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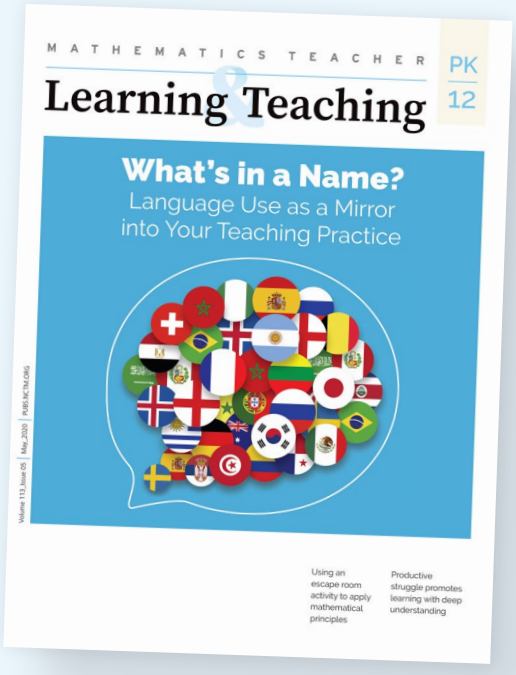

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Discourse Can Create a Learning Culture

When a ninth-grade teacher used discourse moves aligned with responding to students' thinking and explicitly promoting productive dispositions, her students reported having positive experiences.

Kelly Curtis, Katrina Lindo, and Amanda Jansen

When Katrina Lindo taught Integrated Mathematics 1 to ninth- and tenth-grade students, she encountered some students who appeared initially resistant to doing mathematics. Lindo wanted her students to see themselves as capable of making sense of mathematics:

> At the high school level, I think that a lot of students already have a view of themselves as who they are as a student or as a mathematician. With the idea that we're all mathematicians, it's really just about changing their perspective. Because once you believe "Hey, I *am* a mathematician, and all I have to do is try and figure it out . . ." then you get more students that are willing to engage.

Lindo said that being a mathematician is "the process of taking what you know and using what you know to make sense of what you don't know, so we're all mathematicians." We can hear in Lindo's words that she is invested in developing productive identities among all of her students.

We had the opportunity to spend time in Lindo's classroom to discuss with her the classroom discourse that we observed. In this article, we illustrate discourse moves that Lindo used to engage students

with mathematics. She worked to create a culture of learning in her classroom so that students could see themselves as mathematicians. We share these moves together with Lindo's reflections so that readers can consider ways to offer their own students' opportunities to develop productive dispositions and identities.

Lindo's interactions with her students promoted productive dispositions toward mathematics. Productive disposition is the "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (Kilpatrick, Swafford, and Findell 2001, p. 116). Unfortunately, as students move through middle school and high school, their motivation to learn mathematics tends to decline (Chouinard and Roy 2008). It is important to learn what teachers can do to disrupt this trend.

We looked closely at one of Lindo's Integrated Mathematics 1 classes. Lindo identifies as African American. She had six years of teaching experience when we observed her classroom and interviewed her. She now serves her district as a mathematics coach. Her school had more than 2,000 students: 46 percent Black/African American, 3 percent Asian, 24 percent Hispanic/Latinx, 24 percent white, 9 percent English language learners, and 30 percent low income. The students in Lindo's classroom reflected the school's population. Lindo and her students interacted around rich mathematical tasks from the Mathematics Vision Project (MVP) curriculum materials (Hendrickson et al. 2016).

Two types of discourse moves on the teacher's part helped these high school students see themselves as mathematicians: (1) responding to students' thinking and (2) explicit promotion of productive dispositions. These discourse moves are *ongoing* ways of repeatedly interacting with students.

RESPONDING TO STUDENTS' THINKING

How we *respond* to students when they share their mathematical ideas communicates whether and how they experience having a role in developing mathematical knowledge. If we transfer authority to students, they are more likely to see themselves as capable of making sense of mathematics. Lindo responded to students by encouraging in-progress thinking, revoicing student contributions, pressing students to elaborate on their thinking, and advancing students' contributions (see figure 1).

Encourage In-Progress Thinking

Lindo continually invited students to share their thinking about a mathematics problem at any stage of their work. When she asked students to graph $8x + 4y < 16$, some were at a loss about how to begin. She decided to ask them about what made sense to them from a related topic: how to determine intercepts from a linear equation that was written in standard form—by creating a table of x and y values. However, a student, Shani, shared an answer to a different question: how to graph a line once you knew the intercepts, drawing on her prior knowledge about linear relationships. The following is an excerpt from the interaction:

Fig. 1

Encourage in-progress thinking.

