Heat purchase agreements could lower barriers to heat pump adoption

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Abstract

Efficient electric heat pumps have the potential to significantly reduce greenhouse gas emissions from heating and cooling buildings. However, heat pumps' initial costs can be prohibitively high and their lifetime costs are only situationally competitive with incumbent technologies. Here we show that a business model based on heat purchase agreements could lower these barriers to heat pump adoption. In this business model, a user hosts a heat pump owned by an aggregator. The aggregator installs the heat pump at low or no initial cost to the user. The user buys the heat pump's heat or cooling output from the aggregator. The aggregator buys the heat pump's input electricity in the wholesale energy market and sells the flexibility of their aggregate electrical load in ancillary service markets. This paper presents the first economic analysis of heat purchase agreements as a third-party ownership model for electric heat pumps. We derive conditions under which a heat purchase agreement is mutually beneficial to the user and the aggregator. We also provide a method to fairly price heat and cooling. A case study of a typical United States home shows that a heat purchase agreement could more than double the value of a heat pump investment relative to the incumbent business model. The potential impact of this work is to reduce emissions both directly, by accelerating replacement of fossil-fueled or inefficient heating or cooling equipment, and indirectly, by helping power system operators reliably integrate wind and solar generation.

Keywords: heat pumps, aggregation, business models, flexibility, ancillary services, grid-integration of renewables

1. Introduction

One tenth of anthropogenic greenhouse gas emissions are caused by heating and cooling buildings [1]. These emissions could increase sharply in the coming decades, as global demand for heat and cooling is projected to double by 2050 [2]. To avoid the worst consequences of global climate change, deep reductions in greenhouse gas emissions from all economic sectors, including heating and cooling, will likely be necessary [3]. Here we focus on one approach to reducing emissions from heating and cooling: replacing fossil-fueled or inefficient heating or cooling equipment by efficient electric heat pumps.

Electric heat pumps are machines that use electricity to move heat. In summer, heat pumps can provide cooling by moving heat from indoors to outdoors. In winter, heat pumps can provide heating by operating in the reverse direction. Two popular types of heat pumps are *air-source*, which transfer heat to and from the outdoor air, and *ground-source*, which transfer heat to and from the ground. The methods in this paper apply to air- or ground-source electric heat pumps used in new or existing buildings for heating or cooling.

Heat pump technology has advanced significantly in recent years, in terms of both efficiency and cold-weather heating capacity [4]. Meanwhile, the greenhouse gas intensity of electricity has decreased in much of the world, driven by fuel transitions and rising power plant efficiencies [5]. Due to these technological advances, heat pumps now have the potential to reduce emissions from heating and cooling in many settings by half or more [6].

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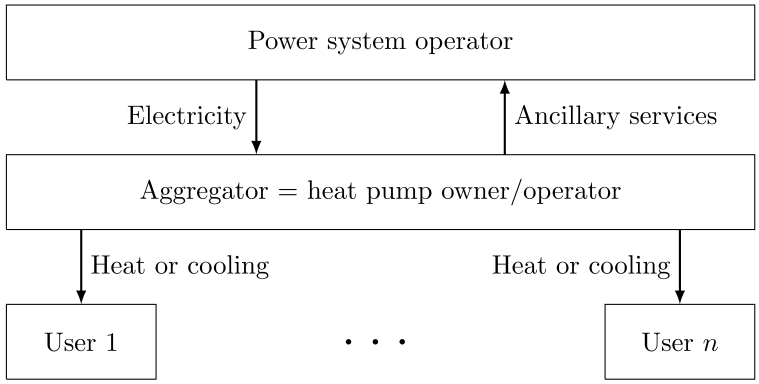

Figure 1: Stakeholders (boxes) and commodity flows (arrows) in the HPA business model. The aggregator buys electricity from the power system operator, sells heat or cooling to the users, and sells ancillary services to the power system operator.

While heat pumps can reduce emissions in many settings, they face several barriers to widespread adoption. Their lifetime costs are not always competitive with incumbent technologies such as natural gas furnaces and central air conditioners [6]. Even when heat pumps' lifetime costs are competitive, the initial costs of procuring and installing them can be prohibitive [7]. Additional barriers to adoption include finding installers who are familiar with modern heat pumps, selecting appropriate heat pump models and sizes, and finding and applying for rebates, tax credits or other incentives [8].

In this paper, we investigate a business model, illustrated in Figure 1, that could lower barriers to heat pump adoption. This business model centers on a *heat purchase agreement* (HPA) between a user and an aggregator. Under an HPA, the user hosts a heat pump owned by the aggregator. The aggregator installs the heat pump at low or no initial cost to the user. Over the duration of the HPA, the user chooses their desired indoor air temperatures; the aggregator operates the heat pump in order to maintain these temperatures. The user pays the aggregator for the heat or cooling the heat pump produces. The aggregator buys the electricity the heat pump consumes and pays for any required maintenance. The aggregator also sells the flexibility of their aggregate electrical load in power system ancillary service markets. (We discuss ancillary services further in Section 2.3.) When the HPA ends, ownership of the heat pump transfers to the user.

HPAs could lower two key barriers to heat pump adoption. First, HPAs could reduce or eliminate the user's initial cost. Second, HPAs could reduce heat pumps' lifetime costs by opening access to wholesale energy and ancillary service markets. The second point is a matter of scale. To keep operations tractable, wholesale market operators typically require market participants to have at least ~100 kW of electric power capacity. This requirement bars individual heat pumps, whose electric power capacities are typically on the order of 1 kW, from participating in wholesale markets. If heat pumps were aggregated by the hundreds or more, however, then the aggregate load would likely meet the requirements for wholesale market participation. The HPA business model provides the needed mechanism for aggregating heat pumps. This could enable an HPA aggregator to buy electricity wholesale, rather than retail, and to sell the flexibility of its aggregate load in ancillary service markets. (An aggregator buying electricity wholesale would likely need to pay additional fees for use of transmission and distribution systems; we quantify these fees in Section 5.1.4.)

1.1. Contributions and potential impact

The central research contribution of this paper is to analyze the economics of the HPA business model for aggregating electric heat pumps. To our knowledge, this paper is the first such analysis in the research literature. Specifically, our economic analyses include (1) deriving conditions under which an HPA is mutually beneficial to the user and the aggregator (Theorem 1 of Section 3.2), and (2) developing a method to fairly price heat and cooling (Algorithm 1 of Section 3.3).

To support these economic analyses, we present a numerical case study of a heat pump HPA for a typical single-family home in the United States. Monte Carlo simulations suggest that in this setting, an HPA

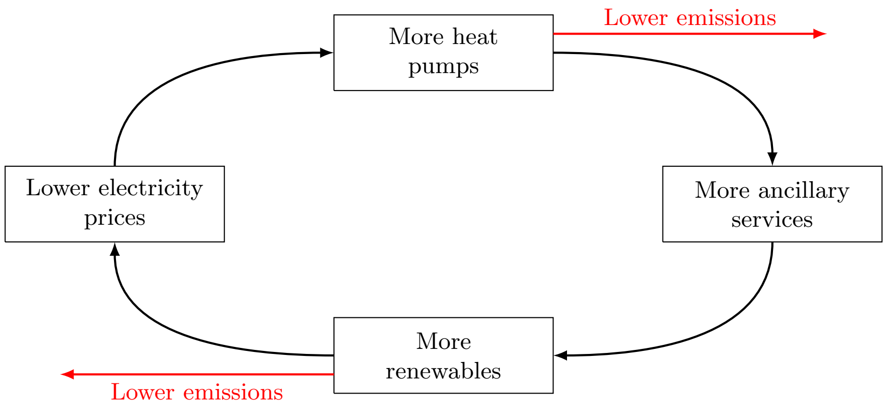

Figure 2: Heat pumps and renewables could form a self-reinforcing cycle, reducing emissions from heating, cooling and electricity generation.

could more than double the total value of a heat pump investment relative to the incumbent business model. About two thirds of the new value comes from electricity price reductions. The remaining one third comes from ancillary service revenues.

The potential impact of the HPA business model is the self-reinforcing cycle illustrated in Figure 2. We now walk through this cycle, beginning at the top.

1. HPAs could improve heat pump economics, accelerating heat pump adoption.
2. An HPA aggregator could use these heat pumps to provide ancillary services, helping power system operators reliably integrate more renewable generation.
3. Integrating more renewable generation could drive down wholesale electricity prices, as has been observed in regions with high shares of wind and solar generation such as Texas [9] and Germany [10, 11].
4. Lower electricity prices could make heat pumps more economically attractive, encouraging further heat pump adoption.

This cycle could reduce emissions directly, by displacing fossil-fueled or inefficient heating and cooling equipment in favor of efficient heat pumps powered by clean electricity. It could also reduce emissions indirectly, by facilitating the transition from fossil-fueled power plants to renewable generation.

