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Sequential predictors for delay-compensating feedback stabilization of bilinear systems with uncertainties[☆]

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ABSTRACT

We construct delay-compensating input-to-state stabilizing feedback controllers for a class of nonlinear control systems that include bilinear systems that have pointwise delays in their inputs. Our new approach for delay compensation does not require constructing or estimating distributed terms in the formulas for the stabilizing control laws. We allow arbitrarily long constant input delays. We illustrate our findings in a power system example.

1. Introduction

This work continues the development (which started, e.g., in [1–3], and [4]) of sequential predictor approaches for compensating for arbitrarily long input delays. Prior results covered dynamics whose right sides satisfy a linear growth condition. Here we cover feedback designs for dynamics for which this linear growth condition is not needed and which therefore can be applied to important bilinear systems that were beyond the scope of previous sequential predictor methods; see [5–7] for the value of bilinear systems.

Our method is motivated by the ubiquity of input delays in many applications, and the bottlenecks that arise when using standard controllers that were not designed to compensate for the input delays; see [8–17]. A natural method for coping with input delays is emulation, which calls for designing a stabilizing feedback that can be applied when the input delays are zero, and where one then calculates a bound on the input delays under which the resulting closed loop system still enjoys desired global asymptotic stability properties; see, e.g., [18]. For cases where the delay bounds from emulation may be too small, other authors explored other input delay compensation methods, including exact predictor and reduction methods. These other methods can compensate for arbitrarily long input delays, but can be complicated to use in practice because their controls are only implicitly defined as solutions of integral equations; see, for instance [19,20], and [21].

This motivated [1] and [2] and other papers on sequential predictors for delay compensation, which normally express the control using values of an auxiliary variable that is viewed as an output of a collection of ordinary differential equations. This collection of equations includes copies of the original system running on multiple time scales, with additional stabilizing terms, making it possible to compensate for arbitrarily long input delays without having any distributed terms in the controls. However, these results required that the right sides of the systems grow linearly in the input and state, which excludes bilinear systems having the form

$$\dot{x}(t) = A(t)x(t) + \sum_{i=1}^{c} u_i(t-h)\big(B_i(t)x(t) + G_i(t)\big) + D(t)\delta(t) \quad (1)$$

with unknown measurable locally essentially bounded functions δ (representing uncertainty), constant delays h, controls $u = (u_1, \dots, u_c)$, and bounded coefficient matrices. Although such systems are often stabilizable using bounded controls, knowing a bound on u is not sufficient to extend previous sequential predictor results to cover bilinear systems (1). This is because the earlier results also need input-to-state stability (or ISS) with respect to measurement uncertainty, and since one must find a bound $\bar{\delta}$ on the supremum of δ using a bound on the measurement uncertainty; see (7), the third part of the proof of Theorem 1, and [3, Assumption 2].

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This calls for the innovations of this work, which eliminates the requirement that the right sides grow linearly in the input and state. These innovations are made possible by our significantly different mathematical analysis, as compared to the study of sequential predictors for linear systems. Our key ingredients include (a) our new Lyapunov–Krasovskii functional construction involving novel uses of Young's inequality and (b) a relaxed condition on the measurement uncertainties in the control (in Assumption 1). This overcomes a longstanding obstacle to building sequential predictors for bilinear systems. See Remarks 1–2 for more on the innovations in our work.

We provide input delay compensating sequential predictors for bilinear systems having the form (1) with continuous coefficient matrices, including ISS with respect to the δ, which were not previously available in the literature. We state and prove a general sequential predictor feedback control result in Sections 3–4. Then Section 5 provides sufficient conditions that facilitate checking our assumptions of our general result. In Section 6, we apply our method to a key class of bilinear systems, which we demonstrate using a power system in Section 7.

2. Definitions and notation

Throughout this paper, the dimensions of the Euclidean spaces are arbitrary unless we note otherwise, and we omit arguments of functions when they are clear. The usual Euclidean norm in $\mathbb{R}^n$ and the induced matrix norm are denoted by $|\cdot|$, and $|\phi|_{\mathcal{I}}$ (resp., $|\phi|_\infty$) is the usual essential supremum of a function ϕ over any interval $\mathcal{I}$ in its domain (resp., its entire domain). Consider a system of the form

$$\dot{X}(t) = \mathcal{F}\big(t, X(t), u_{\mathcal{F}}(t-h), \Delta(t)\big), \tag{2}$$

whose state X, feedback control $u_{\mathcal{F}}$, and unknown Lebesgue measurable locally essentially bounded function Δ are valued in $\mathbb{R}^{n_1}$, $\mathbb{R}^{n_2}$, and $\mathbb{R}^{n_3}$, respectively, where $h > 0$ is a constant delay. Owing to the delay, the solutions of (2) are defined for given initial times $t_0 \geq 0$, initial functions that are defined on an initial interval $\mathcal{I}^0 \subseteq (-\infty, t_0]$ such as $[t_0-h, t_0]$, and functions Δ. We assume that (2) is forward complete, i.e., all such solutions are uniquely defined on $\mathcal{I}^0 \cup [t_0, \infty)$; see Section 3 for our assumptions that ensure this forward completeness property. We use the well known standard classes $\mathcal{KL}$ and $\mathcal{K}_\infty$ of comparison functions from [22, Chapt. 4] and the definition of input-to-state stability (or ISS, which we also use to mean input-to-state stable) for (2); see [23] and [3] for ISS under delays. We use this definition:

Definition 1. For a fixed $u_{\mathcal{F}}$, we say that (2) is ISS with respect to a disturbance set $\mathcal{D} \subseteq \mathbb{R}^{n_3}$ provided there are functions $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}_\infty$ such that for all initial times t_0, initial functions, and choices of the functions Δ that are valued in $\mathcal{D}$, the corresponding solutions of (2) all satisfy $|X(t)| \leq \beta(|X|_{\mathcal{I}^0}, t-t_0) + \gamma(|\Delta|_{[t_0,t]})$ for all $t \geq t_0$.

Let $\mathbb{N} = \{1, 2, \ldots\}$, and $\mathcal{B}_R$ denote the closed ball of any radius $R > 0$ in Euclidean space centered at the origin. For subsets S_1 and S_2 of Euclidean spaces, a function $W : S_1 \times S_2 \to \mathbb{R}^n$ is called locally Lipschitz in its second variable uniformly in its first variable provided: for each constant $R > 0$, there is a constant $L_R > 0$ such that $|W(s_1, s_a) - W(s_1, s_b)| \leq L_R|s_a - s_b|$ for all $s_1 \in S_1$ and all s_a and s_b in $\mathcal{B}_R$. If L_R in the preceding property can be chosen independently of R, then we use the term globally (instead of locally) Lipschitz. We call a $J : [0, \infty) \times \mathbb{R}^n \to [0, \infty)$ uniformly proper and positive definite provided there exist functions $\underline{\gamma} \in \mathcal{K}_\infty$ and $\overline{\gamma} \in \mathcal{K}_\infty$ such that $\underline{\gamma}(|x|) \leq J(t, x) \leq \overline{\gamma}(|x|)$ for all $t \geq 0$ and $x \in \mathbb{R}^n$. We set $\Psi_t(s) = \Psi(t + s)$ for all Ψ, $s \leq 0$, and $t \geq 0$ such that $t+s$ lies in the domain of Ψ. We also use $0_{\ell\times r}$ (resp., I_r) to mean the $\ell \times r$ matrix whose entries are all 0 (resp., the $r \times r$ identity matrix).

3. General result

Before turning to our results on bilinear systems, we provide a novel result for a more general class of systems

$$\dot{x}(t) = f(t, x(t), u(t-h), \delta(t)), \tag{3}$$

whose state x, control u, and unknown Lebesgue measurable locally essentially bounded function δ are valued in $\mathbb{R}^n$, $\mathbb{R}^c$, and $\mathbb{R}^d$ respectively, and $h > 0$ is a constant delay, where we use different notation from (2) in part because the Δ in (2) will not coincide with δ in (3) in Assumption 1 to follow. One difference between the result of this section and [3, Theorem 1] is that here we remove the requirement that the dynamics grow linearly in (x, u), and instead use boundedness conditions on u_s, on the control set, and on the disturbances ϵ and δ; see Remarks 1–2 for more on the significant differences between this work and [3] and about the value added by this work. We assume:

Assumption 1. There are a compact neighborhood $\mathcal{U} \subseteq \mathbb{R}^c$ of $0_{c\times 1}$, a continuous function $u_s : [0, \infty) \times \mathbb{R}^n \to \mathcal{U}$ that is globally Lipschitz in its second variable uniformly in its first variable, and a constant $\bar{\epsilon} > 0$ such that the system

$$\dot{x}(t) = f\big(t, x(t), u_s(t, x(t) + \epsilon(t)), \delta(t)\big) \tag{4}$$

with disturbance $\Delta = (\epsilon, \delta)$ is ISS with respect to the disturbance set $\mathcal{B}_{\bar{\epsilon}} \times \mathbb{R}^d$. Also, $u_s(t, 0_{n\times 1}) = 0_{c\times 1}$ for all $t \in \mathbb{R}$. □