Revoice student contributions.

Press students to elaborate on their thinking.

Advance students' contributions.

Discourse moves are ongoing ways of repeatedly interacting with students.

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Mrs. Lindo: The x (intercept) is four?
Shani: I mean the y (intercept), sorry!
Mrs. Lindo: Ah! So, the four here is on our y-axis?
Shani: Yes, . . . and for two (the x-intercept) we did the same thing . . . and then just drew our line.
Mrs. Lindo: So, let me actually add that on here. We have the x-axis, and we have a positive two on the x and a y with a four. You are making a really strong connection for us between the table and how it got onto the graph, but I am still not sure that I am convinced that everyone in here knows where these numbers came from.

Lindo accepted and acknowledged the student's contribution. Then she cycled back to her goal for the conversation—how to find the intercepts. In this way, she demonstrated to students that they were welcome to share whatever they currently understood, while she managed the discourse so that the class could achieve the mathematical goal of the problem.

One of Lindo's goals was to foster intellectual safety (Krall 2018) by inviting students to share what made sense to them.

> Some students have this fear of "I don't want to be wrong!" or "I don't want to sound like I don't know what I'm talking about." For every idea that surfaces, there are other students who may be thinking the same thing who are unwilling to voice it.

Lindo's effort to welcome in-progress thinking aligned with encouraging rough-draft talk in the mathematics classroom (Jansen et al. 2016), such that all ideas are worth exploring and have the potential to be developed.

Revoice Student Contributions

Revoicing is a talk move that can amplify students' ideas (Chapin, O'Connor, and Anderson 2009). At times, Lindo would repeat a student's idea to make sure everyone heard and understood that student. According to Lindo, repeating information "never hurts." Revoicing supports students as they learn to listen and make sense of their peers' thinking.

Lindo modified an MVP problem to include whole numbers and asked the students to write a system of equations or inequalities that represented the situation:

> Janet needs at least 12 balloons, some purple and some silver. The silver balloons cost \$1, and the purple balloons cost \$2. She has no more than \$20 to spend. (Hendrickson et al. 2016, Lesson 5.8, p. 33)

Her students struggled, so Lindo invited them to share their in-progress thinking:

Mrs. Lindo: So, tell us something that you tried on your paper.
Lela: I did $9p + 2s \leq 12$ and then $18 + 2 = 20$.
Mrs. Lindo: So, 18 dollars plus 2 because 18 for the purple because there were 2 each plus 2 for the silver is?
Lela: Twenty dollars.
Mrs. Lindo: Twenty dollars. So, you kind of picked two numbers that would get you less than 12, and then you calculated their cost to make sure that she used all of her money but that she didn't go over.
Lela: Yep.

Even though Lela's inequality was incorrect (she used p and s as labels rather than as variables), Lindo revoiced Lela's idea to amplify her thinking. She wanted to make sure that all students understood where $18 + 2 = 20$ came from and how it helped Lela make sense of the problem. She then invited students to build on that idea.

Lindo wrote students' thoughts on the board as a written form of revoicing and attributed strategies to students. She said that she would "try to make some notations on the board to try to echo what she's saying so that other students can follow it and see it." Revoicing verbally or through recording demonstrated value for students' ideas.

Press Students to Elaborate on Their Thinking

Lindo would press students to continue to elaborate on their thinking during class discussions (Kazemi 1998) as a form of probing students' thinking (Herbel-Eisenmann, Steele, and Cirillo 2013). After a student made a contribution, she would ask the student to add on to that idea: "Tiana, did you have any other pieces to add on to that?" or "James, was there anything that stood out to you about that second inequality that you noticed?" She would also invite other students to explain a peer's thinking.

She asked students to write a system of equations or inequalities that represented a situation:

> Carlos purchased six dog leashes and six cat brushes for \$45. Clarita purchased

three dog leashes and two cat brushes for $19. (Hendrickson et al. 2016, Lesson 5.8, p. 30)

One student claimed that it was an equation and not an inequality. Lindo then pressed the rest of the class, "Can I have someone just restate or revoice what Jaylen is talking about? She says that this would not be an inequality. *Why* would problem 2 (about Carlos and Clarita) not be an inequality?"

Lindo reflected that pressing students for further explanation helped her to assess students' thinking and gave her insights into how to advance students' thinking.

> It's this idea of how we question students. Sometimes, you need to question a student to surface their thinking and really see where they are. Like, you're assessing. And then there are going to be moments where you want to do probing or pushing, like a question that is going to push their thinking forward.

