



Full length article

Markov chain optimization of repair and replacement decisions of medical equipment

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The cost of repair and maintenance of medical devices can be fairly burdensome to the healthcare industry. Healthcare providers often consider plans such as increasing in-house repairs, using multiple repair service providers, or timely replacement of devices to overcome this issue. This study aims to develop a data-driven Markov Decision Process (MDP) framework based on Discrete-Time Markov Chain (DTMC) model to optimize medical equipment repair and replacement decisions. Effective decision-making on whether to repair or replace is crucial to managing the product lifecycle cost and the costs to healthcare facilities. The study determines the optimal repair or replacement decision based on the product lifecycle data and current product failure status. It utilizes a net present value model to maximize expected value over an infinite time horizon. The study uses a dataset of 24,516 repair and maintenance records of 5,171 individual medical devices of a particular type to extract parameters needed for the optimization model. The dataset provides a rich baseline for analyzing different failures and event modes during product lifespan such as battery-related issues, random failure, preventive maintenance, and physical damage. It further quantifies the chance of moving from one product status to another. The model outcomes are discussed for a particular case. The findings reveal the most frequent reasons for failures and the most economically viable repair and replacement decision for each end-of-use device based on their current condition. Several sensitivity analyses are conducted to clarify the impact of operating revenue and warranty time on the repair or replacement.

1. Background

Repair and maintenance of medical equipment can be quite costly for healthcare facilities (Mummolo et al., 2007). In 2018, the expenditure on US health care was estimated to be \$3.6 trillion, reaching nearly \$6.0 trillion by 2027 (Centers for Medicare and Medicaid Services, 2019). One cost-saving approach that healthcare facilities can take is to make an effective repair and replacement policy. The decision-making on repair and replacement is a traditional problem backing to the 1960s, still popular in many fields (Azevedo et al., 2020, Sheu et al., 2020, Leu and Ying, 2020). The objective of repair and replacement decisions is often to minimize cost while maximizing the operational period (Pan and Thomas, 2010).

The repair and replacement decisions have been studied in the previous literature. To name a new, Abdi and Taghipour (2019) proposed a repair-replacement decision model based on environmental impact factors using a plastic shredder case study (Abdi and Taghipour, 2019). van den Boomen et al. (2020) built Markov Decision Processes (MDP) to

consider uncertainty in prices when deciding on maintenance and replacement of infrastructures (van den Boomen et al., 2020). Azevedo et al. (2020) developed a multi-objective genetic algorithm combined with discrete event simulation using the Generalized Renewal Process for replacement policy problems (Azevedo et al., 2020). Sheu et al. (2020) proposed a replacement policy based on two types of failure arrivals, minor or catastrophic shocks. The preventive replacement occurred when one of the indexes, such as the system's age, the number of occurrence of type 1 failure, and the accumulative damage exceed predefined thresholds (Sheu et al., 2020). Leu and Ying (2020) considered economic assessment to decide the replacement time of hydraulic machinery (Leu and Ying, 2020). Khan et al. (2020) considered factors such as obsolescence, productivity, and cost types to develop a replacement framework for midlife upgrades (Khan et al., 2020). Zheng and Makis (2020) proposed a condition-based maintenance policy based on a semi-Markov decision process (SMDP) for soft and hard failure. When the deterioration reaches the predefined level, the system determines repair or replacement (Zheng and Makis, 2020). Hamed and Al-Eideh (2020) used a random parameters mixed logit model to

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Nomenclature

$P_{ij} = P(j i)$	Transition probability from state i to state j
$f_i(z)$	The probability density function of repair time for a device in state i (failure type i)
$X_{t+1} = j$	The process is in state j at time $t+1$
n_{ij}	The number of occurrences from state i to state j
n	Number of states
$\mathcal{L}(p)$	The log maximum likelihood equation of probabilities of the realization
λ_i	Lagrange multipliers
M	Transition matrix
D_{ij}	Average operating days from state i to state j
$(d_{ij})_k$	The k^{th} record of operating days from state i to state j
PV	Present value
FV	Future value
β	Discount factor
β_{ij}	Discount factor from state i to state j

r	Annual discount rate or rate of return
p	The number of periods
$O_{ij,\delta}$	Operating revenue from state i to state j based on decision δ
δ	decision ($\delta = 1$ means repair; $\delta = 0$ means replacement)
OR	Operating revenue per day
$V(i)$	Net present value (revenue - cost) earned over an infinite number of periods
$r_{i,\delta}$	Cost of decision δ in state i
ARD_i	Average repair or maintenance days for the state i
NC	Replacement costs
WD	Warranty days
RC	Repair costs per day
AIC	Akaike information criterion
BIC	Bayesian information criterion
$\hat{L}$	The maximum value of the likelihood function
k	Number of the probability distribution parameters

determine the repair and replacement of damaged cars caused by traffic accidents (Hamed and Al-Eideh, 2020).

McLaren et al. (2020) highlighted that the understanding of repair in the circular economy literature is limited, and the transformative future-oriented roles of repair are overlooked (McLaren et al., 2020). Wieser and Tröger (2017) explored the customers' behaviors about replacement, repair, and reuse of mobile phones. They revealed that tax benefits and more information about repair could encourage customers to repair broken devices (Wieser and Troeger, 2017). Sabbaghi and Behdad (2018) evaluated consumer decisions on repairing mobile phones and estimated the economic leakage of not repairing devices for both consumers and businesses (Sabbaghi and Behdad, 2018). In another study, Sabbaghi et al. (2016) discussed product repairability's business outcomes and the impact on consumer loyalty and brand recommendations (Sabbaghi et al., 2016). Dabous et al. (2017) compared the life cycle assessment of repair and replacement for a bridge deck case (Dabous et al., 2017). Wursthorn et al. (2010) assessed the environmental impacts of repair and replacement in vehicle maintenance decisions (Wursthorn et al., 2010). He et al. (2017) proposed a model to make the optimal replacement decision for changing the hybrid electric vehicles (He et al., 2017). Stutzman et al. (2017) adopted a numerical analysis model to optimize coal power plant replacement decisions (Stutzman et al., 2017). Mashhadi et al. (2016) investigated customers' experience on product repair with several attributes such as repair cost and repair activity type like replacement (Mashhadi et al., 2016). Vlok et al. (2002) applied a vibration monitoring mechanism to build optimal replacement decisions for circulating pumps in a coal wash plant (Vlok et al., 2002). Although analyzing repair or replacement decisions is common in the previous studies, the literature on the repair and maintenance of medical equipment and the timing of repair or replacement decisions is limited.

Along with repair and replacement decisions of medical devices, Sloan (2007) used a Markov decision model on single-use devices to decide whether to reuse or discard (Sloan, 2007). Several studies on repair and replacement of medical devices consider scores such as Priority Replacement Index (PRI) to determine the rank of replacement order (Mummolo et al., 2007; Basiony, 2013). The PRI is calculated based on several factors, such as mean downtime ratio, life support, and technological obsolescence. Taylor and Jackson (2005) used the Medical Equipment Replacement Score (MERS) system to score the priority replacement for medical devices (Taylor and Jackson, 2005). Taghipour et al. (2011) created a Multi-Criteria Decision Making (MCDM) model to prioritize medical devices for maintenance decisions (Taghipour et al., 2011). Rajasekaran (2005) adopted Equipment Replacement Planning

System (ERPS) to automate replacement planning in hospitals (Rajasekaran, 2005).

Although the studies mentioned above can prioritize repair or replacement decisions, the factors contributing to the score are determined by human subjects instead of observed data. For example, technological obsolescence or the ratio of usage might be too subjective since the medical staff often decides whether a newer device has more advantages (Mummolo et al., 2007). Thus, more data-driven approaches are needed to enhance the effectiveness of repair and replacement decisions.

In the current study, we develop a Markov Chain model to optimize the decision-making process. Prior studies have already used Markov Chain models for repair and replacement decisions. Those studies can be divided into two groups. The first group is studies that use the number of failures and product age. If the two attributes exceed the predefined thresholds, the decision is to replace instead of repair or maintenance (Kapur et al., 1989; Kijima, 1989; Makis and Jardine, 1993; Love et al., 2000). Another group of studies is those that build Markovian deterioration to identify repair or replacement decisions (Pan and Thomas, 2010; Klein, 1962; Kolesar, 1966; Derman, 1963; Ross, 1971). In those studies, the Markov model is built based on the device deterioration level. While the previous studies can make decisions among repair and replacement, they only considered few attributes such as the number of failures, ages, and deterioration. They lack comprehensive empirical datasets to support the analysis and ignored the failure reasons such as battery-related issues, random failure, and physical damage. What distinguishes the current study from the previous literature (Azevedo et al., 2020; Sheu et al., 2020; Leu and Ying, 2020; Khan et al., 2020; Zheng and Makis, 2020; Hamed and Al-Eideh, 2020) is that we consider different types of failure based on product condition beyond just reliability and age deterioration. Also, the cost model is structured based on the product condition derived from lifecycle data, and finally, a real work dataset is employed to show the model's application.

