Dynamic Electromagnetic Exposure Allocation For Rayleigh Fading MIMO Channels

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***Abstract*—Future wearable and portable devices with multiple transmit antennas operating below 6 GHz are constrained by regulatory limitations on the level of electromagnetic radiation a user can be exposed to, measured using the specific absorption rate (SAR). Signaling designs that are optimized to include SAR constraints can improve the performance of uplink transmission. These signaling schemes could include closed-loop beamforming, closed-loop precoding, and space-time coding, which have all been shown to achieve increased rates when optimized as a function of SAR. Previous research addressed SAR constrained optimization only within a single coherence time block. In this paper, we present transmit policies that dynamically allocate user electromagnetic radiation exposure over time. We propose three exposure allocation methods — optimal, uniform, and asymptotic — in the practical case with causal channel state information (CSI), and an on-off transmission approach for the low SAR-to-noise ratio regime. Our results demonstrate that the performance of SAR-aware transmission can be further improved by exploiting frequency and time diversity.**

***Index Terms*—Multiple antenna array, specific absorption rate, convex optimization, dynamic programming**

I. Introduction

Wearable and portable wireless devices, such as smart-bands, smart-watches, smartphones, tablets, and laptops are prolific in modern society. In the coming fifth-generation (5G) era, large numbers of sensors and wirelessly connected devices will form the “internet-of-things” (IoT). A large portion of these IoT devices will be used in close proximity to the human body, including implanted sensors, health monitors, smart-classes, and virtual/augmented reality (VR/AR) devices, etc., forming wireless body area networks (WBAN). These wireless devices and their associated applications hold the potential to greatly improve the life and well being of users.

Wireless devices, however, emit electromagnetic radiation, and devices must satisfy constraints on this radiation. Specific absorption rate (SAR) is the most widely accepted user exposure metric and is used worldwide by government agencies to regulate portable devices operating below 6 GHz. SAR is measured using the amount of power absorbed by the human body per unit mass, with units of Watt/kilogram (W/kg). In the US [1], the peak SAR limitation for partial body exposure, including the human head, is 1.6 W/kg averaged over 1 gram of tissue. Most, if not all, of today’s commercially available portable devices are regulated as single transmit antenna devices, and SAR mitigation is accomplished using advanced hardware designs [2], including applying auxiliary antenna elements, ferrite loading, special high impedance surfaces, and metamaterials. Currently, if a device does not pass the SAR testing, the transmit power is decreased until compliance is achieved.

The difficulty of attaining SAR compliance will increase in the future as commercial transceivers will be equipped with multiple transmit antennas and each additional antenna will increase the maximum potential exposure for a given transmit power [3]–[5]. To allow for higher uplink transmission rates, Long-Term Evolution Advanced (LTE-Advanced) and LTE-Advanced Pro. already support up to eight transmit antenna ports at the user device. Recent research suggests that the near-field SAR limitation will have significant impact on the far-field performance of a wireless communication system with multiple transmit antennas [3]–[6]. Multiple transmit antennas cause large SAR variations as the gain and phase combinations change across the antennas, and regulatory agencies usually test for compliance with the worst-case SAR. A signaling technique that does not consider SAR must reduce the transmit power until the worst-case SAR reading satisfies the constraint, which can significantly degrade the communication link quality. Therefore, it is critical for 5G and beyond systems to employ uplink signals that are optimized with respect to the SAR constraint. Note that with millions of low-cost connected devices in 5G IoT, the usage of SAR-aware techniques will not be limited to cellular uplink transmission. As the exposure from all types of mobile device transmissions are regulated, side-link transmissions, i.e., device-to-device, will also benefit from the proposed SAR-aware techniques.

Two key problems in the development of SAR-aware transmission schemes are: 1) modeling SAR in terms of the transmit signal, and 2) applying these models to incorporate a signal-level SAR constraint into system analysis and design. In the context of SAR modeling, various works have analyzed the dependence of SAR on the transmitted signal via simulations and experiments. In [7], the SAR value is demonstrated to be a sinusoidal function of the phase difference when the second antenna transmits a phase shifted version of the signal sent on the first antenna. Based on the results of [7], a quadratic model of SAR as a function of the transmit signal was first introduced in [3], and further analyzed and validated in [4], [5]. Methods for efficiently determining the parameters needed

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for the quadratic exposure model were proposed in [8]–[10].

Recent research has shown success in developing SAR-aware signaling techniques by applying the SAR model in [3] to directly limit the amount of SAR induced during transmission. In [4], SAR-aware codes designed to mitigate SAR levels are developed and shown to provide a 2.5 dB improvement in probability of error over the well-known Alamouti code in a SAR-constrained channel. In [11] and [12], the optimal beamformer and precoder are formulated for multiple-input multiple-output (MIMO) systems with one SAR constraint, and general SAR-aware precoding optimization is presented in [6]. Work in [13] further extends these results to a multi-user scenario with limited channel information at the transmitter. The results in these works demonstrate that SAR-aware transmission schemes are able to achieve much higher rates than the traditional power back-off techniques while maintaining compliance with exposure constraints.

Previous SAR-aware analyses [3], [5], [6], [11]–[13] only designed the SAR-aware covariance matrix for a single coherence time block. However, regulatory standards allow SAR measurements to be averaged over a relatively long time period before being compared to the enforced limitation for compliance. For example, the Federal Communications Commission (FCC) allows a time period of up to 6 minutes [14]. If the coherence time interval is 100 ms, then there are 3600 coherence time blocks within a maximum test time. Therefore, it is possible and desirable to vary the exposure level per coherence time block to exploit the time diversity available in fading channels. Previous analyses also only focus on designing capacity achieving transmit covariance matrices with SAR limitations for a narrow-band system. With an orthogonal frequency-division multiplexing (OFDM) system, a SAR-aware optimization is required for every subcarrier, and it may be beneficial to consider a joint SAR-aware optimization over multiple subcarriers.

In this paper, we address the problem of dynamically allocating SAR limits over time/frequency to exploit the time/frequency diversity available in MIMO channels. We follow the framework used in [5] to model the SAR value as a quadratic function of the transmitted signal. Much like previous research [15]–[17] focused on optimizing power control subject to a long-term power constraint, we distribute exposure levels over time to maximize the achievable sum rate over multiple coherence time blocks. The proposed SAR allocation method can also be applied in a multi-user broadband OFDM system experiencing frequency-selective fading. Similar to power control on individual subcarriers, we perform SAR management to dynamically share the total user exposure limitation among subcarriers to improve the sum rate. In this work we introduce a metric, named the SAR-to-noise ratio, and show that it is important in characterizing single-user achievable sum rate.