Through the process of being asked to explain more, students' ideas can grow.

Advance Students' Contributions

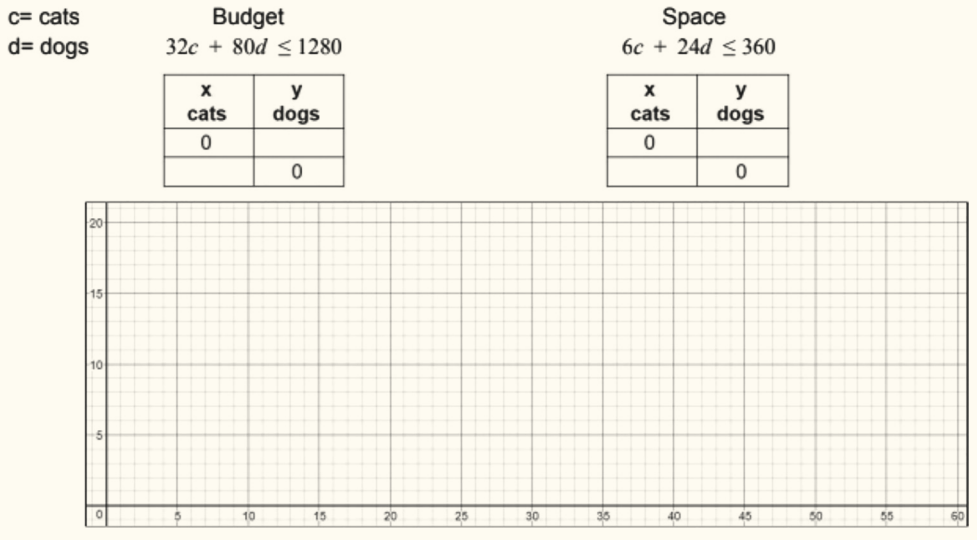
Lindo also attempted to advance students' thinking by connecting students' mathematical ideas to the mathematical goal of the lesson and to one another's ideas. For instance, Lindo presented her students with two inequalities from MVP, which she scaffolded with tables and a coordinate grid (see figure 2) (Hendrickson et al. 2016, Lesson 5.5, p. 17). The original task presented the situations as stories without providing students with the symbolic representation. To focus on the goal of understanding the graphical representation, Lindo displayed the equations of the inequalities during the whole-class discussion. She asked the class to "shout out" what the solution would look like and focused on two specific student responses.

Andrea: Where they are both shaded.
Lily: Where they both connect!
Mrs. Lindo: Lily, it's almost like you're talking about this point of intersection, right? So, if we had equations, not inequalities, our solution would be that point of intersection, but we're looking for a shaded space . . .

Instead of evaluating Lily's response as incorrect, Lindo recognized that Lily noticed the answer to a different question, a solution to a system of linear equations. She highlighted strengths in Lily's thinking by connecting to a relevant mathematics idea. She also advanced Lily's thinking toward Andrea's idea of a shaded space.

Daily, Lindo worked to create safe, collaborative classroom discussions in which students' ideas

Fig. 2



In this scaffolded MVP task, students shade all the points on the coordinate grid that satisfy the constraints.

were foundational in moving mathematics forward. According to Lindo,

> The engagement is really just trying to get the ideas out there, providing an opportunity for students to grapple with those ideas. The hope is that maybe someone else will comment, correct, or put their ideas out there, too, even if they are different.

Such discourse moves have the potential to work together to support students in being mathematical thinkers as well as to help students see themselves as mathematicians.

Some of Lindo's responses to students' thinking align with discourse moves described by Herbel-Eisenmann, Steele, and Cirillo (2013): inviting student participation, revoicing, probing another students' thinking, and creating opportunities to engage with another's reasoning. Some of Lindo's ways of responding were specific examples of these discourse moves. She invited student participation by encouraging in-progress thinking. She probed students thinking by pressing them to explain. She engaged students with another's reasoning by advancing students' contributions or asking them to elaborate on a peer's thinking.

Lindo also enacted an additional set of discourse moves. She talked with her students explicitly about processes of learning and doing mathematics. In doing so, she promoted productive dispositions; her students came to view themselves and their relationships with mathematics more positively.

EXPLICIT PROMOTION OF PRODUCTIVE DISPOSITIONS

Lindo intentionally invited students to see themselves as people who are capable of participating in mathematical activity. She wanted to disrupt a commonly heard statement: "I am not a mathematics person." Figure 3 names discourse moves that Lindo used to explicitly promote productive dispositions toward mathematics.