In addition to the repair decision, the DTMC has shown its success in modeling other decision-making processes. To name a few, Abboud (2001) built a discrete-time Markov model to manage production-inventory cost (Abboud, 2001). Behdad and Thurston (2011) developed a DTMC to identify consumer electronics' optimal upgrade levels (Behdad and Thurston, 2011). Ranjith et al. (2013) discussed timber bridge elements' prediction to predict deterioration degree (Ranjith et al., 2013). Although previous researchers have used Markov models to handle repair and replacement decisions, there is still limited literature on repair and replacement of medical equipment and

factors influencing this domain's decision-making process. The DTMC is a random procedure through the transition from one state to another state and has an important Markov property called "memoryless" (Arif and Shahid, 2018). Also, the MDP framework is constructed based on the four elements including set of states, actions, transition probability, and rewards (Goyal and Grand-Clement, 2018, Zheng and Siami Namin, 2018). When implementing the MDP framework, the MDP framework must be satisfied with Markov property (Zheng and Siami Namin, 2018). Previous researchers applied MDP to different decision-making situations. For example, Marais (2013) used dynamic programming to determine the flows of no failure, repair, and replacement to maximize the present value of net profit (Marais, 2013). Xia (2018) developed the mean-variance optimization problem based on the MDP framework to minimize the variance of system rewards (Xia, 2018). Thodoroff et al. (2018) solved temporal regularization by the MDP framework for different policy evaluation settings (Thodoroff et al., 2018). Roy et al. (2019) proposed a new Reinforcement Learning algorithm by considering an infinite-horizon average reward MDP framework to find optimal policy (Roy et al., 2019).

Table 1 shows the comparison of the existing models and the proposed model in this study on repair and replacement decisions. In the previous studies, the replacement priority decisions on medical devices are often determined by computing PRI. However, when calculating the PRI, some subjective features such as obsolescence and subjective score range of each feature are considered. To overcome the limitations of descriptive decision-making methods, several researchers developed data-driven optimization models to find the optimal decisions. Most researchers built their optimization frameworks based on cost minimization, instead of profit maximization. The cost minimization does not guarantee the maximum profit. Different from previous data-driven optimization models, the proposed model in this study focuses on profit maximization and further incorporates consideration of different failure reasons for a product. Moreover, The proposed cost structure is different from other studies in which the repair and replacement will be determined for each type of failure reason.

This study aims to model and analyze the decision-making process on the timing of repair or replacement of medical devices. The DTMC model has been used to calculate the transition probability from one state to another state. In addition, an MDP framework has been developed with four elements: states, actions, transition probability, and rewards. Among these elements, the transition probability is obtained from the DTMC model. The MDP framework provides the optimal decisions on repair or replacement. Since the MDP framework is constructed from the DTMC model's output, we rename our framework as the MDP-DTMC model. We use a dataset of the repair and maintenance logs of medical devices to estimate the parameters needed in the model. The proposed optimization model is built based on different failure reasons, and the corresponding cost structures are applied in the optimization model. The optimization model provides optimal repair and replacement decisions for each type of failure reason.

Table 1
Comparison of existing models and the current study.

Reference	Method	Subjective Features	Optimization Model	Failure Type	Cost Structure	Real Case	Maximum Profit	Minimum Cost
(Mummolo et al., 2007)	PRI	✓				✓		
(Basiony, 2013)	PRI	✓						
(Taylor and Jackson, 2005)	PRI	✓				✓		
(Taghipour et al., 2011)	PRI	✓		✓		✓		
(Rajasekaran, 2005)	PRI	✓						✓
(Love et al., 2000)	MCM		✓		✓			✓
(Pan and Thomas, 2010)	MCM		✓		✓			✓
(Ranjith et al., 2013)	MCM		✓			✓		
(Marais, 2013)	MDP		✓		✓		✓	
(Sloan, 2007)	MDP		✓		✓	✓		✓
(van den Boomen et al., 2020)	MDP		✓		✓	✓		✓
This study	MDP		✓	✓	✓	✓	✓	

PRI: Priority Replacement Index, MCM: Markov Chain Model, MDP: Markov Decision Processes

The MDP-DTMC model is built based on different types of maintenance reasons. The transition matrix of probabilities and average operating days (operating cycle) used in DTMC have been obtained from the real-world dataset. Finally, an optimization model is framed based on the MDP framework to identify the optimal repair or replacement timing for medical devices. Fig. 1 shows the procedure between the healthcare industry, product dealer, and repair center. The healthcare provider is the primary decision-maker here. Upon receiving a broken product, the failure reason is recorded in their database. If the product fails during the warranty time, the dealer will offer a new replacement at no cost. Otherwise, the healthcare provider needs to decide on repair and replacement. The contribution of this study is to build an optimization framework based on the proposed cost model by considering different types of failure from a real-world dataset.

The remainder of this paper is organized as follows. Section 2 discusses the proposed optimization model. Section 3 provides an overview of the dataset used in this paper. Section 4 summarizes the results. Finally, Section 5 concludes the paper.

2. Methodology

In this section, we first describe the DTMC model. We will then integrate the DTMC process with an optimization framework to determine the optimal repair or replacement decision that maximizes the revenue. Finally, we will discuss a dataset of repair and maintenance of medical devices to estimate the transition probabilities used in the proposed method.

2.1. Discrete-time markov chain (DTMC)

The Markov chain is built based on matrix analysis (Craig and Sendi, 2002), and is a powerful tool to model discrete-time stochastic processes (Ross, 2014). Due to its benefits, it is a popular method for solving problems in different domains such as air quality, suspended sediment concentrations, and flower pollination algorithm (Tsai et al., 2016, He et al., 2017, Chung et al., 2020, Alfa, 2020). Assuming the state space of $\mathcal{S} = \{i_0, i_1, \dots, i_{t-1}, \dots, i, j\}$, the probability that the process moves from state i to state j can be expressed as (Tsai et al., 2016):

$$P\{X_{t+1}=j|X_t=i, X_{t-1}=i_{t-1}, \dots, X_1=i_1, X_0=i_0\} = P_{ij} = P(j|i) \quad (1)$$

where $X_{t+1}=j$ means the process is in state j at time $t+1$, and P_{ij} is the transition probability from state i to state j . Due to the memoryless property of the Markov chain (Li et al., 2017), the transition probability can be expressed as:

$$P\{X_{t+1}=j|X_t=i\} = P_{ij} = P(j|i) \quad (2)$$

In this study, since transition probabilities are unknown, we adopt the maximum likelihood to estimate the transition probabilities by using Lagrange multipliers (Shalizi, 2009). The log maximum likelihood

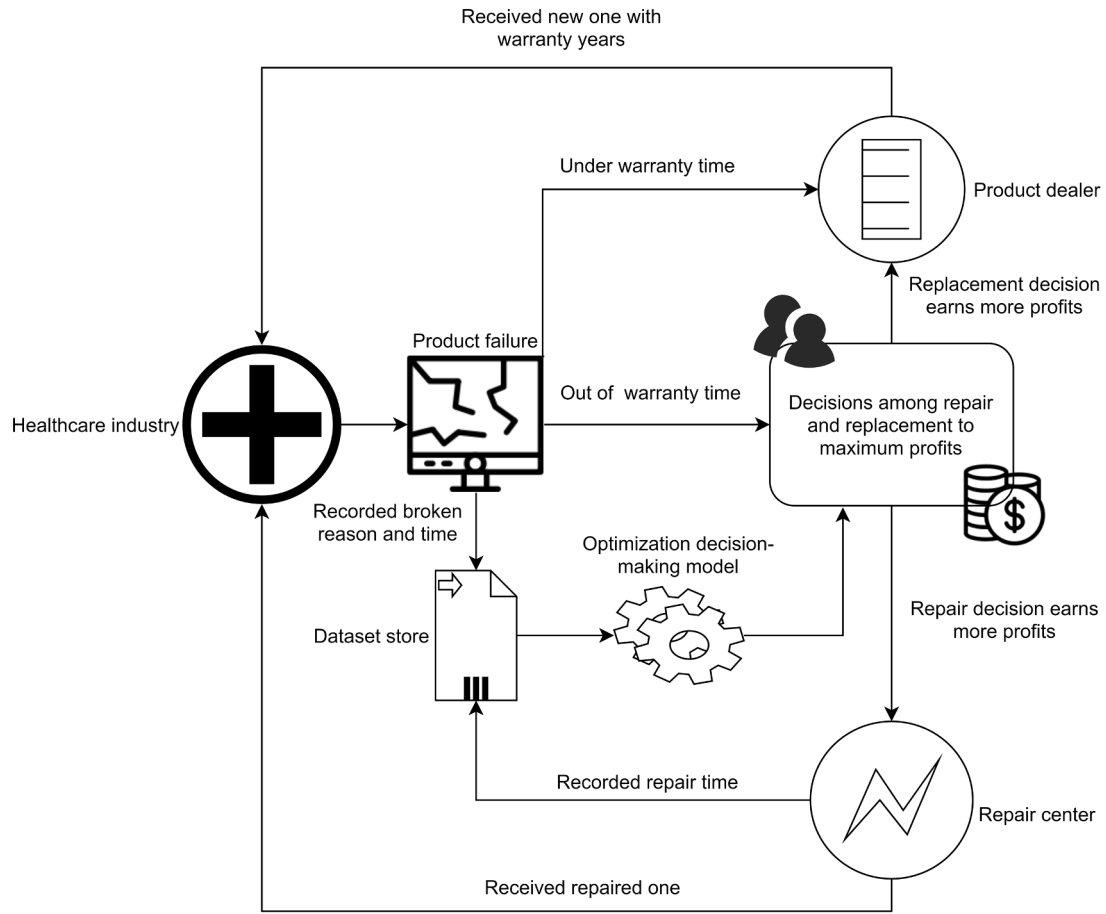


Fig. 1. The relationship between healthcare industry, product dealer, and repair center in this study.

