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A resolvent criterion approach to strong decay of a multilayered Lamé-heat system

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ABSTRACT

We consider a multilayer hyperbolic-parabolic PDE system which constitutes a coupling of 3D thermal – 2D elastic – 3D elastic dynamics, in which the boundary interface coupling between 3D fluid and 3D structure is realized via a 2D elastic equation. Our main result here is one of strong decay for the given multilayered – heat system. That is, the solution to this composite PDE system is stabilized asymptotically to the zero state. Our proof of strong stability takes place in the ‘frequency domain’ and ultimately appeals to the pointwise resolvent condition introduced by Tomilov [23]. This very useful result, however, requires that the semigroup associated with our multilayered FSI system be completely non-unitary (c.n.u.). Accordingly, we firstly establish that the semigroup $\{e^{-\mathcal{A}t}\}_{t \geq 0}$ is indeed c.n.u., in part by invoking relatively recent results of global uniqueness for overdetermined Lamé systems on non-smooth domains. Although the entire proof also requires higher regularity results for some trace terms, this ‘*resolvent criterion approach*’ allows us to establish a ‘classically soft’ proof of strong decay. In particular, it avoids the sort of technical PDE multipliers invoked in [Avalos G, Geredeli PG, Muha B. Wellposedness, spectral analysis and asymptotic stability of a multilayered heat-wavewave system. J Differ Equ. 2020;269:7129–7156].

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KEYWORDS

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1. Introduction

1.1. Description of the problem

The multi-layered PDE models discussed below arise in the context of fluid-structure interaction with composite structures. Such FSI mathematically account for the fact that mammalian veins and arteries are typically composed of various layers of tissues; each layer will manifest its own intrinsic material properties, and are moreover separated from the others by thin elastic laminae; see [1]. Consequently, appropriate FSI will contain an additional PDE which evolves on the boundary interface to account for thin elastic layer.

In what follows we describe the setting and explicit description of the PDE system under the study: Throughout, the fluid geometry $\Omega_f \subseteq \mathbb{R}^3$ will be a Lipschitz, bounded domain with exterior boundary Γ_f . The polyhedral structure domain $\Omega_s \subseteq \mathbb{R}^3$ will be ‘completely immersed’ in Ω_f , with its polygonal boundary faces denoted Γ_j , $1 \leq j \leq K$. If given faces Γ_i and Γ_j satisfy $\Gamma_i \cap \Gamma_j \neq \emptyset$ for $i \neq j$ then the interior dihedral angle between them is in $(0, 2\pi)$ (see Figure 1).

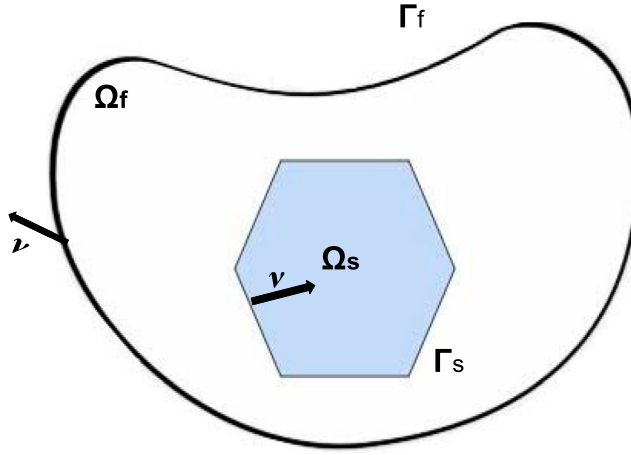


Figure 1. Geometry of the FSI domain.

The boundary interface $\Gamma_s = \partial\Omega_s$ between Ω_f and Ω_s is then the union of said polygonal faces. That is,

$$\Gamma_s = \cup_{j=1}^K \bar{\Gamma}_j.$$

In addition, $\nu(x)$ is the unit normal vector which is outward with respect to Ω_f , and so inwards with respect to Ω_s . The two dimensional vector n_j , $1 \leq j \leq K$, will denote the exterior normal vector with respect to polygonal face Γ_j . With $\{\Omega_s, \Omega_f\}$ as given, the PDE system under consideration is as follows:

$$\begin{cases} u_t - \Delta u = 0 & \text{in } (0, T) \times \Omega_f \\ u|_{\Gamma_f} = 0 & \text{on } (0, T) \times \Gamma_f; \end{cases} \quad (1)$$

$$\begin{cases} \frac{\partial^2}{\partial t^2} h_j - \operatorname{div} \sigma_{\Gamma_s}(h_j) + h_j = \nu \cdot \sigma(w)|_{\Gamma_j} - \frac{\partial u}{\partial \nu}|_{\Gamma_j} & \text{on } (0, T) \times \Gamma_j, \text{ for } 1 \leq l \leq K \\ h_j|_{\partial\Gamma_j \cap \partial\Gamma_l} = h_l|_{\partial\Gamma_j \cap \partial\Gamma_l} & \text{on } (0, T) \times (\partial\Gamma_j \cap \partial\Gamma_l), \text{ for all } 1 \leq l \leq K \text{ such that } \partial\Gamma_j \cap \partial\Gamma_l \neq \emptyset \\ n_j \cdot \sigma_{\Gamma_s}(h_j)|_{\partial\Gamma_j \cap \partial\Gamma_l} = -n_l \cdot \sigma_{\Gamma_s}(h_l)|_{\partial\Gamma_j \cap \partial\Gamma_l} & \text{on } (0, T) \times (\partial\Gamma_j \cap \partial\Gamma_l) \\ \text{for all } 1 \leq l \leq K \text{ such that } \partial\Gamma_j \cap \partial\Gamma_l \neq \emptyset. \end{cases} \quad (2)$$

$$\begin{cases} w_{tt} - \operatorname{div} \sigma(w) = 0 & \text{on } (0, T) \times \Omega_s \\ w_t|_{\Gamma_j} = \frac{\partial}{\partial t} h_j = u|_{\Gamma_j} & \text{on } (0, T) \times \Gamma_j, \text{ for } j = 1, \dots, K \end{cases} \quad (3)$$

$$[u(0), h_1(0), \frac{\partial}{\partial t} h_1(0), \dots, h_K(0), \frac{\partial}{\partial t} h_K(0), w(0), w_t(0)] = [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1]. \quad (4)$$