Since SAR is a time-averaged measure of exposure [14], past systems such as the Global System for Mobile Communications (GSM) have used a simple on-off time-slotted technique to reduce SAR. In this time-slotted approach, a device transmits with a large power intermittently and transmits nothing otherwise. We show how a simple on-off method can be extended to encompass multiple channel coherence blocks in time and achieve good, but suboptimal, sum rate performance in a low SAR-to-noise regime. We apply dynamic programming (DP) algorithms to develop an exposure minimizing on-off method. Dynamic programming [18], [19] has been used in past research to develop optimal online policies for resource allocation and scheduling problems in wireless systems. It is also shown that the proposed on-off method converges to the optimal solution as the SAR-to-noise ratio approaches zero.

For medium-to-high SAR-to-noise ratio scenarios, i.e., when the SAR limit is larger than the noise power, we demonstrate that a uniformly distributed SAR limitation over time has close sum rate performance compared to the optimal SAR distribution strategy with non-causal channel state information (CSI). Furthermore, the uniformly distributed exposure allocation gives the highest sum rate among all blind allocation schemes, due to the concavity of the sum rate objective function. In this paper, we also propose an asymptotic waterfilling algorithm to reduce complexity when optimizing for a large number of coherent time blocks. Simulation results show that our asymptotic waterfilling method achieves a higher sum rate than a uniform SAR allocation.

The remainder of the paper is organized as follows. In Section II, we introduce the quadratic SAR model and the notion of SAR-to-noise ratio. In Section III, we derive the optimal exposure allocation solution with multiple channel uses assuming perfect knowledge of CSI. We reveal that the optimal solution is a modified waterfilling result. In Section IV-A, we show that a uniform exposure allocation is the optimal blind SAR allocation algorithm. In Section IV-B, we propose an asymptotic waterfilling algorithm. A DP-based on-off transmission strategy optimized for a low SAR-to-noise ratio regime is proposed in Section IV-C. In Section V, we present simulation results for the proposed SAR allocation methods. In Section VI, we summarize our work on dynamic exposure allocation.

Notation: In the rest of this paper, all boldface letters indicate matrices (upper case) or vectors (lower case). The notations $\mathrm{tr}(\mathbf{A})$, $|\mathbf{A}|$, $\mathbf{A}^H$, and $\mathrm{rank}\{\mathbf{A}\}$ are the trace, determinant, conjugate transpose, and rank of matrix $\mathbf{A}$, respectively. $\mathbf{A} = \mathrm{diag}\{\mathbf{a}\}$ denotes the diagonal matrix $\mathbf{A}$ with diagonal entries specified by the vector $\mathbf{a}$.

II. System Model and Problem Formulation

In this paper, we consider a point-to-point multiple antenna transmission over T Rayleigh fading channels in time domain. We assume there are $M > 1$ transmit antennas on the portable device and $N \geq 1$ receive antennas at the receiver, e.g., a base station or an associated side-link device. We assume that the input-output relationship of the i-th coherent time block is described by

$$\mathbf{y}_i[k] = \mathbf{H}_i\mathbf{x}_i[k] + \mathbf{n}_i[k],\ \ k = 1, 2, \ldots, K_i, \tag{1}$$

for $i = 1, 2, \ldots, T$, where k is the channel use index. $\mathbf{x}_i[k] \in \mathbb{C}^{M\times 1}$ is the transmit signal in the i-th channel with zero mean and covariance matrix $\mathbb{E}\{\mathbf{x}_i[k]\mathbf{x}_i[k]^H\} = \mathbf{S}_i$. $\mathbf{y}_i[k] \in \mathbb{C}^{N\times 1}$

is the receive signal. The channel matrix $\mathbf{H}_i \in \mathbb{C}^{N\times M}$ is assumed to be constant within K_i channel uses, and coherent channels are assumed to be independent across coherent bands or blocks. We assume that the entries of all channel matrices $\mathbf{H}_i$, $i = 1, 2, \ldots, T$, follow the same distribution of $\mathcal{CN}(0, 1)$. The additive white Gaussian noise vectors $\mathbf{n}_i[k] \in \mathbb{C}^{N\times 1}$, where $k = 1, 2, \ldots, K_i$ and $i = 1, 2, \ldots, T$, are i.i.d. across channel uses, and each noise vector follows the distribution $\mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$. The noise power σ^2 at the receiver is assumed to be a constant.

As we discussed, regulatory agencies require SAR testing and compliance on portable devices sold in most countries. We consider the scenario shown in Fig. 1, where a portable device operates near the body and the user is exposed to electromagnetic radiation. The SAR induced by the device in this operating condition is measured and averaged over volumes of tissue, typically 1 gram or 10 grams. In order to formulate a constraint on the maximum allowable SAR in terms of the transmit covariance, we adopt the quadratic SAR model introduced in [3], [5]. Experimental results in these studies demonstrate that the time-averaged SAR value in a volume V in the human body with one channel use can be modeled as the expectation of a quadratic function of the transmit signal as

$$\mathrm{SAR}_V = \mathbb{E}\{\mathbf{x}[k]^H \mathbf{R}_V \mathbf{x}[k]\} = \mathrm{tr}(\mathbf{R}_V \mathbf{S}), \tag{2}$$

where $\mathbf{S} = \mathbb{E}\{\mathbf{x}[k]\mathbf{x}[k]^H\}$ is the transmit covariance matrix and $\mathbf{R}_V$ is the SAR matrix which fully describes the SAR value's dependence on the transmit signal at the volume V. In this paper, we focus on the optimization of the transmit covariance matrices, and we omit the channel use index k in the rest of the paper.

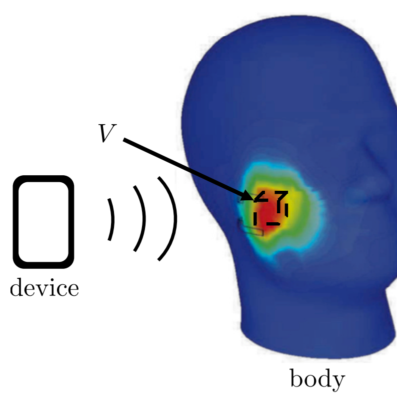

Fig. 1: Schematic of the considered exposure scenario in which a portable device operates in close proximity to the body and exposes the user to electromagnetic radiation. The SAR model in [3]–[5] is adopted to calculate SAR in a volume V inside the body as a function of the transmit signal.