equation of probabilities of the realization can be expressed as:

$$\mathcal{L}(p) = \log P(X_t = x_t) = \log P(X = x_1) + \sum_{i,j} n_{i,j} \log P_{i,j} \quad (3)$$

where X_t is the realization of the random variable at time t , x_t is the observed state from the chain at time t , which is defined as $x_t \equiv x_1, x_2, \dots, x_t$ which shows the history of the chain up to time t for a specific observation, and $n_{i,j}$ is the number of occurrences from state i to state j . Assuming that the dataset has n number of states, the n constraint equations is expressed as:

$$\sum_{j=1}^n P_{i,j} = 1 \quad (4)$$

Based on the n constraint equations for each state, the Lagrange multipliers' equation as the objective function can be represented as:

$$\mathcal{L}(p) - \sum_{i=1}^n \lambda_i \left(\sum_{j=1}^n P_{i,j} - 1 \right) \quad (5)$$

where λ_i is the Lagrange multipliers. Taking derivatives with respect to $P_{i,j}$ from the above equation, the $P_{i,j}$ is as follow:

$$P_{i,j} = \frac{n_{i,j}}{\lambda_i} \quad (6)$$

According to the constraint of Eq. (4), the λ_i can be expressed as:

$$\lambda_i = \sum_{j=1}^n n_{i,j} \quad (7)$$

Now, the transition probabilities can be calculated as:

$$P_{i,j} = P(j | i) = \frac{n_{i,j}}{\sum_{j=1}^n n_{i,j}} \quad (8)$$

Once transition probabilities are calculated, the transition matrix can be derived:

$$M = \begin{bmatrix} P_{1,1} & P_{1,2} & \cdots & P_{1,n} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n-1,1} & P_{n-1,2} & \cdots & P_{n-1,n} \\ P_{n,1} & P_{n,2} & \cdots & P_{n,n} \end{bmatrix} \quad (9)$$

Each device's condition upon failure is defined as a state that includes failure type such as Random Failure, Physical Damage, and others. The states do not have sequential relations. It should be noted that the state of a new device is labeled as "New", where while new devices can transform to other types of failure, there is no transformation from other failure states to the state "New". The transition probabilities between states will be determined based on the number of failures recorded in the repair log available in the whole database. In addition, the average operating days from state i to state j can be expressed as:

$$D_{i,j} = \frac{\sum_{k=1}^{n_{i,j}} (d_{i,j})_k}{n_{i,j}} \quad (10)$$

where $D_{i,j}$ is the average operating days for a device moving from state i to state j , and $(d_{i,j})_k$ is the k^{th} record of operating cycle data from state i to state j . The average operating days gives information about the average operating days from state i to j .

2.2. The optimization framework

An optimization framework is created to identify the optimal repair or replacement decision. The objective function consists of three components: immediate costs, operating revenue, and the expected net present value.

Marais (2013) used a semi-Markov approach to maximize profits under general repair decisions, and considered the expected net present value that includes immediate costs such as repair and replacement cost, operating revenue, and the expected net present value of the next step based on failure times and virtual ages as the objective function (Marais, 2013). Since we will model product lifespan, we need to consider the present and future value of profit. The present value is calculated as:

$$PV = \frac{FV}{(1+r)^p} = \beta \cdot FV \quad (11)$$

where PV is the present value, FV is the future value, r is the annual discount rate or rate of return, p is the number of periods, and β is the discount factor, which can be expressed as $\beta = (1+r)^{-p}$. The present value can be obtained by future value multiplied by a discount factor. The discount factor, the future value as operating revenues, and repair or replacement cost as the immediate cost can be expressed as:

$$\beta_{i,j,\delta} = (1+r)^{-p} = \begin{cases} \left(1 + \frac{r}{100}\right)^{-\left(\frac{D_{i,j}}{365}\right)} & \text{for } \delta = 1 \\ \left(1 + \frac{r}{100}\right)^{-\left(\frac{WD}{365}\right)} & \text{for } \delta = 0 \end{cases} \quad (12)$$

$$O_{i,j,\delta} = \begin{cases} D_{i,j} \cdot OR & \text{for } \delta = 1 \\ WD \cdot OR & \text{for } \delta = 0 \end{cases} \quad (13)$$

$$r_{i,\delta} = \begin{cases} -ARD_i \cdot RC & \text{for } \delta = 1 \\ -NC & \text{for } \delta = 0 \end{cases} \quad (14)$$

where $\beta_{i,j,\delta}$ is the discount factor from state i to state j based on decision δ ($\delta = 1$ means repair; $\delta = 0$ means replacement). If the decision is to replace, the new product can work at least during the warranty days labeled as WD before being failed. Since the annual discount rate is considered, the number of periods can be obtained by dividing it into 365 days. $O_{i,j,\delta}$ is the operating revenue as the future value from state i to state j , and OR is the operating revenue per day based on decision δ . $r_{i,\delta}$ is the immediate cost at the start of each period. Eq. (13) is the future value of operating revenue calculated based on average operating days from state i to state j (repair decision) and warranty days (replacement decision). Eq. (13) is multiplied by the discount factor, presented in Eq. (12), to find the net present value. Eq. (14) shows the immediate cost of repair or replacement decision at the present time, so it is not multiplied by the discount factor.

Marais (2013) applied a semi-Markov decision process combined with discounted cash flow techniques to evaluate the net present value of profit based on the different status of failure using a dynamic programming approach (Marais, 2013). Several assumptions are defined in Marais's model including modeling the system as a semi-Markov decision process, describing the failure state by the number of failures and the product age, and assuming operating revenues among other

parameters. The stochastic deterioration model was defined by the number of failures and product age in which the number of failures and product age defines the probability density function of the first time to failure. Based on the probability density function, the transition probabilities and the expected mean durations for repair and replacement can be computed. Moreover, the cost and revenue flow was determined based on three conditions including no failure, repair, and replace. Marais used a numerical example to show the application of the model by assuming a gamma probability density function for the first time to failure, and defining repair level, annual interest rate, among other parameters. The current study aims to extend the previous model. Inspired by (Marais, 2013) contributions, this study also considers immediate costs, operating revenue, and the expected net present value. However, the current study considers different failure reasons beyond just the number of failures and product age to build the decision-making framework. In addition, the study uses a big dataset of historical repair and maintenance of medical devices as a case study for calculating transition probabilities for different types of failure that may occur to a device. Our model is built based on proposed cost structures in Eq. (12) to (14). Therefore, the value of each decision can be represented as:

$$V(i) = -r_{i,\delta} + \sum_{j=1}^n P(j|i) \beta_{i,j,\delta} O_{i,j,\delta} + \sum_{j=1}^n P(j|i) \beta_{i,j,\delta} V(j) \quad (15)$$

where $V(i)$ is the net present value (revenue - cost), $r_{i,\delta}$ is the cost of repair or replacement, $P(j|i)$ is the transition probability from state i to state j . In Eq. (15), the first item is immediate costs, which depends on repair or replacement. The second item is the operating revenue with the present value. This item is multiplied by transition probability based on the different routes from state i to state j . The average operating days will be different depending on state i and state j . This point will be further discussed in future sections. The third item is the expected net present value of the next future lifecycle over an infinite time horizon. The process is considered as an infinite horizon. Fig. 2 shows the process from state i to j and from state j to k for the reward earned over an infinite number of periods. We presented two periods to clarify the process further. State i , j , and k all represent failure reasons such as Random Failure, Physical Damage, and Battery Related.

State i is the initial state as the start time in period 1. The immediate cost is the repair or replacement cost at state i as the start time. The first operating revenue will start from state i to state j . The operating revenue is multiplied by a discount factor to be the present value in initial state i . The net present value of expected reward in the state i as the start timepoint includes direct cost and operating revenues. The process of the second period is the same as the process of the first period.

The linear programming approach can be applied to the above Markov decision process to solve the optimal expected discounted reward for each state. Finding the best strategy or decision that maximizes Markov chain's expected value over an infinite horizon is equivalent to solving the following linear programming problem Puterman, 1994). Based on Eqs. (12) to (15), the optimization model is as follow:

$$\text{Min} \sum_{j=1, \dots, n} V(j) \quad (16)$$

Subject to

$$V(i) \geq -ARD_i \times RC + \sum_{j=1}^n P(j|i) \left(1 + \frac{r}{100}\right)^{-\left(\frac{D_{i,j}}{365}\right)} (D_{i,j} \times OR) + \sum_{j=1}^n P(j|i) \left(1 + \frac{r}{100}\right)^{-\left(\frac{D_{i,j}}{365}\right)} V(j) \quad (17)$$

for repair decision with $\delta = 1$

$$V(n) \geq -NC + \left(1 + \frac{r}{100}\right)^{-\left(\frac{WD}{365}\right)} (WD \times OR) + \sum_{j=1, \dots, n} P(j|i) \left(1 + \frac{r}{100}\right)^{-\left(\frac{D_{ij}}{365}\right)} V(j) \quad (18)$$

for replacement decision with $\delta = 0$

$$i = 1, 2, \dots, n-1 \quad (19)$$

$$V(1), V(2), \dots, V(n-1), V(n) \geq 0 \quad (20)$$

where ARD_i is average repair or maintenance days for the state i , NC is replacement cost, WD is warranty days, and RC is repair costs per day. $V(1)$ to $V(n-1)$ are the states with repair decision except for New state with replacement decision as $V(n)$. In this study, we describe the state “New” to describe the use of a new product. Therefore, when a broken device is replaced with a new device, the new device does not have any failure record. Therefore, the replacement decision will open a state New as a starting point.