Here, the stress tensors σ and σ_{Γ_s} constitute Lamé systems of elasticity on their respective ‘thick’ and ‘thin’ layers. Namely,

(i) For function v in Ω_s ,

$$\sigma(v) = 2\mu\epsilon(v) + \lambda[I_3 \cdot \epsilon(v)]I_3,$$

where strain tensor $\epsilon(\cdot)$ is given by

$$\epsilon_{ij}(v) = \frac{1}{2} \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right), \quad 1 \leq i, j \leq 3;$$

(ii) Likewise, for function g on polygon Γ_j ,

$$\sigma_{\Gamma_s}(g) = 2\mu_{\Gamma_s}\epsilon_{\Gamma_s}(g) + \lambda_{\Gamma_s}[I_2 \cdot \epsilon_{\Gamma_s}(g)]I_2$$

with

$$(\epsilon_{\Gamma_s})_{ik}(g) = \frac{1}{2} \left(\frac{\partial g_k}{\partial x_i} + \frac{\partial g_i}{\partial x_k} \right), \quad 1 \leq i, k \leq 2.$$

where $\{\mu, \lambda\}$, $\{\mu_{\Gamma_s}, \lambda_{\Gamma_s}\}$ are respective Lamé parameters. We will consider said multi-layered-heat PDE system with initial data (4) drawn from the natural finite energy space $\mathbf{H}$, defined as:

$$\begin{aligned} \mathbf{H} = \Big\{ & [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1] \in L^2(\Omega_f) \times H^1(\Gamma_1) \times L^2(\Gamma_1) \times \dots \\ & \times H^1(\Gamma_K) \times L^2(\Gamma_K) \times H^1(\Omega_s) \times L^2(\Omega_s) \quad , \text{ such that for each } 1 \leq j \leq K : \\ & \text{(i) } w_0|_{\Gamma_j} = h_{0j}; \\ & \text{(ii) } h_{0j}|_{\partial\Gamma_j \cap \partial\Gamma_l} = h_{0l}|_{\partial\Gamma_j \cap \partial\Gamma_l} \text{ on } \partial\Gamma_j \cap \partial\Gamma_l, \text{ for all } 1 \leq l \leq K \text{ such that } \partial\Gamma_j \cap \partial\Gamma_l \neq \emptyset \Big\}. \end{aligned} \quad (5)$$

Here, $\mathbf{H}$ is a Hilbert space with the inner product

$$\begin{aligned} (\Phi_0, \tilde{\Phi}_0)_{\mathbf{H}} = & (u_0, \tilde{u}_0)_{\Omega_f} + \sum_{j=1}^K (\sigma_{\Gamma_s}(h_{0j}), \epsilon_{\Gamma_s}(\tilde{h}_{0j}))_{\Gamma_j} + \sum_{j=1}^K (h_{0j}, \tilde{h}_{0j})_{\Gamma_j} \\ & + \sum_{j=1}^K (h_{1j}, \tilde{h}_{1j})_{\Gamma_j} + (\sigma(w_0), \epsilon(\tilde{w}_0))_{\Omega_s} + (w_1, \tilde{w}_1)_{\Omega_s}, \end{aligned} \quad (6)$$

where

$$\Phi_0 = [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1] \in \mathbf{H}; \quad \tilde{\Phi}_0 = [\tilde{u}_0, \tilde{h}_{01}, \tilde{h}_{11}, \dots, \tilde{h}_{0K}, \tilde{h}_{1K}, \tilde{w}_0, \tilde{w}_1] \in \mathbf{H}. \quad (7)$$

1.2. Main objective and literature

The PDE model (1)–(4) is one amongst a class of coupled PDE systems which have been derived, so as to mathematically describe the interaction between viscous blood flow and the multi-layered vessels through which such flow is transported within a given mammalian species; see e.g. [1, 2]. (See also the following references which generally deal with the mathematical and/or modeling analysis of coupled (single-layered) structure-fluid PDE systems [3–13].)

In this work, we consider the strong stability problem; namely that of ascertaining that the thick elastic-thin elastic-thermal solution components tend asymptotically to the zero state, for given finite energy data in $\mathbf{H}$; see Theorem 3.1. In particular, we investigate whether the dissipa-

tion which emanates only from the thermal component of the coupled system (1)–(4), suffices to strongly stabilize the elastic dynamics, notwithstanding the fact that the three distinct PDE components each evolve on their own respective geometries.

We emphasize here that the domain of the associated thick Lamé–thin Lamé–heat semigroup generator is not compactly embedded into the finite energy space $\mathbf{H}$ —see (8) and (A.i)–(A.iv) below—consequently, a conclusion of strong stability here will not follow from classic PDE control arguments, for which it is essentially sufficient (given an underlying compactness of the resolvent of the associated semigroup generator) to establish weak stability; see e.g. [14]. With reference to such means, the fundamental example in the literature is the strong stability problem for the boundary damped wave equation on a bounded domain (see [15–18]). Again, what allows for said approach is the fact that the semigroup generator of the boundary damped wave equation has domain which is compactly embedded into $H^1 \times L^2$. In the present situation, this avenue of approach is not available.

An analogous result of asymptotic decay was obtained in [19] for a canonical ‘thick’ wave - ‘thin’ wave - heat PDE model. Likewise, as in the present situation, the associated multi-layered structure - heat semigroup generator in [19] does not have compact resolvent. However, in [19], with a view of ultimately invoking the wellknown spectral criterion in [20] for strong stability, the authors were compelled to invoke a PDE multiplier method (in the frequency domain) so as to derive a wave identity for the thick wave PDE component. (Such wave identities for uncoupled dynamics are of course instrumental in establishing uniform stabilization; see [16, 18, 21].) So in some sense, the partial multiplier approach to strong decay in [19] resembles that of [18] for said boundary damped wave equation.