1.2. Paper organization

This paper is organized as follows. We review related research in Section 2. In Section 3, we analyze heat pump economics under the incumbent business model and the HPA model. The economic analysis includes modeling (3.1), derivation of mutual benefit conditions (3.2), and development of a thermal pricing algorithm (3.3). In Section 4, we discuss extensions and variations of the economic analyses. In Section 5, we numerically investigate HPAs for typical United States homes via Monte Carlo simulation. Scenarios and input data are described in Section 5.1; results are discussed in Section 5.2. We conclude in Section 6.

1.3. Notation

Table 1 defines the notation used in this paper. To distinguish between business models and stakeholders, we augment the notation in Table 1 as follows. Symbols with no subscripts correspond to the user's economics under the incumbent business model. Symbols with the subscript u correspond to the user's economics under an HPA. Symbols with the subscript a correspond to the aggregator's economics under an HPA. So, for a few examples, c denotes the user's initial unsubsidized heat pump cost under the incumbent business model,

Table 1: Notation

Symbol	Meaning
c	Initial unsubsidized cost (\$)
s	Initial subsidy (\$)
ρ	Cash flow discount rate (-)
k	Index of discrete time periods (-)
K	Number of discrete time periods (-)
$\pi(k)$	Electricity price in period k (\$/kWh)
$Q_h(k)$	Heat load in period k (kWh)
$Q_c(k)$	Cooling load in period k (kWh)
$\eta_h(k)$	Heating COP in period k (-)
$\eta_c(k)$	Cooling COP in period k (-)
$m(k)$	Maintenance cost in period k (\$)
$\pi_h(k)$	Heat price in period k (\$/kWh)
$\pi_c(k)$	Cooling price in period k (\$/kWh)
$r(k)$	Ancillary service revenue in period k (\$)
v_K	Salvage value (\$)

s_u denotes the initial subsidy received by the user under an HPA, and $m_a(k)$ denotes the aggregator's maintenance cost in period k under an HPA. We also denote the set of real numbers by $\mathbf{R}$, the set of n-dimensional real vectors by $\mathbf{R}^n$, and the expectation operator by $\mathbf{E}$.

2. Literature review

2.1. Incumbent heat pump business model

Today, most heat pumps are sold under a relatively simple business model. In this model, an installer buys a heat pump from a manufacturer and installs it in the user's building. The user pays the installer an initial fee that covers the heat pump, the installation parts and labor, and the installer's overhead and profit. The user owns and operates the heat pump and pays for any required maintenance over its lifetime. The user buys the heat pump's input energy from their electricity provider at retail prices.

The economics of heat pump ownership under the incumbent business model were recently analyzed in [6], [12] and [13]. In [6], Billimoria *et al.* compare heat pumps to competing technologies, such as central air conditioners and space- and water-heating systems fueled by propane, heating oil and natural gas, in three representative locations in the United States. In [12], Nadel compares heat pumps to heating systems fueled by propane and heating oil. Both reports find that under the incumbent business model, heat pumps' lifetime costs are competitive in most situations with heating systems fueled by propane and heating oil, but [6] finds that heat pumps' lifetime costs are rarely competitive with natural gas heating systems. These findings are echoed in [13]. As natural gas is the most prevalent heating fuel in the United States, the analysis in [6] suggests that innovations are needed to make heat pumps economically attractive in this setting. HPAs, which could improve heat pump economics by enabling access to wholesale energy and ancillary service markets, are one such innovation.

2.2. Innovative heat pump business models

HPAs can be found in the marketing materials of several energy companies and in a small number of academic papers, such as [14] and [15]. In both cases, the HPA model arises in the context of district heating systems, where hot water or steam is produced at a central plant and circulated through a distribution network to many users. To our knowledge, this paper is the first academic investigation of HPAs in the context of stand-alone heat pumps serving individual users.

HPAs are conceptually similar to *power purchase agreements*, a business model prevalent in the United States rooftop solar photovoltaic market [16]. In the solar power purchase agreement model, a firm owns solar panels hosted by users; the users buy the power that the panels produce. The primary distinction between HPAs and solar power purchase agreements is that in an HPA, the aggregator draws electricity from the grid and uses it to provide heat or cooling. In a solar power purchase agreement, by contrast, the firm generates electricity from sunlight on site. Power purchase agreements have lowered several key barriers to adoption in rooftop solar markets [17]. A central result of this paper is that HPAs could lower similar barriers to adoption in heat pump markets.

Another relevant ownership model is *property-assessed clean energy financing*. In this model, a user borrows money, typically from a local government or nonprofit agency, and uses it to buy solar panels, windows, insulation, or other energy-efficiency upgrades. The loan is repaid through the user's property taxes. If the user moves before the loan is fully repaid, the remaining debt transfers with the property to the next owner. Property-assessed clean energy financing has accelerated adoption of clean energy technologies in California [18, 19] and Colorado [20]. Adapting this model to heat pumps could lower barriers to adoption, as it would provide users with a source of capital and reduce risks associated with moving. An HPA aggregator could likely benefit from a similar mechanism tying revenues to properties, rather than to individual users.

2.3. Heat pumps, renewables and the grid

The central task of power system operation is to continuously balance electricity supply and demand. This task grows more challenging as power systems integrate more wind and solar generation, which are variable and uncertain [21]. To maintain reliability with increasing shares of wind and solar generation, power system operators will likely need more *ancillary services*. This trend has been observed in California [22], the northwestern United States [23], and other regions with large shares of renewable generation.

Ancillary services are commodities that are traded in power system markets to facilitate the balancing of electricity supply and demand [24]. Most ancillary services involve perturbing power at the system operator's request, at time scales ranging from seconds to tens of minutes. For a recent review of the various ancillary services traded in United States power system markets, we refer the reader to [25].

Ancillary services have traditionally been provided by generators, but controllable loads can also provide them [26]. In particular, recent experiments have demonstrated that individual heat pumps equipped with variable-speed electric motors can rapidly and accurately track requested power perturbations [27, 28]. Another body of work demonstrates through simulation that fixed-speed heat pumps can also provide ancillary services, assuming they are aggregated in sufficient numbers. This body of work includes, for a few examples, studies of load following [29], frequency regulation [30], demand response and peak shaving [31], reserve and power reference tracking [32], and price-based load shifting [33]. The aforementioned papers focus mainly on technical feasibility; estimates of the revenues that heat pumps could earn in ancillary service markets can be found in [34] and [35] for California, Section 5.3 of [36] for the Pennsylvania–Jersey–Maryland region, and [37] for German markets.

2.4. Pricing heat and cooling

One contribution of this paper is a new method to price the heat and cooling that an HPA aggregator sells to its users. The problem of pricing heat has been thoroughly studied in the context of district heating systems, where heat is produced at a central plant and circulated through a distribution network to many users. The context of this paper is a stand-alone heat pump located at the user's building, rather than a district heating system. To provide some background, however, we briefly summarize the two most common pricing methods for district heating here. For a detailed review of these and other pricing methods, we refer the reader to [38].

Some district heating producers are granted monopoly status by their governments and regulated accordingly. In these systems, the regulator typically sets the heat price equal to the producer's reported operating cost plus a reasonable, permitted profit. Other district heating producers operate in competitive markets. In these market-based systems, market operators typically set the heat price equal to the marginal cost of heat, *i.e.*, the system-wide cost of producing an incremental unit of heat. As heat demand and producer efficiencies vary with outdoor temperatures and the time of day, this method results in time-varying heat prices.

The pricing method we propose in this paper differs significantly from pricing methods in district heating systems. As the HPA aggregator we consider is neither a regulated monopoly nor a heat market operator, the pricing methods mentioned above are not applicable. Instead, we propose pricing heat and cooling such that the HPA benefits both the user, by reducing their lifetime costs relative to the incumbent business model, and the aggregator, by providing an attractive return on their initial investment. Our method has flexible input parameters that govern, for two examples, the user's initial cost burden and the rate at which thermal prices increase over time. These input parameters can be decided by the aggregator and the user when they negotiate the HPA contract.

3. Economic analysis

In this section, we analyze the economics of heat pump ownership under both the incumbent business model and an HPA. Physically, the heat pump system is modeled through its heating and cooling coefficients of performance (COPs); the thermal load that the heat pump serves is modeled through its demand for heat and/or cooling. Because the heat pump and thermal load models are quite general, many physical systems may be analyzed using the methods in this section. In particular, the heat pump could be air-source or ground-source; could provide heat, cooling, or both; and could condition space directly or be coupled to a forced-air or hydronic distribution system. The thermal load could represent space heating, space cooling, or water heating; it could be located in a residential, commercial, or industrial building.

3.1. Modeling

3.1.1. Incumbent business model

The incumbent heat pump business model provides a useful benchmark against which an HPA can be compared. In the incumbent business model, a user pays an installer to obtain and install a heat pump. The user may receive an initial subsidy from a government agency or their electricity provider. The user buys the electricity the heat pump consumes and pays for any required maintenance over the heat pump's lifetime.