Assumption 2. The function f is continuous, and is locally Lipschitz in (x, u, δ) uniformly in t, satisfies $f(t, 0_{n\times 1}, 0_{c\times 1}, 0_{d\times 1}) = 0_{n\times 1}$ for all $t \geq 0$, and admits a constant $k > 0$ such that

$$\begin{aligned} &|f(t, z_1, U, \Delta_1) - f(t, z_2, U, \Delta_2)| \\ &\leq k|z_1 - z_2| + k|\Delta_1 - \Delta_2| \end{aligned} \tag{5}$$

holds for all $t \geq 0$, $z_1 \in \mathbb{R}^n$, $z_2 \in \mathbb{R}^n$, $U \in \mathcal{U}$, $\Delta_1 \in \mathbb{R}^d$, and $\Delta_2 \in \mathbb{R}^d$ for the choice of $\mathcal{U}$ from Assumption 1. □

Throughout this paper, we consider any constants $m \in \mathbb{N}$, $\epsilon_* > 0$, $h > 0$, $C_1 \in (0, 2m/h)$, $C_2 > 0$, and $\lambda_a > 0$, and any k and $\bar{\epsilon}$ satisfying Assumptions 1–2, and then we set

$$\begin{aligned} &p = \tfrac{m(4k+\lambda_a)}{2m-hC_1},\ \epsilon_{0,\ell} = \max\left\{1, \tfrac{\overline{C}(1+\lambda_a)h}{m}\right\}, \\ &\overline{C} = \tfrac{p}{C_1}\max\left\{p^2(1+C_2), k^2\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{\lambda_a}{4}\right)\right\}, \\ &\epsilon_0 = \min\left\{2k\left(1-\tfrac{h\overline{C}}{km}(1+\lambda_a)\right), \tfrac{\overline{C}\lambda_a m}{2(h\overline{C}(1+\lambda_a)+m)}\right\}, \\ &\hat{c} = \max\left\{\tfrac{2p^2}{\epsilon_0}, \left(1+\tfrac{1}{C_2}\right)\tfrac{p^3}{2C_1}\lambda_a^\sharp, \tfrac{\epsilon_0}{2}\right\}, \\ &\overline{M} = \tfrac{k^2}{2\lambda_a} + \lambda_a^\sharp\tfrac{phk^2}{2C_1 m}\left(1+\tfrac{1}{C_2}\right),\ \lambda_a^\sharp = 1 + \tfrac{4}{\lambda_a}, \\ &\omega_0 = 1,\ \omega_i = \tfrac{2}{\epsilon_0}\left(\hat{c}\omega_{i-1} + \epsilon_*\right)\ \text{if } 1 \leq i \leq m-1, \\ &\text{and } \bar{\epsilon}_* = \tfrac{1}{\epsilon_{0,\ell}}\min\left\{0.5\epsilon_0, \tfrac{\epsilon_*}{\omega_{m-1}}\right\} \end{aligned} \tag{6}$$

which will all be positive constants under condition (8) of our theorem. The integer m will serve as the number of sequential predictors, and the constants C_i will serve as weighting functions in our Young's inequality applications in our Appendix. In terms of (6) and any constant

$$\bar{\delta} \in \left(0, \frac{\bar{\epsilon}}{m}\sqrt{\frac{\bar{\epsilon}_*}{2\overline{M}\omega_{m-1}}}\right) \tag{7}$$

(which will serve as our bound on δ) and the function $f_0(t, x, u) = f(t, x, u, 0)$, we prove:

Theorem 1. *Let $k > 0$ and $\bar{\epsilon} > 0$ be constants such that (3) satisfies Assumptions 1–2. Assume that*

$$m > \frac{h\overline{C}(1+\lambda_a)}{k}. \tag{8}$$

Consider (3) in closed loop with

$$u(t) = u_s\big(t+h, z_m(t)\big), \tag{9}$$

where z_m is the last n components of the state of the system

$$\begin{cases} \dot{z}_1(t) &= f_0\left(t+\frac{h}{m}, z_1(t), \Phi(t, z_m, 1)\right) \\ & \quad - p[z_1\left(t-\frac{h}{m}\right) - x(t)] \\ \dot{z}_2(t) &= f_0\left(t+\frac{2h}{m}, z_2(t), \Phi(t, z_m, 2)\right) \\ & \quad - p\left[z_2\left(t-\frac{h}{m}\right) - z_1(t)\right] \\ & \vdots \\ \dot{z}_m(t) &= f_0\left(t+h, z_m(t), \Phi(t, z_m, m)\right) \\ & \quad - p\left[z_m\left(t-\frac{h}{m}\right) - z_{m-1}(t)\right] \end{cases} \tag{10}$$

where

$$\Phi(t, z_m, i) = u_s(t+h-h(m-i)/m, z_m(t-h(m-i)/m))$$

for all $t \geq 0$ and $i \in \{1, 2, \ldots, m\}$ and $z_0 = x$. Then there are functions $\beta_d \in \mathcal{KL}$ and $\gamma_d \in \mathcal{K}_\infty$ such that all solutions $(x, z) : [t_0 - 2h, \infty) \to \mathbb{R}^{(m+1)n}$ of the preceding closed loop system, for all Lebesgue measurable essentially bounded functions $\delta : [0, \infty) \to \mathcal{B}_{\bar{\delta}}$ and all initial times $t_0 \geq h/m$, satisfy

$$\begin{aligned} |x(t)| \;\leq\; & \beta_d\left(|x|_{[t_0-2h, t_0+h/m]} + |z|_{[t_0-2h, t_0+h/m]}, t-t_0\right) \\ & + \gamma_d\big(|\delta|_{[t_0,t]}\big) \end{aligned} \tag{11}$$

for all $t \geq t_0$, where $z = (z_1, \ldots, z_m)$. □

Remark 1. Theorem 1 states that it is possible to design a sequence of m predictors such that, when the un-delayed closed-loop system (4) is ISS with respect to the disturbance $\delta(t)$ and the uncertainties $\epsilon(t)$ in the state measurements under the bounds on these functions from Assumption 1, then in the presence of delay, the state of the closed loop system with the predictor remains in a ball whose radius depends on the initial conditions and the bound on $\delta(t)$.

It is tempting to surmise that at least in bilinear cases, we can reduce our analysis of (3) to systems that are globally Lipschitz in the state (which were covered in [3]), by replacing f by the new dynamics f_{new} that is defined by

$$f_{\text{new}}(t, x, u, \delta) = \begin{cases} f(t, x, u, \delta), & \text{if } |u| \leq R \\ f(t, x, uR/|u|, \delta), & \text{if } |u| > R \end{cases} \tag{12}$$

for a bound R on the control u_s. However, this replacement would not address the problems in this work, where there is a restriction on the allowable measurement uncertainties ϵ in Assumption 1 (which makes our assumption less restrictive than in [3], where the ISS assumption is required for all choices of the measurement uncertainties $\epsilon(t)$) and where we must therefore find a bound $\bar{\delta}$ on the allowable uncertainties δ.

Our (less restrictive) condition in Assumption 1 that ϵ remains in a bounded set is called for in order to produce a theorem whose assumptions we can check for bilinear systems; see Lemmas 2–3. However, the price to pay for only considering ϵ's that stay in a bounded set in Assumption 1 is that it calls for the third part of our proof of our theorem; see especially (34) and (37). The requirement that u_s is valued in the compact set $\mathcal{U}$ is used to ensure that (5) is satisfied when U is a control value; see (A.3)–(A.4). The bound on $\mathcal{U}$ in Assumption 2 is needed for the existence of the required k when (2) is bilinear; see (41).

Remark 2. Theorem 1 is also new even when $\delta = 0$, because of its less restrictive condition on the number m of sequential predictors, as compared with the condition

$$m > h(4k+\lambda_a)^{3/2}\sqrt{(2/k)(1+\lambda_a)} \tag{13}$$

from [3]; see Section 7. Our strategy for obtaining our less restrictive lower bound (8) on m is to use the degrees of freedom in the Lyapunov–Krasovskii analysis in the Appendix, where the constants C_1 and C_2 arise from using Young's inequality instead of the triangle inequality. This leads to a different Lyapunov–Krasovskii functional in our analysis and a different p in the sequential predictors, as compared with [3], which used $p = 4k+\lambda_a$. Therefore, although the sequential predictors (10) have the same general structure as earlier sequential predictor designs (consisting of copies of the original system running on different time scales with additional corrective terms), there is considerable novelty in our proof that makes it possible to apply this general structure in our novel setting that includes bilinear systems.

A significant difference between works such as [24] and Theorem 1 is that our theorem yields a control having no distributed terms, based on the computationally cheap sequential predictors (10). While Lyapunov methods can produce conservativeness, we believe that this is the price to pay to compensate for arbitrarily long input delays without using distributed terms that would otherwise have occurred from using standard predictive methods while also handling bilinearities. We can provide a global exponential ISS estimate for the error vector (14) (in (30)), which we can combine with (11) to obtain ISS estimates for the combined variable (x, z), where $z = (z_1, \ldots, z_m)$ is the vector of predictors (using the fact that $z_i(t) = \mathcal{E}_i(t) + \mathcal{E}_{i-1}(t+h/m) + \cdots + \mathcal{E}_1(t+(i-1)h/m) + x(t+ih/m)$ for all $t \geq 0$ and $i \geq 2$). We leave the formulas for comparison functions in the ISS estimate for (x, z) to the reader.