By contrast, we intend in the present work to pursue an approach which eschews the need for deriving analogous energy identities for the thick Lamé solution component of the multilayered-thermal system. Certainly, such Lamé energy identities exist (although of course they are a bit cumbersome) and have been used in the context of PDE boundary stabilization problems—see; e.g. [22] and [4]—however, since the present issue is one of strong, and not uniform decay, it would seem preferable to find a ‘softer’ functional approach—somewhat in the spirit of the aforesaid works on boundary damped wave equation strong decay—at least to the extent possible.

To this end, our strong stability proof here is predicated upon ultimately invoking the resolvent criterion in [23]; see Theorem 3.2. Essentially, in order to infer strong decay of finite energy solutions of (1)–(4), we will show below that the associated thick Lamé - thin Lamé - heat semigroup generator has (non-compact) resolvent which ‘almost everywhere’ obeys a certain strong limit with respect to parameter values in the right half complex plane. In order to avail ourselves of Theorem 3.2, we must as a preliminary step establish that the multilayered structure heat semigroup (besides being a contraction) is also completely non-unitary (c.n.u). In this step, we will need to appeal to the relatively recent global uniqueness (Holmgren’s-type) result for Lamé systems of elasticity; see [24]. Moreover, we will need to recall higher regularity results for uncoupled three dimensional Lamé systems of elasticity on polyhedra; see [25].

We intend, as future work, to investigate uniform decay properties of (1)–(4) – probably taking as our point of departure the canonical multilayered structure – heat system in [19]. Accordingly, we should mention those results of exponential and polynomial decay in the literature for single-layered structure – parabolic PDE models, [26–33].

1.3. Notation

For the remainder of the text norms $\|\cdot\|_D$ are taken to be $L^2(D)$ for the domain D . Inner products in $L^2(D)$ is written $(\cdot, \cdot)$, while inner products $L_2(\partial D)$ are written $\langle \cdot, \cdot \rangle$. The space $H^s(D)$ will denote the Sobolev space of order s , defined on a domain D , and $H_0^s(D)$ denotes the closure of $C_0^\infty(D)$ in the $H^s(D)$ norm which we denote by $\|\cdot\|_{H^s(D)}$ or $\|\cdot\|_{s,D}$. We make use of the standard notation for the trace of functions defined on a Lipschitz domain D , i.e. for a scalar function $\phi \in H^1(D)$, we denote

$\gamma(w)$ to be the trace mapping from $H^1(D)$ to $H^{1/2}(\partial D)$. We will also denote pertinent duality pairings as $(\cdot, \cdot)_{X \times X'}$.

2. Preliminaries

With respect to the above setting, the PDE system given in (1)–(4) may be associated with an abstract ODE in Hilbert space $\mathbf{H}$. To wit, the operator $\mathbf{A} : D(\mathbf{A}) \subset \mathbf{H} \rightarrow \mathbf{H}$ be defined by

$$\mathbf{A} = \begin{bmatrix} \Delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & \cdots & 0 & 0 & 0 & 0 \\ -\frac{\partial}{\partial \nu}|_{\Gamma_1} & (\operatorname{div} \sigma_{\Gamma_s} - I) & 0 & \cdots & 0 & 0 & \nu \cdot \sigma(\cdot)|_{\Gamma_1} & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & I & 0 & 0 \\ -\frac{\partial}{\partial \nu}|_{\Gamma_K} & 0 & 0 & \cdots & (\operatorname{div} \sigma_{\Gamma_s} - I) & 0 & \nu \cdot \sigma(\cdot)|_{\Gamma_K} & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & I \\ 0 & 0 & 0 & \cdots & 0 & 0 & \operatorname{div} \sigma(\cdot) & 0 \end{bmatrix}; \quad (8)$$

$$D(\mathbf{A}) = \left\{ [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1] \in \mathbf{H} : \right.$$

$$(A.i) \quad u_0 \in H^1(\Omega_f), h_{1j} \in H^1(\Gamma_j) \quad \text{for } 1 \leq j \leq K, w_1 \in H^1(\Omega_s);$$

$$(A.ii) \quad (a) \quad \Delta u_0 \in L^2(\Omega_f)$$

$$(b) \quad \operatorname{div} \sigma(w_0) \in L^2(\Omega_s),$$

$$(c) \quad \operatorname{div} \sigma_{\Gamma_s}(h_{0j}) + \nu \cdot \sigma(w_0)|_{\Gamma_j} - \frac{\partial u_0}{\partial \nu}|_{\Gamma_j} \in L^2(\Gamma_j) \quad \text{for } 1 \leq j \leq K;$$

$$(A.iii) \quad u_0|_{\Gamma_f} = 0, \quad u_0|_{\Gamma_j} = h_{1j} = w_1|_{\Gamma_j}, \quad \text{for } 1 \leq j \leq K;$$

$$(A.iv) \quad \text{For } 1 \leq j \leq K :$$

$$(a) \quad h_{1j}|_{\partial \Gamma_j \cap \partial \Gamma_l} = h_{1l}|_{\partial \Gamma_j \cap \partial \Gamma_l} \text{ on } \partial \Gamma_j \cap \partial \Gamma_l, \quad \text{for all } 1 \leq l \leq K \text{ such that } \partial \Gamma_j \cap \partial \Gamma_l \neq \emptyset;$$

$$(b) \quad n_j \cdot \sigma_{\Gamma_s}(h_{0j}) \Big|_{\partial \Gamma_j \cap \partial \Gamma_l} = -n_l \cdot \sigma_{\Gamma_s}(h_{0l}) \Big|_{\partial \Gamma_j \cap \partial \Gamma_l}, \quad \text{for all } 1 \leq l \leq K \text{ such that}$$

$$\partial \Gamma_j \cap \partial \Gamma_l \neq \emptyset \Big\}. \quad (9)$$

With this matrix, if $\Phi(t) = [u(t), h_1(t), \frac{\partial}{\partial t} h_1(t), \dots, h_K(t), \frac{\partial}{\partial t} h_K(t), w(t), w_t(t)]$, and $\Phi_0 = [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1]$ then the solution of (1)–(4) may be written simply as

$$\frac{d}{dt} \Phi(t) = \mathbf{A} \Phi(t); \Phi(0) = \Phi_0. \quad (10)$$

Proceeding along similar lines of approach as in [19], one can obtain the following result of well posedness:

Theorem 2.1: *The operator $\mathbf{A} : D(\mathbf{A}) \subset \mathbf{H} \rightarrow \mathbf{H}$, defined in (8) and (9), generates a C_0 -semigroup of contractions on $\mathbf{H}$. Consequently, the solution $\Phi(t) = [u(t), h_1(t), \frac{\partial}{\partial t} h_1(t), \dots, h_K(t), \frac{\partial}{\partial t} h_K(t), w(t), w_t(t)]$ of (1)–(4), or equivalently (10), is given by*

$$\Phi(t) = e^{\mathbf{A}t} \Phi_0 \in C([0, T]; \mathbf{H}),$$

where $\Phi_0 = [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1] \in \mathbf{H}$.

In fact, the main result of this manuscript is to show that the solution to the system (1)–(4) has a decay to the zero state. To prove this, we firstly need to give the following dissipation estimate from which the decay arises:

Proposition 2.2: *The solution of (1)–(4), or equivalently (10), satisfies the following relation for $0 \leq t_0 \leq t \leq \infty$:*

$$2 \int_{t_0}^t \|\nabla u(\tau)\|_{\Omega_f}^2 d\tau + \|\Phi(t)\|_{\mathbf{H}}^2 = \|\Phi(t_0)\|_{\mathbf{H}}^2 \quad (11)$$

Proof: With respect to the right-hand side of (10) we have, upon taking the $\mathbf{H}$ -inner product with respect to Φ , and then an integration over (t_0, t) ,

$$\begin{aligned} & \int_{t_0}^t (\mathbf{A}\Phi(\tau), \Phi(\tau))_{\mathbf{H}} d\tau \\ &= \int_{t_0}^t (\Delta u, u)_{\Omega_f} d\tau + \sum_{j=1}^K \int_{t_0}^t \left[\left(\sigma_{\Gamma_s} \left(\frac{\partial}{\partial t} h_j \right), \epsilon_{\Gamma_s}(h_j) \right)_{\Gamma_j} + \left(\frac{\partial}{\partial t} h_j, h_j \right)_{\Gamma_j} \right] d\tau \\ & \quad + \sum_{j=1}^K \int_{t_0}^t \left[\left(\operatorname{div} \sigma_{\Gamma_s}(h_j), \frac{\partial}{\partial t} h_j \right)_{\Gamma_j} - (h_j, \frac{\partial}{\partial t} h_j)_{\Gamma_j} \right] d\tau \\ & \quad + \sum_{j=1}^K \int_{t_0}^t \left\langle v \cdot \sigma(w) - \frac{\partial u}{\partial v}, \frac{\partial}{\partial t} h_j \right\rangle_{\Gamma_j} d\tau + \int_{t_0}^t (\sigma(w_t), \epsilon(w))_{\Omega_s} d\tau \\ & \quad + \int_{t_0}^t (\operatorname{div} \sigma(w), \epsilon(w_t))_{\Omega_s} d\tau \\ &= - \int_{t_0}^t (\nabla u, \nabla u)_{\Omega_f} d\tau + \sum_{j=1}^K \int_{t_0}^t \left[\left(\sigma_{\Gamma_s} \left(\frac{\partial}{\partial t} h_j \right), \epsilon_{\Gamma_s}(h_j) \right)_{\Gamma_j} + \left(\frac{\partial}{\partial t} h_j, h_j \right)_{\Gamma_j} \right] d\tau \\ & \quad - \sum_{j=1}^K \int_{t_0}^t \left[\overline{\left(\sigma_{\Gamma_s} \left(\frac{\partial}{\partial t} h_j \right), \epsilon_{\Gamma_s}(h_j) \right)_{\Gamma_j}} + \overline{\left(\frac{\partial}{\partial t} h_j, h_j \right)_{\Gamma_j}} \right] d\tau \\ & \quad + \int_{t_0}^t (\sigma(w_t), \epsilon(w))_{\Omega_s} d\tau - \int_{t_0}^t \overline{(\sigma(w_t), \epsilon(w))_{\Omega_s}} d\tau \\ & \quad + \int_{t_0}^t \left\langle \frac{\partial u}{\partial v}, u \right\rangle_{\partial \Omega_f} d\tau + \sum_{j=1}^K \int_{t_0}^t \left\langle v \cdot \sigma(w) - \frac{\partial u}{\partial v}, \frac{\partial}{\partial t} h_j \right\rangle_{\Gamma_j} d\tau \\ & \quad - \int_{t_0}^t \langle v \cdot \sigma(w), w_t \rangle_{\Gamma_s} d\tau + \sum_{j=1}^K \int_{t_0}^t \left\langle n_j \cdot \sigma_{\Gamma_s}(h_j), \frac{\partial}{\partial t} h_j \right\rangle_{\Gamma_j} d\tau. \end{aligned}$$