In the incumbent business model, the user's net present cost can be written as

$$c - s + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \left[\pi(k) \left(\frac{Q_h(k)}{\eta_h(k)} + \frac{Q_c(k)}{\eta_c(k)} \right) + m(k) \right] - \frac{v_K}{(1+\rho)^K}. \tag{1}$$

Here k indexes discrete time periods, K is the number of periods in the time horizon, c (\$) is the unsubsidized initial cost paid by the user to an incumbent heat pump installer (including parts, labor, and the installer's overhead and profit), s (\$) is the combined subsidy received by the user and the installer, ρ is the rate at which the user discounts future cash flows, and v_K (\$) is the heat pump's salvage value after period K. In period k, $\pi(k)$ (\$/kWh) is the electricity price paid by the user, $Q_h(k)$ (kWh) is the user's heat load, $Q_c(k)$ (kWh) is the user's cooling load, $\eta_h(k)$ is the heat pump's heating COP, $\eta_c(k)$ is the heat pump's cooling COP, and $m(k)$ (\$) is the maintenance cost paid by the user (including parts, labor, and the maintainer's overhead and profit).

3.1.2. Heat purchase agreement

Under an HPA, the user pays the aggregator an initial unsubsidized cost c_u (\$). A key feature of an HPA is that it can reduce the user's initial cost c_u far below the user's initial cost c under the incumbent business model. In each period k over the heat pump's lifetime, the user buys heat and cooling from the aggregator at prices $\pi_h(k)$ (\$/kWh) and $\pi_c(k)$ (\$/kWh), respectively. To account for the user's initial cost reduction, their heat and cooling prices under the HPA will generally be higher than the effective heat and cooling prices under the incumbent business model, which are simply the price of electricity divided by the heat pump's heating and cooling COPs. In other words, when comparing a typical HPA to the incumbent business model, the user would see an initial cost reduction ($c_u < c$) in exchange for increased heat prices ($\pi_h(k) > \pi(k)/\eta_h(k)$) and cooling prices ($\pi_c(k) > \pi(k)/\eta_c(k)$).

The user's net present cost under an HPA can be written as

$$c_u - s_u + \sum_{k=1}^{K} \frac{\pi_h(k)Q_h(k) + \pi_c(k)Q_c(k)}{(1+\rho)^k} - \frac{v_K}{(1+\rho)^K}, \tag{2}$$

where s_u (\$) is the subsidy received by the user under the HPA. The net present value of the HPA to the user is the difference between the user's net present cost under the incumbent business model and the user's net present cost under the HPA. Mathematically, the net present value of the HPA to the user can be written as

$$\begin{aligned} v_u(\pi_h, \pi_c, c_u) := c - s - (c_u - s_u) + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \Bigg[\left(\frac{\pi(k)}{\eta_h(k)} - \pi_h(k) \right) Q_h(k) \\ + \left(\frac{\pi(k)}{\eta_c(k)} - \pi_c(k) \right) Q_c(k) + m(k) \Bigg]. \end{aligned} \tag{3}$$

The price vectors on the left-hand side of Equation (3) are defined as $\pi_h := (\pi_h(1), \ldots, \pi_h(K)) \in \mathbf{R}^K$ and $\pi_c := (\pi_c(1), \ldots, \pi_c(K)) \in \mathbf{R}^K$.

From the aggregator's perspective, the initial HPA transaction involves paying an initial unsubsidized cost c_a (\$) to obtain and install the heat pump, then receiving the initial payment c_u from the user. The aggregator's initial cost c_a includes hardware, labor and overhead, but no profit. In each period k over the heat pump's lifetime, the aggregator buys the heat pump's electricity at price $\pi_a(k)$ (\$/kWh), pays the maintenance costs $m_a(k)$ (\$) (again including parts, labor and overhead, but no profit), earns revenue $r(k)$ (\$) in ancillary service markets, and sells heat and cooling to the user at prices $\pi_h(k)$ and $\pi_c(k)$, respectively. Mathematically, the net present value of the HPA to the aggregator can be written as

$$\begin{aligned} v_a(\pi_h, \pi_c, c_u) := c_u - c_a + s_a + \sum_{k=1}^{K} \frac{1}{(1+\rho_a)^k} \Bigg[\left(\pi_h(k) - \frac{\pi_a(k)}{\eta_h(k)} \right) Q_h(k) \\ + \left(\pi_c(k) - \frac{\pi_a(k)}{\eta_c(k)} \right) Q_c(k) - m_a(k) + r(k) \Bigg]. \end{aligned} \tag{4}$$

Here s_a (\$) is the initial subsidy received by the aggregator and ρ_a is the rate at which the aggregator discounts future cash flows.

The user's and aggregator's net present values under the HPA can be written more compactly as

$$\begin{aligned} v_u(x) &= U - a_u^\top x \\ v_a(x) &= a_a^\top x - L. \end{aligned} \tag{5}$$

Here $x := (\pi_h, \pi_c, c_u) \in \mathbf{R}^{2K+1}$ is a vector of price variables. The upper limit $U \in \mathbf{R}$ and lower limit $L \in \mathbf{R}$ are defined as

$$\begin{aligned} U &:= c - s + s_u + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \left[\pi(k) \left(\frac{Q_h(k)}{\eta_h(k)} + \frac{Q_c(k)}{\eta_c(k)} \right) + m(k) \right] \\ L &:= c_a - s_a + \sum_{k=1}^{K} \frac{1}{(1+\rho_a)^k} \left[\pi_a(k) \left(\frac{Q_h(k)}{\eta_h(k)} + \frac{Q_c(k)}{\eta_c(k)} \right) + m_a(k) - r(k) \right]. \end{aligned} \tag{6}$$

The coefficient vectors $a_u \in \mathbf{R}^{2K+1}$ and $a_a \in \mathbf{R}^{2K+1}$ are defined as

$$a_u := \begin{bmatrix} Q_h(1)/(1+\rho) \\ \vdots \\ Q_h(K)/(1+\rho)^K \\ Q_c(1)/(1+\rho) \\ \vdots \\ Q_c(K)/(1+\rho)^K \\ 1 \end{bmatrix}, \quad a_a := \begin{bmatrix} Q_h(1)/(1+\rho_a) \\ \vdots \\ Q_h(K)/(1+\rho_a)^K \\ Q_c(1)/(1+\rho_a) \\ \vdots \\ Q_c(K)/(1+\rho_a)^K \\ 1 \end{bmatrix}. \tag{7}$$

In general, input variables such as future thermal loads and electricity prices are not known exactly in advance. To reflect the uncertainty in the input data, we view U and L as random variables and a_u and a_a as random vectors. We denote their expected values by $\bar{U}$, $\bar{L}$, $\bar{a}_u$ and $\bar{a}_a$, respectively. With this notation, the user's and aggregator's expected net present values under the HPA can be written as

$$\begin{aligned} \mathbf{E}\, v_u(x) &= \bar{U} - \bar{a}_u^\top x \\ \mathbf{E}\, v_a(x) &= \bar{a}_a^\top x - \bar{L}. \end{aligned} \tag{8}$$

Here $\mathbf{E}$ denotes the expectation operator.

3.2. Mutual benefit condition

In this section, we derive conditions under which the HPA is mutually beneficial to the user and the aggregator. We begin in Section 3.2.1 by defining 'mutually beneficial' in precise mathematical terms. We then state and prove a theorem establishing necessary and sufficient conditions for mutual benefit. A key term in this theorem is the total value of the HPA, which we define and discuss in Section 3.2.2.

3.2.1. Mutual benefit theorem

Definition 1. *The HPA is* mutually beneficial under x *if* $\mathbf{E}\, v_u(x) \geq 0$ *and* $\mathbf{E}\, v_a(x) \geq 0$.

Assumption 1. *The user and aggregator discount future cash flows at the same rate:* $\rho = \rho_a$.

Theorem 1. *Suppose Assumption 1 holds. Then the HPA is mutually beneficial under x if and only if*

1. $\bar{U} \geq \bar{L}$, *and*
2. $\bar{a}_u^\top x = \theta \bar{L} + (1-\theta)\bar{U}$ *for some* $\theta \in [0, 1]$.

Furthermore, the expected net present values under x are $\mathbf{E}\, v_u(x) = \theta(\bar{U} - \bar{L})$ *and* $\mathbf{E}\, v_a(x) = (1-\theta)(\bar{U} - \bar{L})$.

Proof. By definition, the HPA is mutually beneficial under x if x is a solution to the following feasibility problem:

$$\begin{array}{ll} \text{find} & x \in \mathbf{R}^{2K+1} \\ \text{subject to} & \mathbf{E}\, v_u(x) \geq 0 \\ & \mathbf{E}\, v_a(x) \geq 0. \end{array} \tag{9}$$

If $\rho = \rho_a$, then $\bar{a}_u = \bar{a}_a$, and an equivalent problem is the following:

$$\begin{array}{ll} \text{find} & x \in \mathbf{R}^{2K+1} \\ \text{subject to} & \bar{L} \leq \bar{a}_u^\top x \leq \bar{U}. \end{array} \tag{10}$$

Let x be a solution to Problem (9). Then x is also a solution to Problem (10). Problem (10) must therefore be feasible, meaning $\bar{U} \geq \bar{L}$. As $\bar{L} \leq \bar{a}_u^\top x \leq \bar{U}$, there must exist a $\theta \in [0, 1]$ such that $\bar{a}_u^\top x = \theta \bar{L} + (1-\theta)\bar{U}$. This proves one direction of the 'if and only if' in Theorem 1. For the other direction, suppose $\bar{U} \geq \bar{L}$ and

x satisfies $\bar{a}_u^\top x = \theta \bar{L} + (1-\theta)\bar{U}$ for some $\theta \in [0,1]$. Then $\bar{L} \leq \bar{a}_u^\top x \leq \bar{U}$, meaning x is a solution to Problem (10). By the equivalence of Problems (9) and (10), x is also a solution to Problem (9).