Fig. 3

Promote a view of students as mathematicians.

Promote ongoing sense making.

Emphasize that learning takes time.

Promote intentional listening.

Mrs. Lindo used these discourse moves to explicitly promote productive dispositions toward mathematics in her class of ninth and tenth graders.

Promote a View of Students as Mathematicians

When Lindo asked students to share their strategies, she would highlight how their approaches aligned with the "work of a mathematician." When Mario, a student, tried to graph an inequality in standard form (similar to the example of Shani under the previous heading Encourage In-Progress Thinking), he tried to graph the inequality by testing x- and y-values that would make the inequality true. Instead of telling Mario that he was approaching the problem incorrectly, Lindo instead decided to amplify his mathematical practices by noting that he was "trying to make sense of it, which I love. Being a mathematician is all about sense making." Lindo's effort to identify when students were being mathematicians appeared to help her students see themselves in that light.

Promote Ongoing Sense Making

In Lindo's classroom, she explicitly encouraged students to continually make more and more sense of mathematics. Lindo said, "I really wish, that as mathematicians, not only are we sense making, but we can persevere through something that maybe we haven't figured out yet." After a class discussion, she would offer an opportunity for revising individual thinking by saying, "If you need to add on or change your responses, do so at this time."

During the whole-class discussion about using intercepts to graph an inequality in standard form (see the inequality under the heading Encourage In-Progress Thinking), Lindo said, "See if we can all make sense of this together. Tiana, you can add on to what Chanel said or tell us what you remember about this problem." In this way, Lindo encouraged multiple ways to participate. She reflected on this by saying, "I actually give them the choice to add on to someone else's thinking and build off of that. But I also don't want to discourage any other engagement." Sharing what students generally think or remember about the problem was considered valuable by Lindo for students to continue thinking together.

Emphasize That Learning Takes Time

Lindo wrestled with a relatable dilemma: How can she give her students enough time to grapple with mathematical ideas but also cover all the mathematical content? She would often tell students how long they had to work on an activity and set a timer. However,

depending on what she saw from the students' work as she moved around the room, she sometimes prolonged the activity. During a whole-class discussion, when students would share solutions, Lindo would explain why they spent extended time on the activity, making statements such as, "That was way more time than I intended to spend, but you had a lot of really good questions. This is really the opportunity where you should be asking those questions and making sense of it."

It was important to Lindo to normalize that learning mathematics can take time:

> If you don't give them enough time, then they are not prepared to build on, and then it discourages students from trying. Then it's like, "Well, she's moving on, and I haven't finished thinking about it."

In part, time mattered to Lindo because grappling with mathematics is an important part of the practice of doing mathematics, but the district required certain content coverage. She chose to regularly be transparent and explicit with students regarding pacing. Sometimes, she would move on to a new topic or task, even if the entire class did not seem to have mastered what was at hand, for the sake of pacing. She acknowledged to students that it was OK to move forward with the class even if they had not finished or completely understood. But she often strategically chose to prioritize following the pace of the students, and this can communicate to students that taking time to make sense of mathematics is valuable.

Promote Intentional Listening

Lindo explicitly made statements to support students with intentional listening to one another during mathematics class. She brought the class together to talk about a warm-up problem that she had created (see figure 4).

When the class devolved into some side conversations, she encouraged listening by directing attention to a student's contribution: "We have a few other voices that are on, so, everyone, you are listening to James as he explains what he noticed and was thinking about that second inequality." She thought that listening to peers and considering what was shared was an important part of making sense of mathematics. Lindo reflected on this by saying that she would focus on "just really hearing those ideas and making sure that everyone else is accountable for listening to them."

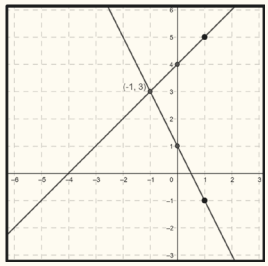
Students often ask teachers, "Is my answer correct?" This question presents a challenge to teachers if they want to share authority with their students.

Fig. 4

Shade the feasible region on the graph provided using the system of inequalities below

$y \leq -2x + 1$

$y \geq 1x + 4$



When the class got off the topic of a warm-up problem and into some side conversations, the teacher directed their attention to a student's contribution.

