Network-Constrained Stackelberg Game for Pricing Demand Flexibility in Power Distribution Systems

Nayara Aguiar, *Graduate Student Member, IEEE*, Anamika Dubey, *Member, IEEE*, and Vijay Gupta, *Senior Member, IEEE*

***Abstract*—We propose a network-constrained Stackelberg game framework to set energy prices for flexible consumers in a distribution grid. In this set-up, an aggregator acts as the leader, setting energy prices for each node, and price-responsive consumers are the followers, adjusting their demand according to the price charged. We show that this problem has an equilibrium in which the optimal demands can be written as a function of the Lagrange multipliers of the problem. For each node, voltage and current shadow costs have a cumulative effect that depends both on the upstream path to the substation and on the downstream demand level. We compare the Stackelberg solution to a centralized approach which maximizes social welfare. Our analysis reveals that, although the system-level optimal demand is higher in the centralized case, some individual nodes have higher consumption in the Stackelberg game. This counter-intuitive result cannot be observed in network-free formulations commonly adopted in game-theoretic works on demand-side management, where a centralized approach benefits every individual consumer. Numerical studies on an IEEE 123-bus feeder provide a system-level and a node-level analysis of this problem, highlighting the effect of network constraints on the optimal demands, and comparing the Stackelberg and the centralized solutions.**

***Index Terms*—Demand response, Stackelberg game, power distribution system optimization, distribution LMP.**

I. Introduction

The DISTRIBUTION grid has been traditionally composed of passive, inelastic electricity consumers who are charged a flat energy price by retailers. However, this scenario has been rapidly changing with the increasing adoption of distributed energy resources (DERs) and the possibility to enhance grid flexibility by leveraging consumer price-responsiveness through demand response programs [1]. These programs have been shown to decrease the effective cost of serving customers, thus improving economic efficiency in electricity markets [2]. Further, demand-side flexibility can help integrate variable renewable generation [3], and serves as an alternative to grid expansion [4]. This new paradigm led to studies which propose the use of distribution locational marginal prices (DLMPs) for the provision of economic signals that effectively quantify the marginal cost of supplying electricity to each node across the distribution grid.

In this paper, we formulate and analyze a demand management problem which considers an aggregator who is responsible for purchasing energy in the wholesale market and supplying it to consumers in a radial distribution grid. The consumers are price-sensitive, and adjust their demand according to the energy price charged. Branch flow equations characterize the underlying network and the DLMPs are calculated while considering network constraints and how consumers respond to prices. The interactions between consumers and the aggregator are modeled as a Stackelberg game in which the aggregator sets the nodal prices, and consumers decide on their demand based on this signal. Further, we compare the results of this Stackelberg game to the ones achieved when social welfare is maximized instead. We show that, from a system-level perspective, the aggregated demand of consumers is larger with a centralized approach; however, if consumers are evaluated individually, we see that they can be better off in the Stackelberg game. This counter-intuitive result only arises when the network constraints are considered, and thus network-free formulations may fail to capture such cases.

Most contributions in the literature on DLMPs have leveraged a centralized formulation to calculate these prices. In [5], an iterative method to solve a three-phase dc optimal power flow (DCOPF) problem is proposed considering price-sensitive demand. Electrical vehicle optimal charging is studied in [6], where the DCOPF is also used. However, this power flow approximation assumes voltage variations are negligible, and thus do not capture the importance of voltage regulation in the distribution grid. Voltage constraints are considered in [7], where an ac optimal power flow problem is formulated and analyzed for an unbalanced distribution grid, but demand elasticity is not modeled. Other works have explored the use of DLMPs for congestion management, loss reduction, and voltage improvement [8], [9].

Previous works on demand-side management have proposed game-theoretic approaches to provide consumers with price signals that incentivize them to adjust their consumption levels. The interactions between consumers and the agent responsible for supplying energy to the distribution grid are modeled as a Stackelberg game in numerous works,

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Nayara Aguiar and Vijay Gupta are with the Department of Electrical Engineering, University of Notre Dame, IN 46556 USA (e-mail: ngomesde@nd.edu; vgupta2@nd.edu).

Anamika Dubey is with the School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA 99164 USA (e-mail: anamika.dubey@wsu.edu).

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such as [10], [11], [12]. By modeling multiple retailers, [10] incorporates company-side competition for the supply of energy. The Pareto front that characterizes the trade-off between retailer profit and consumer surplus is characterized in [11], and the analysis in [12] indicates that consumers can be incentivized to reshape their demand to contribute towards shaving peak demand. However, this related stream of work has largely focused on the game theory aspect of the problem, and ignored power flow constraints and their effect on the task of pricing consumers in the distribution grid.

Recent efforts towards incorporating network constraints in this demand management problem include [13], in which the feasibility of a community storage energy in the distribution grid is studied, and [14], where the use of customer-owned storage is coordinated so that the per-user economic benefit is maximized. Both works use the linearized branch flow equations to impose voltage limits. Our formulation uses the convex relaxation for the branch flow equations proposed in [15] instead, and thus we are also able to incorporate the effects of thermal limits. Further, unlike these works, we leverage the concept of DLMPs to solve for nodal prices that reflect the shadow costs of the network constraints, which have been shown to provide proper economic signals to incentivize investments in distributed resources [16], [17]. We also provide analytical results which highlight how the optimal demand in each node depends on these shadow prices, providing an intuitive understanding of these effects that cannot be attained through numerical analysis only.

The major contributions of this work are two-fold. First, we formulate a demand-side management problem as a network-constrained Stackelberg game. An aggregator acts as the leader, moving first to set the prices to be charged for each consumer. Price-responsive consumers then follow and adjust their consumption level according to the price signal. We show that this game has an equilibrium and characterize the optimal demands as a function of the shadow costs of the voltage and thermal constraints, represented by the Lagrange multipliers of the problem. Our analysis reveals that, for each node, these costs propagate through the upstream path to the substation and are also closely related to the power flow going downstream to this node. Second, we compare the solution to this problem to two commonly used approaches: a network-free formulation, which may lead to consumption levels that violate network constraints, and a centralized problem, whose optimal solution yields a higher system-level demand, but that is in detriment of the utility of some individual consumers.

The remainder of this paper is organized as follows. Section II presents the network model, and formulates the optimization problems. Some considerations on a network-free formulation are discussed in Section III. The constrained problem is analyzed in Section IV, and Section V presents our numerical case studies. Lastly, Section VI summarizes our findings and identifies directions for future work.

II. Problem Formulation

We model a radial distribution network in which flexible consumers choose their demand level based on the price set by an aggregator. In turn, this aggregator is tasked with purchasing and distributing the energy demanded from these flexible consumers. We begin by defining the power flow equations in the distribution grid, which are taken into account by the aggregator when setting the prices. Then, the optimization problems of the aggregator and consumers are formulated using both a Stackelberg game and a centralized approach.