For the final statement, we have

$$\begin{aligned} \mathbf{E}\, v_u(x) &= \bar{U} - \bar{a}_u^\top x \\ &= \bar{U} - \theta \bar{L} - (1-\theta)\bar{U} \\ &= \theta(\bar{U} - \bar{L}) \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mathbf{E}\, v_a(x) &= \bar{a}_u^\top x - \bar{L} \\ &= \theta \bar{L} + (1-\theta)\bar{U} - \bar{L} \\ &= (1-\theta)(\bar{U} - \bar{L}). \end{aligned} \tag{12}$$

■

Theorem 1 implies that the expected total value of the HPA is $\bar{U} - \bar{L}$. In the extreme case of $\theta = 1$, the user receives all of the value: $\mathbf{E}\, v_u(x) = \bar{U} - \bar{L}$ and $\mathbf{E}\, v_a(x) = 0$. At the other extreme of $\theta = 0$, the aggregator receives all of the value: $\mathbf{E}\, v_a(x) = \bar{U} - \bar{L}$ and $\mathbf{E}\, v_u(x) = 0$. For intermediate values of θ, the user receives $100\theta\%$ of $\bar{U} - \bar{L}$ and the aggregator receives the remaining $100(1-\theta)\%$. The parameter $\theta \in [0,1]$ in Theorem 1 can therefore be interpreted as the user's share of the expected total value of the HPA.

3.2.2. Interpretation of the total value

Definition 2. *The total value of the HPA is*

$$\begin{aligned} v &:= U - L \\ &= \; c - s - (c_a - s_u - s_a) + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \Bigg[m(k) - m_a(k) + r(k) \\ &\quad + (\pi(k) - \pi_a(k)) \left(\frac{Q_h(k)}{\eta_h(k)} + \frac{Q_c(k)}{\eta_c(k)} \right) \Bigg]. \end{aligned} \tag{13}$$

The total value v can be decomposed into four components:

1. the initial subsidized cost reduction,

$$c - s - (c_a - s_u - s_a), \tag{14}$$

2. the net present maintenance cost reduction,

$$\sum_{k=1}^{K} \frac{m(k) - m_a(k)}{(1+\rho)^k}, \tag{15}$$

3. the net present electricity cost reduction,

$$\sum_{k=1}^{K} \frac{\pi(k) - \pi_a(k)}{(1+\rho)^k} \left(\frac{Q_h(k)}{\eta_h(k)} + \frac{Q_c(k)}{\eta_c(k)} \right), \tag{16}$$

   and

4. the net present ancillary service revenue,

$$\sum_{k=1}^{K} \frac{r(k)}{(1+\rho)^k}. \tag{17}$$

The total value of the HPA can be viewed as the 'money on the table' that can be shared between the user and the aggregator. Importantly, the total value includes whatever profit the incumbent firm makes on the initial heat pump installation and any ongoing maintenance. To see this, we recall that the user's initial cost under the incumbent business model, c, includes not only the heat pump, parts, and labor, but also the incumbent installer's profit. By contrast, the aggregator's initial cost c_a does *not* include any profit to the aggregator. Similarly, the user's maintenance cost $m(k)$ includes the incumbent firm's profit, while the aggregator's maintenance cost $m_a(k)$ does not. Therefore, the total value of an HPA will likely be larger in regions where incumbent firms command higher profit margins on heat pump installation and maintenance. The aggregator could add further value through economies of scale by specializing in heat pumps and buying them in bulk, thereby reducing c_a and increasing the first component of v.

The third and fourth components of v represent genuinely new value created by the aggregation mechanism inherent in the HPA business model. This value stems from the aggregator's access to wholesale energy and ancillary service markets. By buying electrical energy at wholesale prices, rather than retail, the aggregator could reduce electricity costs significantly. Similarly, the aggregator's access to ancillary service markets provides a new revenue stream that is inaccessible to individual heat pumps without aggregation. We quantify the energy cost reductions and ancillary service revenues in Section 5.

3.3. Pricing heat and cooling

Theorem 1 implies that we can solve Problem (9) by solving

$$\bar{a}_u^\top x = \theta \bar{L} + (1-\theta)\bar{U}. \tag{18}$$

This is one linear equation in the $2K+1$ unknown elements of $x = (\pi_h, \pi_c, c_u)$, so it has infinitely many solutions. We view c_u as a fixed parameter chosen by the aggregator. Determining (π_h, π_c) then requires $2K-1$ more independent equations. One option is to require the prices to escalate at a given rate $\beta \geq 0$ from each period to the next, for example to keep up with inflation or rising electricity prices. This requirement imposes another $2K-2$ equations:

$$\begin{aligned} \pi_h(k+1) &= (1+\beta)\pi_h(k),\ k = 1, \dots, K-1 \\ \pi_c(k+1) &= (1+\beta)\pi_c(k),\ k = 1, \dots, K-1. \end{aligned} \tag{19}$$

Determining the prices requires one more independent equation. One option is to relate the heat price to the cooling price by selecting a $\gamma \geq 0$ and setting

$$\pi_h(k) = \gamma \pi_c(k),\ k = 1, \dots, K. \tag{20}$$

It is straightforward to show that any $K-1$ of these K equations are linearly dependent with the $2K-2$ escalation equations, so it suffices to pick one. A fully-determined system, therefore, is

$$\begin{aligned} \bar{a}_u^\top x &= \theta \bar{L} + (1-\theta)\bar{U} \\ \pi_h(k+1) &= (1+\beta)\pi_h(k),\ k = 1, \dots, K-1 \\ \pi_c(k+1) &= (1+\beta)\pi_c(k),\ k = 1, \dots, K-1 \\ \pi_h(1) &= \gamma \pi_c(1), \end{aligned} \tag{21}$$

with c_u viewed as a fixed parameter. Algorithm 1 summarizes the proposed method for pricing heat and cooling.

4. Extensions and variations

4.1. Comparisons to incumbent technologies

In Section 3, we compared ownership of a heat pump under the incumbent business model to an HPA for the same heat pump. Also of interest is the comparison between (a) an HPA for a heat pump, and (b)

Algorithm 1 Thermal pricing.

input: $\bar{a}_u \in \mathbf{R}^{2K+1}$, $\bar{L} \in \mathbf{R}$, $\bar{U} \in \mathbf{R}$, $c_u \in \mathbf{R}$, $\theta \in [0,1]$, $\beta \geq 0$, $\gamma \geq 0$
  if $\bar{U} \geq \bar{L}$
    return the solution (π_h, π_c) to the linear system (21)
  else
    return infeasible
  end if

ownership under the incumbent business model of incumbent technologies such as air conditioners or natural gas furnaces or boilers.

Theorem 1 and Algorithm 1 extend readily to such comparisons. Under incumbent ownership of incumbent technologies, the user's net present cost is

$$\tilde{c} - \tilde{s} + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \left[\frac{\tilde{\pi}(k) Q_h(k)}{\tilde{\eta}_h(k)} + \frac{\pi(k) Q_c(k)}{\tilde{\eta}_c(k)} + \tilde{m}(k) \right] - \frac{\tilde{v}_K}{(1+\rho)^K}, \tag{22}$$

assuming the incumbent cooling system is powered by electricity. Here $\tilde{c}$ (\$) is the unsubsidized initial cost paid by the user to an incumbent installer (including parts, labor, and the installer's overhead and profit), $\tilde{s}$ (\$) is the combined subsidy received by the user and the installer for the incumbent technologies, and $\tilde{v}_K$ (\$) is the salvage value of the incumbent technologies after period K. In period k, $\tilde{\pi}(k)$ (\$/kWh) is the incumbent heating fuel price paid by the user, $\tilde{\eta}_h(k)$ is the incumbent heating system's COP, $\tilde{\eta}_c(k)$ is the incumbent cooling system's COP, and $\tilde{m}(k)$ (\$) is the maintenance cost paid by the user for the incumbent technologies (including parts, labor, and the maintainer's overhead and profit).