A SAR-aware transmission incorporates the SAR constraint into transmit signaling design. For a given relative position between a device and the testing area of the body, the SAR constraint is captured by limiting the maximum SAR measurement over multiple volumes, yielding the maximum channel capacity analysis problem under both SAR and transmit power constraints as

$$\begin{aligned}
\text{(PB)} \quad & \max_{\mathbf{S}\succeq\mathbf{0}} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}\mathbf{H}^H\right| \\
& s.t.\ \mathrm{tr}(\mathbf{S}) \le P, \\
& \qquad \max_{j=1,2,\ldots,J}\{\mathrm{tr}(\mathbf{R}_{V_j}\mathbf{S})\} \le Q,
\end{aligned}$$

where P is the transmit power constraint, $V_1, V_2, \ldots, V_J$ are the testing volumes, and Q is the SAR threshold. Various SAR limitations are set for different body areas, such as whole body, partial body, hands, wrists, feet, and ankles, etc., therefore, multiple SAR matrices are often required to characterize all constraints imposed on a device. We note that the analysis performed in this paper only depends on the quadratic structure of the SAR constraints and not on the values of the SAR matrices nor on the specific exposure scenario. Therefore, we omit the dependence of the SAR matrix on the testing volume V_j and denote the j-th SAR matrix as $\mathbf{R}_j$. The SAR-aware transmit covariance matrix maximizes the rate of a SAR-constrained wireless channel, and it can be found as the optimal solution of the optimization problem

$$\begin{aligned}
\text{(PC)} \quad & \max_{\mathbf{S}\succeq\mathbf{0}} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}\mathbf{H}^H\right| \\
& s.t.\ \mathrm{tr}(\mathbf{S}) \le P, \\
& \qquad \mathrm{tr}(\mathbf{R}_j\mathbf{S}) \le Q_j, j = 1, 2, \ldots, G.
\end{aligned}$$

We consider a transmit power constraint P and G SAR constraints $\{Q_j\}_{j=1,2,\ldots,G}$ on a portable device. In [6], results show that the above problem can be solved with a modified waterfilling process.

In [6], it is shown that the effect of each quadratic SAR constraint in the optimization problem can be modeled as an effective channel correlation at the transmitter. Therefore, the tightness of the SAR limitation is dependent not only on the constraint value Q_j, but also on the norm of $\mathbf{R}_j$. For example, a portable device with a SAR matrix of $10\mathbf{R}$ faces a much tighter SAR constraint than another portable device with a SAR matrix of $\mathbf{R}$ assuming both have the same SAR limit Q. Hence, we define the normalized SAR constraint as

$$\mathrm{tr}(\tilde{\mathbf{R}}_j\mathbf{S}) \le \tilde{Q}_j = \frac{M}{\mathrm{tr}(\mathbf{R}_j)} Q_j, \tag{3}$$

where $\tilde{\mathbf{R}}_j = \frac{M}{\mathrm{tr}(\mathbf{R}_j)}\mathbf{R_j}$ is the normalized SAR matrix with $\mathrm{tr}(\tilde{\mathbf{R}}_j) = M$. The transmit power constraint can be regarded as a special normalized SAR constraint. We also introduce the normalized SAR-to-noise ratio (NSarNR) as

$$\mathrm{NSarNR}_j = \frac{\tilde{Q}_j}{\sigma^2} = \frac{MQ_j}{\sigma^2\,\mathrm{tr}(\mathbf{R}_j)} \tag{4}$$

to characterize the tightness of a SAR constraint on a device with the SAR matrix $\mathbf{R}_j$. We discuss how the low, medium, and high NSarNR regimes correspond to the relative performance of the proposed transmission schemes in Section V. The NSarNR is a generalization of the power-to-noise ratio if $\tilde{\mathbf{R}}_j = \mathbf{I}$, however, the normalized SAR matrix $\tilde{\mathbf{R}}_j$ is not an identity matrix in general [3], [5].

In the next section, we first address the joint SAR-aware optimization with perfect CSI, and we demonstrate that the

SAR-aware transmission can achieve much higher sum rate compared to the conventional power optimization method without SAR consideration. The proposed optimal SAR allocation method can be used in user exposure management over subcarriers in an OFDM system. It also provides an upper bound on the performance of the SAR allocation methods with only causal CSI. In Section IV, we then focus on the dynamic SAR allocation over coherence time blocks with causal CSI.

III. Exposure Allocation with Perfect CSI

The main idea of exposure allocation is to design covariance matrices $\{\mathbf{S}_i\}_{i=1,2,\ldots,T}$ which maximize the sum rate while complying with total power and total SAR constraints over the T channels, rather than power and SAR constraints placed on each time block. By considering the former constraints, we are able to allocate larger portions of the power and SAR budgets when the channel is "good" and smaller portions when the channel is "bad." As previously mentioned, constraints placed over the T channels are realistic because SAR readings are time-averaged when determining device compliance. In this section, we formulate and study the exposure allocation problem when the system has perfect CSI. The device may or may not have access to the SAR matrices required to formulate the SAR constraint, so we address both of these scenarios.

If a device has no knowledge of the SAR matrices, it needs to first perform transmit power control over T channels by solving the problem

$$\begin{aligned}(\mathrm{PF}-1)\quad &\max_{\mathbf{S}_i\succeq\mathbf{0}} \sum_{i=1}^{T}\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right| \\ &s.t. \ \sum_{i=1}^{T}\mathrm{tr}(\mathbf{S}_i)\le PT,\end{aligned}$$

where P is the average power constraint over T channels, and it is assumed all channel matrices $\{\mathbf{H}_i\}_{i=1,2,\ldots,T}$ are perfectly known at the transmit device. Using the transmit covariance matrices $\{\mathbf{S}_i\}$, the transmit power allocation on each band is then

$$P_i=\mathrm{tr}(\mathbf{S}_i), i=1,2,\ldots,T. \tag{5}$$

Because the device does not have access to the SAR matrices, it has to satisfy the worst-case compliance limitation, i.e.,

$$\begin{aligned}\sum_{i=1}^{T}\max_{\mathrm{tr}(\mathbf{S}_i)=P_i}\mathrm{tr}(\mathbf{R}_j\mathbf{S}_i)&=\sum_{i=1}^{T}r_{j,1}\,\mathrm{tr}(\mathbf{S}_i)\\ &\le Q_jT, j=1,2,\ldots,G,\end{aligned} \tag{6}$$

where Q_j is the average SAR constraint for T channels and $r_{j,1}$ is the maximum eigenvalue of the SAR matrix $\mathbf{R}_j$. If the SAR constraint is not met, the transmit power must be reduced. Hence, the SAR constraints are equivalently power constraints dictating that

$$\sum_{i=1}^{T}\mathrm{tr}(\mathbf{S}_i)\le\frac{Q_jT}{r_{j,1}}. \tag{7}$$

Therefore, the resulting transmit covariance matrices after power back-off are the solutions for the problem

$$\begin{aligned}(\mathrm{PF}-2)\quad &\max_{\mathbf{S}_i\succeq\mathbf{0}} \sum_{i=1}^{T}\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right| \\ &s.t. \ \sum_{i=1}^{T}\mathrm{tr}(\mathbf{S}_i)\le\min\left(PT,\frac{Q_1T}{r_{1,1}},\ldots,\frac{Q_GT}{r_{G,1}}\right).\end{aligned}$$