A. Single-Phase Branch Flow Equations

The distribution grid is modeled as a radial network represented by a graph $\mathcal{G}(\mathcal{N}, \mathcal{E})$ with $\mathcal{N}$ buses and $\mathcal{E}$ edges. For each edge $(i, j) \in \mathcal{E}$ connecting buses i and j, let $z_{ij} = r_{ij} + jx_{ij}$ denote the complex impedance of the line, $S_{ij} = P_{ij} + jQ_{ij}$ be the apparent power flow from i to j, and I_{ij} denote the complex line current. For each node $i \in \mathcal{N}$, let V_i be the complex voltage and $s_i = p_i + jq_i$ be the net apparent power demand. Further, let $v_i := |V_i|^2$ and $l_{ij} := |I_{ij}|^2$. The single-phase branch flow equations for a radial distribution network represented by $\mathcal{G}(\mathcal{N}, \mathcal{E})$ are:

$$P_{ij} = \sum_{k:j\to k} P_{jk} + r_{ij}l_{ij} + p_j \quad \forall i \in \mathcal{N} \tag{1}$$

$$Q_{ij} = \sum_{k:j\to k} Q_{jk} + x_{ij}l_{ij} + q_j \quad \forall i \in \mathcal{N} \tag{2}$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + \left(r_{ij}^2 + x_{ij}^2\right)l_{ij} \ \forall i \in \mathcal{N} \tag{3}$$

$$v_i l_{ij} = P_{ij}^2 + Q_{ij}^2 \quad \forall ij \in \mathcal{E}. \tag{4}$$

B. Stackelberg Game

We formulate the problem as a Stackelberg game in which the aggregator is the leader and sets energy prices for the consumers, while the consumers are followers and respond to the price signal received to decide on how much energy to demand. The demand profile in the network must satisfy physical laws, and thus the branch flow equations are included as constraints of the aggregator's problem. However, the quadratic equality constraint (4) makes the problem non-convex. Therefore, we instead utilize the relaxed constraint

$$v_i l_{ij} \geq P_{ij}^2 + Q_{ij}^2 \quad \forall ij \in \mathcal{E}. \tag{5}$$

Because our problem formulation focuses on the demand response management for consumers which do not inject power in the distribution grid, there are no reverse power flows and the nodal voltages will only drop (and not rise) across the network. Thus, upper voltage bounds will never be binding, and our problem satisfies the sufficient conditions established in [15] for exactness of the convex relaxation presented. We highlight that the conditions for exactness can still hold in the presence of voltage raises. Thus, as long as the upper voltage limit is not binding, reactive power injections from capacitor banks can be incorporated in the model without loss of generality. We refer the reader to [15], [18], [19] for a more detailed discussion about this relaxation.

The aggregator's utility function is given by his own profit

$$u_a = \sum_{i\in\mathcal{N}} \pi_i^* p_i - \lambda_{DA}\left(\sum_{i\in\mathcal{N}} p_i + \sum_{(i,j)\in\mathcal{E}} r_{ij}l_{ij}\right). \tag{6}$$

The first term in (6) corresponds to payments received from consumers for the energy supplied, where π_i^* is the inverse demand function of the consumer at node i.[1] The second term is the cost of purchasing energy in the wholesale market at a price λ_{DA}. Note that not only the demand from consumers, but also the power losses are considered in this cost. We consider the aggregator to be a price-taker in the day-ahead market, and thus λ_{DA} is assumed to be fixed and known. This assumption follows other works which consider the participation of demand response [20], wind [21], and storage resources [22] in the market.

The aggregator's problem is to maximize his own profit subject to network constraints and operational voltage and thermal limits, as well as considering the consumers' demand bounds and response to the prices set:

$$\max_{P_{ij},Q_{ij},v_i,l_{ij},p_i} \quad u_a \tag{7}$$

$$\text{s.t.} \quad (1)-(3),\ (5)$$

$$0.95^2 \leq v_i \leq 1.05^2 \quad \forall i \in \mathcal{N} \tag{7a}$$

$$l_{ij} \leq I_{ij,rated}^2 \quad \forall ij \in \mathcal{E} \tag{7b}$$

$$0 \leq p_i \leq \overline{P}_i \quad \forall i \in \mathcal{N} \tag{7c}$$

$$\pi_i^* = \pi_i\big(p_i^*\big) \quad \forall i \in \mathcal{N}. \tag{7d}$$

The inverse demand function (7d) maximizes the utility of consumer i, who solves the following problem:

$$\max_{p_i} \quad u_c^i = \gamma_i \ln(\alpha_i + p_i) - \pi_i p_i \tag{8}$$

$$\text{s.t.} \quad 0 \leq p_i \leq \overline{P}_i \tag{8a}$$

The utility function u_c^i of each consumer i incorporates their utility from consuming p_i, given by the first term in (8), and their payment corresponding to this demand. The logarithmic function has been shown to lead to proportional fairness [23], and has been previously used to model consumer-side utility in demand response problems [10], [24], [25]. The parameters γ_i and α_i are particular of each consumer, and thus these agents need not have a homogeneous behavior. These parameters reflect the consumer's valuation of his demand, and can also be interpreted as the consumer's willingness to forgo consumption. We later show that these parameters can be seen as a measure of consumer flexibility. We let $\gamma_i > 0$ and $\alpha_i \geq 1$, so that the utility from consuming is always non-negative.

C. Centralized Problem

Consider a scenario in which a central planner is tasked with deciding the demand levels for the distribution grid (and, consequently, the nodal prices). This decision is made so that the social welfare, given by the sum of utilities of each agent, is maximized. Similarly to the Stackelberg game, the network constraints and operational limits also need to be satisfied, and we used the relaxed constraint (5). Let the social welfare be

$$U = u_a + \sum_{i\in\mathcal{N}} u_c^i$$
$$= \sum_{i\in\mathcal{N}} \gamma_i \ln(\alpha_i + p_i) - \lambda_{DA}\left(\sum_{i\in\mathcal{N}} p_i + \sum_{(i,j)\in\mathcal{E}} r_{ij} l_{ij}\right). \tag{9}$$

Then, the centralized problem is defined as:

$$\max_{P_{ij},Q_{ij},v_i,l_{ij},p_i} \quad U \tag{10}$$
$$\text{s.t.} \quad (1)-(3),\ (5)$$
$$(7a)-(7c)$$

We note that, when summing the utilities of the consumers and the aggregator to form the social welfare function (9), the payments made from consumers to the aggregator constitute an internal transfer, and thus cancel out. Then, this objective only retains the terms which quantify the customers' benefit from consuming and the cost of purchasing energy in the wholesale market.