After the above adjustment to the user's net present cost, the user's net present value under the HPA becomes

$$\begin{aligned} \tilde{v}_u(\pi_h, \pi_c, c_u) := \tilde{c} - \tilde{s} - (c_u - s_u) + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \Bigg[\left(\frac{\tilde{\pi}(k)}{\tilde{\eta}_h(k)} - \pi_h(k) \right) Q_h(k) \\ + \left(\frac{\pi(k)}{\tilde{\eta}_c(k)} - \pi_c(k) \right) Q_c(k) + \tilde{m}(k) \Bigg] + \frac{v_K - \tilde{v}_K}{(1+\rho)^K}. \end{aligned} \tag{23}$$

The upper bound U becomes

$$\tilde{U} := \tilde{c} - \tilde{s} + s_u + \sum_{k=1}^{K} \frac{1}{(1+\rho)^k} \left[\frac{\tilde{\pi}(k) Q_h(k)}{\tilde{\eta}_h(k)} + \frac{\pi(k) Q_c(k)}{\tilde{\eta}_c(k)} + \tilde{m}(k) \right] + \frac{v_K - \tilde{v}_K}{(1+\rho)^K}. \tag{24}$$

The aggregator's net present value v_a and the lower bound L are unchanged. Theorem 1 and Algorithm 1 can therefore be applied to comparisons with incumbent technologies simply by changing U to $\tilde{U}$.

4.2. Internalizing the cost of greenhouse gas emissions

When comparing heat pumps to incumbent technologies as in Section 4.1, greenhouse gas emissions can be an important factor. To account for greenhouse gas emissions, we introduce a price π_g (\$/kg), where 'kg' refers to the mass of emitted CO_2-equivalent. The cost of greenhouse gas emissions from fuel use can then be internalized by redefining

$$\begin{aligned} \pi(k) &\leftarrow \pi(k) + \pi_g \mu(k) \\ \tilde{\pi}(k) &\leftarrow \tilde{\pi}(k) + \pi_g \tilde{\mu}(k). \end{aligned} \tag{25}$$

Here $\mu(k)$ (kg/kWh) and $\tilde{\mu}(k)$ (kg/kWh) are the greenhouse gas intensities of electricity and of the incumbent heating fuel, respectively, in period k. Similarly, the embodied emissions (*e.g.*, from manufacturing and shipping) of the heat pump and the incumbent hardware can be internalized by redefining

$$\begin{aligned} c_a &\leftarrow c_a + \pi_g M \\ \tilde{c} &\leftarrow \tilde{c} + \pi_g \tilde{M}, \end{aligned} \tag{26}$$

where M (kg) and $\tilde{M}$ (kg) are the masses of CO_2-equivalent embodied in the heat pump and the incumbent hardware, respectively.

4.3. Unequal discount rates

If the user and aggregator discount future cash flows at different rates, *i.e.*, if $\rho \neq \rho_a$, then Theorem 1 does not hold in general. In this case, heat and cooling can be priced by solving the following problem:

$$\begin{array}{ll} \text{maximize} & \phi\,\mathbf{E}\,v_u(x) + (1-\phi)\,\mathbf{E}\,v_a(x) \\ \text{subject to} & \mathbf{E}\,v_u(x) \geq 0 \\ & \mathbf{E}\,v_a(x) \geq 0 \\ & Ax = b. \end{array} \tag{27}$$

Here the equation $Ax = b$ encodes constraints on the prices, such as those discussed in Section 3.3. The tunable parameter $\phi \in [0, 1]$ governs the trade-off between the net present values of the user and the aggregator. An equivalent statement of this problem is the following:

$$\begin{array}{ll} \text{minimize} & [\phi\bar{a}_u - (1-\phi)\bar{a}_a]^\top x \\ \text{subject to} & \bar{a}_u^\top x \leq \bar{U} \\ & \bar{a}_a^\top x \geq \bar{L} \\ & Ax = b. \end{array} \tag{28}$$

A solution to this linear program can be computed efficiently using standard algorithms such as the simplex method or interior-point methods.

4.4. Price-elastic thermal loads

Throughout Section 3, we assumed that the user's heating and cooling loads were price-inelastic. We made this assumption because it simplified analysis significantly. In some settings, however, a portion of the users' thermal load is discretionary and may be sensitive to thermal prices [39]. If the user's thermal load depends on the thermal prices, then Theorem (1) no longer holds in general and adaptations may be necessary.

For heating, for example, the term

$$\left(\frac{\pi(k)}{\eta_h(k)} - \pi_h(k)\right) Q_h(k) \tag{29}$$

in the user's net present value becomes

$$\frac{\pi(k)Q_h^0(k)}{\eta_h(k)} - \pi_h(k)Q_h(\pi_h(k)) - \mu_h(Q_h^0(k)) + \mu_h(Q_h(\pi_h(k))). \tag{30}$$

Here $Q_h^0(k)$ is the heat the user would use at heat price $\pi(k)/\eta_h(k)$, the effective heat price the user would pay under incumbent ownership. With slight abuse of notation, we now view $Q_h : \mathbf{R} \to \mathbf{R}$ as a function mapping heat prices into heat loads. The function $\mu_h : \mathbf{R} \to \mathbf{R}$ describes the utility (*e.g.*, thermal comfort) that the user derives from heat; $\mu_h(Q_h^0(k))$ is the user's utility under incumbent ownership and heat price $\pi(k)/\eta_h(k)$, while $\mu_h(Q_h(\pi_h(k)))$ is the user's utility under the HPA and heat price $\pi_h(k)$.

In the aggregator's net present value, the term

$$\left(\pi_h(k) - \frac{\pi_a(k)}{\eta_h(k)}\right) Q_h(k) \tag{31}$$

becomes

$$\left(\pi_h(k) - \frac{\pi_a(k)}{\eta_h(k)}\right) Q_h(\pi_h(k)). \tag{32}$$

The cooling terms in the net present values change identically, but the net present values are otherwise unchanged.

If the expressions (30) and (32) and their cooling counterparts are concave functions of $\pi_h(k)$, then Problem (9) remains computationally tractable. More precisely, Problem (9) can be transformed into a convex feasibility problem. Such problems can typically be solved in polynomial time using interior-point methods, as implemented in software such as the CVX [40, 41] and YALMIP [42] toolboxes. Whether a version of Theorem (1) can be reproduced in the case of elastic thermal loads depends on the structure of the functions Q_h, μ_h, and their cooling counterparts. Analytical progress could likely be made by linearizing about the nominal prices of heat and cooling, $\pi(k)/\eta_h(k)$ and $\pi(k)/\eta_c(k)$, but we leave such investigations for future work.

5. Case study

In this section, we compare the economics of one heat pump under two ownership models – the incumbent business model and an HPA – using Monte Carlo simulation. We discuss the input data in Section 5.1 and present numerical results in Section 5.2. We use Monte Carlo simulation because (a) we prefer a stochastic framework to a deterministic one, and (b) exact mathematical analysis is intractable due to the number of uncertain input variables and the nonlinearity of the underlying equations. We prefer a stochastic framework because there is significant uncertainty associated with input parameters such as equipment costs, thermal loads, COPs, maintenance requirements, electricity prices, and ancillary service revenues. Neglecting these uncertainties through a strictly deterministic analysis could give rise to misleading results. While this case study is specific to the United States, the ideas and methods in this paper could be applied wherever heat pumps are sold and aggregated end-use devices are permitted to participate in wholesale electricity markets.

5.1. Input data

5.1.1. Heat pump

The device we model is a reversible air-source heat pump that provides hot or cold air directly to the space it serves, rather than to distribution ducts. In the heating and cooling industry, heat pumps of this type are typically referred to as *ductless mini-splits*. Modern ductless mini-splits are highly efficient, can provide both heat and cooling, and retain useful heating capacity even in very cold weather.

We consider a time span of $K = 12$ years in one-year time steps. The twelve-year duration coincides with the warranties available from several major heat pump manufacturers. In field studies of ductless mini-splits, such as [43], [44] and [45], seasonal heating COPs typically vary from 2.6 to 2.9. Seasonal cooling COPs are significantly higher: typically 3.7 to 4.3 [45]. To generate $\eta_h(1), \ldots, \eta_h(K)$ and $\eta_c(1), \ldots, \eta_c(K)$, we draw uniformly and independently from these ranges.

5.1.2. Heat and cooling loads

We model the heat and cooling load of one floor of a typical home in the United States. We base the heat and cooling loads on two resources from the United States Department of Energy: the Residential Energy Consumption Survey [46] and the Building America house simulation protocols [47]. Specifically, we calculate heat and cooling loads by multiplying the national-average annual heat and cooling energy end-uses per household from [46] by a typical heating efficiency of 0.7 and cooling seasonal COP of 2.67, respectively, based on [47]. We then divide by the national-average heated and cooled floor areas per household from [46].

This process yields estimated heat and cooling load intensities, respectively, of 44.6 kWh/m^2 and 43.3 kWh/m^2. For 80 m^2 of conditioned floor area – roughly one floor of a typical United States home [46] – this calculation gives heat and cooling loads, respectively, of 3600 kWh and 3450 kWh. As loads vary from year to year, we perturb these values by $\pm 10\%$ (uniformly distributed and independently) to generate $Q_h(1), \ldots, Q_h(K)$ and $Q_c(1), \ldots, Q_c(K)$.