Remark 3. Like in [3], our requirement $t_0 \geq h/m$ in Theorem 1 is used in our Lyapunov–Krasovskii analysis but can be relaxed. While the main result of Mazenc and Malisoff [3] contains suprema over $[t_0 - h, t_0 + h/m]$ on the right side of (11) instead of $[t_0 - 2h, t_0 + h/m]$, we use $2h$ instead of h to allow the special case where $m = 1$. Moreover, we can use the method from [3, Section V] (with its requirement $U \in \mathbb{R}^c$ replaced by $U \in \mathcal{U}$) to replace $[t_0 - 2h, t_0 + h/m]$ by $[t_0 - 2h, t_0]$ in the final estimate (11).

4. Proof of Theorem 1

Throughout the proof, all inequalities and equalities should be understood to hold for all $t \geq t_0$ and $t_0 \geq h/m$ along all solutions of the closed loop system from the statement of the theorem, unless otherwise indicated. Recalling our definition $z_0 = x$, we use the error variables

$$\begin{aligned} & \mathcal{E} = (\mathcal{E}_1, \ldots, \mathcal{E}_m), \text{ where} \\ & \mathcal{E}_i(t) = z_i(t) - z_{i-1}(t+h/m) \text{ for } i = 1, \ldots, m. \end{aligned} \tag{14}$$

The rest of the proof has three parts.

First Part: Lyapunov–Krasovskii Functionals for $\mathcal{E}_i$. We use

$$\hat{\mu}(\mathcal{E}_{i,t}) = \frac{1}{2}|\mathcal{E}_i(t)|^2 + \int_{t-2h/m}^{t} |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \tag{15}$$

for $i = 1, 2, \ldots, m$ and the following lemma (which we prove in the Appendix, and where $\mathcal{E}_{i,t}$ is the ith component of $\mathcal{E}_t$ for each i):

Lemma 1. *Consider the functions $\nu(\mathcal{E}_i) = \frac{1}{2}|\mathcal{E}_i|^2$ and*

$$\begin{aligned} & \mu(\mathcal{E}_{i,t}) \\ & = \nu(\mathcal{E}_i(t)) + \overline{C}(1+\lambda_a)\int_{t-2h/m}^{t}\int_s^t \nu(\mathcal{E}_i(\ell))\mathrm{d}\ell \mathrm{d}s \\ & \text{and } \tilde{\mu}(\mathcal{E}_{i,t}) = \mu(\mathcal{E}_{i,t}) + \int_{t-2h/m}^{t} |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \end{aligned} \tag{16}$$

for $i = 1, 2, \ldots, m$. Then, the inequalities

$$\dot{\mu}(\mathcal{E}_{1,t}) \leq -\epsilon_0 \tilde{\mu}(\mathcal{E}_{1,t}) + \overline{M}|\delta|^2_{[t,t+h/m]} \quad (17)$$

and

$$\begin{aligned} \dot{\mu}(\mathcal{E}_{i,t}) \leq & -\tfrac{\epsilon_0}{2}\tilde{\mu}(\mathcal{E}_{i,t}) + \tfrac{p^2}{\epsilon_0}|\mathcal{E}_{i-1}(t)|^2 \\ & + \left(1 + \tfrac{1}{C_2}\right)\tfrac{p^3}{2C_1}\lambda_a^\sharp \int_{t-h/m}^t |\mathcal{E}_{i-1}(\ell)|^2 \mathrm{d}\ell \end{aligned} \quad (18)$$

hold for all $t \geq h/m$ and $i \in \{2, \ldots, m\}$. □

The proof that ϵ_0 from (6) satisfies the requirements from Lemma 1 uses the fact that the constant $\epsilon_{0,\ell}$ from (6) is such that

$$\begin{aligned} \mu(\mathcal{E}_{i,t}) \leq & \tfrac{1}{2}|\mathcal{E}_i(t)|^2 \\ & + \overline{C}(1+\lambda_a)\tfrac{2h}{m}\tfrac{1}{2}\int_{t-2h/m}^t |\mathcal{E}_i(\ell)|^2 \mathrm{d}\ell \\ \leq & \ \epsilon_{0,\ell}\hat{\mu}(\mathcal{E}_{i,t}) \end{aligned} \quad (19)$$

for $i = 1, \ldots, m$ and all $t \geq h/m$. We also use (19) later in the proof below. From our choices (6) of our constants, it follows from Lemma 1 that for all $i \in \{2, \ldots, m\}$ and $t \geq h/m$, we have

$$\dot{\mu}(\mathcal{E}_{i,t}) \leq -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{i,t}) + \hat{c}\hat{\mu}(\mathcal{E}_{i-1,t}). \quad (20)$$

Second Part: ISS Estimate for $\mathcal{E}$ Dynamics. We next show that with the constants ω_i from (6), the function

$$\mu_m^\sharp(\mathcal{E}_t) = \sum_{j=1}^m \omega_{m-j}\mu(\mathcal{E}_{j,t}) \quad (21)$$

is an ISS Lyapunov–Krasovskii functional for the $\mathcal{E}$ dynamics with the disturbance δ. We use induction and the partial sums

$$\mu_r^\sharp(\mathcal{E}_t) = \mu(\mathcal{E}_{m,t}) + \omega_1\mu(\mathcal{E}_{m-1,t}) + \cdots + \omega_r\mu(\mathcal{E}_{m-r,t}) \quad (22)$$

for $r = 1, \ldots, m-1$ when $m \geq 2$. Using the fact that

$$\mu_1^\sharp(\mathcal{E}_t) = \mu(\mathcal{E}_{m,t}) + (2/\epsilon_0)(\hat{c} + \epsilon_*)\mu(\mathcal{E}_{m-1,t}) \quad (23)$$

and (20), we get

$$\begin{aligned} & \dot{\mu}_1^\sharp \leq \\ & -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m,t}) - \epsilon_*\hat{\mu}(\mathcal{E}_{m-1,t}) + \hat{c}\tfrac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\hat{\mu}(\mathcal{E}_{m-2,t}) \end{aligned} \quad (24)$$

holds if $m > 2$ and $t \geq h/m$. On the other hand, for $m = 2$, we can use (17) to verify that

$$\dot{\mu}_1^\sharp \leq -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{2,t}) - \epsilon_*\hat{\mu}(\mathcal{E}_{1,t}) + \tfrac{2}{\epsilon_0}(\hat{c}+\epsilon_*)\overline{M}|\delta|^2_{[t,t+h/m]} \quad (25)$$

for all $t \geq h/m$. By induction, it follows that

$$\begin{aligned} \dot{\mu}_m^\sharp \leq & -\tfrac{\epsilon_0}{2}\hat{\mu}(\mathcal{E}_{m,t}) \\ & - \epsilon_* \sum_{j=1}^{m-1}\hat{\mu}(\mathcal{E}_{m-j,t}) + \omega_{m-1}\overline{M}|\delta|^2_{[t,t+h/m]} \end{aligned} \quad (26)$$

for all $t \geq h/m$ and $m > 1$. Moreover, (19) gives

$$\omega_{m-i}\mu(\mathcal{E}_{i,t}) \leq \epsilon_{0,\ell}\omega_{m-i}\hat{\mu}(\mathcal{E}_{i,t}) \quad (27)$$

for $i = 1, \ldots, m$ and all $t \geq h/m$, and $1 \leq \omega_i \leq \omega_{i+1}$ for $i = 0, \ldots, m-2$ and $m \geq 2$, since $\hat{c} \geq \epsilon_0/2$.

It follows from (19) and (26) and our choice of $\bar{\epsilon}_*$ in (6) that we have

$$\dot{\mu}_m^\sharp \leq -\bar{\epsilon}_*\mu_m^\sharp(\mathcal{E}_t) + \omega_{m-1}\overline{M}|\delta|^2_{[t,t+h/m]} \quad (28)$$

for all $t \geq h/m$. Applying the method of variation of parameters to (28) (by multiplying it through by $e^{\bar{\epsilon}_* t}$ and integrating the result on $[t_0, t]$ for any $t_0 \geq h/m$), we obtain a constant $c_a > 0$ such that

$$\begin{aligned} \tfrac{1}{2}|\mathcal{E}(t)|^2 \leq & \ \mu_m^\sharp(\mathcal{E}_t) \\ \leq & \ c_a e^{\bar{\epsilon}_*(t_0-t)}|\mathcal{E}|^2_{[t_0-2h,t]} + \tfrac{\omega_{m-1}\overline{M}|\delta|^2_{[t_0,t+h/m]}}{\bar{\epsilon}_*} \end{aligned} \quad (29)$$

for all $t \geq t_0$. By multiplying (29) through by 2 and using the subadditivity of the square root (to upper bound the square root of the two right side terms), it follows that

$$\begin{aligned} & |\mathcal{E}(t)| \leq \\ & e^{0.5\bar{\epsilon}_*(t_0-t)}\sqrt{2c_a}|\mathcal{E}|_{[t_0-2h,t_0]} + \sqrt{\tfrac{2\omega_{m-1}\overline{M}}{\bar{\epsilon}_*}}|\delta|_{[t_0,t+h/m]} \end{aligned} \quad (30)$$

holds for the $\mathcal{E}$ dynamics for all $t \geq t_0$ and $t_0 \geq h/m$.