III. Analysis on a Network-Free Case

Game-theoretic studies involving the demand management of price-responsive consumers have typically used network-free formulations. Following that, we first consider an unconstrained scenario in which the operational constraints (7a)–(7b), and the branch flow equations (1)-(3), (5) are ignored. This is equivalent to a network-free or single-node case.

Theorem 1: In the Stackelberg game, the aggregator charges each consumer i with the energy price

$$\pi_i^* = \begin{cases} \sqrt{\lambda_{DA}\gamma_i/\alpha_i} & \text{if } \lambda_{DA} > \gamma_i\alpha_i/\big(\alpha_i+\overline{P}_i\big)^2 \\ \gamma_i/\big(\alpha_i+\overline{P}_i\big) & \text{otherwise.} \end{cases} \tag{11}$$

Each consumer responds with a non-zero consumption level if $\pi_i^* < \gamma_i/\alpha_i$, i.e., whenever $\lambda_{DA} < \gamma_i/\alpha_i$. Further, when $\lambda_{DA} < \gamma_i\alpha_i/(\alpha_i+\overline{P}_i)^2$, consumer i's demand will be binding at the maximum value.

Proof: This problem is solved using backwards induction. First, the consumer-side problem (8) is solved. For each consumer i, the utility function u_c^i is concave in the decision variable p_i. The Lagrangian function for this problem is

$$\mathcal{L}_c^i = \gamma_i \ln(\alpha_i + p_i) - \pi_i p_i + \underline{\lambda}_i p_i + \overline{\lambda}_i\big(\overline{P}_i - p_i\big), \tag{12}$$

leading to the KKT conditions

$$\frac{\partial \mathcal{L}_c^i}{\partial p_i} = \frac{\gamma_i}{\alpha_i + p_i} - \pi_i + \underline{\lambda}_i - \overline{\lambda}_i = 0 \tag{13}$$

$$\underline{\lambda}_i p_i = 0 \tag{14}$$

$$\overline{\lambda}_i(\overline{P}_i - p_i) = 0 \tag{15}$$

$$\underline{\lambda}_i, \overline{\lambda}_i \geq 0 \tag{16}$$

If the consumer's demand p_i is non-binding, we must have $\underline{\lambda}_i = \overline{\lambda}_i = 0$. It follows from (13) that $p_i^* = \gamma_i/\pi_i - \alpha_i$, which

[1] We remark that the use of index i to refer to consumers does not preclude the presence of multiple consumers at the same node. In that scenario, sub-indices can be created to refer to each customer at the same node, at the expense of more notation. Alternatively, these customers can be included in the formulation by incorporating "phantom" branches of zero impedance where necessary, so that each customer has their own "node".

must satisfy the bounds (7c). By checking the cases in which demand is binding, we can find that

$$p_i^*(\pi_i) = \begin{cases} \overline{P}_i & \text{if } \pi_i < \gamma_i/\big(\alpha_i + \overline{P}_i\big) \\ \gamma_i/\pi_i - \alpha_i & \text{if } \gamma_i/\big(\alpha_i + \overline{P}_i\big) \le \pi_i < \gamma_i/\alpha_i \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

is the demand function of consumer i. For $0 \le p_i^* \le \overline{P}_i$, the corresponding inverse demand function can be written as

$$\pi_i\big(p_i^*\big) = \gamma_i/\big(\alpha_i + p_i^*\big). \tag{18}$$

The power loss cost in the aggregator's utility (6) is zero due to the network-free formulation. This function can be shown to be concave in p_i upon substitution of the inverse demand function (18). Considering only constraints (7c) and (7d),

$$\mathcal{L}_a = \sum_{i \in \mathcal{N}} \left[\frac{\gamma_i p_i}{\alpha_i + p_i} - \lambda_{DA} p_i + \underline{\mu}_i p_i + \overline{\mu}_i (\overline{P}_i - p_i) \right] \tag{19}$$

is the Lagrangian function, and the KKT conditions are

$$\frac{\partial \mathcal{L}_a}{\partial p_i} = \frac{\gamma_i \alpha_i}{(\alpha_i + p_i)^2} - \lambda_{DA} + \underline{\mu}_i - \overline{\mu}_i = 0 \ \forall i \tag{20}$$

$$\underline{\mu}_i p_i = 0 \ \forall i \tag{21}$$

$$\overline{\mu}_i (\overline{P}_i - p_i) = 0 \ \forall i \tag{22}$$

$$\underline{\mu}_i, \overline{\mu}_i \ge 0 \ \forall i. \tag{23}$$

Suppose the aggregator chooses non-binding demands. Then, the multipliers $\underline{\mu}_i = \overline{\mu}_i = 0 \ \forall i$ and, from (20),

$$p_i^* = \sqrt{\gamma_i \alpha_i / \lambda_{DA}} - \alpha_i \tag{24}$$

is the optimal demand level for each consumer i. This demand is only positive if $\lambda_{DA} < \gamma_i/\alpha_i$, and becomes binding at $p_i^* = \overline{P}_i$ for $\lambda_{DA} < \gamma_i \alpha_i / (\alpha_i + \overline{P}_i)^2$. Using (18), we find that this demand can be enforced through the price (11). ■

This result shows that consumers with lower γ_i/α_i require lower prices to start consuming, and thus can be seen as less flexible. Further, if consumers are homogeneous (i.e., have the same γ_i/α_i and $\overline{P}_i$), then they are charged the same price, as expected for a network-free model.

Theorem 2: In the centralized case, consumers are charged

$$\Pi_i^* = \begin{cases} \lambda_{DA} & \text{if } \lambda_{DA} > \gamma_i/\big(\alpha_i + \overline{P}_i\big) \\ \gamma_i/\big(\alpha_i + \overline{P}_i\big) & \text{otherwise,} \end{cases} \tag{25}$$

and respond with a positive demand if $\lambda_{DA} < \gamma_i/\alpha_i$. When demands are not binding, consumers are charged the day-ahead energy price, and thus no rent is extracted by the central planner. In this case, the centralized price is always lower than that charged in the Stackelberg case.