The medical devices dataset provides us the information of transition probabilities between failure types (states). Also, it provides the chance of moving from “New” to any other failure types. So, it is assumed that the New state is equivalent to a new device or replacement decision. Also, since we did not have access to the device acquisition time, we assume that a new device can work at least during the warranty time before it is failed.

According to the optimization model, the $V(i)$ is calculated for each state i . When one of the repair states among $n-1$ types has a value greater than the value of state n (replacement), the decision is to repair, and vice versa.

2.3. Frequency analysis

According to MarketsandMarkets, the global infusion pump market will reach near \$16 billion by 2023 from near \$12 billion in 2018

(MarketsandMarkets 2021). The case study described in this paper is based on the dataset provided for a medical device labeled as Device A, which is applied in infusion pumps and multitherapy. The dataset provides 24,516 records of repair and maintenance logs of Device A over a 10-year time horizon from 2008 to 2018. Device A has 23 different maintenance reasons, such as battery-related issues, random failure, and physical damage. To further understand the device's repair and maintenance needs, frequency analysis has been conducted to analyze the failure count, time to repair (TTR), and failure count per 72 hours to analyze different maintenance reasons. The TTR is the interval time between the repair start time and completion time, and failure count per 72 hours is the number of failures in 72 hours for each maintenance reason.

The first step is to fit the best probability distribution to the factors mentioned above. Previous studies have analyzed failure datasets using different distributions. For example, Patil (2019) uses Normal, Lognormal, and Weibull distributions to analyze the hardware and software failure data (Patil, 2019). Lamprea et al. (2019) used Weibull for the mean time between failure of marine gas turbines (Lamprea et al., 2019), and Sukhwani et al. (2016) used Gamma distribution for the software reliability analysis (Sukhwani et al., 2016). Wessels (2007) adopted Weibull and Exponential distribution to analyze reliability (Wessels, 2007). This study selected five distributions, including Gamma, Normal, Lognormal, Weibull, and Exponential, to analyze the dataset.

The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are used to fit the best probability distribution to the failure count, TTR, and failure count per 72 hours. The best-fitted probability distribution has the minimum AIC and BIC values compared to other distributions. AIC or BIC for identifying the best-fitted probability distribution is common in the literature (Mutua, 1994; Haddad and Rahman, 2011; Rahman et al., 2013; Alam et al., 2018). The AIC is based on the maximum entropy principle and is given by (Akaike, 1998):

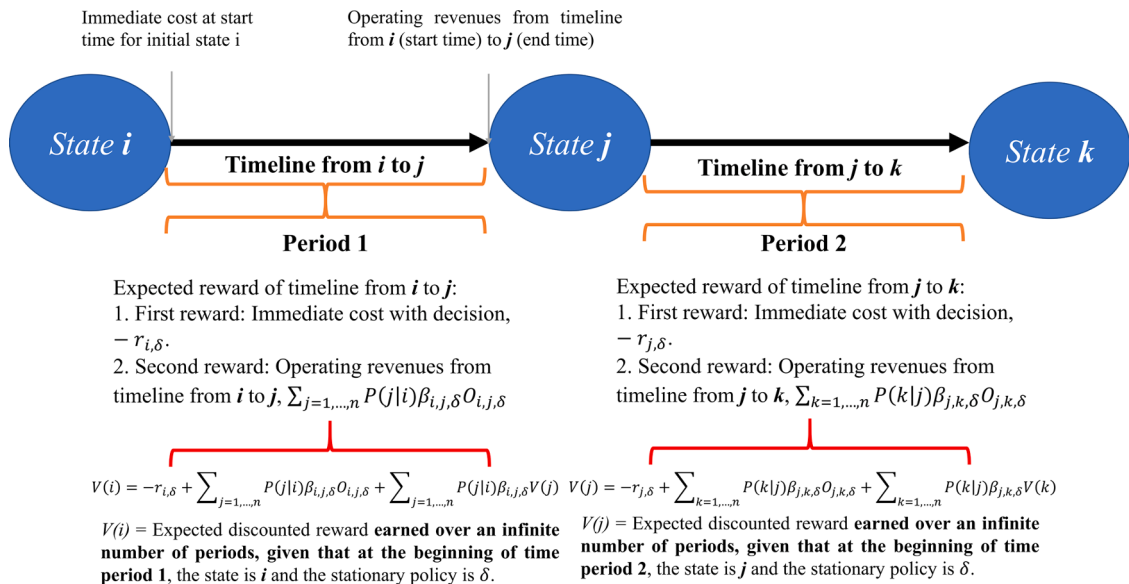


Fig. 2. The relationship between periods 1 and 2 for initial state i , state j , and state k in periods 1 and 2.

Table 2

The description of dataset fields.

#	Data Attribute	Description
1	WO Number	The work order of repair
2	Type Code	Product type
3	First Asset Number	Property unique number
4	First Asset Description	Function of product
5	First Asset Manufacturer Name	Maintenance manufacturer
6	First Asset Model Number	Product name
7	First Asset Serial Number	Property unique serial number
8	Date Created	Repair start time
9	Completed Date	Repair completion time
10	Determination Description	The reason for repair

$$AIC = 2k - 2\ln(\hat{L}) \quad (21)$$

where $\hat{L}$ denotes the maximum value of the likelihood function and k is the number of the probability distribution parameters fitted to real data. The best-fitted distribution has a minimum value of AIC. The BIC has a similar equation to the AIC and is based on the Bayesian framework (Schwarz, 1978):

$$BIC = \ln(n)k - 2\ln(\hat{L}) \quad (22)$$

where n denotes the sample size. Section 3 provides further information on the dataset and the outcomes of frequency analysis.

3. Background of dataset

Although the prior studies (Mummolo et al., 2007, Taylor and Jackson, 2005, Taghipour et al., 2011, Rajasekaran, 2005) investigated repair decisions, the factors contributing to the score are determined by human subjects instead of observed data. In this study, we investigated 24,516 repair and maintenance records from 2008 to 2018 for a medical device labeled as Device A, and we have 5,171 individual products. The dataset is provided by one of the largest healthcare providers in the US. The function of Device A is infusion pumps and multitherapy for medical purposes with the price of \$895. Because we do not have the first active time of each individual product, we estimate that the healthcare industry buys 2,585 Device A every 5 years, assuming that the minimum operation time of Device A is 5 years. It seems that the healthcare industry needs to spend \$462,715 ($895 \times 2585/5$) each year. Also, the average number of repair and maintenance records per year is 2,452. If the healthcare industry decided to replace whatever type of failure, the cost for purchasing new Device A will be \$2,194,540 (895×2452) every year. Since the healthcare industry reuses Device A for five years, they do not need to spend the replacement cost each year.

Each repair record has 10 data attributes, as shown in Table 2. First Asset Number and First Asset Model Number provide sufficient information to identify an individual product's unique code and the product's name. The First Asset Description provides information on the function

purposes like infusion pumps and multitherapy. The attributes Date Created and Completed Date show the start time and the completion time for each repair record. Finally, the Determination Description provides the record of repair and maintenance reasons.

Table 3 shows 23 failure types identified from Determination Description, the number of records of each type, and the average days for repair or maintenance. The first twenty-second states are derived from 24,516 records. The final 'New' state is defined by us to describe the use of a new device. This state somehow reflects the point that among the 24,516 records, we have 5,171 individual products. Each individual product has only one "New" state reflecting the use of a new product. The 23 states are independent and do not have sequential relations. For example, if the current failure reason is Random Failure, the next failure can be any one of the states. The transition probabilities reflect the chance of moving from the current state to the next once based on the percentage of failure transitions observed in the dataset. Table 3 shows that Physical Damage, Random failure, and No problem Found are the most frequent failure types. Also, the average time of repair for PM Failed, Damaged Beyond Repair, and Applicable is the highest. For example, the average time of repairing a Device A failed due to PM Failed is 3 months.

4. Case study and results

4.1. Outcomes of frequency analysis

According to Section 2.3, five distributions, including Gamma, Normal, Lognormal, Weibull, and Exponential distributions, will be used to fit the frequency analysis of the failure count, TTR, and failure count per 72 hours. The best-fitted probability distributions are shown in Table 4. Some states are not shown in Table 4 as we did not have sufficient data for them. Most of the best-fitted probability distributions are Weibull and Gamma. The fitted distributions help us calculate different probability values. For example, if a device has a battery-related failure, by employing the TTR on Gamma distribution, we can calculate the likelihood that it may take over 15 days to repair the device. The five distributions fitted to the data have been commonly used in the literature for reliability analyses (Patil, 2019, Lampreia et al., 2019, Sukhwani et al., 2016, Wessels, 2007).

