

High-dimensional frequency-bin tomography with random measurements

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Abstract: Utilizing electro-optic modulation and pulse shaping for random measurements, we reconstruct the full density matrix of biphoton frequency combs for entangled qudits up to $d = 5$. Our method relies on simple experimental settings and can be applied to any frequency-bin quantum system. © 2021 The Author(s)

Encoding quantum information in equispaced frequency bins has emerged as a novel scheme for photonic quantum information processing [1]. Generation of high-dimensional frequency-entangled states, usually in the form of biphoton frequency combs (BFCs), can be as simple as pumping a nonlinear, resonant structure. A conventional joint spectral intensity (JSI) measurement can easily reveal tens of correlated frequency modes, but such a measurement cannot provide information regarding the phase coherence across frequency-bin pairs. To reconstruct the density matrix of the BFC state requires active mixing of frequency bins [2–4] or ultrahigh temporal resolution [5] such that projective measurements other than the computational basis can be realized. One method coherently mixes different frequency components of the BFC via a Fourier-transform pulse shaper and electro-optic phase modulator (EOM) [2,3]. By properly setting the amplitude and phase mask on the shaper and the modulation voltage on the EOM, one can filter out sidebands populated by equal contributions from all input frequency modes. An alternative solution exploits the functionality of a quantum frequency processor [1], which consists of a concatenation of multiple EOMs and pulse shapers, to realize near-deterministic quantum gates for tomography based on multi-outcome measurements. Nevertheless, both methods face roadblocks en route to higher dimensions, as mixing multiple frequency bins *equally* is never a trivial task: aggressive amplitude filtering of the input state is inevitable in the first approach (to balance mixing weights), while the number of elements required for arbitrary QFP operations (scaling linearly with dimensionality d) limits the maximum dimensionality possible with current setups. Indeed, the highest dimension of a fully characterized BFC hitherto is a two-quqart state ($d = 4$) [2,5].

Here we propose an alternative solution to this problem. Instead of attempting to mix all frequency bins according to a prescribed pattern, we apply a series of random operations to the input biphotons and follow with JSI measurements. At each measurement setting, the frequency-resolved outputs consist of different superpositions of the input bins, with the weightings determined by the EO modulation depth. Though these measurements are not mutually unbiased, they are sufficiently random to probe all elements of the density matrix, and we can utilize state-of-the-art Bayesian statistical methods [7] to obtain a realistic estimate of the density matrix. Figure 1(a) illustrates our proposed method. In this work, we utilize an etalon and pulse shaper to carve a broadband down-conversion spectrum into a 40 GHz-spaced BFC, with a total of d frequency bin pairs. We then impose a set of random spectral phases onto the signal and idler bins (a total of $2d$ phases) and drive the EOM with a 40 GHz sinusoidal voltage, resulting in a temporal phase modulation $e^{i\delta \sin \Delta\omega t}$. The strength of the modulating RF field δ is chosen in a random order from 20 values preselected uniformly at random from $\delta \in [0, 2.5]$ rad, with the maximum set by the amount of RF power we can generate at this frequency. Figure 1(b) provides examples of JSI measurement results for $d = 4$. In the absence of EO modulation ($\delta = 0$), the signal-idler photon pairs are completely anticorrelated due to energy conservation. When the EOM is turned on, the frequency correlations be-

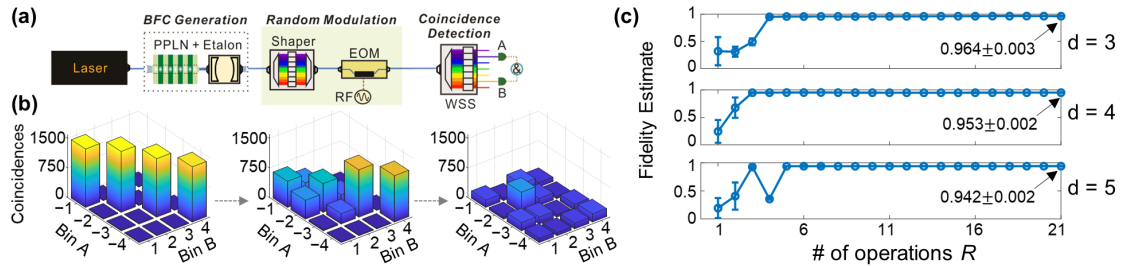


Fig. 1. (a) Experimental setup. (b) Examples of JSI measurements in the presence of random modulation (left to right): $\delta = 0, 0.7$, and 2.3 rad. (c) Convergence plots for experimental data sets.