To promote intentional listening, a teacher may turn this question back to the class: Do we agree or disagree with this solution?" During an instance when Lindo responded to a student's thinking to encourage in-progress thinking, she said, "I don't want you to say yes or no. I want to see what Tiana remembers." Lindo moved away from the role of evaluator of students' knowledge. Instead, she encouraged students to make sense of one another's ideas. One of a teacher's roles, according to Hintz, Tyson, and English (2018), is to engage in supportive listening, which is "when teachers are listening to and for ways to support students to listen to one another, and so they learn to consider and learn from perspectives other than their own" (p. 3).

REPORTS FROM MRS. LINDO'S STUDENTS

We gave Lindo's students a survey to learn about how they experienced her classroom (see figure 5). A subset of students and their parents gave consent to share responses from this survey ($N = 12$), which was 44.4 percent of the class. Results indicate generally productive dispositions among these students (see table 1 and figure 6).

It appears that these students responded to their teacher's effort to encourage and support her students in many positive ways. Her students experienced a sense of competence, as well as recognizing the support provided by their teacher. They experienced learning as understanding over memorizing.

We notice that these students may still be learning to build on their peers' ideas. This result demonstrates the challenge with such a process because we noted that Lindo made much effort to help students see

Fig. 5

For each item, circle the number that most accurately reflects your experience in this math class:

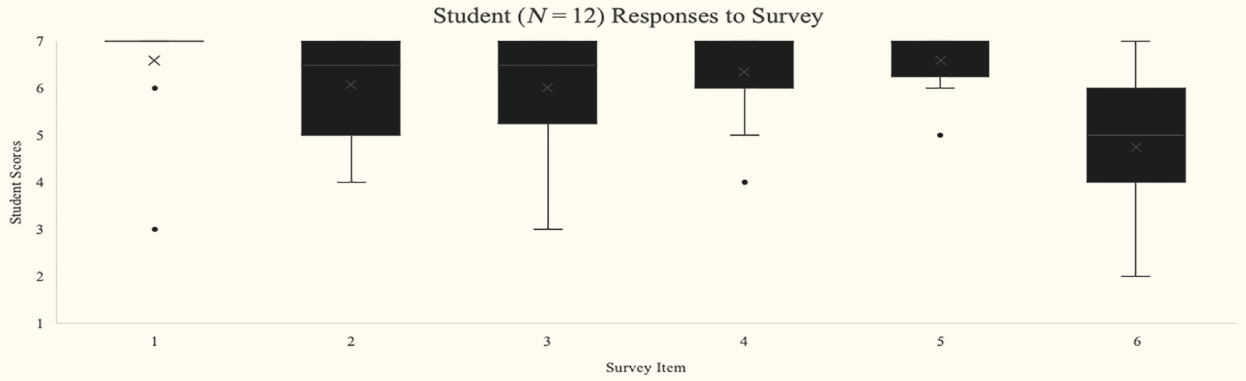
	Never True	Rarely True	Occasion-ally True	About half the time	Often True	Very often True	Always True
1. My teacher believes I can do well in math.	1	2	3	4	5	6	7
2. I believe I can do well in math	1	2	3	4	5	6	7
3. My math teacher gives us plenty of opportunities to understand new math ideas.	1	2	3	4	5	6	7
4. My math teacher tries to understand how I see things before suggesting a new strategy.	1	2	3	4	5	6	7
5. My math teacher wants us to understand our work, not just memorize it.	1	2	3	4	5	6	7
6. I build on others' ideas during my math class.	1	2	3	4	5	6	7

The co-authors gave Mrs. Lindo's class a student survey.

Table 1 Student Responses to Survey

Survey Item	Median	Range
1. My teacher believes that I can do well in mathematics.	7	3–7
2. I believe that I can do well in mathematics.	6.5	4–7
3. My mathematics teacher gives us plenty of opportunities to understand new mathematics ideas.	6.5	3–7
4. My mathematics teacher tries to understand how I see things before suggesting a new strategy.	7	4–7
5. My mathematics teacher wants us to understand our work, not just memorize it.	7	5–7
6. I build on others' ideas during my mathematics class.	5	2–7

Fig. 6



A subset of the class gave the authors permission to share the survey results.

the value of one another's ideas and the connections among them. Although students had opportunities to listen intentionally and build on peers' ideas, they may need even more support to do so.

CONCLUSION

A teacher has many ways to use discourse moves to give high school students opportunities to develop productive dispositions and identities (see figures 1 and 3). Lindo's classroom offers us a glimpse of some of those discourse moves, and her reflections reveal her intentions that underlie these moves. As Lindo does, we encourage teachers to identify their own classroom discourse moves, to reflect on their interactions with students using these moves, and to seek students' input about the effectiveness of those moves.

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