Proof: Consider the optimization problem (10) without the network constraints. The Lagrangian function is

$$\mathcal{L}_c = \sum_{i \in \mathcal{N}} \big[\gamma_i \ln(\alpha_i + p_i) - \lambda_{DA} p_i + \underline{\mu}_i^c p_i + \overline{\mu}_i^c \big(\overline{P}_i - p_i\big) \big], \tag{26}$$

and therefore we have the KKT conditions

$$\frac{\partial \mathcal{L}_c}{\partial p_i} = \frac{\gamma_i}{\alpha_i + p_i} - \lambda_{DA} + \underline{\mu}_i^c - \overline{\mu}_i^c = 0 \ \forall i \tag{27}$$

$$\underline{\mu}_i^c p_i = 0 \ \forall i \tag{28}$$

$$\overline{\mu}_i^c (\overline{P}_i - p_i) = 0 \ \forall i \tag{29}$$

$$\underline{\mu}_i^c, \overline{\mu}_i^c \ge 0 \ \forall i. \tag{30}$$

For non-binding demands, $\underline{\mu}_i^c = \overline{\mu}_i^c = 0 \ \forall i$. Then, we use (27) to find $p_i^* = \gamma_i/\lambda_{DA} - \alpha_i$. Similarly to the Stackelberg game, this demand is only positive if $\lambda_{DA} < \gamma_i/\alpha_i$. The demands will be binding at the maximum value when prices drop to $\lambda_{DA} < \gamma_i/(\alpha_i + \overline{P}_i)$. From the consumer's inverse demand function (18), we find that the optimal non-binding demand for the centralized case can be accomplished by charging $\Pi_i^* = \lambda_{DA} \ \forall i$. Comparing to the Stackelberg case, we find that

$$\pi_i^* = \sqrt{\Pi_i^* \gamma_i / \alpha_i}, \tag{31}$$

from which we have the following possibilities:

1) Case $\pi_i^* = \Pi_i^*$: Equality holds if $\lambda_{DA} = \gamma_i/\alpha_i$. In this case, demand is zero for consumer i both in the Stackelberg game and in the centralized case.
2) Case $\pi_i^* < \Pi_i^*$: Condition holds if $\lambda_{DA} > \gamma_i/\alpha_i$, which is not low enough to incentivize consumption, also leading to zero demand for consumer i.
3) Case $\pi_i^* > \Pi_i^*$: Condition holds if $\lambda_{DA} < \gamma_i/\alpha_i$. In this case, consumer i will respond with positive demand both in the Stackelberg game and in the centralized case.

Thus, for positive non-binding demand levels, $\pi_i^* > \Pi_i^*$. ■

Since consumers are charged a lower price in the centralized case, it follows that, in a network-free problem, the demand of each *individual* consumer is higher in a centralized set-up. We later show that this is not necessarily what occurs when network constraints are considered, and some consumers may have a higher demand in the Stackelberg game. Our results also imply that the centralized problem leads to a higher incentive to consume, increasing the *aggregated* demand level in the grid. This will be shown to also hold in the networked case. We remark that, when network constraints are not considered, the price signals calculated may prompt consumers to exercise demand levels that cannot be supported by the network, causing undesired congestion or voltage violations. Hence the importance of a network-constrained formulation.

IV. Constrained Distribution Network Analysis

In this section, all the power flow and operational constraints for the radial distribution system are considered.

A. System-Level Analysis

We begin by evaluating our problem from a system-wide perspective. The following result compares the solutions to the Stackelberg game and the centralized set-up while considering the aggregated optimal demand of the consumers.

Proposition 1: The aggregated utility of consumers $\sum_{i \in \mathcal{N}} u_c^i$ at the Stackelberg game solution is upper bounded by their optimal aggregated utility in the centralized case.

Proof: Let $\overline{\mathbf{p}} := [\overline{p}_1, \ldots, \overline{p}_{|\mathcal{N}|}]$ be the optimal demand levels at the solution for the centralized problem (10). Proof follows from the chain of inferences below:

1) Aggregator's Optimal Utility: Since

- the constraints in the centralized problem (10) are the same as those in the aggregator's problem in the Stackelberg game (7), and

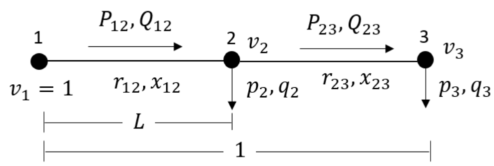

Fig. 1. Three-node network used in Example 1.

- the decision variables for both problems are the same,

these problems have the same feasible set. With that, because $\overline{\mathbf{p}}$ is feasible in the centralized case, then it must also be feasible in the Stackelberg game. Thus, since the aggregator maximizes his own utility, he will not choose any demand level $\hat{\mathbf{p}} \neq \overline{\mathbf{p}}$ which yields a lower utility for him than the one he can achieve with $\overline{\mathbf{p}}$. It follows that the aggregator's optimal utility in the Stackelberg game is lower bounded by his utility in the centralized case.

2) Optimal Social Welfare: By definition, the social welfare achieves its highest level at the solution of the centralized problem (10). Thus, the social welfare in the Stackelberg game can be at most the optimal centralized social welfare.

3) Aggregated Consumers' Optimal Utility: Combining the previous two points, it follows that the aggregated utility of the consumers in the Stackelberg game is upper bounded by their optimal aggregated utility in the centralized case. ■

Proposition 1 implies that the overall demand in the centralized case sets an upper bound on this aggregated demand in the Stackelberg game. However, we show next that this system-level observation may not hold for each individual consumer.

B. Node-Level Analysis

In the previous sections, we showed that (i) in a network-free model, all consumers are charged less, and thus have higher demand in the centralized case, and (ii) in a network-constrained model, the aggregated demand level in the Stackelberg game is upper bounded by that in the centralized case. We now analyze the optimal demand of each node individually and how the network constraints affect consumer consumption. We begin by presenting an example which illustrates that, in the presence of a network, some nodes may have higher demand in the centralized case, while others consume more in the Stackelberg game.

Example 1: For the network in Fig. 1, let L be the distance between nodes 1 (substation) and 2, and the total line length be 1. Assume $v_1 = 1p.u.$ and that the grid is loaded enough that node 3 has a binding lower voltage limit, i.e., $v_3 = 0.95^2 p.u.$ Let the consumers at nodes 2 and 3 have $\gamma = 35$ and $\alpha = 2$. Assume $r = 0.0174p.u.$ per unit length to be the line resistance. For simplicity, set the reactive power consumption to zero and consider the linearized branch flow equations:

$$P_{ij} = \sum_{k:j\to k} P_{jk} + p_j \ \forall i \in \{1, 2, 3\} \quad (32)$$

$$Q_{ij} = \sum_{k:j\to k} Q_{jk} + q_j \ \forall i \in \{1, 2, 3\} \quad (33)$$

$$v_j = v_i - 2\big(r_{ij}P_{ij} + x_{ij}Q_{ij}\big) \ \forall i \in \{1, 2, 3\} \quad (34)$$

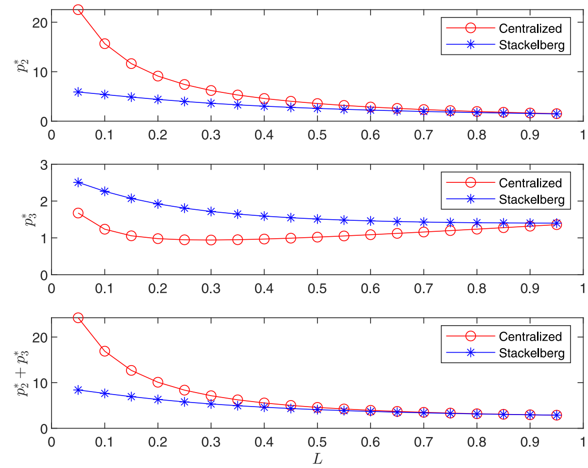

Fig. 2. Optimal demands for the three-node example.