5.1.3. Incumbent economics

In the incumbent business model, the user pays an incumbent installer to procure and install the heat pump. The user's initial cost c includes the heat pump, associated hardware, the manufacturer's hardware warranty, and the incumbent installer's labor, overhead, and profit. Installed costs for heat pumps vary by location, between installers in a given location, and over time. In our experience, publicly-available data on installed costs are very limited. We base our initial costs on conversations with industry experts, the practitioner and consumer commentary in [48], and the cost estimates available online at homewyse.com. Based on these sources, we generate c uniformly from the range of \$4000–4500. We assume that an initial subsidy of $s = \$500$ is available, *e.g.*, from a utility or government agency.

Over the heat pump's lifetime, the user buys the heat pump's electricity at their utility's retail price. For the first year's electricity price, we use the 2016 United States average residential retail price, $\pi(1) = 13.1$ ¢/kWh. This is the most recent data available from the United States Energy Information Administration [49]. To generate $\pi(2), \dots, \pi(K)$, we assume a 1% annual inflation rate. This is consistent with the inflation rate of the United States average electricity price from 2006–2016 [50]. We treat the user's electricity prices as deterministic.

The user pays an incumbent firm for any maintenance the heat pump requires. Maintenance payments include the incumbent firm's labor, overhead, and profit. They do not include hardware, which we assume is covered by the manufacturer's warranty, the cost of which is included in the user's initial cost c. We assume that maintenance is needed twice over the heat pump's lifetime, in years four and eight. The user's maintenance costs $m(4)$ and $m(8)$ are independent and uniformly distributed on the range \$250–350; $m(k) = 0$ for all other k.

5.1.4. HPA economics

In the HPA business model, the aggregator procures, installs and maintains the heat pump. The aggregator's capital cost c_a and maintenance costs $m_a(k)$, like the user's costs c and $m(k)$ under the incumbent business model, include hardware, labor, and overhead. Unlike the user's incumbent costs, however, the aggregator's costs do not include the incumbent firm's profit. In this case study, we assume that the aggregator's installation and maintenance costs equal the incumbent firm's *internal* installation and maintenance costs. We do not assume that the aggregator reduces installation or maintenance costs relative to the incumbent firm through economies of scale or other efficiencies. For this reason, we set $c_a = (1 - \lambda)c$ and $m_a = (1 - \lambda)m$, where λ is the incumbent firm's profit margin. We generate λ uniformly from the range of 15–20%, corresponding to a 17.5% mean profit margin for the incumbent installer. This profit margin range is based on the discussion in [48]. We assume that the same \$500 subsidy available under the incumbent business model is available under the HPA; the aggregator receives the subsidy ($s_a = \$500$ and $s_u = 0$).

We model an aggregator with a large fleet of heat pumps. The heat pumps' aggregate electrical load is sufficiently large that the aggregator can participate in wholesale electricity markets. This allows the aggregator to (a) buy electricity wholesale, rather than retail, and (b) sell ancillary services.

The average United States wholesale electricity price was 2.9 ¢/kWh in 2016; this is the most recent data available from [49]. While the aggregator buys electricity at wholesale prices in this range, they must also pay for electricity transmission and distribution. Transmission and distribution fees vary over time and space, but are typically 1–2 ¢/kWh for transmission and 3–4 ¢/kWh for distribution [51]. Electricity retailers may also be subject to taxes and other fees; to account for these, we add another 1–2 ¢/kWh to the aggregator's electricity price. We therefore draw $\pi_a(1)$ uniformly from the range of 7.9–10.9 ¢/kWh. We assume that $\pi_a(1), \dots, \pi_a(K)$ inflate at the same 1% annual rate as the retail electricity prices $\pi(1), \dots, \pi(K)$.

The provision of ancillary services from heat pumps is currently an active research area. Early estimates of annual ancillary service revenues from heat pumps can be found in [34] and [35] for California, Section 5.3 of [36] for the eastern United States, and [37] for Germany. For reversible variable-speed heat pumps of the size modeled here, estimated annual revenues range from \$25–75 [36]. To generate the annual ancillary service revenues $r(1), \dots, r(K)$, we draw uniformly and independently from the \$25–75 range.

The \$25–75 range for annual ancillary service revenue is based on selling regulation and contingency reserve in the Pennsylvania–Jersey–Maryland Interconnection, the largest power system in the United States.

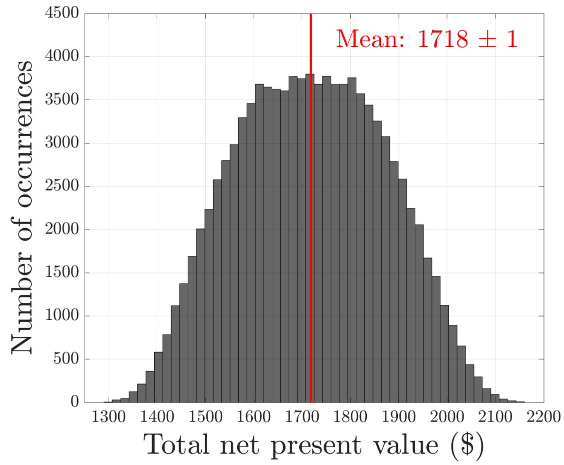

Figure 3: Histogram of the total value of the HPA. The total value is nonnegative in all scenarios, implying that in the case studied here, the HPA could almost certainly be mutually beneficial to the user and aggregator.

This market has an energy-neutral regulation product, wherein the requested upward power perturbations balance downward power perturbations over any half-hour. Furthermore, contingency reserve in this market is primarily a capacity product: demand-side reserve providers are paid for their ongoing capacity to rapidly curtail load, but are rarely called upon to actually curtail. From 2013–2018, for example, contingency reserves were dispatched only 22 times per year, with an average deployment duration of 12 minutes (see Figure 4.6 of [36]). Because these regulation and reserve products require little energy exchange with the grid, heat pumps can provide them without disrupting or discomforting building occupants. This claim is supported through simulations in Sections 5.1 and 5.2 of [36] and through experiments in [27, 28]. The negligible energy exchange from regulation and contingency reserve also implies that in this case study, ancillary service provision is essentially decoupled from energy consumption.

The $25–75 revenue range is computed in [36] through an array of years-long simulations at sub-hourly time steps. These simulations use real weather data. They cover heating and cooling seasons, when heat pumps operate quite frequently, as well as shoulder seasons, when heat pumps are usually turned off and incapable of providing ancillary services. Mathematical models of the simulated heat pumps' thermal power capacities and COPs are fit to manufacturer data. Capacities and COPs vary with indoor and outdoor air temperatures; COPs also vary with the heat pumps' part-load ratios.

Under an HPA, the user buys the heat pump's thermal energy output from the aggregator. We price heat and cooling using Algorithm 1. We assume that the user receives the heat pump at no initial cost ($c_u = 0$), the aggregator receives the full value of the HPA while the user breaks even with respect to the incumbent business model ($\theta = 0$), the heat and cooling prices are equal ($\gamma = 1$), and the prices escalate at a 1% annual rate ($\beta = 0.01$). We investigate the dependence of the prices on these parameters in Section 5.2.2. We assume that the user and aggregator discount future cash flows at the same rate of $\rho = \rho_a = 10\%$.

5.2. Results and discussion

In this section, we discuss the numerical results of 10^5 Monte Carlo simulations using the input data described in Section 5.1. With this sample size, all estimates presented in the sequel are accurate to at least three significant digits.

5.2.1. Total value estimation

The total value of an HPA can be interpreted as the 'money on the table' that can be shared between the user and the aggregator. How precisely the total value is shared depends on how heat and cooling are priced. Per Theorem 1, an HPA can be mutually beneficial only if the expected total value is nonnegative.

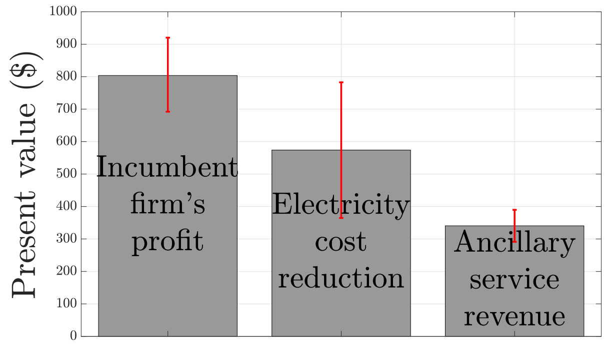

Figure 4: Decomposition of the total value of the HPA. Bar heights are sample averages; error bars span the 5th to 95th percentiles. The left bar is the profit a firm would make from installation and maintenance under the incumbent business model. The center and right bars represent new value added by the HPA's aggregation mechanism.

Figure 3 shows a histogram of the total value. The key message of this figure is that in the simulated scenario, there is money on the table. The 95% confidence interval on the expected total value is \$1718±1. In all simulations, the total value was above \$1301. This implies that an HPA is almost certain to be mutually beneficial in the simulated scenario, provided heat and cooling are priced in a manner consistent with Theorem 1.