The optimal solution for (PF-2) is a typical waterfilling result over all T channels. Suppose the i-th channel matrix, $\mathbf{H}_i$, has a singular value decomposition (SVD) given by

$$\mathbf{H}_i=\mathbf{U}_i\mathbf{\Gamma}_i^{1/2}\mathbf{V}_i^H,\ i=1,2,\ldots,T, \tag{8}$$

where $\mathbf{\Gamma}_i=\mathrm{diag}\{g_{i,1},g_{i,2},\ldots,g_{i,m}\}$ with $g_{t,1}\ge g_{t,2}\ge\ldots\ge g_{t,M}\ge0$ and $m=\min(M,N)$. The optimal transmit covariance matrices are obtained by waterfilling as

$$\mathbf{S}_i=\mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^H,\ i=1,2,\ldots,T, \tag{9}$$

where $\mathbf{\Lambda}_i=\mathrm{diag}\{p_{i,1},p_{i,2},\ldots,p_{i,m}\}$ with $p_{i,j}=\left(1/\lambda-\sigma^2/g_{i,j}\right)^+$ for all $i=1,2,\ldots,T$. The quantity $1/\lambda$ is the global water-level satisfying

$$\begin{aligned}&\sum_{i=1}^{T}\sum_{j=1}^{m}\left(\frac{1}{\lambda}-\frac{\sigma^2}{g_{i,j}}\right)^+\\ &\quad=\min\left(PT,\frac{Q_1T}{r_{1,1}},\ldots,\frac{Q_GT}{r_{G,1}}\right).\end{aligned} \tag{10}$$

If the transmit device has SAR information, it can perform the SAR-aware optimization over channels with SAR constraints, i.e.,

$$\begin{aligned}(\mathrm{PF}-3)\quad &\max_{\mathbf{S}_i\succeq\mathbf{0}} \sum_{i=1}^{T}\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right| \\ &s.t. \ \sum_{i=1}^{T}\mathrm{tr}(\mathbf{S}_i)\le PT,\\ &\qquad \sum_{i=1}^{T}\mathrm{tr}(\mathbf{R}_j\mathbf{S}_i)\le Q_jT,\ j=1,2,\ldots,G.\end{aligned}$$

The optimal solution for $T=1$ is presented in [6]. However, the optimal transmit covariance matrices in the case $T>1$ can be found in a similar manner. To provide a closed-form solution for the transmit covariance matrix, define

$$\mathbf{K}=\mu\mathbf{I}+\sum_{j=1}^{G}\lambda_j\mathbf{R_j}. \tag{11}$$

where $\lambda_j, j=1,2,\ldots,G$, and μ are optimal dual variables for (PF-3). Assume that matrix $\mathbf{H}_i\mathbf{K}^{-1/2}$, referred to as an *effective channel* for the i-th channel, has an SVD

$$\mathbf{H}_i\mathbf{K}^{-1/2}=\mathbf{U}_i\mathbf{\Gamma}_i^{1/2}\mathbf{V}_i^H, \tag{12}$$

where $\mathbf{\Gamma}_i=\mathrm{diag}\{g_{i,1},g_{i,2},\ldots,g_{i,m}\}$ with $g_{i,1}\ge g_{i,2}\ge\ldots\ge g_{i,m}\ge0$. The optimal transmit covariance matrix $\mathbf{S}_i$ is formulated as

$$\mathbf{S}_i=\mathbf{K}^{-1/2}\mathbf{V}_i\mathbf{\Lambda}_i\mathbf{V}_i^H\mathbf{K}^{-1/2}, \tag{13}$$

where $\mathbf{\Lambda}_i = \mathrm{diag}\{p_{i,1}, p_{i,2}, \ldots, p_{i,m}\}$ with $p_{i,j} = \left(1 - \sigma^2/g_{i,j}\right)^+$ for $i = 1, 2, \ldots, T$ and $j = 1, 2, \ldots, m$. The optimization problem (PF-3) is convex, so it can be effectively solved using optimization toolboxes [20], [21]. The proposed optimal SAR allocation method with perfect CSI can also be applied for SAR management over subcarriers in OFDM systems by modifying the algorithm to distribute SAR limitations over subcarriers instead of coherence time blocks. Compared to individually optimizing each subcarrier, the proposed joint optimization over subcarriers provides better performance and also reduces complexity in OFDM systems.

IV. Exposure Allocation with Causal CSI

Assumption of perfect CSI implies non-causal CSI for SAR allocation over time. Therefore, a realistic online dynamic SAR allocation method which only relies on the past and current channel matrices $\{\mathbf{H}_j\}_{j \le i}$ at the i-th coherence time block is required. Since we assume that there is no temporal correlation among the channel matrices of different blocks, only the current channel $\mathbf{H}_i$ is relevant to the decision making at the i-th time block. The allocation method must also track the power and SAR budget in the previous $i-1$ time blocks to find the optimal allocation for the i-th time block. Consequently, a practical dynamic SAR allocation policy ϕ is defined as a mapping from the current CSI $\mathbf{H}_i$, the past covariance matrices $\{\mathbf{S}_j\}_{j<i}$, and the collection of constraint values $\mathbf{q}$ to the transmit covariance matrix $\mathbf{S}_i$, i.e.,

$$\phi : (\mathbf{H}_i, \{\mathbf{S}_j\}_{j<i}, \mathbf{q}) \mapsto \mathbf{S}_i,\ i = 1, 2, \ldots, T, \tag{14}$$

where $\mathbf{q} = [Q_0, Q_1, \ldots, Q_G]$. We denote $Q_0 = P$.