Recognizing that $r_{12} = Lr$ and $r_{23} = (1 - L)r$, equations (31)–(33) can be used to find the demand at nodes 2 and 3 as a function of the voltage at node 2 and the distance L:

$$p_2 = \frac{(1 - v_2) - (1 - 0.95^2)L}{2L(1 - L)r}, \ p_3 = \frac{v_2 - 0.95^2}{2(1 - L)r}. \quad (35)$$

For non-negative demands (35), we must have $0.95^2 \le v_2 \le 1-(1-0.95^2)L = V_H$. Given L, we can find v_2 that maximizes the objective for the centralized case and the Stackelberg game, which in turn determines the optimal demands p_2^* and p_3^*. For that, we substitute the expressions (35) in the social welfare (9) and in the aggregator's utility (6)[2] to solve

$$\max_{0.95^2 \le v_2 \le V_H} U(v_2), \quad \max_{0.95^2 \le v_2 \le V_H} u_a(v_2). \quad (36)$$

Both functions are concave within the interval $0.95^2 \le v_2 \le V_H$. The results in Fig. 2 show that (i) node 2 (top) benefits more in the centralized case, especially when closer to the substation, (ii) node 3 (center) consumes more in the Stackelberg game, and (iii) the centralized aggregated demand is lower bounded by that of the Stackelberg game (bottom). Further, as $L \to 1$, both nodes become equally distant from the substation and the solutions for both cases tend to approximate.

Remark 1: The linearized model used in Example 1 has been shown to provide good approximation of the branch flow equations (e.g., in [26]). However, it does not factor in the current constraints. Since our goal is to understand the role of both voltage and current constraints on optimal demands, we use the nonlinear model in our main analysis.

Motivated by Example 1, Theorem 3 formulates the optimal demands of individual nodes when the nonlinear branch flow equations (1)-(3), (5) are considered.

Theorem 3: Let $\mathcal{P}_i$ denote the set of all edges on the upstream path from node i to the substation. The optimal demand of consumer i in the Stackelberg game is

$$p_i^* = \sqrt{\frac{\gamma_i\alpha_i}{\lambda_{DA} + 2\sum_{(i,j)\in\mathcal{P}_i}\Big(r_{ij}\nu_j^s + P_{ij}\xi_j^s\Big) - \underline{\mu}_i^s + \overline{\mu}_i^s}} - \alpha_i, \quad (37)$$

[2]The Stackelberg game first substitutes the inverse demand function (18).

and the solution in the centralized case is given by

$$p_i^* = \frac{\gamma_i}{\lambda_{DA} + 2\sum_{(i,j)\in\mathcal{P}_i}\left(r_{ij}\nu_j^c + P_{ij}\xi_j^c\right) - \underline{\mu}_i^c + \overline{\mu}_i^c} - \alpha_i, \tag{38}$$

where $\nu_j^{(.)}$, $\xi_j^{(.)}$, $\underline{\mu}_j^{(.)}$, $\overline{\mu}_j^{(.)}$ are the Lagrange multipliers of the voltage equality constraint (3), the relaxed inequality constraint (5), and the demand lower and upper bounds (7c) for node j, respectively, and the superscripts indicate if the centralized (c) or Stackelberg (s) set-up are considered.

Proof: The Lagrangian for the centralized problem is given by

$$\begin{aligned}\mathcal{L} = &\sum_{i\in\mathcal{N}}\gamma_i \ln(\alpha_i + p_i) - \lambda_{DA}\left(\sum_{i\in\mathcal{N}} p_i + \sum_{(i,j)\in\mathcal{E}} r_{ij}l_{ij}\right)\\ &- \sum_{i\in\mathcal{N}}\Pi_i\left(P_{ij} - \sum_{k:j\to k}P_{jk} - r_{ij}l_{ij} - p_j\right)\\ &- \sum_{i\in\mathcal{N}}\kappa_i\left(Q_{ij} - \sum_{k:j\to k}Q_{jk} - x_{ij}l_{ij} - q_j\right)\\ &- \sum_{i\in\mathcal{N}}\nu_i\Big(v_j - v_i + 2\big(r_{ij}P_{ij} + x_{ij}Q_{ij}\big) - \Big(r_{ij}^2 + x_{ij}^2\Big)l_{ij}\Big)\\ &+ \sum_{i\in\mathcal{N}}\xi_i\Big(v_i l_{ij} - P_{ij}^2 - Q_{ij}^2\Big) + \epsilon_i\Big(I_{ij,rated}^2 - l_{ij}\Big)\\ &+ \sum_{i\in\mathcal{N}}\underline{\rho}_i\Big(v_i - 0.95^2\Big) + \overline{\rho}_i\Big(1.05^2 - v_i\Big)\\ &+ \sum_{i\in\mathcal{N}}\underline{\mu}_i p_i + \overline{\mu}_i\big(\overline{P}_i - p_i\big).\end{aligned} \tag{39}$$

Thus, the KKT conditions for stationarity are

$$\frac{\partial\mathcal{L}}{\partial P_{ij}} = \Pi_i - \Pi_j - 2r_{ij}\nu_j - 2P_{ij}\xi_j = 0\ \forall i \tag{40}$$

$$\frac{\partial\mathcal{L}}{\partial Q_{ij}} = \kappa_i - \kappa_j - 2x_{ij}\nu_j - 2Q_{ij}\xi_j = 0\ \forall i \tag{41}$$

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial l_{ij}} &= -\lambda_{DA}r_{ij} + \Pi_i r_{ij} + \kappa_i x_{ij} + \nu_i\Big(r_{ij}^2 + x_{ij}^2\Big)\\ &\quad + v_i\xi_i - \epsilon_i = 0\ \forall i\end{aligned} \tag{42}$$

$$\frac{\partial\mathcal{L}}{\partial v_i} = -\nu_i + \sum_{(i,j)\in\mathcal{E}}\big(\nu_j + \xi_j l_{ij}\big) + \underline{\rho}_i - \overline{\rho}_i = 0\ \forall i \tag{43}$$

$$\frac{\partial\mathcal{L}}{\partial p_i} = \gamma_i/(\alpha_i + p_i) - \lambda_{DA} + \Pi_i + \underline{\mu}_i - \overline{\mu}_i = 0\ \forall i. \tag{44}$$