Third Part: ISS-Like Estimate for Closed Loop x Dynamics. We show how our new variable

$$\mathcal{E}^\sharp(t) = \sum_{\ell=0}^{m-1}\mathcal{E}_{m-\ell}\big(t + \ell\frac{h}{m} - h\big) \quad (31)$$

can be viewed as a measurement error added to the state $x(t)$ in the feedback control in the closed loop system from the statement of our theorem, which will allow us to apply Assumption 1 with $\epsilon = \mathcal{E}^\sharp$.

To this end, we first choose a constant $\lambda_* \in (0, 1)$ that satisfies

$$\bar{\delta} = \lambda_*\frac{\bar{\epsilon}}{m}\sqrt{\frac{\bar{\epsilon}_*}{2\omega_{m-1}\overline{M}}}, \quad (32)$$

which exists by (7). Since $1/\lambda_* > 1$, the exponential ISS condition in (30) then yields a constant $\mathcal{T} > 0$ such that

$$|\mathcal{E}(t)| \leq \frac{1}{\lambda_*}\sqrt{\frac{2\omega_{m-1}\overline{M}}{\bar{\epsilon}_*}}\bar{\delta} \quad (33)$$

for all $t \geq t_0 + \mathcal{G}_\delta$ and such that we also have

$$|\mathcal{E}^\sharp(t)| \leq m|\mathcal{E}|_{[t-h,t-h/m]} \leq \tfrac{m}{\lambda_*}\sqrt{\tfrac{2\omega_{m-1}\overline{M}}{\bar{\epsilon}_*}}\bar{\delta} = \bar{\epsilon} \quad (34)$$

for all $t \geq t_0 + h + \mathcal{G}_\delta$, where

$$\mathcal{G}_\delta = \mathcal{T}(|x|_{[t_0-2h,t_0+h/m]} + |z|_{[t_0-2h,t_0+h/m]}), \quad (35)$$

by (32) and our condition $|\delta|_\infty \leq \bar{\delta}$; a formula for the required constant $\mathcal{T}$ can be deduced from the fact that $\ln(1+r) \leq r$ for all $r \geq 0$.

On the other hand, using the fact that

$$\begin{aligned} & z_m(t) = \mathcal{E}_m(t) + z_{m-1}(t + h/m), \\ & z_{m-1}(t) = \mathcal{E}_{m-1}(t) + z_{m-2}(t + h/m), \\ & \ldots, \text{ and } z_1(t) = \mathcal{E}_1(t) + x\left(t + \tfrac{h}{m}\right) \end{aligned} \quad (36)$$

all hold for all $t \geq 0$, it follows (e.g., by induction on m) that $z_m(t) = \mathcal{E}^\sharp(t+h) + x(t+h)$. Hence, (3) in closed loop with (9) is

$$\dot{x}(t) = f\left(t, x(t), u_s\left(t, x(t) + \mathcal{E}^\sharp(t)\right), \delta(t)\right). \quad (37)$$

Then (34) allows us to use Assumption 1 with $\epsilon = \mathcal{E}^\sharp$ along all solutions of the closed loop system and all $t \geq t_0 + h + \mathcal{G}_\delta$.

In fact, the last part of the proof of Mazenc and Malisoff [3, Theorem 1] with its initial time t_0 replaced by $t_0^\sharp = t_0 + G_0$ allows us to find functions $\beta_b \in \mathcal{KL}$ and $\gamma_b \in \mathcal{K}_\infty$ such that, for all solutions of the closed loop system of our theorem, and for all $t \geq t_0^\sharp$ and $t_0 \geq h/m$, we have

$$\begin{aligned} & |x(t)| \leq \\ & \beta_b(|x^\sharp|_{[t_0-2h,t_0^\sharp+h/m]}, t - t_0 - G_0) + \gamma_b(|\delta|_{[t_0,t]}), \end{aligned} \quad (38)$$

where $G_0 = \mathcal{G}_\delta + h$ and $x^\sharp = (x, z)$. On the other hand, Assumptions 1–2 provides a constant $\bar{L} > 0$ (that is independent of the initial condition) such that $|\dot{x}^\sharp(t)| \leq \bar{L}\left(|x^\sharp|_{[t-h,t]} + |\delta|_{[t_0,t]}\right)$ when $t \geq t_0 \geq 0$. Integrating the preceding bound for $|\dot{x}^\sharp(t)|$, and applying Gronwall's inequality to the function $|x^\sharp|_{[t-h,t]}$, we get a constant $c_b > 0$ (which is also independent of the initial condition) so that

$$\begin{aligned}|x^\sharp(t)| &\le |x^\sharp|_{[t-h,t]} \le e^{c_b\mathcal{G}_\delta}c_b\hat{G}_0\left(|x^\sharp|_{[t_0-h,t_0]}+|\delta|_{[t_0,t]}\right)\\ &\le e^{c_b\mathcal{G}_\delta}c_b\left(\hat{G}_0|x^\sharp|_{[t_0-h,t_0]}+\bar{\delta}\mathcal{G}_\delta+2h|\delta|_{[t_0,t]}\right)\\ &\le \mathcal{M}(|x^\sharp|_{[t_0-2h,t_0+h/m]})+\mathcal{L}(|\delta|_{[t_0,t]})\end{aligned} \tag{39}$$

for $t\in[t_0,t_0^\sharp+h/m]$, with $\hat{G}_0=\mathcal{G}_\delta+2h$, $\mathcal{M}(s)=2c_bse^{2c_b\mathcal{T}s}(\mathcal{T}(s+\bar{\delta})+2h(1+\bar{\delta}c_b\mathcal{T}))$, and where $\mathcal{L}(s)=2c_bhs$, and where we used $2hc_be^{c_b\mathcal{T}s}|\delta|_{[t_0,t]}\le 2hc_b(c_b\mathcal{T}se^{c_b\mathcal{T}s}+1)|\delta|_{[t_0,t]}\le 2\bar{\delta}hc_b^2\mathcal{T}e^{c_b\mathcal{T}s}s+2hc_b|\delta|_{[t_0,t]}$ (which is a consequence of the bound $e^r\le re^r+1$ for $r\ge 0$) and

$$|x|_{[t_0-2h,t_0+h/m]}+|z|_{[t_0-2h,t_0+h/m]}\le 2|x^\sharp|_{[t_0-2h,t_0+h/m]} \tag{40}$$

and $|\delta|_\infty\le\bar{\delta}$. Using (39) to upper bound the first argument of β_b in (38), and then using the fact that $\beta_b(r_1+r_2,r_3)\le\beta_b(2r_1,r_3)+\beta_b(2r_2,0)$ for all nonnegative r_1, r_2, and r_3, it follows that we can upper bound the first right side term of (38) by $\beta_b(\mathcal{M}^\sharp(|x^\sharp|_{[t_0-2h,t_0+h/m]}),t-t_0^\sharp)+\beta_b(2\mathcal{L}(|\delta|_{[t_0,t]}),0)$ with $\mathcal{M}^\sharp(s)=\max\{s,2\mathcal{M}(s)\}$. Hence, by separately considering times $t\in[t_0,t_0^\sharp]$ and $t>t_0^\sharp$, we conclude that we can satisfy the requirements of Theorem 1 with $\beta_d(s,t)=\max\{\mathcal{M}(s)e^{2\mathcal{T}s+h-t},\beta_b(\mathcal{M}^\sharp(s),\max\{t-2\mathcal{T}s-h,0\})\}$ and $\gamma_d(s)=\max\{\mathcal{L}(s),\beta_b(2\mathcal{L}(s),0)+\gamma_b(s)\}$.