According to the best-fitted distributions, Fig. 3 to Fig. 5 reveals the probabilities on failure count, TTR, and failure count per 72 hours, respectively. The number listed on each bar means the probability (%) of that property. The X of $P(X \geq 1)$ is used as a general random variable for failure count, TTR, and failure count per 72 hours. For example, the $P(X \geq 1)$ for failure count shows the probability that the number of failures is greater or equal to 1. For TTR, it shows the probability that the repair time is greater or equal to 10 days. And for failure count per 72 hours, it shows the probability that the number of failures per 72 hours is greater or equal to 1.

In Fig. 3, three states have higher probabilities among other states,

Table 3

The description of determination description.

#	Determination Description (State)	Count	Average Days of Repair/Maintenance	#	Determination Description (State)	Count	Average Days of Repair/Maintenance
1	Accessory Problem	496	8	13	PM Related	1309	10
2	Applicable	11	19	14	Prior To Clinical Use - Failed	1	13
3	Battery Related	103	6	15	Project	12	1
4	Damaged Beyond Repair	2	28	16	Random Failure	7348	11
5	Environment Related	499	7	17	Random Software Failure	241	11
6	Modification/Upgrade	37	5	18	Rechargeable Battery Failure	1	0
7	Network Problem	62	2	19	Repeat Repair	10	7
8	No Problem Found	6579	8	20	Setup Related	27	4
9	Not Applicable	4	1	21	Tech Support	153	10
10	Physical Damage	7384	8	22	Use Error	188	6
11	PM Failed	1	102	23	New	5171	0
12	PM Passed	48	8				

Table 4

The best-fitted probability distribution for each state.

State	Failure Count	TTR (10 days)	Failure count per 72 hours
Accessory Problem	Gamma	Lognorm	Gamma
Battery Related	Gamma	Gamma	Gamma
Environment Related	Weibull	Weibull	Gamma
Modification/Upgrade	Weibull	Gamma	Gamma
Network Problem	Weibull	Weibull	Gamma
No Problem Found	Weibull	Lognorm	Expon
Physical Damage	Weibull	Lognorm	Weibull
PM Passed	Weibull	Weibull	Gamma
PM Related	Gamma	Weibull	Weibull
Random Failure	Lognorm	Weibull	Weibull
Random Software Failure	Gamma	Weibull	Weibull
Tech Support	Weibull	Gamma	Gamma
Use Error	Weibull	Gamma	Gamma

including No problem found, Physical Damage, and Random Failure. The question is whether to repair or replace Device A when encountering these issues. The decision model in Section 4.3 will be used.

Fig. 4 shows the results for TTR. In $P(X \geq 1)$, Modification/Upgrade, PM Passed, and Tech Support are states with a higher chance of TTR more than 10 days. In the case of $P(X \geq 2)$ to $P(X \geq 4)$, PM Passed, and Tech Support are higher among others. For $P(X \geq 5)$, the PM Passed is the most frequent state. Higher probabilities on TTR mean higher repair cost and ultimately higher chance of being replaced instead of repair.

Fig. 5 shows the results for the failure count per 72 hours. The reason to use per 72 hours is that we attempt to observe the frequency by moderate time-scale instead of tight scales like one day or long scales like a week. For $P(X \geq 1)$ to $P(X \geq 5)$, three states, including No problem found, Physical Damage, and Random Failure, have higher probabilities among other states.

The frequency analysis analyzes metrics such as failure count, TTR, and failure count per 72 hours. The analyses provide an overview of the situation and help us quantify the transition probabilities for the DTMC process in the next section.

According to Fig. 4, the average TTR can be computed as:

$$\text{average TTR} = \left[\sum_{i=1, \dots, n} \int_0^{\infty} z f_i(z) dz \right] / n \quad (23)$$

where $f_i(z)$ is the probability density function of the repair time for a product with failure state i . The repair time can range from 0 to infinity. The integral part shows the average repair time for each state i . Each state has a different average repair time. After computing each state's average repair time, the grand average TTR will be calculated by

$$V(i) \geq -ARD_i \times 20 + \sum_{j=1}^{23} P(j | i) \left(1 + \frac{3}{100} \right)^{-\left(\frac{D_{ij}}{365} \right)} (D_{ij} \times OR) + \sum_{j=1}^{23} P(j | i) \left(1 + \frac{3}{100} \right)^{-\left(\frac{D_{ij}}{365} \right)} V(j) \quad (25)$$

for repair decision with $\delta = 1$

$$V(23) \geq -895 + \left(1 + \frac{3}{100} \right)^{-\left(\frac{WD}{365} \right)} (WD \times OR) + \sum_{j=1}^{23} P(j | i) \left(1 + \frac{3}{100} \right)^{-\left(\frac{D_{ij}}{365} \right)} V(j) \quad (26)$$

for replacement decision with $\delta = 0$

averaging n states. Based on Eq. (23), the average TTR for Device A is 18 days. Assuming that the total repair cost for Device A is 40% of the original purchase price of \$895. Thus, the repair cost per day is around \$20 which is 40% of \$895 divided by 18 days. The \$20 repair cost per day is an input to the optimization model.

4.2. Probability transition matrix and the average operating cycle

The transition probabilities and the average operating day are calculated using Eqs. (8) and (10). The results are summarized in Figs. 6, and 7.

As listed in Fig. 6, all current states will move into No Problem Found, Physical Damage, and Random Failure with higher probabilities, as we expected in Section 4.1. For example, if the current state is Battery Related, after repairing, the next state will be No Problem Found, Physical Damage, and Random Failure with a probability of 0.19, 0.22, and 0.39.

Fig. 7 shows the average operating time for Device A before having any issue in the next state. None means that the dataset does not have any record from the current state to the next state. The dataset also does not have records from the current state New to the next state as the dataset does not include the operating start time. Regardless of the operating start time, the information on the probability from state New to the next state can be evaluated since the dataset has records on the first repair order. With the first repair order info, we can evaluate the transition from a new product to the first repair order. Thus, it is possible to evaluate the transition probabilities from the state New to other states. The replacement decision will open the state “New” as the starting point of using a new product.

According to Fig. 7, states such as Damaged Beyond Repair, Prior To Clinical Use – Failed, and Rechargeable Battery Failure do not have transition information from them to other states. These three states do not have enough data, as shown in Table 3. Also, after repairing or maintenance, Device A can operate for a long time. For example, if the repair reason is Battery Related, it can work up to 960 days after repairing the device. In contrast, in some cases like Repeat Repair, the device can work for 2 or 3 months.

4.3. Specific case for the optimization model

The model has been run for a specific case, as summarized in Table 5. A 3% discount rate is considered. OR is assumed to be \$1 to \$30 per day, RC to be \$20 per day, and WD to be 1, 2, and 3 years. Besides, the prices for a new Device A is \$895. Using Eqs. (16) to (20), the optimization model is formulated:

$$\text{Min} \sum_{j=1, \dots, 23} V(j) \quad (24)$$

Subject to

$$i = 1, 2, \dots, 21, 22 \quad (27)$$

$$V(1), V(2), \dots, V(22), V(23) \geq 0 \quad (28)$$

$$OR = 1, 2, \dots, 21, 30 \quad (29)$$

$$WD = 1, 2, 3 \quad (30)$$

The information of $V(1)$ to $V(22)$ is described in Table 3. $V(23)$ represents the state “New”. After calculating each state’s profit, the optimal decision of repair or replacement can be determined.

4.4. Results and sensitivity analysis

The transition probabilities are used as inputs to the optimization model. The outcome of the optimization model gives us the profit of each state, $V(1)$ to $V(23)$. If the profit of state i , $V(i)$, is greater than the state New, $V(23)$, the decision is to repair, otherwise, to replace. For example, the Network Problem’s value of \$ 321,811 is less than the value of state New with \$325,437. Thus, the decision will be to replace it with a new Device A.

Fig. 8 shows the optimal decision in each state along with each state’s expected value, for the case of \$20 repair cost and \$30 operating revenue per day. The repair decision is shown by the red bar and the replacement decision by the blue bar. If a state’s value is greater than the value of state New (the yellow bar), the decision is to repair. Otherwise, to replace.

Fig. 9 shows a sensitivity analysis on warranty durations when the repair cost is \$20 per day. For some states such as Damaged Beyond Repair, Prior To Clinical Use – Failed, and Rechargeable Battery Failure, since there is not sufficient data as discussed in Section 4.1, the decision always is to replace, as shown in Fig. 8. However, if more data is

recorded for these three states, the optimal decision may change. The white color in Fig. 9 shows the uncertain decision due to limited data. Also, the optimization model shows if the value of $V(1)$ to $V(22)$ is less than $V(23)$, the decision is to replace it.

Two factors of operating revenue per day and warranty have been used for conducting sensitivity analyses. The operating revenue varies from \$1 to \$30 per day, and the warranty varies from 1 year to 3 years. The repair costs are \$20 per day based on the discussion in Section 4.1. In Fig. 9 (a), the x-axis corresponds to each state as defined in Table 3, the y-axis is the operating revenue from \$1 to \$30 per day, and the z-axis is warranty time from 1 to 3 years. The repair decision is shown with red color, the replacement with blue color.