The complementary slackness conditions are given by

$$\xi_i\Big(v_i l_{ij} - P_{ij}^2 - Q_{ij}^2\Big) = 0\ \forall i \tag{45}$$

$$\epsilon_i\Big(I_{ij,rated}^2 - l_{ij}\Big) = 0\ \forall i \tag{46}$$

$$\underline{\rho}_i\big(v_i - 0.95^2\big) = 0\ \forall i \tag{47}$$

$$\overline{\rho}_i\big(1.05^2 - v_i\big) = 0\ \forall i \tag{48}$$

$$\underline{\mu}_i p_i = 0\ \forall i \tag{49}$$

$$\overline{\mu}_i\big(\overline{P}_i - p_i\big) = 0\ \forall i. \tag{50}$$

Further, the problem needs to satisfy the equality and inequality constraints of the original problem, and the multipliers for the inequality constraints must be positive. From (44), we have

$$p_i^* = \frac{\gamma_i}{\lambda_{DA} - \Pi_i - \underline{\mu}_i + \overline{\mu}_i} - \alpha_i, \tag{51}$$

which can be rewritten as in (38) by isolating Π_i from (40) and recognizing that its expression can be written recursively to become independent of any other Π_j. For that, we move from node i to the substation to find that

$$\Pi_i = -2\sum_{(i,j)\in\mathcal{P}_i}\big(r_{ij}\nu_j + P_{ij}\xi_j\big), \tag{52}$$

where $\mathcal{P}_i$ is the set of all edges on this upstream path. The solution to the Stackelberg game can be found in a similar manner, but using (6) as the objective function and letting the inverse demand function be given by (18). ■

The results in Theorem 3 reveal the effect of the network constraints on the optimal demand of consumers. The term $2\sum_{(i,j)\in\mathcal{P}_i}(r_{ij}\nu_j^{(.)} + P_{ij}\xi_j^{(.)})$ back-propagates the effect of voltage and current constraints from each node to the substation, indicating that consumers with the same willingness to change their consumption may receive different price signals depending on their location in the network. We highlight that these location-dependent results are not only related to the upstream path from each node to the substation, but it also implicitly depends on the nodes downstream from the consumer considered, as their demand level influences the shadow costs of the constraints and the power flow injection in the node where the consumer is located.

V. Case Study

We consider a single-phase IEEE 123-bus system operating at 2.4kV with a total of 85 load nodes with flexible consumers. To efficiently solve the relaxed convex problems presented in Section II for the centralized and the Stackelberg cases, we let the initial condition be the solution of an equivalent linearized optimal power flow model. Without loss of generality, we let $\lambda_{DA} = 1$ for ease of comparison with the prices that emerge in the distribution network, and we neglect the upper bound in consumer demand. Further, we assume the voltage at the substation to be 1 p.u., and henceforth we utilize the per unit system with 1000 as the base kVA.

A. Network-Free Case

We first simulate a case in which the branch flow equations and the corresponding operational constraints are not considered. For this case, the flexibility ratio γ_i/α_i of each consumer i was randomly selected between 15 and 20. The results in Fig. 3 show that the price charged for each consumer i depends on the flexibility ratio γ_i/α_i of each consumer in the Stackelberg case. On the other hand, the solution to the centralized case is such that the consumers are charged the same price, which is equal to the day-ahead energy price. We further note that the pricing scheme in the centralized case leads to higher variance in the optimal demand levels, as compared to the Stackelberg case. Lastly, the utility of each individual consumer is higher

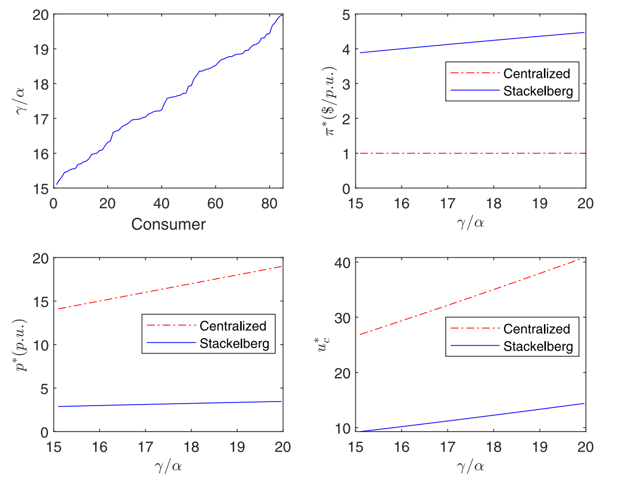

Fig. 3. Network-free case. Flexibility level of consumers (top left), energy price charged (top right), optimal demand for each consumer (bottom left), and consumers' individual utilities at the optimal solution (bottom right).

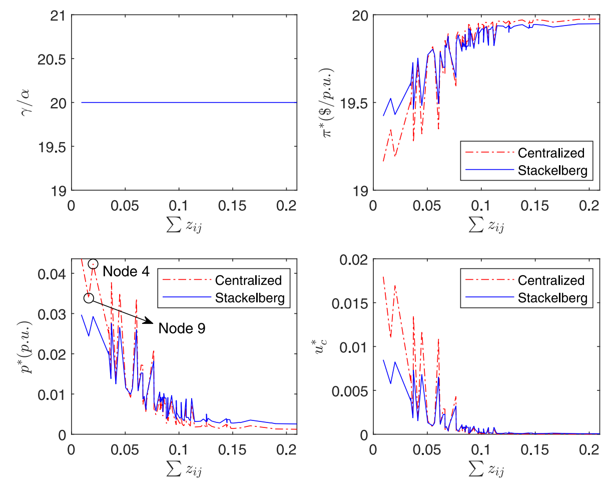

Fig. 4. Centralized and Stackelberg solutions as a function of the impedance distance from each node to the substation, $\sum z_{ij}$. Consumer flexibility levels (top left), price charged at each node (top right), individual optimal demands (bottom left), and consumers' individual optimal utilities (bottom right).

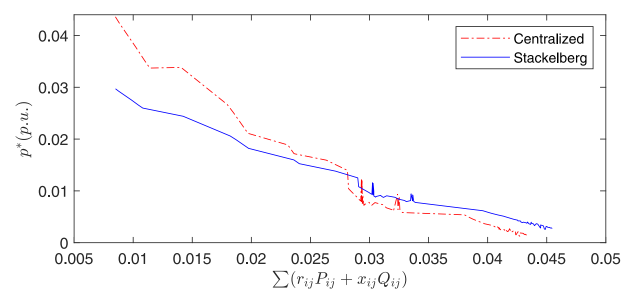

Fig. 5. Optimal demands as a function of the accumulated voltage drop due to power flows from the substation to each node, $\sum(r_{ij}P_{ij} + x_{ij}Q_{ij})$.

in the centralized case. We also note that consumers with lower γ_i/α_i are charged less; however, they consume less than other consumers that are more flexible, leading to a utility that is lower than what can be achieved with higher flexibility levels.