Invoking the BCs in (A.iii) and (A.iv), imposed on the structure-structure-heat variables, we then have

$$\begin{aligned} & \int_{t_0}^t (\mathbf{A}\Phi(\tau), \Phi(\tau))_{\mathbf{H}} d\tau \\ &= - \int_{t_0}^t \|\nabla u\|_{\Omega_f}^2 d\tau + 2i \operatorname{Im} \sum_{j=1}^K \int_{t_0}^t \left[\left(\sigma_{\Gamma_s} \left(\frac{\partial}{\partial t} h_j \right), \epsilon_{\Gamma_s}(h_j) \right)_{\Gamma_j} + \left(\frac{\partial}{\partial t} h_j, h_j \right)_{\Gamma_j} \right] d\tau. \quad (12) \end{aligned}$$

Applying this relation to the RHS of the relation,

$$\int_{t_0}^t \left(\frac{d}{d\tau} \Phi(\tau), \Phi(\tau) \right)_{\mathbf{H}} d\tau = \int_{t_0}^t (\mathbf{A}\Phi(\tau), \Phi(\tau))_{\mathbf{H}} d\tau$$

we obtain the desired estimate. ■

3. Main result: strong stability via resolvent criterion

This section is devoted to addressing the issue of asymptotic behavior of the solution whose existence – uniqueness is guaranteed by Theorem 2.1. In this regard, we show that the system given in (1)–(4) is strongly stable in the finite energy space $\mathbf{H}$. Our main result is as follows:

Theorem 3.1: *Finite energy solutions of the multilayered-heat PDE system (1)–(4), or equivalently (10) decay strongly to zero. That is to say, the solution of (1)–(4), with initial data $\Phi_0 \in \mathbf{H}$, satisfies*

$$\lim_{t \rightarrow 0} \|\Phi(t)\|_{\mathbf{H}} = 0.$$

To prove Theorem 3.1, in contrast to the approach taken in [19] which is geared to invoke the wellknown spectral criteria for stability in [20], we will adopt instead a resolvent-based methodology. In particular, we will ultimately appeal to the following theorem (see [34, Theorem 8.7] or [23, p. 75–76], see also [35].)

Theorem 3.2: *Let A generate a C_0 –semigroup of completely, non-unitary contractions on a Hilbert space H . If there exists a dense set $M \subset H$ such that*

$$\lim_{\alpha \rightarrow 0^+} \sqrt{\alpha} R(\alpha + i\beta; A)x = 0 \quad (13)$$

for every $x \in M$ and almost every $\beta \in \mathbb{R}$, then the semigroup is strongly stable. Here, R is the resolvent operator of the generator A defined by $R(\alpha + i\beta; A) = ((\alpha + i\beta)I - A)^{-1}$.

Remark 3.3: We recall that an operator $L \in \mathcal{L}(H)$ is completely non-unitary (c.n.u) if the trivial subspace is the only subspace of H which reduces L to a unitary operator (see, e.g. [14]).

Remark 3.4: Pointwise resolvent criterion such as those in Theorem 3.2 are formulated to yield insight on how behavior of a semigroup generator resolvent, as its ‘frequency domain’ parameter tends to the imaginary axis, allows for the inference of asymptotic decay. For example, given Hilbert space H , if $A : H \rightarrow H$ generates a bounded C_0 -semigroup, then, analogous to (13) of the manuscript, if

$$\lim_{\alpha \rightarrow 0^+} \sqrt{\alpha} R(\alpha + i\beta; A) = 0$$

for all x in some dense set of H , and for all $\beta \in \mathbb{R}$, then as argued on page 18 of [34], necessarily $i\mathbb{R} \cap \sigma(A) = \{\emptyset\}$. Consequently, from the spectral criteria in [20] and [36], one concludes strong stability of $\{e^{At}\}_{t \geq 0}$. One will have similar implications for ‘c.n.u.’ resolvent criteria in Theorem 3.2.

Lemma 3.5: *The given elastic-elastic-heat semigroup $\{e^{At}\}_{t \geq 0}$ is completely non-unitary (c.n.u).*

Proof: With reference to problem (1)–(4), assume that initial data Φ_0 is drawn from an invariant subspace $W \subset \mathbf{H}$ on which the operator family $\{e^{At}\}_{t \geq 0}$ is unitary. Then from the expression (10) and the dissipative relation (11) of Proposition 2.2, we have that the heat component of (1)–(4) satisfies

$$u = 0 \quad \text{on } (0, T) \times \Omega_f. \quad (14)$$

Consequently from the BC’s in (3) we have

$$w_t|_{\Gamma_s} = 0 \quad \text{on } (0, T) \times \Gamma_s, \quad (15)$$

$$\frac{\partial}{\partial t} h_j = 0 \quad \text{in } (0, T) \times \Gamma_j \text{ for } j = 1, \dots, K. \quad (16)$$

Differentiating the thin elastic equations in (2), and subsequently invoking (14) and (16) we have in turn

$$v \cdot \sigma(w_t)|_{\Gamma_s} = 0 \quad \text{on } (0, T) \times \Gamma_s. \quad (17)$$

If we now make the change of variable $v \equiv w_t$, then from (3), (15) and (17), we have that v satisfies the overdetermined Lamé system

$$\begin{cases} v_{tt} - \operatorname{div} \sigma(v) = 0 & \text{on } (0, T) \times \Omega_s \\ v|_{\Gamma_s} = v \cdot \sigma(v)|_{\Gamma_s} = 0 & \text{on } (0, T) \times \Gamma_s. \end{cases} \quad (18)$$

Consequently, from the Holmgren's type result in [24]¹ (see also [37]) we get for $T > 0$ sufficiently large,

$$v = 0 \quad \text{OR} \quad w_t = 0 \quad \text{in } (0, T) \times \Omega_s, \quad (19)$$

and so

$$w = \text{constant} \quad \text{in } (0, T) \times \Omega_s. \quad (20)$$

From the compatibility condition between thick and thin elastic displacements in (5), we then have from (20) that

$$h_j = \text{constant} \quad \text{in } (0, T) \times \Gamma_j, \quad \text{for } 1 \leq j \leq K.$$

Applying this consequence to the thin elastic equation in (2) and further invoking (14), (16) and (20) we get

$$h_j = 0 \quad \text{in } (0, T) \times \Gamma_j, \quad \text{for } 1 \leq j \leq K. \quad (21)$$

Finally, (20), (21) and said compatibility condition between thick and thin elastic displacements, imposed in the natural energy space $\mathbf{H}$, yield

$$w = 0 \quad \text{on } (0, T) \times \Omega_s. \quad (22)$$

To conclude, (14)–(16), (21) and (22) yield on $(0, T)$

$$e^{At} \Phi_0 = 0, \quad \Phi_0 \in W$$

so necessarily $W = \{0\}$. This finishes the proof of Lemma 3.5. ■

Before embarking on our proof of strong stability, we recall the following regularity result for the homogeneous boundary value problem involving the Lamé system of elasticity on polyhedron Ω_s . (We are not aiming here for great generality.)