According to Fig. 9, the decision to repair or replace depends on the product condition. For example, if the product is in State 2, the decision is to replace it. The main factor contributing to the replacement is the lengthy repair time. The repair time for State 2 is 19 days. However, the repair time is not the only contributing factor. For example, although the PM Failed state has the highest repair time among all states, as operating revenue per day increases, the decision will be changed from replacement to repair. With probability 1, the device will move from the PM Failed state to the PM Related state. The PM Related state can generate more profit than state New. Thus, if the failure state is PM Failed, the decision will be to repair.

In Fig. 9, as operating revenue per day increases, four situations may happen. The first situation is to always repair, regardless of operating revenue per day. For example, in State 8, “No Problem Found”, the decision is to repair. With minor repairs such as clear up or maintenance, Device A with state No Problem Found can still generate profits. The second situation is to always replace, regardless of operating revenue per day. One example is State 2.

The third situation is when the decision changes from repair to

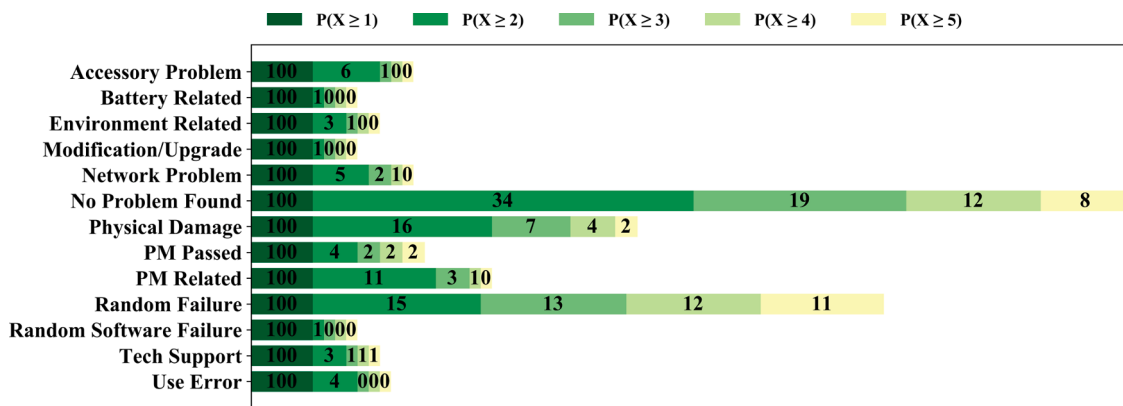


Fig. 3. The probability (%) of failure count considering the best-fitted PDF: $P(X \geq 1)$ to $P(X \geq 5)$.

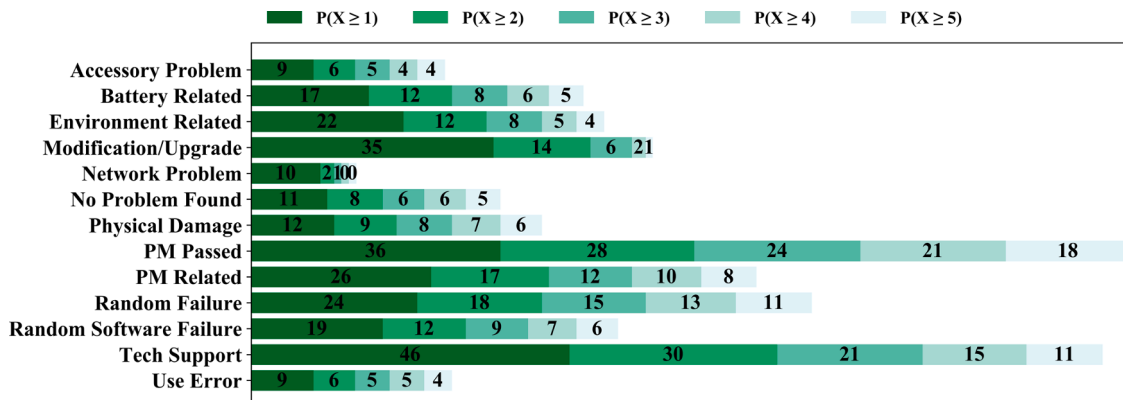


Fig. 4. The probability (%) of TTR (10 days) considering the best-fitted PDF: $P(X \geq 1)$ to $P(X \geq 5)$.

replacement as operating revenue per day or warranty time increases. Table 6 shows the sensitivity analysis based on the operating revenue per day to highlight the operating revenue threshold. If operating revenue per day changes by 20%, from \$5 to \$6, the decision will change from repair to replacement. The profits increase by \$10,904 (from \$49,209 to \$60,113) and by \$11,053 (from \$49,115 to \$60,168) for Network Problem and New, respectively.

In Table 6, regression equations show the operating revenue per day for each state. Overall, by increasing the operating revenue per day, the decision switches from repair to replacement. Also, different warranty times influence the optimal decision as shown in Fig. 10. The profits of the Use Error state do not change among different warranty years, but of State New changes depending on different warranty years. By increasing

warranty years, the decision will change from repair to replacement. Also, the profits increase when the warranty time increases by 50% (from 2 years to 3 years).

The fourth situation is when the optimal decision switches from replacement to repair, as shown in Table 6. If operating revenue per day is less than \$16, the decision is to replace when the state is the PM Failed. When operating revenue per day increases by 6%, from \$15 to \$16, the optimal decision switches from replacement to repair. Also, the fourth situation is not sensitive to the warranty years. By increasing warranty years, some state's decision will change from repair to replacement, but no decision will be changed from replacement to repair.

The four situations are summarized in Table 7. Under different

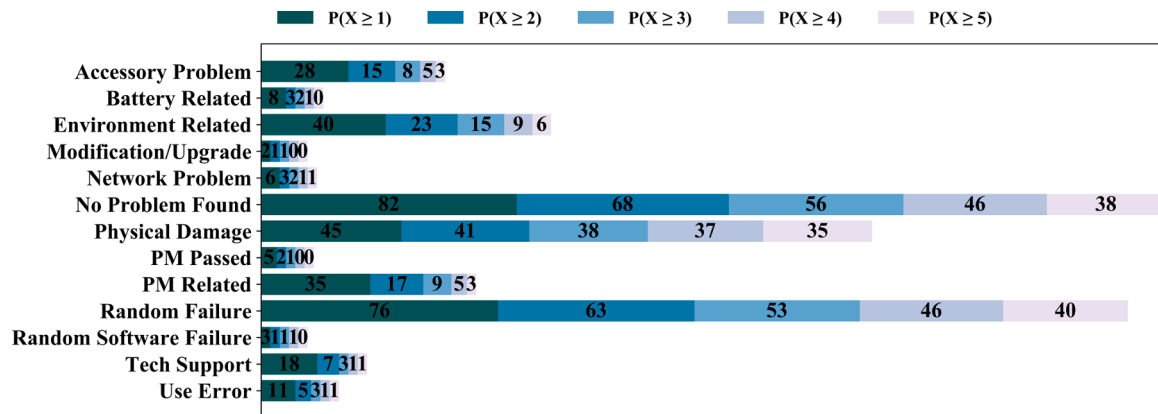


Fig. 5. The probability (%) of failure count per 72 hours considering the best-fitted PDF: $P(X \geq 1)$ to $P(X \geq 5)$.

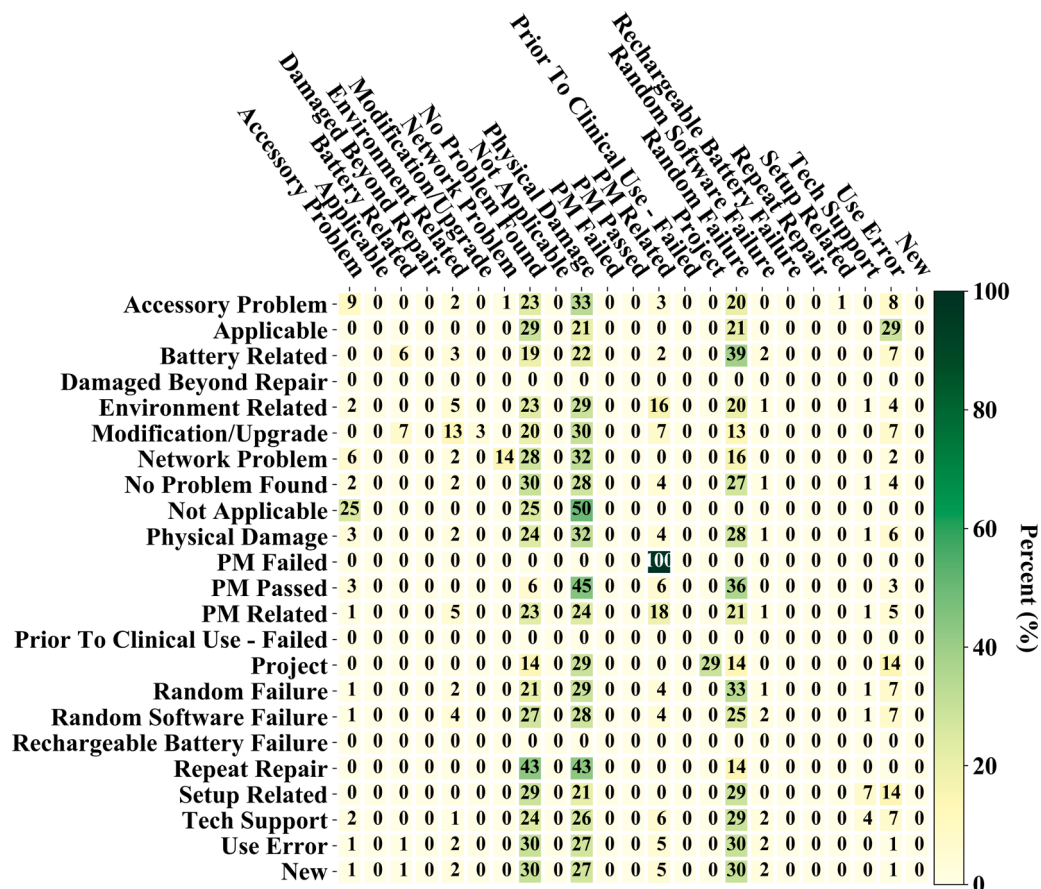


Fig. 6. The transition matrix of probabilities from 2008 to 2018 (y-axis is current state; x-axis is next state).