B. Constrained Case: System-Level Analysis

We consider a case with $I^2_{ij,rated} = 1p.u.\ \forall ij \in \mathcal{E}$, and homogeneous consumers with flexibility ratio $\gamma_i/\alpha_i = 20\ \forall i$. Thus, any differences in the optimal demand for each flexible consumer and the price charged at each node are due to the network constraints. Fig. 4 presents the results for this network-constrained case as a function of the impedance distance from each node with a flexible consumer to the substation. We note that not all the findings from the network-free case are transferable to this problem. For example, prices are not always lower in the centralized case, and thus some consumers may be better off in the Stackelberg game. These counter-intuitive results indicate that, although more tractable, network-free formulations to price consumer flexibility may fail to capture some important features of the underlying distribution network, and economic losses may ensue.

Overall, it can be observed that the centralized formulation favors consumers that are closer to the substation, while the Stackelberg game seeks flexibility from nodes that are further away due to the rent-seeking behavior of the aggregator. However, because of the costs associated with power losses and voltage drops, the flexibility demanded from a certain node depends on how much supply is being delivered to the nodes located downstream to it. As a consequence, we observe the variations of the curves in Fig. 4 as the impedance distance is increased, although the decreasing trend in the optimal demand is maintained. For example, node 4 has a higher optimal demand than node 9, although it is at a larger impedance distance from the substation. A closer look at the network topology shows that node 4 is a leaf node, whereas node 9 is in the main feeder and almost the entire grid lies downstream to it. Fig. 5 shows the optimal demands for the same case, but plotted as a function of the accumulated voltage drop from the substation to each node. The smoother curve observed indicates that the results are indeed dependent on the overall power flows that reach each node to be delivered to consumers downstream to them.

To further investigate this case from a system-level perspective, we analyze how the utility of the aggregator and the aggregated utility of the consumers change as the thermal limit becomes less strict. Fig. 6 presents these results for the aggregator in the top figure, and for the aggregated consumers in the bottom one. The bar plots show the difference in utility between the Stackelberg and the centralized case. The aggregator's utility in the Stackelberg game is never below what he earns in the centralized case, whereas the opposite occurs for the sum of consumers' utilities. These observations corroborate the discussions in Section IV.

When the thermal limit surpasses $I^2_{rated} = 10p.u.$, the Stackelberg game achieves an equilibrium in which thermal limits are not binding. This means the aggregator cannot further increase his utility by allowing consumers to increase consumption. Thus, from this point forward, the Stackelberg game solution remains unchanged even if the thermal limit is further increased. However, social welfare can still be increased by seeking flexibility from consumers in detriment of the aggregator's utility. With that, the demand level in the centralized case keeps increasing as the thermal limit is relaxed, leading to a decrease in the aggregator's utility and increase in the aggregated consumer utility until

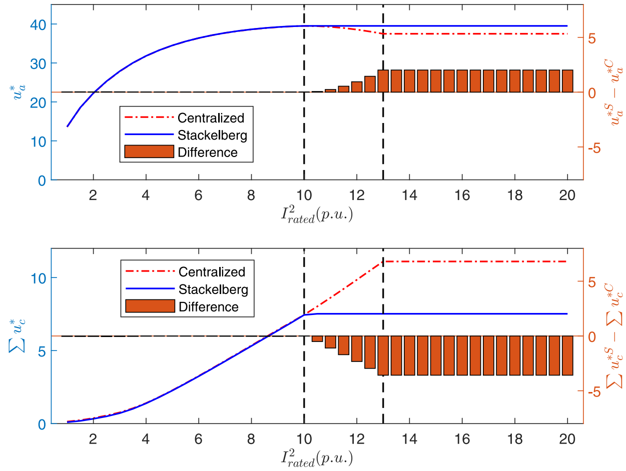

Fig. 6. Aggregator utility (top) and aggregated consumer utility (bottom) as thermal limit is relaxed for the Stackelberg and centralized cases. The bars show the difference between these utilities in the Stackelberg and the centralized case. The left-most (right-most) vertical dashed line indicates the thermal limit at which the Stackelberg (centralized) solution ceases to have a binding thermal limit.

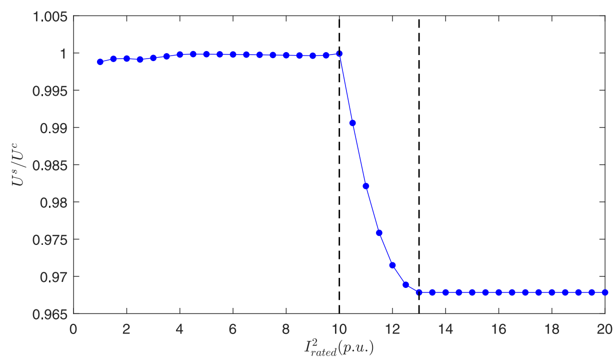

Fig. 7. Sub-optimal social welfare ratio as thermal limit is relaxed. The left-most (right-most) vertical dashed line indicates the thermal limit at which the Stackelberg (centralized) solution ceases to have a binding thermal limit.

$I_{rated}^2 = 13p.u.$, from which social welfare can no longer be improved.

The sub-optimality of the social welfare at the Stackelberg equilibrium for this case study is presented in Fig. 7, which shows the ratio of this value and the optimal social welfare from the centralized problem. As previously observed in Fig. 6, the solutions of both problems are very close while there exists binding thermal limits. However, as discussed in Section IV, the social welfare in the Stackelberg equilibrium cannot be higher than that achieved in the centralized case. When thermal limits cease to be binding in the Stackelberg problem, the sub-optimality ratio starts dropping due to the continued increase of the social welfare in the centralized case, until this case also achieves a solution without binding thermal limits, stabilizing the sub-optimality ratio.

C. Constrained Case: Node-Level Analysis

When individual consumers are evaluated, we note that their optimal demands are closely related to their location in the network. To illustrate this effect, we compare the results for two nodes which are at a similar impedance distance from the substation, nodes 4 and 9, but whose positions in the network relative to other nodes are distinct.