5. Checking our assumptions

The growth requirement (5) from Assumption 2 holds for our bilinear systems (1) for any bounded neighborhood $\mathcal{U}\subseteq\mathbb{R}^c$ of the origin and any bounded continuous functions A, D, B_i, and G_i for each i. This follows by picking

$$k=\max\left\{|A|_\infty+\bar{U}\sum_{i=1}^{c}|B_i|_\infty,|D|_\infty\right\} \tag{41}$$

for any bound $\bar{U}$ on the elements of $\mathcal{U}$. However, it is less trivial to check Assumption 1, so we next present sufficient conditions for Assumptions 1–2 to hold for some u_s. We specialize the sufficient conditions from this section to bilinear systems in the next section. We prove the following, whose condition (a) differs from a standard Lyapunov decay condition because α_0 is not required to be positive definite:

Lemma 2. *Let f in (3) admit a compact neighborhood $\mathcal{U}\subseteq\mathbb{R}^c$ of the origin and a constant $k>0$ such that the requirements from Assumption 2 hold. Let $\bar{\omega}>0$ be a constant such that $[-\bar{\omega},\bar{\omega}]^c\subseteq\mathcal{U}$. Assume that there are a C^1 function $V:\mathbb{R}^{n+1}\to[0,\infty)$, a continuous $\alpha_0:\mathbb{R}^n\to[0,\infty)$, a function $\gamma_*\in\mathcal{K}_\infty$, and C^1 functions $M_i:\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}$ that are locally Lipschitz in the second variable uniformly in the first variable for $i=1,2,\dots,c$ such that:*

(a) the inequality

$$\dot{V}\le-\alpha_0(x(t))+\sum_{i=1}^{c}u_iM_i(t,x(t))+\gamma_*(|\delta(t)|) \tag{42}$$

holds along all solutions of $\dot{x}(t)=f(t,x(t),u,\delta(t))$ for all $t\ge 0$ and each vector $u\in\mathcal{U}$;

(b) the functions

$$\alpha_0(x)+\sum_{i=1}^{c}|M_i(t,x)|\ \text{ and }\ V(t,x) \tag{43}$$

are uniformly proper and positive definite; and

(c) the functions $M_i^*(t,x)=(\partial M_i/\partial x)(t,x)/(1+M_i^2(t,x))$ are bounded on $\mathbb{R}^{n+1}$ for $i=1,\dots,c$.

Choose any positive value

$$\bar{L}_*\ge\sup\{|M_i^*(t,x)|:(t,x)\in\mathbb{R}^{n+1},1\le i\le c\}. \tag{44}$$

Then Assumption 1 is satisfied for any constant

$$\bar{\epsilon}\in\left(0,\frac{\pi}{2\bar{L}_*}\right) \tag{45}$$

and $u_s=-\frac{2\bar{\omega}}{\pi}(\arctan(M_1),\dots,\arctan(M_c))$. □

Proof. For each tuple $(t,x)\in\mathbb{R}\times\mathbb{R}^n$, each $\epsilon\in\mathbb{R}^n$, and each $i\in\{1,2,\dots,c\}$, we can apply the Fundamental Theorem of Calculus to the function

$$\mathcal{M}_i^{t,x,\epsilon}(\lambda)=\arctan(M_i(t,x+\lambda\epsilon)) \tag{46}$$

on the interval $[0,1]$ to verify that

$$\begin{aligned}&|\arctan(M_i(t,x+\epsilon))-\arctan(M_i(t,x))|\\ &=|\mathcal{M}_i^{t,x,\epsilon}(1)-\mathcal{M}_i^{t,x,\epsilon}(0)|\ \le\ \int_0^1|\dot{\mathcal{M}}_i^{t,x,\epsilon}(s)|ds\\ &=\ \int_0^1|M_i^*(t,x+s\epsilon)\epsilon|ds\ \le\ \bar{L}_*|\epsilon|\end{aligned} \tag{47}$$

and so also

$$\begin{aligned}&-\arctan(M_i(t,x+\epsilon))M_i(t,x)\\ &\le-\arctan(M_i(t,x))M_i(t,x)+\bar{L}_*|\epsilon||M_i(t,x)|.\end{aligned} \tag{48}$$

Fixing constants $w_*>0$ and $\delta_*\in(0,1)$ such that $\bar{\epsilon}=\delta_*\pi/(2\bar{L}_*)$ (which exist by (45)) and such that $\arctan(s)\ge(\delta_*+1)\pi/4$ for all $s\ge w_*$ (which exists because $\lim_{s\to+\infty}\arctan(s)=\pi/2$), and any $(t,x)\in\mathbb{R}^{n+1}$, $i\in\{1,2,\dots,c\}$, and $\epsilon\in\mathcal{B}_{\bar{\epsilon}}$, we consider two cases:

Case 1. $|M_i(t,x)|\le w_*$. To cover this case, we fix a constant $c_0>0$ such that $\arctan(s)\ge c_0s$ for all $s\in[0,w_*]$. Then we can use the fact that arctan is an odd function to upper bound the right side of (48) by $-c_0M_i^2(t,x)+\bar{L}_*|\epsilon||M_i(t,x)|\le-\frac{1}{2}c_0M_i^2(t,x)+\frac{1}{2c_0}\bar{L}_*^2|\epsilon|^2$, where we used Young's inequality $ab\le c_0a^2/2+b^2/(2c_0)$ with $a=|M_i(t,x)|$ and $b=\bar{L}_*|\epsilon|$ to upper bound $\bar{L}_*|\epsilon||M_i(t,x)|$.

Case 2. $|M_i(t,x)|>w_*$. In this case, we can use the fact that arctan is nondecreasing on $[0,\infty)$ and odd to upper bound the right side of (48) by $-(\pi/4)(\delta_*+1)|M_i(t,x)|+\bar{L}_*|\epsilon||M_i(t,x)|\le-(1-\delta_*)(\pi/4)|M_i(t,x)|$, by our choices of δ_* and w_*.

Combining the previous two cases, we conclude that for all choices of the functions δ and ϵ from Assumption 1, the time derivative of V along all solutions of (4) satisfies

$$\begin{aligned}&\dot{V}\le-\left\{\alpha_0(x)+\sum_{i=1}^{c}\mathcal{G}_i(t,x)\right\}+\gamma_*(|\delta|)+\tfrac{\bar{\omega}}{c_0\pi}\bar{L}_*^2|\epsilon|^2\\ &\text{where }\mathcal{G}_i(t,x)=\\ &\tfrac{2\bar{\omega}}{\pi}\min\left\{(c_0/2)M_i^2(t,x),(1-\delta_*)\tfrac{\pi}{4}|M_i(t,x)|\right\}\end{aligned} \tag{49}$$

for each i and $t\ge 0$. Recalling our assumption (b), we conclude that the sum in curly braces in (49) is uniformly proper and positive definite. Therefore, V is an ISS Lyapunov function for this closed loop system (as defined, e.g., in [22, Chapter 4]) for disturbances (ϵ,δ) valued in $\mathcal{B}_{\bar{\epsilon}}\times\mathbb{R}^d$, giving the ISS property of Assumption 1. □

Remark 4. We can replace the formulas $\arctan(M_i(t,x))$ in Lemma 2 by $\sigma_i(M_i(t,x))$ for any functions $\sigma_i:\mathbb{R}\to\mathbb{R}$ that satisfy: σ_i is a bounded C^1 strictly increasing odd function, $\lim_{s\to 0}\sigma_i(s)/s>0$, and $\sigma_i'(M_i(t,x))(\partial M_i/\partial x)(t,x)$ is a bounded function for $i=1,\dots,c$. Then Lemma 2 remains true if we replace $\pi/2$ in (45) by $\max_i|\sigma_i|_\infty$, replace the M_i^* formulas by $M_i^*(t,x)=\sigma_i'(M_i(t,x))(\partial M_i/\partial x)(t,x)$, and replace $2/\pi$ in the u_s formula by $1/\sup_s\sigma_i(s)$, by a similar proof. □

6. Application to bilinear systems

This special case of Lemma 2 covers bilinear systems, and is obtained by specializing Lemma 2 to the case where $M_i(t,x) = 2(x^\top P(t)B_i(t)x + x^\top P(t)G_i(t))$ using a quadratic Lyapunov function $V(t,x) = x^\top P(t)x$:

Lemma 3. *Let $A : \mathbb{R} \to \mathbb{R}^{n\times n}$, $D : \mathbb{R} \to \mathbb{R}^{n\times d}$, and $B_i : \mathbb{R} \to \mathbb{R}^{n\times n}$ and $G_i : \mathbb{R} \to \mathbb{R}^n$ for $i = 1,\dots,c$ be bounded matrix valued continuous functions. Assume that there are a function $\gamma_* \in \mathcal{K}_\infty$, constants $c_i \geq 0$, and a C^1 bounded function $P : \mathbb{R} \to \mathbb{R}^{n\times n}$ such that $P^\top(t) = P(t)$ for all $t \in \mathbb{R}$ and such that the following hold with $V(t,x) = x^\top P(t)x$:*

(a) along all solutions of $\dot{x} = A(t)x + D(t)\delta$, we have

$$\dot{V} \leq -\sum_{i=1}^{n} c_i x_i^2(t) + \gamma_*(|\delta(t)|) \tag{50}$$

at all times $t \geq 0$;

(b) the functions $V(t,x)$ and

$$\sum_{i=1}^{n} c_i x_i^2 + \sum_{i=1}^{c} \left|x^\top P(t)B_i(t)x + x^\top P(t)G_i(t)\right| \tag{51}$$

are uniformly proper and positive definite; and

(c) the functions

$$\mathcal{H}_i(t,x) = \frac{2(x^\top(P(t)B_i(t)+B_i^\top(t)P(t))+G_i^\top(t)P(t))}{1+4(x^\top P(t)B_i(t)x+x^\top P(t)G_i(t))^2} \tag{52}$$

are bounded for $i = 1,\dots,c$.