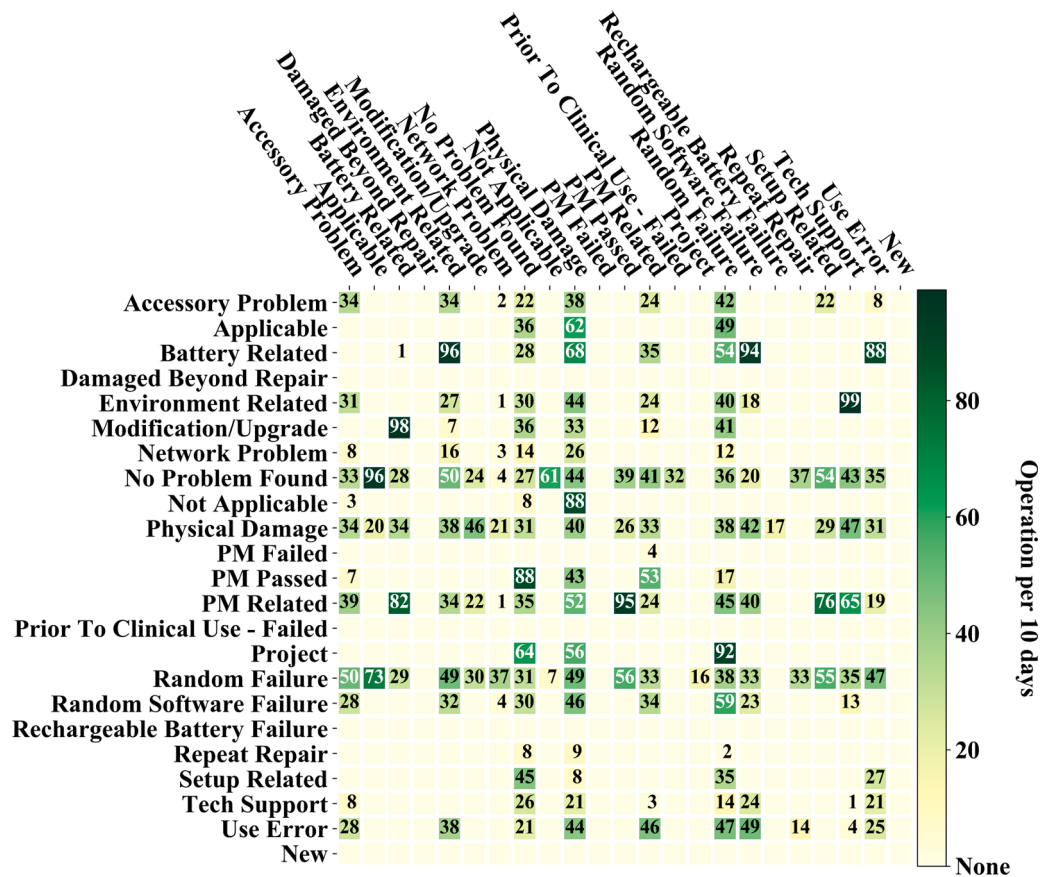


Fig. 7. The transition matrix for average operating days from 2008 to 2018 (y-axis is current state; x-axis is next state).

Table 5
The parameters for the specific case.

Parameter	Values	Remarks
r	3%	Annual discount rate
OR	\$1 to \$30 per day	Operating revenue per day
RC	\$20 per day	Repair costs per day
WD	1 to 3 years	Warranty days
NC	\$895	Replacement costs

operating revenue and warranty times, the optimal decision can be determined.

4.5. Practical implications and limitations

The results help healthcare providers minimize their operational costs and maximize the expected revenue by setting proper repair and replacement policies. In addition, proper decisions reduce the waste generation rate by considering repair instead of replacement to extend the product lifespan. Furthermore, the outcomes can help decision-

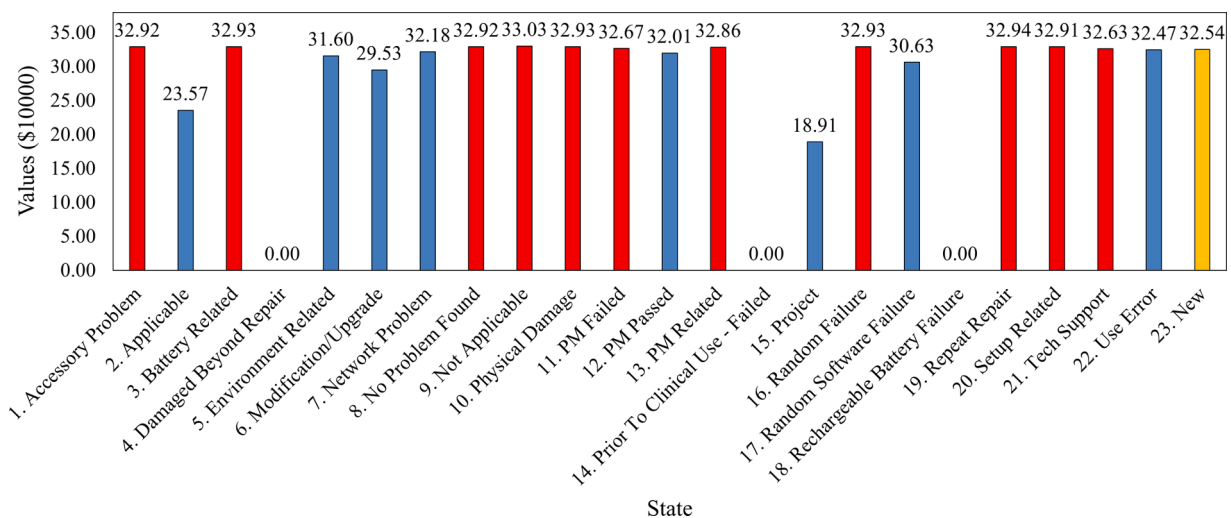


Fig. 8. Decision-making results under \$20 repair costs and \$30 operating revenue per day for 1 year warranty (repair decision with red bar, and replacement decision with blue bar).