If the device does not know the SAR matrices, similar to problem (PF-2), the transmitted signal must be designed to simultaneously satisfy all exposure requirements, meaning that the results from mapping ϕ should satisfy

$$\sum_{i=1}^{T} \mathrm{tr}(\mathbf{S}_i) \le \min\left(PT, \frac{Q_1 T}{r_{1,1}}, \ldots, \frac{Q_G T}{r_{G,1}}\right). \tag{15}$$

A practical allocation policy without SAR information is said to be optimal if it maximizes the expected sum rate of all T time blocks, i.e., ϕ is the optimal solution among all possible mappings for the problem

$$\begin{aligned}(\mathrm{PT}-1) \quad & \max_{\phi}\ \mathbb{E}_{\{\mathbf{H}_1,\ldots,\mathbf{H}_T\}}\left\{\sum_{i=1}^{T} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right|\right\} \\ & s.t. \sum_{i=1}^{T} \mathrm{tr}(\mathbf{S}_i) \le \min\left(PT, \frac{Q_1 T}{r_{1,1}}, \ldots, \frac{Q_G T}{r_{G,1}}\right), \\ & \mathbf{S}_i = \phi(\mathbf{H}_i, \{\mathbf{S}_j\}_{j<i}, \mathbf{q}),\ i = 1, 2, \ldots, T.\end{aligned}$$

Similarly, an optimal online allocation policy with access to the SAR matrices can be found by solving

$$\begin{aligned}(\mathrm{PT}-2) \quad & \max_{\phi}\ \mathbb{E}_{\{\mathbf{H}_1,\ldots,\mathbf{H}_T\}}\left\{\sum_{i=1}^{T} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right|\right\} \\ & s.t. \sum_{i=1}^{T} \mathrm{tr}(\mathbf{S}_i) \le PT, \\ & \sum_{i=1}^{T} \mathrm{tr}(\mathbf{R}_j\mathbf{S}_i) \le Q_j T, j = 1, 2, \ldots, G, \\ & \mathbf{S}_i = \phi(\mathbf{H}_i, \{\mathbf{S}_j\}_{j<i}, \mathbf{q}),\ i = 1, 2, \ldots, T.\end{aligned}$$

Optimal online policies for both (PT-1) and (PT-2) are difficult to find. The expectation in (PT-1) and (PT-2)'s objective functions cannot be decomposed because the sum power and SAR constraints make the rate function of each time block correlated with others. However, the sum-rate performance of the optimal policy can be upper bounded by the sum rate of the optimal off-line solution with non-causal CSI and lower bounded by the sum rate from uniform SAR allocation. Our simulation results show that the upper and lower bound for dynamic SAR allocation over time are close to each other for medium-to-high NSarNR regime.

In the following subsections, we consider three exposure allocation strategies for systems with causal CSI. The uniform allocation algorithm serves as a close lower bound to the optimal solution at medium-to-high NSarNR. It simply distributes the SAR limitation evenly across time, and solves the SAR-constrained MIMO transmission problem for each time slot. The asymptotic waterfilling algorithm reduces the computational complexity by taking a large portion of the optimization offline. It also provides better performance in low-to-medium NSarNR region than uniform allocation. The on-off SAR allocation has the lowest complexity, however, it serves as a good approximation only in very low NSarNR region.

A. *Optimal Blind Algorithm: Uniform Allocation*

We first consider a blind allocation policy in which the amount exposure allowed per coherence time block is determined without CSI. A blind SAR allocation algorithm first assigns instantaneous SAR constraint values $\{Q_{i,j}\}_{j=0,1,\ldots,G}$ to time block i without considering the current channel matrix $\mathbf{H}_i$, then it finds the optimal transmit covariance matrices in each time block based on $Q_{i,j}$'s. Therefore, a blind mapping ϕ is a cascade of two mappings: $\phi(\mathbf{H}_i, \{\mathbf{S}_j\}_{j<i}, \mathbf{q}) = \phi_2(\mathbf{H}_i, \phi_1(\{\mathbf{S}_j\}_{j<i}, \mathbf{q}))$ with

$$\phi_1 : (\{\mathbf{S}_j\}_{j<i}, \mathbf{q}) \mapsto \mathbf{q}_i,\ i = 1, 2, \ldots, T, \tag{16}$$

where $q_i = [P_i, Q_{i,1}, \ldots, Q_{i,G}]$ are the collection of instantaneous constraint values and

$$\phi_2 : (\mathbf{H}_i, \mathbf{q}_i) \mapsto \mathbf{S}_i. \tag{17}$$

The optimal ϕ_2 with and without SAR information is already analysed and presented in [6]. Therefore, optimizing over a blind allocation ϕ is equivalent to optimizing over ϕ_1, which allocates the SAR budget for each time block.

With a blind allocation method, the objective functions in (PT-1) and (PT-2) are greatly simplified by decomposing the expectation as

$$\begin{aligned}(\mathrm{PT-b}) \quad & \max_{\phi_1} \sum_{i=1}^{T} \mathbb{E}_{\mathbf{H}_i}\left\{\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{H}_i\mathbf{S}_i\mathbf{H}_i^H\right|\right\} \\ & s.t.\ \mathbf{q}_i=\phi_1(i,\mathbf{q}),\ i=1,2,\ldots,T, \\ & \qquad \sum_{i=1}^{T} q_{i,j}\le Q_j,\ j=0,1,\ldots,G, \\ & \qquad \mathbf{S}_i=\phi_2(\mathbf{H}_i,\mathbf{q}_i),\ i=1,2,\ldots,T.\end{aligned}$$

We show that among all blind allocation algorithms ϕ_1, the uniform exposure allocation over time is optimal with or without SAR information due to the concavity of the sum-rate function. Under uniform exposure allocation, the limitation values are given by $Q_{i,j} = Q_j$ for $i = 1,2,\ldots,T$ and $j = 0,1,\ldots,G$. By definition, we have

$$\mathbf{S}=\phi_2(\mathbf{H},\mathbf{q})=\max_{\substack{\mathbf{S}\succeq\mathbf{0}\\ \mathrm{tr}(\mathbf{R}_j\mathbf{S})\le Q_j\ \forall j}} \log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HSH}^H\right| \tag{18}$$

with or without the SAR-awareness at portable devices.

Denote

$$\begin{aligned}B(\mathbf{q}) &= \mathbb{E}_{\mathbf{H}}\left\{\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HSH}^H\right|_{\mathbf{S}=\phi_2(\mathbf{H},\mathbf{q})}\right\} \\ &= \mathbb{E}_{\mathbf{H}}\left\{\max_{\substack{\mathbf{S}\succeq\mathbf{0}\\ \mathrm{tr}(\mathbf{R}_j\mathbf{S})\le Q_j\ \forall j}} \log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HSH}^H\right|\right\}. \end{aligned}\tag{19}$$

Function $B(\mathbf{q})$ is a concave function of the constraint variables P and Q_j, $j = 0,1,\ldots,G$. The proof can be found in the Appendix. In addition, the following lemma gives an upper bound for the objective function of a blind allocation.

