

STUDENTS' CONCEPTIONS OF SUBSTITUTION

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Substitution is a key idea that is woven throughout the mathematics curriculum. In secondary school, substitution is described as an interchangeability of equal numbers, and then as a method for finding solutions of systems of equations. In university, substitution is used as a means to recognize familiar structures in Integral Calculus. Despite its prevalence in mathematics, there is little research on substitution, especially on students' understanding of substitution. This work aims to investigate students' meanings for substitution, and how they use it.

We draw on Tall and Vinner's (1981) ideas of concept definition and concept image to explore students' meanings of substitution through their personal definitions of substitution, what they identify as substitution, and how they perform substitution. In this presentation, we report on elementary algebra students' responses to questions about substitution. Data comes includes written responses to multiple-choice and open-ended questions and transcripts from clinical interviews across multiple semesters at a community college.

Through a combination of thematic and conceptual analysis, we categorized students' thinking about substitution and what features appeared to impact how they enact it. We found that students often identify substitution as a process of replacement of one mathematical object for another but differ in the generality of the mathematical objects that they consider (e.g., strictly as the replacement of a number for a variable versus replacement of any expression for another expression). Students further differed in whether or not they thought that substitution entailed equivalence of the objects being replaced. When performing substitution (e.g., substituting $x + 1$ for y in $2y^2$), we found that students' activity was heavily based on their understanding of the structure of the expression where the substitution is taking place (the unified 'pieces' of $2y^2$). In addition to other findings, we elaborate on the mental processes that students engage in when performing substitution and synthesize our findings with the notion of substitution equivalence (Wladis et al., 2020).

References

- Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*. <https://doi.org/10.1007/BF00305619>
- Wladis, C., Offenholley, K., Jaffe, E., Dawes, D., Thakkar, N., Beiting, M., & Griffith, S. (2020). A proposed framework of student thinking around substitution equivalence: Structural versus operational views. *Proceedings of the 23rd Annual Conference on Research in Undergraduate Mathematics Education*.