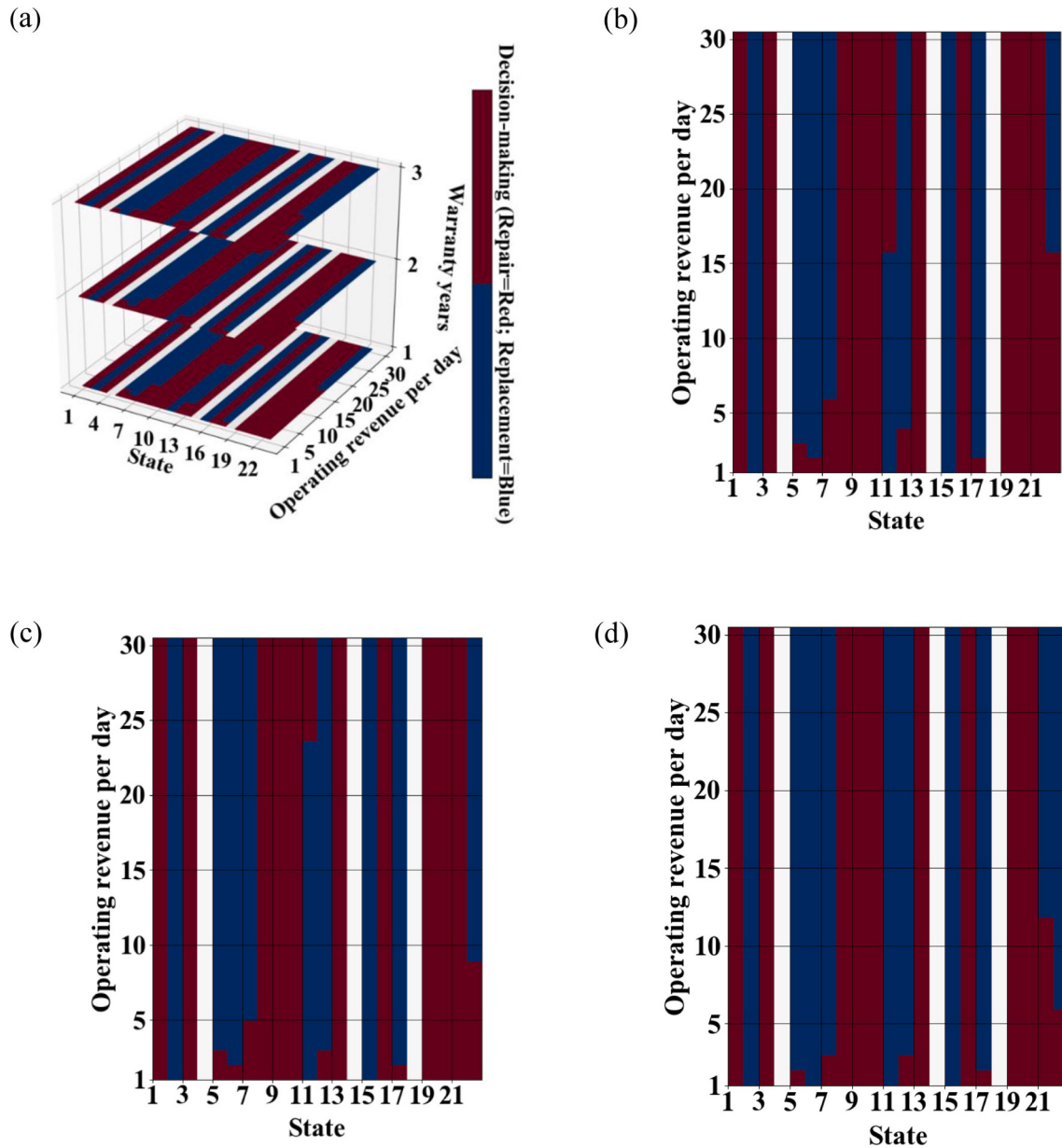


Fig. 9. Decision-making results (repair with red color, replacement with blue color, and an uncertain decision due to limited data with white color) under \$20 repair costs per day: (a) 1 to 3 (z-axis) year warranty, (b) 1-year warranty, (c) 2-year warranty, and (d) 3-year warranty.

Table 6

Sensitivity analysis on the relationship between operating revenue per day (OR) and net present value (V) under \$20 repair cost per day and a 1-year warranty for Network Problem (repair decision) and state New (replacement decision) as a case from repair to replacement decision; and PM Failed (repair decision) and New (replacement decision) as a case from replacement to repair decision.

Example of switching from repair to replacement decision			Example of switching from replacement to repair decision		
Operating revenue per day (\$)	Network Problem (\$)	New (\$)	Operating revenue per day (\$)	PM Failed (\$)	New (\$)
1	5593	4904	11	115033	115433
2	16497	15956	12	126173	126485
3	27401	27009	13	137313	137538
4	38305	38062	14	148453	148591
5	49209	49115	15	159594	159644
6	60113	60168	16	170734	170697
7	71017	71221	17	181874	181750
8	81921	82274	18	193014	192803
9	92826	93327	19	204154	203856
10	103730	104380	20	215294	214909
Linear regression	$V=10904OR-5311$	$V=11053OR-6149$	Linear regression	$V=11140(OR-10)+103893$	$V=11053(OR-10)+104380$

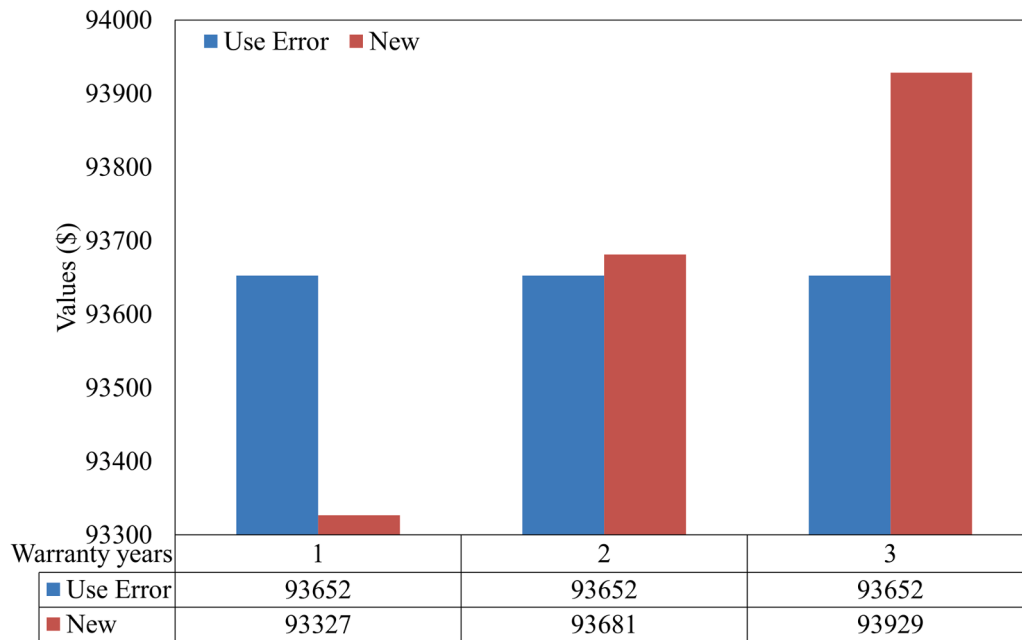


Fig. 10. Sensitivity analysis for warranty year with Use Error state (repair decision) and New state (replacement decision) under 20\$ repair costs per day and 9\$ operating revenue per day.

Table 7

Four classes for the decisions based on sensitivity analysis.

Class	Decision	Illustration	Sensitive condition
1	Repair forever	Regardless of operating revenue per day and warranty year, the decision is to repair.	Not sensitive to operating revenue per day and warranty year.
2	Replacement forever	Regardless of operating revenue per day and warranty year, the decision is to replace it.	Not sensitive to operating revenue per day and warranty year.
3	From repair to replacement	With lower operating revenue per day and warranty year, the decision is to repair. However, by increasing both variables, the optimal decision is to replace.	Sensitive for operating revenue per day and warranty year.
4	From replacement to repair	With lower operating revenue per day, the optimal decision is to replace, and with higher revenue, the decision is to repair.	Sensitive for operating revenue per day.

makers with labor scheduling and efficient operational planning of repair technicians. Finally, the research outcomes can reveal the demand for repair services and help healthcare providers determine whether to outsource repair services or use in-house resources to reduce operational costs.

The outcomes of the model are not limited to medical equipment. They can be applied to other cases such as home appliances, manufacturing machinery, and consumer electronics, to name a few. Overall, the emergence of data collection technologies such as IoT and distributed ledger technologies makes it easier to track individual devices over their entire lifespan and collect lifecycle data like the timing of repair needs and failure events. Therefore, analytical models such as the proposed optimization model help use the collected data toward more sustainable lifecycle engineering.

The proposed model and the corresponding case study have several limitations. For example, the lack of data on different failure reasons such as Damaged Beyond Repair, Prior To Clinical Use–Failed, and

Rechargeable Battery Failure. The dataset quality is another limitation as some records such as the repair start and completion times lack consistency and accuracy. Also, we have analyzed only one type of medical device. However, healthcare providers often deal with hundreds of devices, and they make repair and replacement decisions considering a family of products or product-service bundle available to them. Also, other considerations such as healthcare safety standards, rules, and regulations may influence the decision to repair or replace specific medical equipment.

5. Conclusion

In order to address the cost burden of repair and maintenance of medical devices, this study develops a decision-making framework to help healthcare providers decide on the optimal timing of repair or replacement of medical devices. A discrete-time Markov Chain model is used to model the device lifecycle, and an optimization framework is developed to determine the repair or replace decision. The transition probability obtained from the DTMC model is used in an MDP framework, where the MDP framework provides the optimal repair or replacement decisions. Three factors, including the immediate cost of repair or replacement, operating revenues, and future profits, are considered to form the objective function. A dataset of 24,516 records of repair and maintenance of a medical device from 2008 to 2018 is used to show the application of the model. The dataset was used to identify failure modes and calculate the transition probabilities needed in the optimization model. A frequency analysis and several sensitivity analyses are conducted to elaborate the results further.

The outcomes of the frequency analysis reveal the most frequent failure reasons for infusion pumps in the current dataset, including No problem found, Physical Damage, and Random Failure. The results of the sensitivity analysis show that the repair and replacement decision is influenced by operating revenue per day and warranty times. Four classes of decisions, including always repair, always replacement, switch from repair to replacement, and from replacement to repair, are presented.

This study can be extended in several ways. First, defining the timing of repair or replacement decision is influenced by other factors such as safety standards, regulation, production functionalities, complexity, and new technologies coming to the market. Second, artificial intelligence

and machine learning tools can be used to analyze the product reliability data and provide further insights into Markov chain models. Third, the Markov chain model can be extended to consider the dynamic nature of the problem. Fourth, forecasting models can be used to estimate better the future value of repair or replacement decisions based on economic consequences and environmental and social outcomes such as health equity and public access to the health services, resulting from repair and reuse of medical devices. Finally, comparison with other extant models can further validate the performance of the proposed model.

CRedit authorship contribution statement

Hao-yu Liao: Conceptualization, Methodology, Writing – original draft. **Willie Cade:** Conceptualization. **Sara Behdad:** Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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