Lemma 4.1. *For a blind allocation problem, the objective function $B(\mathbf{q})$ is bounded above by*

$$\frac{1}{T}\sum_{i=1}^{T} B(\mathbf{q}_i) \le B\left(\frac{1}{T}\sum_{i=1}^{T}\mathbf{q}_i\right). \tag{20}$$

Moreover, the upper bound is achieved when $P_i = P$ and $Q_{i,j} = Q_j$ for all $i = 1,2,\ldots,T$ and $j = 0,1,\ldots,G$.

As a result of Lemma 4.1, the uniform exposure allocation over coherence time blocks is the optimal blind allocation policy. The uniform allocation is a temporal counterpart of the spatial isotropic transmission for MIMO systems [22]. It is well-known that if no CSI at the transmitter (CSIT) is available, the optimal transmit covariance matrix is a scaled identity matrix, which evenly divides the radiation power in all orthogonal directions. Hence, if the temporal SAR allocation does not consider CSI at each time block, the uniform limitation distribution over time should be optimal. Our simulations show that uniform allocation gives almost the same performance as the optimal allocation in a medium-to-high NSarNR regime.

With uniform SAR and power allocation, the rate performance difference with and without knowledge of the SAR matrices comes from the ϕ_2 mapping. From [6], the SAR-aware transmission gives considerable rate increase over the conventional method without SAR knowledge. The uniform SAR allocation method has a simple decision on placing power or SAR limitations on each time block, but it still requires finding the optimal SAR-aware transmit covariance matrix for every block based on the allocated limitation values. In the next section, we introduce a SAR allocation method based on asymptotic analysis by solving the allocation problem with $T \to \infty$. The benefit of the proposed asymptotic allocation is that a large portion of optimization can be taken offline. Furthermore, our simulation results demonstrate that it has the same sum-rate performance as the optimal allocation method in the medium-to-high NSarNR range, and it outperforms uniform allocation in the low NSarNR regime. Moreover, the asymptotic method also works well with a small number of coherence time blocks.

B. Asymptotic waterfilling algorithm

The optimal allocation result of (PF-2) is a form of waterfilling. Hence, if we know the optimal dual variables μ and $\{\lambda_j\}_{j=1,2,\ldots,G}$, then the optimal covariance matrices $\{\mathbf{S}_i\}_{i=1,2,\ldots,T}$ are easily found. The optimal dual variables, however, are dependent not only on the power and SAR constraint values, but also on the channel matrices $\{\mathbf{H}_i\}_{i=1,2,\ldots,T}$. In this section, we show that the optimal dual variables converge to their asymptotic approximations as $T \to \infty$.

As $T \to \infty$, the allocation policy ϕ should only depend on the current CSI $\mathbf{H}_i$, so we drop the dependence on the time index i. Since there are infinitely many channel realizations, the sum rate maximization problem under sum-power and sum-SAR constraints in (PF-3) should be rewritten as a maximization of the expected rate under expected power and SAR constraints, i.e.,

$$\begin{aligned}(\mathrm{PA}) \quad & \max_{\phi} \mathbb{E}_{\mathbf{H}}\left\{\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HSH}^H\right|\right\} \\ & s.t.\ \mathbb{E}_{\mathbf{H}}\{\mathrm{tr}(\mathbf{S})\}\le P, \\ & \qquad \mathbb{E}_{\mathbf{H}}\{\mathrm{tr}(\mathbf{R}_j\mathbf{S})\}\le Q_j,\ j=1,2,\ldots,G, \\ & \qquad \mathbf{S}=\phi(\mathbf{H}).\end{aligned}$$

The Lagrangian of (PA) is formulated as

$$\begin{aligned}L(\mathbf{S},\boldsymbol{\lambda},\mu) = \mathbb{E}_{\mathbf{H}}\Bigg\{&\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HSH}^H\right| - \mu(\mathrm{tr}(\mathbf{S})-P) \\ &- \sum_{j=1}^{G}\lambda_j(\mathrm{tr}(\mathbf{R}_j\mathbf{S})-Q_j)\Bigg\}. \end{aligned}\tag{21}$$

The optimal dual variable of (PA) is the solution of the dual problem, expressed as

$$(PA-D) \quad \min_{\substack{\lambda_i\ge 0\\ \mu\ge 0}} \max_{\mathbf{S}=\phi(\mathbf{H})} L(\mathbf{S},\boldsymbol{\lambda},\mu). \tag{22}$$

Given any positive dual variables $\{\lambda_j\}_{j=1,2,\ldots,G}$, and μ, we can find the optimal transmit covariance matrix based on the channel matrix by waterfilling on the effective channel

$\mathbf{HK}^{-1/2}$ with $\mathbf{K} = \mu\mathbf{I} + \sum_{j=1}^{G} \lambda_j \mathbf{R}_j$. Suppose $\mathbf{HK}^{-1/2}$ has an SVD given by

$$\mathbf{HK}^{-1/2} = \mathbf{U}\mathbf{\Gamma}^{1/2}\mathbf{V}^H, \tag{23}$$

where $\mathbf{\Gamma} = \text{diag}\{g_1, g_2, \ldots, g_m\}$ with $g_1 \geq g_2 \geq \ldots \geq g_m \geq 0$. The transmit covariance matrix that maximizes the Lagrangian is given by

$$\mathbf{S} = \phi(\mathbf{H}) = \mathbf{K}^{-1/2}\mathbf{V}\mathbf{\Lambda}\mathbf{V}^H\mathbf{K}^{-1/2}, \tag{24}$$

where $\mathbf{\Lambda} = \text{diag}\{p_1, p_2, \ldots, p_m\}$ with $p_i = \left(1 - \sigma^2/g_i\right)^+$.