Figure 4 breaks down the \$1718 sample-average total value illustrated in Figure 3 into three components. As discussed in Section 3.2.2, the total value consists in general of four components: the initial cost reduction, maintenance cost reduction, electricity cost reduction, and ancillary service revenue. In Figure 4, however, we combine the initial cost reduction and net present maintenance cost reduction into a single component labeled 'Incumbent firm's profit'. We do this because in this case study, the initial cost reduction and maintenance cost reduction are exactly equal to the profit that the incumbent firm would earn on installation and maintenance under the incumbent business model. In practice, an HPA aggregator might reduce installation and maintenance costs through economies of scale or other efficiencies, but we do not model those effects in this case study. Instead, we focus on the electricity cost reduction and ancillary service revenue, which represent genuinely new value added by the HPA business model's aggregation mechanism.

In Figure 4, the incumbent firm's profit, the electricity cost reduction, and the ancillary service revenue comprise 47%, 33%, and 20%, respectively, of the \$1718 sample-average total value. The striking result illustrated in Figure 4 is that the new value added by the HPA is more than double the incumbent firm's profit. Roughly two thirds of the new value comes from accessing the lower electrical energy prices available on wholesale, rather than retail, markets. The remaining one third comes from selling ancillary services. This result may interest heat pump companies, as it highlights the value they could add by aggregating heat pumps in sufficient numbers to access wholesale energy and ancillary service markets.

Figure 5 shows the sample-average discounted cash flows (black bars) over time for the user (left plot) and the aggregator (right plot), as well as the cumulative sums of the cash flows (red curves). The final values of the red curves are the net present values of the HPA to the user and aggregator; with the value of $\theta = 0$ used in this case study, the user's net present value is zero while the aggregator's is the total value of the HPA, \$1718. For the user, the cash flow in year $k = 0$ is the avoided initial cost relative to the incumbent business model: about \$3750 in this case study. While the user begins with an initial avoided cost, the aggregator begins with an initial investment of about \$3000 to procure and install the heat pump. The aggregator recovers this initial investment over time by selling heat, cooling and ancillary services. The aggregator breaks even after six years.

5.2.2. Thermal pricing

As discussed in Section 5.2.1, there is significant money on the table in the simulated scenario. How this money is shared between the user and the aggregator depends on how heat and cooling are priced. The

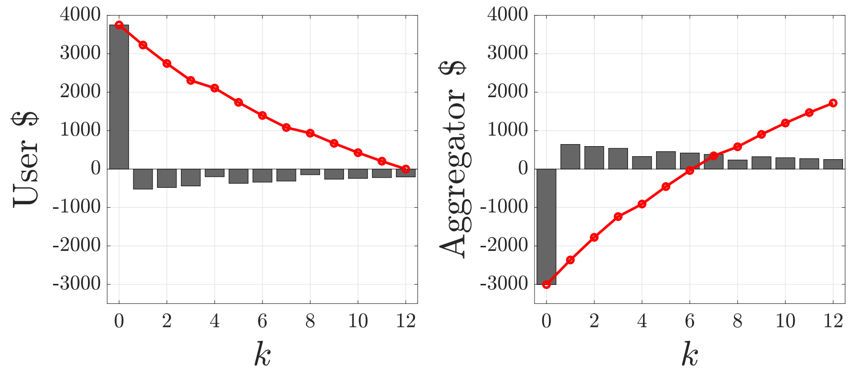

Figure 5: Sample-average discounted cash flows (black bars) for the user (left) and aggregator (right). The red curves are the cumulative sums of the discounted cash flows. The final values of the red curves are the net present values of the HPA to the user and aggregator. In this case study, the user and aggregator break even in years twelve and six, respectively.

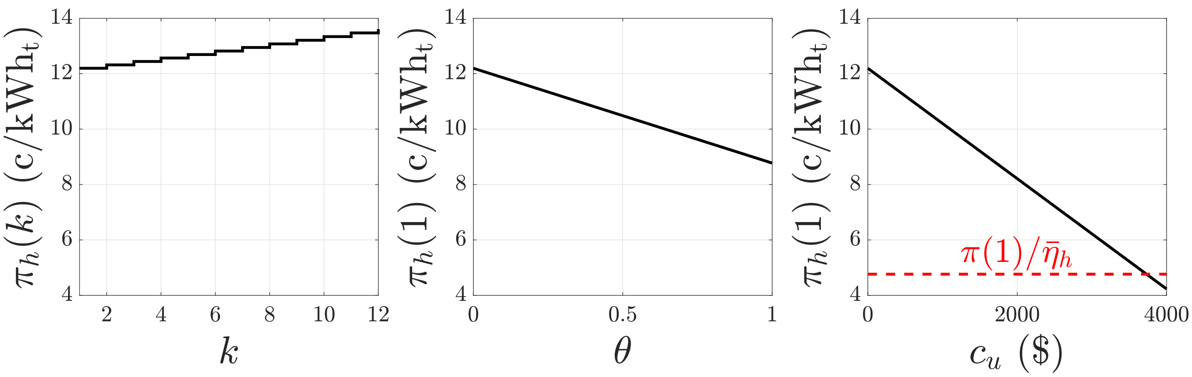

Figure 6: Left: variation of the heat price $\pi_h(k)$ with the time index k. Center: variation of the first heat price, $\pi_h(1)$, with θ, the user's share of the expected total value of the HPA. Right: variation of $\pi_h(1)$ with c_u, the initial cost paid by the user to the aggregator.

lower the thermal prices, the larger the user's share of the total value. Figure 6 shows the variation of the heat prices with the time index k (left plot), the user's share θ of the expected total HPA value (center plot), and the user's initial payment c_u to the aggregator (right plot). The prices of heat and cooling are equal in these simulations.

The left plot shows the variation of $\pi_h(k)$ with the year index k. In this plot, θ and c_u are held fixed at their nominal values of zero. The initial price of about 0.12 \$/kWh is more than double the effective thermal price that the user would pay under the incumbent business model, $\pi(1)/\bar{\eta}_h = 0.048$ \$/kWh (depicted in red in the right plot). The slow increase over time is a constraint imposed in Algorithm 1 through the choice of a 1% annual escalation rate. This provides a means of ensuring that thermal revenues keep pace with inflation.

The center plot shows the variation of the first heat price, $\pi_h(1)$, with θ. In this plot, c_u is held fixed at its nominal value of zero. The first heat price varies from about 0.08 \$/kWh for a nonprofit aggregator ($\theta = 1$) to 0.12 \$/kWh in the case where the user breaks even relative to the incumbent business model ($\theta = 0$). The subsequent heat prices $\pi_h(2), \ldots, \pi_h(K)$, which are not pictured in the center plot, increase over $\pi_h(1)$ in a trajectory similar to the left plot.

The right plot shows the variation of $\pi_h(1)$ with c_u. In this plot, θ is held fixed at its nominal value of zero. This plot demonstrates that the user may trade a higher initial payment for lower thermal payments over time. For c_u above about \$3700, the user pays less for heat than they would pay under the incumbent business model, despite paying a lower initial cost than the mean initial incumbent cost of \$4250. The

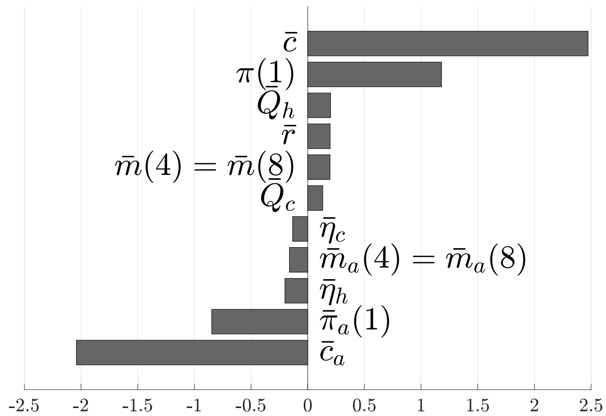

Figure 7: Sensitivity of the expected total value $\bar{v}$ to the input parameters. The horizontal axis is the percent variation in $\bar{v}$ caused by a one percent variation in the parameter.

aggregator can afford this due to their access to wholesale energy and ancillary service markets.

5.2.3. Sensitivity analysis

The results presented in Section 5.2.1-5.2.2 are specific to the input parameter distributions given in Section 5.1. Figure 7 shows the sensitivity of the primary simulation output – the expected total value of the HPA, $\bar{v}$ – to variations in the means of the input parameter distributions. In Figure 7, the horizontal axis is the percent reduction in expected total value caused by a one percent increase in the input parameter, with the other parameters held fixed at their nominal values. For example, a one percent increase in $\bar{c}$, the user's mean initial cost under the incumbent business model, causes a 2.5 percent increase in $\bar{v}$. A negative sensitivity indicates that $\bar{v}$ is decreasing in the parameter in question.

