WEBSTER, *A linearized viscous, compressible flow-plate interaction with non-dissipative coupling*, J. Math. Anal. Appl., 477 (2019), pp. 334–356.

- [10] G. AVALOS AND R. TRIGGIANI, *The coupled PDE system arising in fluid-structure interaction. I. Explicit semigroup generator and its spectral properties*, in Fluids and Waves, Contemp. Math. 440, AMS, Providence, RI, 2007, pp. 15–54.
- [11] G. AVALOS AND R. TRIGGIANI, *Semigroup wellposedness in the energy space of a parabolic-hyperbolic coupled Stokes-Lamé PDE of fluid-structure interactions*, Discrete Contin. Dyn. Syst., 2 (2009), pp. 417–447.
- [12] G. AVALOS, R. TRIGGIANI, AND I. LASIECKA, *Heat-wave interaction in 2 or 3 dimensions: Optimal decay rates*, J. Math. Anal. Appl., 437 (2016), pp. 782–815.
- [13] L. BOCIU, D. TOUNDYKOV, AND J. P. ZOLÉSIO, *Well-posedness analysis for a linearization of a fluid-elasticity interaction*, SIAM J. Math. Anal., 47 (2015), pp. 1958–2000, <https://doi.org/10.1137/140970689>.
- [14] V. V. BOLOTIN, *Nonconservative Problems of the Theory of Elastic Stability*, Macmillan, New York, 1963.
- [15] A. BUFFA, M. COSTABEL, AND D. SHEEN, *On traces for $\mathbf{H}(\text{curl}, \Omega)$ in Lipschitz domains*, J. Math. Anal. Appl., 276 (2002), pp. 845–867.
- [16] A. J. CHORIN AND J. E. MARSDEN, *A Mathematical Introduction to Flow Mechanics Vol. 3*, Springer, New York, 1990.
- [17] I. CHUESHOV, I. LASIECKA, AND J. WEBSTER, *Flow-plate interactions: Well-posedness and long-time behavior*, Discrete Contin. Dyn. Syst. Ser. S, 7 (2014), pp. 925–965.
- [18] I. CHUESHOV AND I. RYZHKOVA, *Wellposedness and long time behavior for a class of fluid-plate interaction models*, in IFIP Conference on System Modeling and Optimization, Springer, Berlin, Heidelberg, 2011, pp. 328–337.
- [19] I. CHUESHOV, *Dynamics of a nonlinear elastic plate interacting with a linearized compressible viscous flow*, Nonlinear Anal., 95 (2014), pp. 650–665.
- [20] I. CHUESHOV, *Interaction of an elastic plate with a linearized inviscid incompressible fluid*, Comm. Pure Appl. Anal., 13 (2014), pp. 1459–1778.
- [21] H. B. DA VEIGA, *Stationary motions and incompressible limit for compressible viscous flows*, Houston J. Math., 13 (1987), pp. 527–544.
- [22] M. DAUGE, *Stationary Stokes and Navier–Stokes systems on two- or three-dimensional domains with corners, Part I: Linearized equations*, SIAM J. Math. Anal., 20 (1989), pp. 74–97, <https://doi.org/10.1137/0520006>.
- [23] M. DAUGE, *Elliptic Boundary Value Problems on Corner Domains*, Lecture Notes in Math. 1341, Springer-Verlag, New York, 1988.
- [24] M. DAUGE, *Regularity and Singularities in Polyhedral Domains. The Case of Laplace and Maxwell Equations*, Slides d’un mini-cours de 3 heures, Karlsruhe, 2008, https://perso.univ-rennes1.fr/monique.dauge/publis/Talk_Karlsruhe08.pdf.
- [25] E. DOWELL, *A Modern Course in Aeroelasticity*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2004.
- [26] P. G. GEREDELI, *A time domain approach for the exponential stability of a nondissipative linearized compressible flow-structure PDE system*, Math. Methods Appl. Sci., 44 (2021), pp. 1326–1342, <https://doi.org/10.1002/mma.6833>.
- [27] D. S. JERISON AND C. E. KENIG, *The Neumann problem on Lipschitz domains*, Bull. Amer. Math. Soc. (N.S.), 4 (1981), pp. 203–207.
- [28] Y. KAGEI, *Decay estimates on solutions of the linearized compressible Navier-Stokes equation around a parallel flow in a cylindrical domain*, Kyushu J. Math., 69 (2015), pp. 293–343.
- [29] J. E. LAGNESE, *Boundary Stabilization of Thin Plates*, Stud. Appl. Numer. Math. 10, SIAM, Philadelphia, 1989, <https://doi.org/10.1137/1.9781611970821>.
- [30] I. LASIECKA AND R. TRIGGIANI, *Control Theory for Partial Differential Equations: Volume 1, Abstract Parabolic Systems: Continuous and Approximation Theories*, Cambridge University Press, Cambridge, UK, 2000.
- [31] W. C. H. MCLEAN, *Strongly Elliptic Systems and Boundary Integral Equations*, Cambridge University Press, Cambridge, UK, 2000.
- [32] B. MUHA AND S. CANIC, *Existence of a weak solution to a nonlinear fluid-structure interaction problem modeling the flow of an incompressible, viscous fluid in a cylinder with deformable walls*, Arch. Ration. Mech. Anal., 207 (2013), pp. 919–968.
- [33] J. NEČAS, *Direct Methods in the Theory of Elliptic Equations*, translated by Gerard Tronel and Alois Kufner, Springer, New York, 2012.
- [34] A. PAZY, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Appl. Math. Sci. 44, Springer, New York, 1983.
- [35] A. VALLI, *On the existence of stationary solutions to compressible Navier-Stokes equations*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 4 (1987), pp. 99–113.