Therefore, the maximum of the Lagrangian is computed as

$$\max_{\mathbf{S}=\phi(\mathbf{H})} L(\mathbf{S}, \boldsymbol{\lambda}, \mu) = \left(\mu P + \sum_{j=1}^{G} \lambda_j Q_j\right) + \int_0^\infty \left(\log\left(\frac{x}{\sigma^2}\right)^+ - \left(1 - \frac{\sigma^2}{x}\right)^+\right) f_g(x)dx. \tag{25}$$

where $f_g(x)$ is the marginal PDF of the unordered singular values of $\mathbf{HK}^{-1/2}$ which is expressed as [23]

$$f_g(x) = \frac{|\mathbf{K}|^N}{M} \sum_{i=1}^{M}\sum_{j=1}^{M} \mathcal{D}(i,j) x^{N-M+j-1} e^{(-xk_{M-i+1})} \times \left(\prod_{\ell=1}^{M}(N-\ell)! \prod_{t<\ell}^{M}(k_{M-\ell+1} - k_{M-t+1})\right)^{-1} \tag{26}$$

where $k_1 \geq k_2 \geq \ldots \geq k_M$ are the eigenvalues of $\mathbf{K}$. $\mathcal{D}(i,j)$ is the (i,j)-th cofactor of the following matrix $\mathbf{D}$ with entries

$$\mathbf{D}_{\ell,t} = \frac{(N-M+t-1)!}{(k_{M-\ell+1})^{N-M+t}}. \tag{27}$$

Minimizing over the maximum Lagrangian gives the optimal dual variables, and the optimal transmit covariance matrix follows the mapping $\mathbf{S} = \phi(\mathbf{H})$.

When the asymptotic allocation is applied over a finite time block of length T, the remaining power and SAR budget must be carefully tracked to maintain compliance. Define the remaining budget value as $Q_{i,j}^r$ for the i-th time block and j-th SAR constraint, where $Q_{1,j}^r = Q_j T$ and $Q_{i+1,j}^r = Q_{i,j}^r - Q_{i,j}$. We denote $Q_{i,j}$ as the actual constraint value set for the i-th time block and j-th SAR constraint. At the i-th time block, if each of the resulting SAR allocation $Q'_{i,j} = \text{tr}(\mathbf{R}_j\mathbf{S})$ from (PA-D) is smaller than the remaining SAR budget $Q_{i,j}^r$, then we keep the allocation result $\mathbf{S}$ and $Q_{i,j} = Q'_{i,j}$. If any of the resulting $Q'_{i,j}$ from (PA-D) is larger than the remaining SAR budget $Q_{i,j}^r$, then we set the constraint limitations for the i-th time block as $Q_{i,j}^r$. The transmit covariance matrix is recalculated based on the new constraint values. Moreover, for the last coherence block T, we should always set $Q_{T,j} = Q_{T,j}^r$.

One special case is when there is only one SAR constraint as the active constraint on the portable device. In this case the optimal dual variable λ is given by the following lemma.

Lemma 4.2. *Given only one active SAR constraint Q, the optimal dual variable λ is the solution of the equation*

$$\int_0^\infty \left(\frac{1}{\lambda} - \frac{\sigma^2}{x}\right)^+ f_g(x)dx = Q. \tag{28}$$

The previous result is obtained by considering the water-filling solution of (27) in the case that $\mu = 0$. Although it requires a significant effort to find the optimal dual variables, these calculations can be kept offline. A table of optimal dual variables and constraint values can be pre-computed and stored in the device. In each coherence time block, the device only needs to find the optimal transmit covariance matrix based on the channel matrix. Our simulation results demonstrate that the asymptotic waterfilling algorithm can achieve good performance in exposure allocation even with small T, and it outperforms the uniform allocation in the low-SAR, low-power regime.

C. Low NSarNR with On-Off SAR allocation

Many future WBAN systems may operate commercially in the low NSarNR regime. For example, wireless earphones and VR/AR headsets have transceivers right next to the user's head. Moreover, devices that are commonly used by children may be designed (or regulated in the future) to expose the users to minimal radiation. Although no regulatory metric has been announced for the above-6 GHz bandwidth yet, millimeter-wave systems (mmWave) are expected to induce large peak SAR values [24], [25].

In [24], the peak SAR measurement of a mmWave system is shown to be much larger than those in microwave frequencies because the penetration depth at a high frequency band is very shallow. A peak SAR value of 22 W/kg is measured at 60 GHz, while the same system operating at 2 GHz would only give a peak SAR value of 0.4 W/kg. This means that there is a roughly 17dB (55 times higher) gain in the SAR measurement for a mmWave system over a sub-6 GHz system. As a result, if mmWave systems are regulated by SAR measurements, then these systems may face very tight SAR constraints. Moreover, outdoor mmWave channel measurements show that mmWave channels are highly directional with few propagation paths. Diverting the beam alignment for lower SAR may lead to disastrous loss in rate performance. It might be desirable to manage mmWave SAR readings by exploiting temporal diversity.

The approach considered in this paper can also be applied to mitigate exposure in mmWave systems. From [24], [26], [27], it can be shown that both power density and SAR constraints can be expressed in the form $\text{tr}(\mathbf{RS})$ for an appropriate exposure matrix $\mathbf{R}$. Assuming a far-field model, the exposure matrix for power density is given by

$$\mathbf{R}_{\mathsf{PD}} = \frac{\mathbf{a}_T\mathbf{a}_T^H}{4\pi D^2}, \tag{29}$$

where a_T denotes the transmitter array steering vector and D is the distance from the transmitter array to the point of contact of interest. Limited penetration of millimeter wave signals in human tissue yields a simple relationship between power density and SAR at the tissue surface, and it is simple to show that the exposure matrix R_{surf} for surface SAR is given by $R_{\mathsf{surf}} = kR_{\mathsf{PD}}$ for some scalar $k \in \mathbb{R}$. Relatively large exposure measurements in the mmWave induce tight constraints on portable devices, thus we focus on the low SAR-to-noise regime in this paper.

In a low NSarNR regime, the waterfilling results can be well approximated as "temporal beamforming," meaning the device should transmit with the maximum allowable exposure constraint values in a single coherence time block, e.g., an "on-off" SAR allocation method. The remaining question is when to turn on the transmission in the given time period. If the device does not know its SAR matrices, then it should pick the best channel with the largest beamforming gain. Suppose, $g_{i,1}$ is the largest singular value of the i-th channel $\mathbf{H}_i$, and $g_{i_0,1}$ is the largest among all $g_{i,1}$, i.e.,

$$g_{i_0,1} = \max_{0 \le i \le T} g_{i,1}. \tag{30}$$

Then the optimal allocation policy without SAR knowledge should transmit at the i_0-th time block with all available constraint value PT, and Q_jT, $j = 1, 2, \ldots, G$. The transmit covariance matrix is then formulated as

$$\mathbf{S} = \min\left(PT, \frac{Q_1T}{r_{1,1}}, \ldots, \frac{Q_GT}{r_{1,G}}\right) \mathbf{v}_1\mathbf{v}_1^H, \tag{31}$$

where $\mathbf{v}_1$ is the maximum right singular vector of the channel matrix $\mathbf{H}_{i_0}$.