The total value of the HPA is increasing in the capital and maintenance costs under the incumbent business model, $\bar{c}$ and $\bar{m}$. This implies that HPAs have more value in regions where incumbent installation and maintenance firms are either less efficient or command higher profit margins. Conversely, the total value is decreasing in the aggregator's initial cost $\bar{c}_a$ and maintenance costs $\bar{m}$. This implies that HPAs add more value if the aggregator can reduce their initial cost and maintenance costs, for example by specializing in heat pumps, installing many of them, and taking advantage of economies of scale.

The total value is increasing in the thermal loads $\bar{Q}_h$ and $\bar{Q}_c$, increasing in the retail electricity price π, and decreasing in the wholesale electricity price $\bar{\pi}_a$. This stands to reason, as a portion of the total value comes from buying energy wholesale and selling it retail. More value is added when the volume of energy delivered is larger and the difference between retail and wholesale prices is larger. The total value is also increasing in the ancillary service revenue $\bar{r}$, meaning that the HPA adds more value if the aggregator provides more ancillary service capacity, or provides capacity at times when ancillary service prices are higher.

The total value is decreasing in the heating and cooling COPs, $\bar{\eta}_h$ and $\bar{\eta}_c$. This may run counter to intuition, which suggests that more efficient heat pumps should add more value. However, we recall that we defined the total value of the HPA relative to ownership of the same heat pump under the incumbent business model. The HPA adds value by enabling electricity to be bought at wholesale, rather than retail, prices. All else being equal, a more efficient heat pump will use less energy, decreasing the value added by accessing wholesale energy markets. Furthermore, we note that the sensitivities to $\bar{\eta}_h$ and $\bar{\eta}_c$ in Figure 7 were computed without changing the initial costs $\bar{c}$ and $\bar{c}_a$. In practice, the COPs and initial costs are coupled, as more efficient heat pumps typically have higher initial costs.

The fact that the total value of the HPA is decreasing in the heat pump's COPs raises an important question: Is the aggregator incentivized to install inefficient heat pumps? The answer to this question is somewhat complicated. While the total value is decreasing in the COPs, the aggregator's net present value (defined in Equation 4) is increasing in η_h and η_c. From the aggregator's perspective, then, more efficient heat pumps are likely preferable. Complications arise, however, from two factors. First, the aggregator's net present value is decreasing in their initial cost c_a, which is likely to be larger for more efficient heat pumps. Second, the aggregator's net present value is increasing in their ancillary service revenue r, which is likely to be smaller for more efficient heat pumps. This is because more efficient heat pumps use less electricity and therefore have less flexibility to offer for ancillary services. In our opinion, the question of selecting the best heat pump in this context is subtle, and best answered through numerical optimization. One method for optimal heat pump selection in this context can be found in Chapter 4 of [36].

5.2.4. Who should aggregate HPAs?

A key message of Figure 7 is that the total HPA value is quite sensitive to the initial costs and the electricity prices, and comparatively insensitive to the other parameters. This observation suggests that two types of businesses – heat pump manufacturers and electric utilities – could make particularly effective HPA aggregators.

Heat pump manufacturers have an advantage over other potential HPA aggregators in terms of the initial cost c_a. In this case study, we assumed that the aggregator's initial cost was equal to the user's initial cost under the incumbent business model, minus the incumbent installer's profit. Implicit in this assumption is that the aggregator, like the incumbent installer, buys the heat pump and associated hardware from a manufacturer. An aggregator manufacturing their own heat pumps, by contrast, would see significantly lower hardware costs and higher total values. For example, if the mean of c_a reduces by \$500 (14%), then the expected total value of the HPA increases by 29%, from \$1718 to \$2218. In addition to their initial cost advantage, heat pump manufacturers also have the advantage of designing the heat pump's hardware and software. This advantage could be key to developing (a) heat pump controllers that provide high-quality ancillary services, and (b) sensors that accurately measure heat pumps' thermal power output. Both of these technologies are important enablers of the HPA business model.

Electric utilities have an advantage over other potential HPA aggregators in terms of the electricity price π_a. In the simulations in this section, we assumed that the aggregator paid a 2.9 ¢/kWh wholesale price for electricity, plus 1–2 ¢/kWh for transmission, 3–4 ¢/kWh for distribution, and 1–2 ¢/kWh for taxes and other fees. An electric utility, by contrast, would own their own distribution network, so could avoid distribution fees. This could reduce the aggregator's initial electricity price by about 36%, to the range of 4.9–6.9 ¢/kWh. We re-ran 10^5 simulations with $\pi_a(1)$ drawn uniformly from this range (and all other parameters held at their nominal values). The expected total value of the HPA increased by 32%, from \$1718 to \$2259. In addition to their electricity price advantage, electric utilities also have the advantages of (a) existing relationships with energy end-users, and (b) in-house expertise in financing, purchasing, installing and maintaining energy equipment.

5.2.5. Influence of assumptions

We now examine three important assumptions underlying this case study and consider their potential impact on the results. First, we assume that the aggregator earns \$25–75 per heat pump per year by providing ancillary services. This revenue range comes from the array of high-fidelity simulations described in Section 5.3 of [36]. We note, however, that these simulations use historical ancillary service prices, and do not attempt to predict how ancillary service prices might change in the future. Ancillary service prices may well rise in coming years as power systems integrate more renewable generation. Furthermore, in this case study we do not consider other ways to monetize the flexibility of a fleet of heat pumps, such as selling curtailment-based demand response services to electric utilities, or arbitraging real-time energy prices in wholesale markets. For these reasons, the ancillary service revenue estimates in this case study may be somewhat conservative.

Second, we assume that the aggregator buys electricity on the wholesale energy market and ships it to the user. In addition to wholesale energy prices, we include costs of 4–6 ¢/kWh to account for the aggregator's use of transmission and distribution infrastructure. Electricity retailers are often subject to additional taxes and fees, above and beyond the costs of energy, transmission and distribution. We include an additional

1–2 ¢/kWh to account for these taxes and fees, but this is an imprecise estimate. In practice, the taxes and fees beyond distribution and transmission charges will vary from region to region, and could be higher than 1–2 ¢/kWh. Furthermore, the aggregator might face price risk on the wholesale energy market, as energy buyers are typically required to provide day-ahead load forecasts to the market operator. If the actual load deviates from the forecasted load, then the difference is typically settled at real-time energy prices, which are highly variable. For these reasons, the energy cost reduction estimates in this case study may be a somewhat aggressive.

Third, we assume that the aggregator's initial cost of procuring and installing the heat pump is the same as the incumbent installer's internal cost. This assumption may be conservative, as incumbent installers often carry wide varieties of mechanical equipment and install comparatively low volumes of heat pumps. An HPA aggregator, by contrast, might specialize in heat pumps, install many of them, and take advantage of bulk discounts and other economies of scale. Economies of scale are not modeled in this case study.

6. Conclusion

In this paper, we investigated a model of third-party heat pump ownership based on heat purchase agreements. This business model could lower barriers to adoption by (a) reducing or eliminating the user's initial cost, and (b) increasing the net present value of the heat pump investment. We derived necessary and sufficient conditions for a heat purchase agreement to benefit both the user and the aggregator. We also developed a method to fairly price heat and cooling. The potential impact of the heat purchase agreement business model is to reduce greenhouse gas emissions both directly, by accelerating replacement (or avoiding installation) of fossil-fueled or inefficient thermal equipment, and indirectly, by providing ancillary services that facilitate the integration of renewable generation into power systems.

We conclude by highlighting three opportunities for future work. First, the assumption that thermal loads are price-inelastic could be relaxed. (We discussed this extension briefly in Section 4.4.) While some amount of heat or cooling is essential in most climates, a portion of some users' thermal loads is discretionary and potentially sensitive to thermal prices. A basic feature of heat purchase agreements is that, relative to the incumbent business model, the user trades a lower initial cost for higher thermal prices. If the user consumes less heat or cooling than expected, then the aggregator's return on their initial investment may be lower than expected. To mitigate this risk, heat purchase agreements in district heating systems often contain minimum-use clauses, wherein the user commits to buying a certain amount of heat each year. While minimum-use clauses could also be used for stand-alone heat pumps, it is possible that preferable solutions for this setting could be developed.

Second, the effect of the heat purchase agreement business model on heat pump adoption rates could be investigated. While we demonstrated that heat purchase agreements could lower key barriers to adoption, we did not attempt to model or quantify the effect on adoption rates. In [8], Snape *et al.* use agent-based modeling to investigate the influence of various factors on heat pump adoption rates under the incumbent business model in the United Kingdom. A similar approach could be applied to the heat purchase agreement business model to provide insights into potential adoption rates and the associated emission reductions.

Third, other devices could be simulated. The case study in this paper involved an air-source heat pump used for space heating and cooling. However, the heat purchase agreement business model, as well as the theorem and pricing algorithm in Section 3, could also be applied to ground-source heat pumps and to heat pump water heaters. As both of these technologies have relatively high initial costs, but also high efficiencies, it is possible that heat purchase agreements could accelerate their adoption and further reduce greenhouse gas emissions.

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