Choose a positive value $\mathcal{H}_* \geq \sup\{|\mathcal{H}_i(t,x)| : (t,x) \in \mathbb{R}^{n+1}, 1 \leq i \leq c\}$. Then, for any constants $\bar{\omega} > 0$ and $\bar{\epsilon} \in (0, \pi/(2\mathcal{H}_*))$, and with the feedback

$$\begin{aligned} & u_s(t,x) = -\bar{\omega}(\arctan(M_1(t,x)),\dots,\arctan(M_c(t,x))) \\ & \text{where } \; M_i(t,x) = 2\big(x^\top P(t)B_i(t)x + x^\top P(t)G_i(t)\big), \end{aligned} \tag{53}$$

the bilinear system (1) satisfies Assumptions 1–2. □

The preceding results are novel, even in the special case where the coefficient matrices in (1) are constant. To illustrate Lemma 3 in the constant coefficients case, we consider the case where the coefficient matrices in (1) and P are

$$A = \begin{bmatrix} A_0 & 0_{n_a\times n_b} \\ 0_{n_b\times n_a} & 0_{n_b\times n_b} \end{bmatrix}, \; P = \begin{bmatrix} P_0 & 0_{n_a\times n_b} \\ 0_{n_b\times n_a} & P_1 \end{bmatrix}, \tag{54}$$

$$D = \begin{bmatrix} D_1 \\ 0_{n_b\times d} \end{bmatrix}, \; B_i = \begin{bmatrix} B_{i1} & B_{i2} \\ B_{i3} & B_{i4} \end{bmatrix}, \text{ and } G_i = \begin{bmatrix} G_{i1} \\ G_{i2} \end{bmatrix} \tag{55}$$

for $i = 1,\dots,c$ for any n_a and $n_b = n - n_a$, where A_0 is Hurwitz and $P_0 \in \mathbb{R}^{n_a\times n_a}$ and $P_1 \in \mathbb{R}^{n_b\times n_b}$ are symmetric positive definite matrices, and

$$P_0 A_0 + A_0^\top P_0 = -I_{n_a}, \tag{56}$$

and where the upper sub-matrices in the block matrices D, B_i, and G_i consist of n_a rows. Using the triangle inequality, it follows that along all solutions of $\dot{x} = Ax + D\delta$, we have

$$\dot{V} = -|x_a|^2 + 2x_a^\top P_0 D_1 \delta \leq -\tfrac{1}{2}|x_a|^2 + 2|P_0|^2|D_1|^2|\delta|^2 \tag{57}$$

where x_a denotes the first n_a components of x. It follows that we can satisfy requirement (a) of Lemma 3 using $c_i = 0.5$ if $1 \leq i \leq n_a$ and $c_i = 0$ if $n_a < i \leq n$ and $\gamma_*(s) = 2|P_0|^2|D_1|^2 s^2$. Hence, if we let x_b denote the last n_b components of x, then condition (b) of Lemma 3 will also be satisfied provided

$$\mathcal{N}(x_b) = \sum_{i=1}^{c} \left|x_b^\top P_1 B_{i4} x_b + x_b^\top P_1 G_{i2}\right| \tag{58}$$

is proper and positive definite. This produces the following sufficient condition for condition (b) of Lemma 3 to hold when the coefficient matrices are constant: There is an index $i \in \{1,\dots,c\}$ such that $P_1 G_{i2} = 0$ and such that $P_1 B_{i4}$ is either negative definite or positive definite. We next illustrate the constant coefficient case in a significant power system dynamic involving inverters.

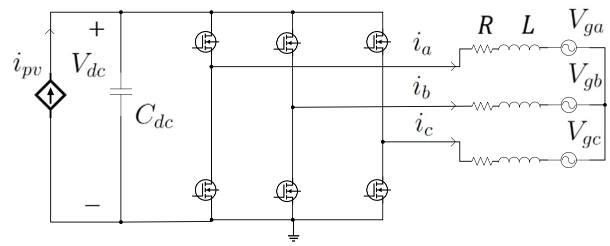

Fig. 1. Grid-connected three-phase PV converter.

7. Power systems application

We study a grid connected three-phase photovoltaic (PV) inverter shown in Fig. 1. The PV source is modeled using a voltage controlled current source; see [25]. Let i_{pv} denote the current value of the PV source, and i_a, i_b, and i_c and V_{ga}, V_{gb}, and V_{gc} denote the three-phase power grid currents and voltages, respectively. We let L and R denote the aggregated inductance and resistance of the output filter (which are needed for attenuating switching harmonics), transformers, and transmission lines connected to the output terminal of the converter, respectively, V_{dc} denote the voltage at the input terminal of the converter, and C_{dc} denote the dc-link capacitor required to maintain the input voltage steady. Using Park's transformation (e.g. from [25, Appendix 2]) to transform the three-phase power grid currents and voltages from the abc reference frame into the synchronously rotating $dq0$ reference frame gives

$$\begin{cases} L\frac{di_d}{dt} &= d_d V_{dc} - Ri_d + \omega L i_q - V_{gd} \\ L\frac{di_q}{dt} &= d_q V_{dc} - Ri_q - \omega L i_d - V_{gq} \\ C_{dc}\frac{dV_{dc}}{dt} &= i_{pv} - d_d i_d - d_q i_q \end{cases} \tag{59}$$

as our PV converter model, where ω is the angular frequency of the power grid voltage, and (V_{gd}, V_{gq}) and (i_d, i_q) are the d-q components of the power grid voltage and current in the $dq0$ frame, respectively; and d_d and d_q are corresponding controls associated with the switching states of the PV inverter. The constants L, R, ω, C_{dc}, and i_{pv} are positive, and the constants V_{gd} and V_{gq} are nonnegative.

We next apply Lemma 3 to a bilinear error dynamics corresponding to (59). Choose any reference values $I_d^* \geq 0$, $I_q^* \geq 0$, and $V_{dc}^* > 0$ for the states i_d, i_q, and V_{dc} respectively that satisfy $R(I_d^*)^2 + R(I_q^*)^2 + I_d^* V_{gd} + I_q^* V_{gq} = i_{pv} V_{dc}^*$, so I_d^* and I_q^* are not both zero. The corresponding steady state reference control values $D_d = (RI_d^* - \omega L I_q^* + V_{gd})/V_{dc}^*$ and $D_q = (RI_q^* + \omega L I_d^* + V_{gq})/V_{dc}^*$ then satisfy $RI_d^* = D_d V_{dc}^* + \omega L I_q^* - V_{gd}$, $RI_q^* = D_q V_{dc}^* - \omega L I_d^* - V_{gq}$, and $0 = i_{pv} - D_d I_d^* - D_q I_q^*$. Choosing the error state variables $x_1 = i_d - I_d^*$, $x_2 = i_q - I_q^*$, and $x_3 = V_{dc} - V_{dc}^*$ and the new control variables $u_1 = d_d - D_d$ and $u_2 = d_q - D_q$, the preceding relations produce the error equations

$$\begin{cases} \dot{x}_1(t) &= \frac{1}{L}\big[D_d x_3(t) + u_1(t-h)(x_3(t) + V_{dc}^*) \\ & \quad -Rx_1(t) + \omega L x_2(t)\big] \\ \dot{x}_2(t) &= \frac{1}{L}\big[D_q x_3(t) + u_2(t-h)(x_3(t) + V_{dc}^*) \\ & \quad -Rx_2(t) - \omega L x_1(t)\big] \\ \dot{x}_3(t) &= \frac{1}{C_{dc}}\big[-D_d x_1(t) - u_1(t-h)(x_1(t) + I_d^*) \\ & \quad -D_q x_2(t) - u_2(t-h)(x_2(t) + I_q^*)\big], \end{cases} \tag{60}$$

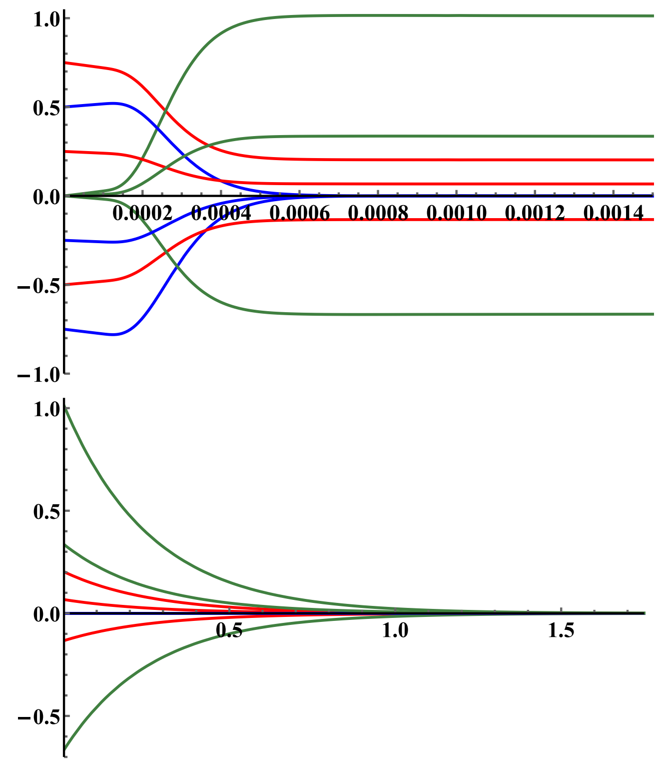

Fig. 2. First (Red), Second (Blue), and Third (Green) Components of (60), with Initial States $(0.75, -0.75, 0)$, $(-0.5, 0.5, 0)$, and $(0.25, -0.25, 0)$, On Time Intervals $[0, 0.0015]$ (Top Panel) and $[0.0015, 1.75]$ (Bottom Panel). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

which is a special case of (1) with $\delta = 0$. The requirements of Lemma 3 are satisfied with $n = 3$, $c = 2$, the diagonal matrix $P = 0.5\text{diag}\{L, L, C_{dc}\}$, $c_1 = c_2 = R$, $c_3 = 0$, $M_1(t, x) = V^*_{dc}x_1 - I^*_d x_3$, $M_2(t, x) = V^*_{dc}x_2 - I^*_q x_3$, and $\mathcal{H}_* = \max\{|(V^*_{dc}, I^*_d)|, |(V^*_{dc}, I^*_q)|\}$. Therefore, Theorem 1 applies.