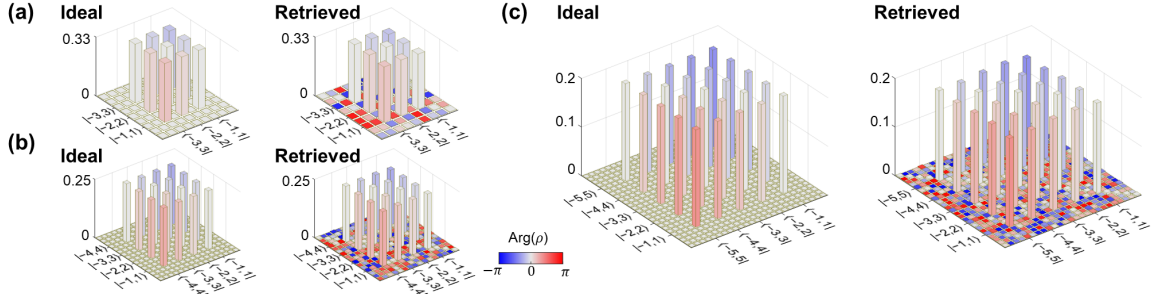


Fig. 2. Density matrix plots of the ideal and retrieved for (a) $d = 3$, (b) $d = 4$, and (c) $d = 5$ BFC state. The bar heights represent the absolute values of ρ , and colors represent the phases.

come more complex as photons are scattered into many frequency modes, and the JSIs are no longer diagonal [6]. We intentionally leave a ~ 300 GHz gap between the signal and idler photons, such that we can apply strong EO modulation without fear of the signal photon jumping over into the idler's modes, and vice versa.

We consider a total of R measurement settings ($R = 1, 2, \dots, 21$), the first of which is the traditional JSI measurement (i.e., EOM off), and the rest of which correspond to random spectral and temporal phase modulation applied by the shaper and the EOM. For each measurement setting, we collect coincidences for d^2 combinations and disregard any events outside of the original subspace. We then employ Bayesian analysis to find the possible input states giving rise to the measurement results. Specifically, we construct a Poissonian likelihood function $\mathcal{L}_D(c, \mathbf{x}) = \prod_{s=1}^{Rd^2} e^{-cp_s(\mathbf{x})} [cp_s(\mathbf{x})]^{N_s}$, where c denotes the mean biphoton counts over all possible outcomes (including those unobserved), s specifies a particular combination of measurement setting and frequency bins—an ordered triple $(r(s), m(s), n(s))$ — N_s is the measured number of coincidences between frequency mode ω_m (idler photon) and mode ω_n (signal photon) at the r -th measurement setting, and $p_s(\mathbf{x}) = \langle \omega_m, \omega_n | U_r \rho(\mathbf{x}) U_r^\dagger | \omega_m, \omega_n \rangle$ is the corresponding outcome probability.

The effect of the pulse shaper and EOM on the two-photon state is given by $U_r = U_r^{(I)} \otimes U_r^{(S)}$, where $U_r^{(I)}$ and $U_r^{(S)}$ describe the operations applied to idler and signal photons, respectively. We follow the procedures in Ref. [7] to parameterize the density matrix $\rho(\mathbf{x})$ to assure physicality, and assume a uniform prior on $\mathbf{x}$; for the biphoton flux c , we take the prior as a normal distribution of mean C_0 and standard deviation $0.1C_0$, setting C_0 as the sum of all counts in the first JSI measurement. Numerically sampling the posterior formed by the product of likelihood and prior, we can then estimate the mean and standard deviation of any function of interest, such as state fidelity and logarithmic negativity $E_{\mathcal{N}}$, an upper bound on distillable entanglement [8].

Figure 1(c) plots the evolution of state fidelity for the estimated BFC state as the number of measurements increases. We compute the fidelity with respect to the d -dimensional Bell state with additional spectral phase accumulated after the equivalent of 20 m single-mode fiber. In all cases, the fidelity converges within $R \sim 10$ measurements. For $d = 5$, we report a fidelity $\mathcal{F} = (94.2 \pm 0.2)\%$ and $E_{\mathcal{N}} = 2.237 \pm 0.002$ ebits (theoretical maximum is 2.32 for $d = 5$), with the retrieved Bayesian mean density matrix plotted in Fig. 2(c). We notice that both the absolute value and the phase of the density matrix elements align well with the theory—the only discrepancy being the inconsequential phase values of the near-zero off-diagonal elements. These results showcase the highest dimension of a fully reconstructed density matrix in experimental frequency-bin encoding.

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