Proposition 3.6: *Suppose $z \in H^1(\Omega_s)$ satisfies the BVP*

$$\begin{cases} -\operatorname{div} \sigma(z) = f \in L^2(\Omega_s) \\ z|_{\Gamma_s} = 0 \quad \text{on } \Gamma_s. \end{cases}$$

Then z has the higher regularity

$$\|z\|_{H^{\frac{3}{2}+\epsilon}(\Omega_s)} + \sum_{j=1}^K \|v \cdot \sigma(z)|_{\Gamma_j}\|_{H^\epsilon(\Gamma_j)} \leq C \|f\|_{\Omega_s}. \quad (23)$$

Proof: The fact that $z \in H^{\frac{3}{2}+\epsilon}(\Omega_s)$ comes immediately from [25, Theorem 4.5.1, p. 140, see also the remark on p. 149]. (See also Theorem 4.5 of [38].) Moreover, given a point on boundary face Γ_j , let unit (tangent) vectors τ_1, τ_2 –we neglect the index j here– be such that $\{v(x), \tau_1(x), \tau_2(x)\}$ constitutes an orthonormal basis of $\mathbb{R}^3$. Therewith, one can compute outright –see e.g. Proposition A.1 (iii) of [29]– the expression for $j = 1, 2, \dots, K$:

$$\begin{aligned} v \cdot \sigma(z)|_{\Gamma_j} &= \lambda \left[\frac{\partial z}{\partial v} \cdot v + \frac{\partial z}{\partial \tau_1} \cdot \tau_1 + \frac{\partial z}{\partial \tau_2} \cdot \tau_2 \right] v \\ &\quad + 2\mu \frac{\partial z}{\partial v} + \mu \left[\left(\frac{\partial z}{\partial \tau_2} \cdot v \right) - \left(\frac{\partial z}{\partial v} \cdot \tau_2 \right) \right] \tau_2 \\ &\quad + \mu \left[\left(\frac{\partial z}{\partial \tau_1} \cdot v \right) - \left(\frac{\partial z}{\partial v} \cdot \tau_1 \right) \right] \tau_1. \end{aligned} \quad (24)$$

To deal with RHS, we recall the known bounded Sobolev trace maps for a polyhedron (see e.g. Theorem 6.9 (i), page 43 of [39]):

$$\begin{aligned} z \in H^{\frac{3}{2}+\epsilon}(\Omega_s) &\rightarrow z|_{\Gamma_j} \in H^{1+\epsilon}(\Gamma_j), \quad j = 1, 2, \dots, K \\ z \in H^{\frac{3}{2}+\epsilon}(\Omega_s) &\rightarrow \frac{\partial z}{\partial v}|_{\Gamma_j} \in H^\epsilon(\Gamma_j), \quad j = 1, 2, \dots, K. \end{aligned} \quad (25)$$

Applying these maps to RHS of (24), and invoking said continuous mapping $f \rightarrow z$ in (23) now completes the proof. ■

Proof of Theorem 3.1 – Strong Stability

Having established that the multi-layered structure-heat PDE contraction semigroup $\{e^{At}\}_{t \geq 0}$ is c.n.u by Lemma 3.5, we can make use of the resolvent criterion given in Theorem 3.2. To this end, we define the operator $A_D : D(A_D) \subset L^2(\Omega_s) \rightarrow L^2(\Omega_s)$ via

$$\begin{cases} A_D f = -\operatorname{div} \sigma(f) \\ D(A_D) = \{f \in H_0^1(\Omega_s) : -\operatorname{div} \sigma(f) \in L^2(\Omega_s)\}. \end{cases} \quad (26)$$

With respect to this self-adjoint operator with compact inverse, we denote

$$S \equiv \{\beta \in \mathbb{R} : \beta^2 \text{ is an eigenvalue of } A_D : D(A_D) \subset L^2(\Omega_s) \rightarrow L^2(\Omega_s)\}. \quad (27)$$

In order to invoke the resolvent criterion in Theorem 3.2, we will establish that the thick elastic-thin elastic-heat generator obeys the strong limit (13) for all $\beta \in \mathbb{R} \setminus (S \cup \{0\})$. To this end, with $\alpha > 0$ and fixed $\beta \in \mathbb{R} \setminus (S \cup \{0\})$, we consider the resolvent equation

$$[(\alpha + i\beta)I - \mathbf{A}] \Phi = \Phi_0^* \quad (28)$$

where the solution $\Phi = [u_0, h_{01}, h_{11}, \dots, h_{0K}, h_{1K}, w_0, w_1] \in D(\mathbf{A})$ and the data $\Phi_0^* = [u_0^*, h_{01}^*, h_{11}^*, \dots, h_{0K}^*, h_{1K}^*, w_0^*, w_1^*] \in \mathbf{H}$. From the definition of $D(\mathbf{A})$, this abstract equation can be written explicitly as

$$(\alpha + i\beta)u_0 - \Delta u_0 = u_0^* \quad \text{in } \Omega_f \quad (29)$$