Grid-tie inverters are widely employed for grid integration of renewable energy sources. For realizing any control scheme for the grid-tie inverter, voltage and current waveforms are measured and samples are sent to a digital signal processor (DSP). Sampling takes place at a fixed frequency. However, due to limited processing capability of DSPs and control scheme complexity, there is a delay between the time sample measurements are received and the time control actions are made by the DSP. Common sampling frequencies in grid-tie inverters are 10 to 20 kHz. As for digital implementation delay, it is a common assumption to consider it equal to the sampling period. Thus, we have considered a delay of 100 micro sec ($= 1/10$ kHz, i.e., the reciprocal of the sampling frequency) in this paper.

In Fig. 2, we plot Mathematica simulations for the state $x = (x_1, x_2, x_3)$ of (60) using the control from Theorem 1 with 0 initial states for each z_i, u_s from Lemma 3, the preceding choices, and (41) with $\bar{U} = \sqrt{2}\bar{\omega}(\pi/2)$. We choose $h = 0.0001$, $L = 0.015$, $V_{gd} = 230$, $V^*_{dc} = 400$, $R = 0.2$, $\omega = 120\pi$, $V_{gq} = 0$, $C_{dc} = 0.0015$, $I^*_d = 80$, $I^*_q = 0$, $\lambda_a = 0.1$, $\bar{\omega} = 1$, $m = 3$, and D_d, D_q, and i_{pv} from our formulas above, which satisfy the requirements from Theorem 1 with the choices $C_1 = 1.38/h$ and $C_2 = 0.4$.

We expressed the convergence in two phases in Fig. 2, to show the qualitatively different performance on the interval $[0, 0.0015]$ (during which only two of the error states converge closely to the 0 equilibrium) and the second phase during the interval $[0.0015, 1.75]$ (when all states convergence to the equilibrium). The preceding simulations show the good performance of our control design. Moreover, they illustrate the reduction in the number m of required sequential predictors made possible by Theorem 1, because if we had instead used the formula (13) for the lower bound on m with all other parameter values kept the same, then we would have required $m \geq 5$ sequential predictors, instead of allowing $m = 3$.

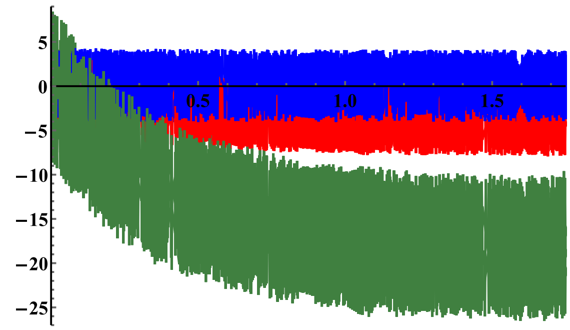

Fig. 3. First (Red), Second (Blue), and Third (Green) Components of (60), with Initial States $(0.75, -0.75, 0)$, $(-0.5, 0.5, 0)$, and $(0.25, -0.25, 0)$, On Time Interval $[0, 1.75]$ without Delay Compensation. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

While the components of our initial states at time $t_0 = 0$ range from -0.75 to 0.75, the corresponding $x_i(t)$ components are valued in $[-0.65, 1]$ by time 0.0015, and this explains why the starting values of the $x_i(t)$'s are contained in $[-0.65, 1]$ in the bottom panel which starts at time 0.0015. While h is small, it is significant relative to the system dynamics, and we can compensate for any constant $h > 0$. For example, if we increase h to $h = 0.001$, then our assumptions are satisfied with $m = 31$ and with all other constants kept the same. On other hand, for this larger h choice, the condition (13) would have required $m \geq 47$ sequential predictors, so again we see the improvement made possible by Theorem 1, which in this case is a 34% reduction (from 47 to 31) in the required number m of sequential predictors. The practical value of using $m = 31$ sequential predictors is the reduction in computational burden, as compared with traditional predictor methods that yield distributed terms and which therefore produce an infinite dimensional analysis.

It is tempting to surmise that since the delay h is small, the system would exhibit good performance even without predictors. However, this would not be correct, because our predictive approach still improves on the control performance compared to what we would have obtained without delay compensation. This is illustrated in our Mathematica simulation in Fig. 3, where we replaced the components of the last sequential predictor z_3 in the controls by the corresponding components of x (by simulating (60) with the controls $u_1(t-h) = -\arctan(V^*_{dc}x_1(t-h) - I^*_d x_3(t-h))$ and $u_2(t-h) = -\arctan(V^*_{dc}x_2(t-h) - I^*_q x_3(t-h))$ and the parameter values stated above), which corresponds to not compensating for the delay, and kept everything else the same as the first simulation. Since Fig. 3 illustrates the lack of convergence in the absence of delay compensation, it also helps motivate our methods.

8. Conclusions

We presented a new sequential predictor approach to feedback stabilization under arbitrarily long constant input delays which can be applied to bilinear systems that violate the usual linear growth conditions. Compared with other delay compensation approaches, potential advantages include that the closed-loop systems satisfy ISS without using distributed terms in the control that were present in exact predictor approaches. In future work, we hope to explore applications to large scale networked systems as in [15] and extensions for reaction–diffusion PDEs as in [16]. We will also study cases where there are different delays in different components of the input, which may call for bilinear analogs of the predictor structures from [2] having different sets of chain predictors corresponding to the different input delays.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix. Proof of Lemma 1

Using the definition of f_0 gives

$$\begin{aligned}&\dot{\mathcal{E}}_1(t) = -p\mathcal{E}_1\left(t-\tfrac{h}{m}\right)\\&+f_0\left(t+\tfrac{h}{m}, z_1(t), u\left(t-\tfrac{h(m-1)}{m}\right)\right)\\&-f\left(t+\tfrac{h}{m}, x\left(t+\tfrac{h}{m}\right), u\left(t-\tfrac{h(m-1)}{m}\right), \delta\left(t+\tfrac{h}{m}\right)\right)\end{aligned}\tag{A.1}$$

$$\begin{aligned}&\text{and } \dot{\mathcal{E}}_i(t) = -p\mathcal{E}_i\left(t-\tfrac{h}{m}\right)+p\mathcal{E}_{i-1}(t)\\&+f_0\left(t+i\tfrac{h}{m}, z_i(t), u\left(t-\tfrac{h(m-i)}{m}\right)\right)\\&-f_0\left(t+i\tfrac{h}{m}, z_{i-1}\left(t+\tfrac{h}{m}\right), u\left(t-\tfrac{h(m-i)}{m}\right)\right)\end{aligned}\tag{A.2}$$

when $i>1$. We first study the $\mathcal{E}_1$-subsystem (A.1).

The Fundamental Theorem of Calculus yields

$$\begin{aligned}&\dot{\mathcal{E}}_1(t) = -p\mathcal{E}_1(t)+p\int_{t-\frac{h}{m}}^{t}\dot{\mathcal{E}}_1(\ell)\mathrm{d}\ell\\&+f_0\left(t+\tfrac{h}{m}, z_1(t), u\left(t-\tfrac{h(m-1)}{m}\right)\right)\\&-f\left(t+\tfrac{h}{m}, x\left(t+\tfrac{h}{m}\right), u\left(t-\tfrac{h(m-1)}{m}\right), \delta\left(t+\tfrac{h}{m}\right)\right).\end{aligned}\tag{A.3}$$

Then Assumption 2 and Young's inequality give

$$\begin{aligned}\dot{\nu}(t) &\le -p|\mathcal{E}_1(t)|^2+p\int_{t-\frac{h}{m}}^{t}\mathcal{E}_1(t)^\top\dot{\mathcal{E}}_1(\ell)\mathrm{d}\ell\\&\quad+|\mathcal{E}_1(t)|k\left(\left|z_1(t)-x\left(t+\tfrac{h}{m}\right)\right|+\left|\delta\left(t+\tfrac{h}{m}\right)\right|\right)\\&=(k-p)|\mathcal{E}_1(t)|^2+p\int_{t-\frac{h}{m}}^{t}\mathcal{E}_1(t)^\top\dot{\mathcal{E}}_1(\ell)\mathrm{d}\ell\\&\quad+k|\mathcal{E}_1(t)|\left|\delta\left(t+\tfrac{h}{m}\right)\right|\\&\le(k-p)|\mathcal{E}_1(t)|^2+k|\mathcal{E}_1(t)|\left|\delta\left(t+\tfrac{h}{m}\right)\right|\\&\quad+p\int_{t-\frac{h}{m}}^{t}\left[\tfrac{C_1}{2}|\mathcal{E}_1(t)|^2+\tfrac{1}{2C_1}|\dot{\mathcal{E}}_1(\ell)|^2\right]\mathrm{d}\ell\\&=\left(k-p+\tfrac{phC_1}{2m}\right)|\mathcal{E}_1(t)|^2+\tfrac{p}{2C_1}\int_{t-\frac{h}{m}}^{t}|\dot{\mathcal{E}}_1(\ell)|^2\mathrm{d}\ell\\&\quad+k|\mathcal{E}_1(t)|\left|\delta\left(t+\tfrac{h}{m}\right)\right|.\end{aligned}\tag{A.4}$$