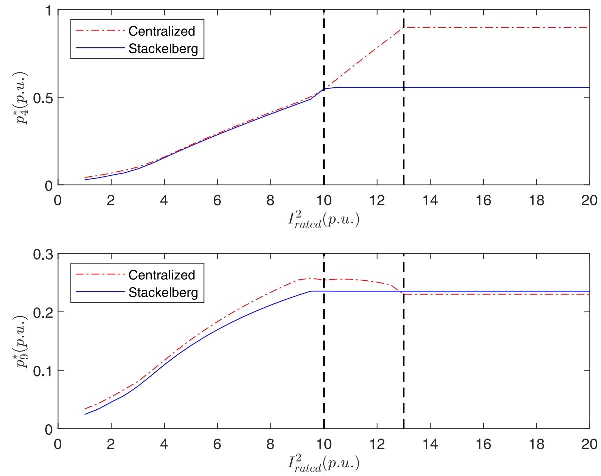

Fig. 8. Optimal demands for nodes 4 (top) and 9 (bottom) as thermal limit is relaxed. The left-most (right-most) vertical dashed line indicates the thermal limit at which the Stackelberg (centralized) solution ceases to have a binding thermal limit.

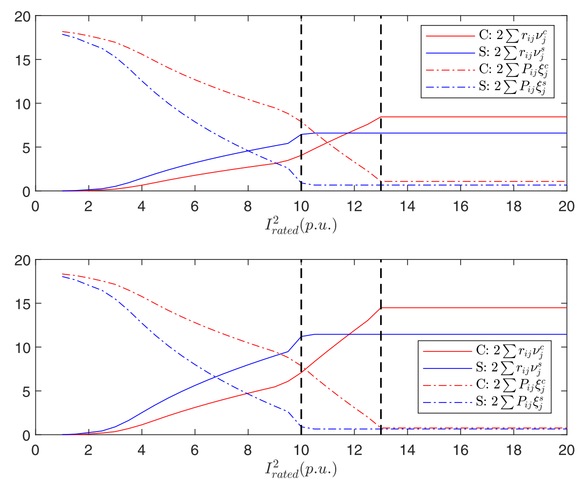

Fig. 9. Comparison of terms that influence optimal demand for centralized (C) and Stackelberg (S) cases for nodes 4 (top) and 9 (bottom). The left-most (right-most) vertical dashed line indicates the thermal limit at which the Stackelberg (centralized) solution ceases to have a binding thermal limit.

Fig. 8 shows the optimal demands for these nodes as the thermal limit is relaxed. The system-level results showed that the centralized formulation keeps increasing the overall demand in the grid after $I_{rated}^2 > 10p.u.$, while the Stackelberg game ceases to supply at this moment. From the individual optimal demands in Fig. 8, we note that node 4 benefits from the increase in overall demand provided in the centralized case, while node 9 observes a decrease in their optimal demand.

Lastly, we evaluate the voltage term $2\sum_{(i,j)\in\mathcal{P}_i} r_{ij}\nu_j^{(.)}$ and the current term $2\sum_{(i,j)\in\mathcal{P}_i} P_{ij}\xi_j^{(.)}$ presented in Theorem 3 as modifiers of the optimal demands in the network-constrained case. Fig. 9 shows these terms individually for nodes 4 and 9, with the solid (dashed) lines corresponding to the voltage (current) term, and red (blue) referring to the centralized (Stackelberg) case. We first observe that the term that relates to the current constraint has a similar effect on the optimal demand at both nodes, being higher in the centralized case due to the push for higher system-level demand, and decreasing as the thermal limit is relaxed. This may be due to the fact that the only line whose thermal limit becomes binding in this network is the one connecting the substation to the first node in the grid, and this congestion affects all nodes in a similar way. On the other hand, the voltage-related term rises more significantly for node 9, leading to lower optimal demand for this node as compared to node 4. This can be explained by the position of these nodes in the network. Since 4 is a leaf node, its voltage has less influence in the overall supply distribution in the network than node 9's voltage, which is located on the main feeder and has several nodes downstream.

VI. Conclusion

We proposed a network-constrained Stackelberg game for a demand-side management problem. We showed how the optimal demands of price-responsive consumers change depending on their flexibility level and on their location on the grid. Our analysis compared this framework with a centralized approach, revealing that, although the aggregated demand in the centralized case will never be below that of the Stackelberg game, some individual consumers may benefit from the game-theoretic set-up.

The current formulation focused on a scenario with one aggregator only, and thus no market competition was present in the distribution grid. Extending this formulation to consider the presence of multiple aggregators who compete to supply to consumers constitutes an interesting direction for future work. Further, since demand uncertainty was not in the scope of this paper, possible mismatches between the supply purchased in the day-ahead market and the actual consumer demand were not considered in the formulation. Thus, avenues for future work also include modeling demand as a stochastic variable, so that the aggregator must also participate in the real-time market to settle supply-demand mismatches.

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Nayara Aguiar (Graduate Student Member, IEEE) received the B.Sc. degree in electrical engineering from the Federal University of Campina Grande, Brazil, in 2016, and the M.S. degree in electrical engineering from the University of Notre Dame in 2018, where she is currently pursuing the Ph.D. degree with the Department of Electrical Engineering. Her current research interests include design and analysis of electricity markets in the presence of intermittent renewable energy generation, and demand response. She was a recipient of the 2019 Patrick and Jana Eilers Graduate Student Fellowship for Energy Related Research Electrical Engineering from the Center for Sustainable Energy at Notre Dame.

Anamika Dubey (Member, IEEE) received the M.S.E and Ph.D. degrees in electrical and computer engineering from the University of Texas at Austin in 2012 and 2015, respectively. She is currently an Assistant Professor with the School of Electrical Engineering and Computer Science, Washington State University, Pullman, WA, USA. Her research is focused on the operation and planning of modern power distribution systems characterized by distributed energy resources and responsive loads for improved efficiency and resilience. She is a recipient of the National Science Foundation CAREER Award. She is the current a Secretary of IEEE Distribution Systems Analysis Subcommittee and IEEE University Education Subcommittee, and serves as a PES Chapter Chair for the IEEE Palouse Section. She serves as the Associate Editor for IEEE TRANSACTIONS ON POWER SYSTEMS, IEEE POWER ENGINEERING LETTERS, and IEEE ACCESS.

Vijay Gupta (Senior Member, IEEE) received the B.Tech. degree in electrical engineering from Indian Institute of Technology Delhi, Delhi, and the M.S. and Ph.D. degrees in electrical engineering from the California Institute of Technology. He is a Professor with the Department of Electrical Engineering, University of Notre Dame, having joined the Faculty in January 2008. Prior to joining Notre Dame, he also served as a Research Associate with the Institute for Systems Research, University of Maryland, College Park. His research and teaching interests are broadly in distributed decision making. He received the 2018 Antonio Ruberti Award from IEEE Control Systems Society, the 2013 Donald P. Eckman Award from the American Automatic Control Council, and an 2009 National Science Foundation CAREER Award.