For $1 \leq j \leq K$:

$$\begin{cases} (\alpha + i\beta)h_{0j} - h_{1j} = h_{0j}^* & \text{in } \Gamma_j \\ (\alpha + i\beta)h_{1j} - \operatorname{div} \sigma_{\Gamma_s}(h_{0j}) + h_{0j} + \frac{\partial u_0}{\partial v} - v \cdot \sigma(w_0) = h_{1j}^* & \text{in } \Gamma_j \end{cases} \quad (30)$$

$$h_{0j}|_{\partial\Gamma_j \cap \partial\Gamma_l} = h_{0l}|_{\partial\Gamma_j \cap \partial\Gamma_l} \text{ on } (\partial\Gamma_j \cap \partial\Gamma_l), \quad \text{for all } 1 \leq l \leq K \text{ such that } \partial\Gamma_j \cap \partial\Gamma_l \neq \emptyset \quad (31)$$

$$n_j \cdot \sigma_{\Gamma_s}(h_{0j})|_{\partial\Gamma_j \cap \partial\Gamma_l} = -n_l \cdot \sigma_{\Gamma_s}(h_{0l})|_{\partial\Gamma_j \cap \partial\Gamma_l} \text{ on } (\partial\Gamma_j \cap \partial\Gamma_l), \quad \text{for all } 1 \leq l \leq K \text{ such that}$$

$$\partial\Gamma_j \cap \partial\Gamma_l \neq \emptyset \quad (32)$$

$$w_1 = (\alpha + i\beta)w_0 - w_0^* \text{ in } \Omega_s \quad (33)$$

$$-\beta^2 w_0 - \operatorname{div} \sigma(w_0) = -(\alpha^2 + 2i\alpha\beta)w_0 + (\alpha + i\beta)w_0^* + w_1^* \text{ in } \Omega_s \quad (34)$$

$$[(\alpha + i\beta)w_0 - w_0^*]|_{\Gamma_j} = h_{1j} = u_0|_{\Gamma_j} \text{ on } \Gamma_j. \quad (35)$$

Throughout, take $0 < \alpha < M_0$, for some positive constant M_0 , and we will give the proof in the following steps:

Step I (A static dissipation relation): Taking the $(\cdot, \cdot)$ -inner product of both sides of (28) with respect to Φ , we have

$$\alpha \|\Phi\|_{\mathbf{H}}^2 + i\beta \|\Phi\|_{\mathbf{H}}^2 - (\mathbf{A}\Phi, \Phi)_{\mathbf{H}} = (\Phi_0^*, \Phi)_{\mathbf{H}}.$$

Proceeding as in the proof of Proposition 2.2, we obtain

$$\begin{aligned} \alpha \|\Phi\|_{\mathbf{H}}^2 + i\beta \|\Phi\|_{\mathbf{H}}^2 + \|\nabla u_0\|_{\Omega_f}^2 &= (\Phi_0^*, \Phi)_{\mathbf{H}} + 2i \sum_{j=1}^K \operatorname{Im}(\sigma_{\Gamma_s}(h_{1j}), \epsilon_{\Gamma_s}(h_{0j}))_{\Gamma_j} \\ &\quad + 2i \sum_{j=1}^K \operatorname{Im}(h_{1j}, h_{0j})_{\Gamma_j} + 2i \operatorname{Im}(\sigma(w_1), \epsilon(w_0))_{\Omega_s} \end{aligned}$$

or

$$\alpha \|\Phi\|_{\mathbf{H}}^2 + \|\nabla u_0\|_{\Omega_f}^2 = \operatorname{Re}(\Phi_0^*, \Phi)_{\mathbf{H}}. \quad (36)$$

Invoking the boundary conditions (35) and the Sobolev Trace Theorem, we then have for $1 \leq j \leq K$

$$[(\alpha + i\beta)w_0 - w_0^*]|_{\Gamma_j} = h_{1j} = u_0|_{\Gamma_j} = \sqrt{\mathcal{O}(|(\Phi_0^*, \Phi)_{\mathbf{H}}|)} \text{ on } \Gamma_j. \quad (37)$$

Step II (An estimate for the thick elastic displacement): We start here by defining the ‘Dirichlet’ map $D : L^2(\Omega_s) \rightarrow L^2(\Omega_s)$ via

$$Dg = v \iff \begin{cases} \operatorname{div} \sigma(v) = 0 & \text{in } \Omega_s \\ v = g & \text{on } \Gamma_s \end{cases} \quad (38)$$

By the Lax–Milgram Theorem, and a subsequent integration by parts with respect to (38), we have

$$D \in \mathcal{L}\left(H^{\frac{1}{2}}(\Gamma_s), H^1(\Omega_s)\right); v \cdot \sigma(D(\cdot)) \in \mathcal{L}\left(H^{\frac{1}{2}}(\Gamma_s), H^{-\frac{1}{2}}(\Gamma_s)\right). \quad (39)$$

With this mapping, if we now let

$$z = w_0 - \frac{i}{\beta} D([\alpha w_0 - u_0 - w_0^*]|_{\Gamma_s}), \quad (40)$$

then from (34), we have that z solves the BVP:

$$\begin{cases} -\beta^2 z - \operatorname{div} \sigma(z) = -i\beta D([\alpha w_0 - u_0 - w_0^*]|_{\Gamma_s}) - (\alpha^2 + 2i\alpha\beta)w_0 + (\alpha + i\beta)w_0^* + w_1^* & \text{in } \Omega_s \\ z = 0 & \text{on } \Gamma_s. \end{cases}$$

Since β^2 is not an eigenvalue of A_D (defined in (26)), we then have

$$z = (\beta^2 - A_D)^{-1} \left[i\beta D([\alpha w_0 - u_0 - w_0^*]|_{\Gamma_s}) + (\alpha^2 + 2i\alpha\beta)w_0 - (\alpha + i\beta)w_0^* - w_1^* \right] \quad \text{in } \Omega_s.$$

Estimating RHS by means of (36), (39), (37) and the Sobolev Trace Theorem, we then have

$$\|z\|_{\Omega_s} = \mathcal{O} \left(\sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right). \quad (41)$$