Next note that (A.1) gives the following for all $\ell\ge 0$:

$$\begin{aligned}&|\dot{\mathcal{E}}_1(\ell)|\\&\le p\left|\mathcal{E}_1\left(\ell-\tfrac{h}{m}\right)\right|+\Big|f_0\left(\ell+\tfrac{h}{m}, z_1(\ell), u\left(\ell-\tfrac{h(m-1)}{m}\right)\right)\\&-f\left(\ell+\tfrac{h}{m}, x\left(\ell+\tfrac{h}{m}\right), u\left(\ell-\tfrac{h(m-1)}{m}\right), \delta\left(\ell+\tfrac{h}{m}\right)\right)\Big|\\&\le p\left|\mathcal{E}_1\left(\ell-\tfrac{h}{m}\right)\right|+k\left(|\mathcal{E}_1(\ell)|+\left|\delta\left(\ell+\tfrac{h}{m}\right)\right|\right),\end{aligned}$$

by Assumption 2. Hence, Young's Inequality gives

$$\begin{aligned}|\dot{\mathcal{E}}_1(\ell)|^2 &\le (1+C_2)p^2|\mathcal{E}_1(\ell-h/m)|^2\\&\quad+\left(1+\tfrac{1}{C_2}\right)k^2\left(|\mathcal{E}_1(\ell)|^2+|\delta(\ell+h/m)|^2\right.\\&\quad\left.+2|\mathcal{E}_1(\ell)||\delta(\ell+h/m)|\right)\\&\le(1+C_2)p^2|\mathcal{E}_1(\ell-h/m)|^2\\&\quad+\left(1+\tfrac{1}{C_2}\right)k^2\left(\left(1+\tfrac{\lambda_a}{4}\right)|\mathcal{E}_1(\ell)|^2\right.\\&\quad\left.+(1+4/\lambda_a)|\delta(\ell+h/m)|^2\right)\end{aligned}$$

for all $t\ge 0$. Therefore, (A.4) gives

$$\begin{aligned}\dot{\nu}(t) &\le \left(k-p+\tfrac{phC_1}{2m}\right)|\mathcal{E}_1(t)|^2+k|\mathcal{E}_1(t)||\delta(t+h/m)|\\&\quad+\tfrac{p^3(1+C_2)}{2C_1}\int_{t-2h/m}^{t-h/m}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell\\&\quad+\tfrac{pk^2}{2C_1}\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{\lambda_a}{4}\right)\int_{t-h/m}^{t}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell\\&\quad+\tfrac{pk^2}{2C_1}\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{4}{\lambda_a}\right)\int_{t}^{t+h/m}|\delta(\ell)|^2\mathrm{d}\ell\end{aligned}\tag{A.5}$$

for all $t\ge\frac{h}{m}$. Since Young's Inequality also gives

$$k|\mathcal{E}_1(t)||\delta(t+h/m)|\le\tfrac{\lambda_a}{2}|\mathcal{E}_1(t)|^2+\tfrac{k^2}{2\lambda_a}|\delta(t+h/m)|^2,\tag{A.6}$$

we can use (A.6) to upper bound the last term in (A.5) and our choice of p from (6) and finally our choice of ν to get

$$\begin{aligned}\dot{\nu}(t) &\le -k|\mathcal{E}_1(t)|^2+\tfrac{p^3(1+C_2)}{2C_1}\int_{t-2h/m}^{t-h/m}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell\\&\quad+\tfrac{pk^2}{2C_1}\left(1+\tfrac{1}{C_2}\right)\left(1+\tfrac{\lambda_a}{4}\right)\int_{t-h/m}^{t}|\mathcal{E}_1(\ell)|^2\mathrm{d}\ell\\&\quad+\bar{M}|\delta|^2_{[t,t+h/m]}\\&\le -2k\nu(\mathcal{E}_1(t))+\overline{C}\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell\mathrm{d}\ell\\&\quad+\bar{M}|\delta|^2_{[t,t+h/m]},\end{aligned}\tag{A.7}$$

where $\bar{M}$ is defined in (6).

Recalling our choice of $\mu(\mathcal{E}_{1,t})$ from (16), it follows that for all $t\ge h/m$, we have

$$\begin{aligned}&\tfrac{d}{dt}\mu(\mathcal{E}_{1,t})\le-2k\nu(\mathcal{E}_1(t))+\bar{M}|\delta|^2_{[t,t+h/m]}\\&+\overline{C}\left(\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell+\tfrac{2h(1+\lambda_a)}{m}\nu(\mathcal{E}_1(t))\right)\\&-\overline{C}(1+\lambda_a)\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell.\end{aligned}\tag{A.8}$$

This gives

$$\begin{aligned}&\tfrac{d}{dt}\mu(\mathcal{E}_{1,t})\le 2k\left[-1+\tfrac{h\overline{C}}{km}(1+\lambda_a)\right]\nu(\mathcal{E}_1(t))\\&-\lambda_a\overline{C}\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell+\bar{M}|\delta|^2_{[t,t+h/m]}.\end{aligned}\tag{A.9}$$

Therefore, our condition (8) from our theorem and our choice of ϵ_0 in (6), combined with the bound

$$\tilde{\mu}(\mathcal{E}_{1,t})\le\nu(\mathcal{E}_1(t))+2\left(1+\tfrac{h\overline{C}(1+\lambda_a)}{m}\right)\int_{t-2h/m}^{t}\nu(\mathcal{E}_1(\ell))\mathrm{d}\ell,$$

give (17) along all trajectories of the $\mathcal{E}_1$ dynamics.

Similarly, since there is no δ in the z system, (A.2) and the relation $2rs\le\lambda_a r^2/4+4s^2/\lambda_a$ for all $r\ge 0$ and $s\ge 0$ give

$$\begin{aligned}|\dot{\mathcal{E}}_i(\ell)|^2 &\le (1+C_2)p^2|\mathcal{E}_i(\ell-h/m)|^2\\&\quad+\left(1+\tfrac{1}{C_2}\right)(k|\mathcal{E}_i(\ell)|+p|\mathcal{E}_{i-1}(\ell)|)^2\\&\le(1+C_2)p^2|\mathcal{E}_i(\ell-h/m)|^2\\&\quad+\left(1+\tfrac{1}{C_2}\right)k^2(1+\lambda_a/4)|\mathcal{E}_i(\ell)|^2\\&\quad+\left(1+\tfrac{1}{C_2}\right)p^2(1+4/\lambda_a)|\mathcal{E}_{i-1}(\ell)|^2\end{aligned}\tag{A.10}$$

for any $i \in \{2, 3, \ldots, m\}$. This implies that the function $\mu(\mathcal{E}_{i,t})$ from (18) satisfies the following for all $t \geq h/m$:

$$\begin{array}{rcl} \frac{d}{dt}\mu(\mathcal{E}_{i,t}) & \leq & -\epsilon_0\tilde{\mu}(\mathcal{E}_{i,t}) + p|\mathcal{E}_i(t)||\mathcal{E}_{i-1}(t)| \\ & & +C_2^\sharp \int_{t-h/m}^t |\mathcal{E}_{i-1}(\ell)|^2 \mathrm{d}\ell \\ & \leq & -\epsilon_0\tilde{\mu}(\mathcal{E}_{i,t}) + \frac{\epsilon_0}{4}|\mathcal{E}_i(t)|^2 + \frac{p^2}{\epsilon_0}|\mathcal{E}_{i-1}(t)|^2 \\ & & + C_2^\sharp \int_{t-h/m}^t |\mathcal{E}_{i-1}(\ell)|^2 \mathrm{d}\ell \\ & \leq & -\frac{\epsilon_0}{2}\tilde{\mu}(\mathcal{E}_{i,t}) + \frac{p^2}{\epsilon_0}|\mathcal{E}_{i-1}(t)|^2 \\ & & + C_2^\sharp \int_{t-h/m}^t |\mathcal{E}_{i-1}(\ell)|^2 \mathrm{d}\ell, \end{array} \quad (A.11)$$

where the second inequality used Young's inequality and

$$C_2^\sharp = \left(1 + \frac{1}{C_2}\right) \frac{p^3}{2C_1} (1 + 4/\lambda_a)\,, \quad (A.12)$$

which proves the lemma.

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