A MODEL OF STUDENTS' CONCEPTIONS OF EQUIVALENCE

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In mathematics education, much research has focused on studying how students think about the equals sign, but equality is just one example of the larger concept of equivalence, which occurs extensively throughout the K-16 mathematics curriculum. Yet research on how students think about broader notions of equivalence is limited. We present a model of students' thinking that is informed by Sfard's theories of the Genesis of Mathematical Objects, in which she distinguishes between operational versus structural thinking (e.g., 1995), which we conceptualize as a continuum rather than a binary categorization. Sfard also describes a *pseudostructural* conception, in which the objects that a student conceptualizes are not the reification of a process. We combine Sfard's theory with a categorization of the source of students' definitions, where *stipulated* definitions are given a priori and can be explicitly consulted when determining whether something fits the definitions, while *extracted* definitions are constructed from repeated observation of usage (Edwards & Ward, 2004). We combine these theories with inductive coding of data (open-ended questions, multiple-choice questions, and cognitive interviews) collected from thousands of students enrolled in a range of mathematics classes in college in the US, to generate categories of students' thinking around equivalence. We see this model as a tool for analysing students' work to better understand how students conceptualize equivalence. With this model we hope to begin a conversation about how students tend to conceptualize equivalence at various levels, as well as the ways in which equivalence is or is not explicitly addressed currently in curricula and instruction, and what consequences this might have for students' conceptions of equivalence.

	Operational Thinking ←→	Structural Thinking
Extracted Definition of Equivalence	**Pseudo-Process View:** Students see equivalence as a computational process, and their approaches come from extracted rather than stipulated definitions. Definitions of equivalence are typically non-standard, ill-defined, and/or unstable.	**Pseudo-Object View:** Students are able to identify/generate equivalent objects by drawing on structure, instead of reverting to explicit computation; but criteria for equivalence are not the reification of a process. Objects are typically extracted rather than based on stipulated definitions; definitions of equivalence are typically non-standard, ill-defined, and/or unstable.
Stipulated Definition of Equivalence	**Process View:** Students see equivalence as a process governed by stipulated rules. Such students can often calculate in the correct order, but this may not translate to an ability to use stipulated definitions to recognize equivalent objects.	**Object View:** Students are able to think about equivalent objects by considering syntactic structure, without reverting to explicit processes to determine equivalence, by drawing on stipulated rather than solely extracted definitions of equivalence.

Figure 1: Model of Students' Thinking about Equivalence of Mathematical Objects

References

Edwards, B., & Ward, M. (2004). Surprises from mathematics education research: Student (mis)use of mathematical definitions. *American Mathematical Monthly*, *111*(5), 411–424

Sfard, A. (1995). The development of algebra: Confronting historical and psychological perspectives. *Journal of Mathematical Behavior*, *14*(1), 15-39.

2020. In NNN (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. XX-YY. Khon Kaen, Thailand: PME.