In turn, by the higher regularity result in Proposition 3.6, we have

$$\begin{aligned} & \|z\|_{H^{\frac{3}{2}+\epsilon}(\Omega_s)} + \sum_{j=1}^K \left\| \nu \cdot \sigma(z)|_{\Gamma_j} \right\|_{H^\epsilon(\Gamma_j)} \\ & \leq C \left\| \beta^2 z - i\beta D([\alpha w_0 - u_0 - w_0^*]|_{\Gamma_s}) - (\alpha^2 + 2i\alpha\beta)w_0 + (\alpha + i\beta)w_0^* + w_1^* \right\|_{\Omega_s}. \end{aligned}$$

Appealing to estimate (23) and (41) (and once more (36), (39), (37) and Sobolev Trace Theory), we have

$$\|z\|_{H^{\frac{3}{2}+\epsilon}(\Omega_s)} + \sum_{j=1}^K \left\| \nu \cdot \sigma(z)|_{\Gamma_j} \right\|_{H^\epsilon(\Gamma_j)} \leq \mathcal{O} \left(\sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right). \quad (42)$$

Now, invoking the decomposition

$$w_0 = z + \frac{i}{\beta} D([\alpha w_0 - u_0 - w_0^*]|_{\Gamma_s}),$$

we combine (36), (42), (39) and (37) to have

$$\|w_0\|_{H^1(\Omega_s)} + \sum_{j=1}^K \left\| \nu \cdot \sigma(w_0)|_{\Gamma_j} \right\|_{H^{-\frac{1}{2}}(\Gamma_s)} \leq \mathcal{O} \left(\sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right). \quad (43)$$

Step III (An estimate for the thin elastic displacement): For $1 \leq j \leq K$, we multiply both sides of the thin elastic equation in $(30)_2$ by h_{0j} , followed by an integration over Γ_j , and integration by parts. Summing the resulting expressions gives

$$\begin{aligned} & \sum_{j=1}^K \left[(\sigma_{\Gamma_s}(h_{0j}), \epsilon_{\Gamma_s}(h_{0j}))_{\Gamma_j} + \|h_{0j}\|_{\Gamma_j}^2 \right] - \sum_{j=1}^K \langle n_j \cdot \sigma_{\Gamma_s}(h_{0j}), h_{0j} \rangle_{\Gamma_j} \\ & = -(\alpha + i\beta)(u_0, w_0)_{\Omega_s} + \langle \nu \cdot \sigma(w_0), w_0 \rangle_{\Gamma_s} - \left\langle \frac{\partial u_0}{\partial \nu}, w_0 \right\rangle_{\Gamma_s} + \sum_{j=1}^K (h_{1j}^*, h_{0j})_{\Gamma_j}. \end{aligned} \quad (44)$$

(Here, in canceling the thin layer boundary terms on $\partial\Gamma_j$, we are invoking the boundary conditions in (31)–(32).) Now, with respect to the normal derivative $\frac{\partial u_0}{\partial \nu}|_{\Gamma_s}$ of the thermal component, we can integrate by parts so as to deduce the trace estimate

$$\begin{aligned} \left\| \frac{\partial u_0}{\partial \nu} \right\|_{H^{-\frac{1}{2}}(\Gamma_s)} & \leq C \left[\|\nabla u_0\|_{\Omega_f} + \|\Delta u_0\|_{\Omega_f} \right] \\ & = C \left[\|\nabla u_0\|_{\Omega_f} + \|(\alpha + i\beta)u_0 - u_0^*\|_{\Omega_f} \right]. \end{aligned}$$

Invoking the estimate (36) now gives

$$\left\| \frac{\partial u_0}{\partial \nu} \right\|_{H^{-\frac{1}{2}}(\Gamma_s)} = \mathcal{O} \left(\sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right). \quad (45)$$

Applying this estimate, along with relation (36), and (43) for $\{w_0, \nu \cdot \sigma(w_0)|_{\Gamma_s}\}$ (and using again the Sobolev Trace Theorem), we get

$$\sqrt{\sum_{j=1}^K \left[(\sigma_{\Gamma_s}(h_{0j}), \epsilon_{\Gamma_s}(h_{0j}))_{\Gamma_j} + \|h_{0j}\|_{\Gamma_j}^2 \right]} = \mathcal{O} \left(\sqrt{\|\Phi_0^*\|_{\mathbf{H}} \|\Phi\|_{\mathbf{H}}} + \sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right) \quad (46)$$

Moreover, applying (36) and the estimate (43) to the resolvent relation (33), we have

$$\|w_1\|_{\Omega_s} = \mathcal{O} \left(\sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right). \quad (47)$$

Now, if we combine (36), (43), (46) and (47), we then have

$$\|\Phi\|_{\mathbf{H}} \leq C_{0,\beta} \left(\sqrt{\|\Phi_0^*\|_{\mathbf{H}} \|\Phi\|_{\mathbf{H}}} + \sqrt{|(\Phi_0^*, \Phi)_{\mathbf{H}}|} + \|\Phi_0^*\|_{\mathbf{H}} \right).$$

Invoking finally $|ab| \leq \delta a^2 + C_\delta b^2$ for $\delta > 0$, we arrive at (after rescaling $\delta > 0$),

$$(1 - C_\beta) \|\Phi\|_{\mathbf{H}} \leq C_{\beta,\delta} \|\Phi_0^*\|_{\mathbf{H}}.$$

This gives the required strong limit in (13) for all $\Phi_0^* \in \mathbf{H}$ and all $\beta \in \mathbb{R} \setminus (S \cup \{0\})$. This completes the proof of Theorem 3.1.

Note

1. In [24], the geometry was assumed to be C^1 . However the details of proof apply readily to piecewise C^1 —domains. Indeed, Holmgren's uniqueness will hold for Lamé systems on Lipschitz domains; see [40].

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