If SAR matrices are also available, then a portable device can calculate the i-th effective channel matrix as $\mathbf{H}_i\mathbf{K}_i^{-1/2}$ for time slot i, $i = 1, 2, \ldots, T$. Since $\mathbf{K}_i = \mu_i\mathbf{I} + \sum_{j=1}^{G} \lambda_{i,j}\mathbf{R}_j$, where μ_i and $\lambda_{i,j}$ are the optimal dual variables given the channel matrix $\mathbf{H}_i$, we have that $\mathbf{K}_i$ is a function of the channel matrix $\mathbf{H}_i$. In the low NSarNR regime, the optimal covariance matrix obtained from waterfilling for time slot i is in fact a beamforming solution,

$$\mathbf{S}_i = \mathbf{K}_i^{-1/2}\mathbf{v}_{i,1}\left(1 - \frac{\sigma^2}{g_{i,1}}\right)\mathbf{v}_{i,1}^H\mathbf{K}_i^{-1/2}, \tag{32}$$

where $g_{i,1}$ is the largest singular value of the matrix $\mathbf{H}_i\mathbf{K}_i^{-1/2}$, and $\mathbf{v}_{i,1}$ is the corresponding dominate right singular vector. Assume $g_{i_0,1} = \max_i g_{i,1}$ is the largest beamforming gain among all effective channel, then the optimal SAR-aware "on-off" algorithm in the low NSarNR regime has its covariance matrix expressed as

$$\mathbf{S} = \mathbf{K}_{i_0}^{-1/2}\mathbf{v}_{i_0,1}\left(1 - \frac{\sigma^2}{g_{i_0,1}}\right)\mathbf{v}_{i_0,1}^H\mathbf{K}_{i_0}^{-1/2}. \tag{33}$$

The problem is to decide when the transmission should be "on" when the transmitter only has knowledge of the current CSI. In this paper, we use dynamic programming (DP) to optimize the decision on the best "on" time block to transmit in the low NSarNR regime. As in a typical DP problem, the payoff functions for each time block are formulated through backward induction and Bellman's equation [18]. For the last coherence block T, it is clear that the device should just transmit if there is any SAR budget left. Therefore the payoff function is expressed as

$$J_T(\mathbf{q}, \mathbf{H}) = s(\mathbf{q}, \mathbf{H}), \tag{34}$$

where $s(\mathbf{q}, \mathbf{H})$ is the maximum receive signal-to-noise ratio (SNR), given channel matrix $\mathbf{H}$ and constraint values $\mathbf{q}$. In the low NSarNR regime, the receive SNR is a better performance metric than the rate, since practical modulation and coding schemes usually require the receive SNR to be higher than certain a threshold. If a device has no SAR information, then

$$s(\mathbf{q}, \mathbf{H}) = \frac{g_1}{\sigma^2}\min\left(PT, \frac{Q_1T}{r_{1,1}}, \ldots, \frac{Q_jT}{r_{j,1}}\right), \tag{35}$$

while if the device has access to the SAR matrices, then

$$s(\mathbf{q}, \mathbf{H}) = \frac{g_1}{\sigma^2} - 1. \tag{36}$$

For the i-th block, the DP algorithm compares two possible decisions: transmit now or wait. If the device transmits in the current time block, the receive SNR is $s(\mathbf{q}, \mathbf{H})$. If the device waits and saves the transmission opportunity for future time blocks, the expected future payoff is $\mathbb{E}_{\mathbf{H}}\{J_{i+1}(\mathbf{q}, \mathbf{H})\}$. Obviously, the algorithm should choose the one with highest payoff, i.e., Bellman's equation is formulated as

$$J_i(\mathbf{q}, \mathbf{H}) = \max\{s(\mathbf{q}, \mathbf{H}), \mathbb{E}_{\mathbf{H}}\{J_{i+1}(\mathbf{q}, \mathbf{H})\}\}. \tag{37}$$

Note that the payoff function $J_i(\cdot)$ also depends on $\{\mathbf{R}_j\}_{j=1,2,\ldots,G}$. We hide the dependence on these variables, since they are assumed to be constant during all T time blocks.

We first find the DP solution for devices without information on the SAR matrices. For the last coherence time block T

$$\begin{aligned} J_T(\mathbf{q}, \mathbf{H}) &= s(\mathbf{q}, \mathbf{H}) \\ &= \frac{g_1}{\sigma^2}\min\left(PT, \frac{Q_1T}{r_{1,1}}, \ldots, \frac{Q_jT}{r_{j,1}}\right). \end{aligned} \tag{38}$$

For block $t = T - 1$, we have

$$\begin{aligned} J_{T-1}(\mathbf{q}, \mathbf{H}) &= \max\left\{s(\mathbf{q}, g_1), \mathbb{E}_{\mathbf{H}}\left\{J_T(\mathbf{q}, \mathbf{H})\right\}\right\} \\ &= \max\left\{\frac{g_1}{\sigma^2}\tilde{Q}, \frac{\mathbb{E}_{\mathbf{H}}\{g_1\}}{\sigma^2}\tilde{Q}\right\} \\ &= \begin{cases} \frac{\mathbb{E}_{\mathbf{H}}\{g_1\}\tilde{Q}}{\sigma^2}, & g_1 \le \mathbb{E}_{\mathbf{H}}\{g_1\} \\ \frac{g_1\tilde{Q}}{\sigma^2}, & g_1 > \mathbb{E}_{\mathbf{H}}\{g_1\}. \end{cases} \end{aligned} \tag{39}$$

where $\tilde{Q} = \min\left(PT, \frac{Q_1T}{r_{1,1}}, \ldots, \frac{Q_jT}{r_{j,1}}\right)$. Therefore, at time block $i = T-1$, if the current channel matrix has a maximum singular value g_1 larger than the expected largest singular value for time block T, given as $\mathbb{E}_{\mathbf{H}}\{g_1\} = \gamma_{T-1}$, then transmit in the current time block. Otherwise, the device should wait and transmit in the T-th time block. The resulting channel gain for the $T-1$-th time block in DP problem is

$$\tilde{g}_{T-1} = \begin{cases} \gamma_{T-1}, & g_1 \le \gamma_{T-1} \\ g_1, & g_1 > \gamma_{T-1}. \end{cases} \tag{40}$$

Let $\gamma_T = 0$, then the payoff function for the i-th block, $i = 1, 2, \ldots, T$, is

$$J_i(\mathbf{g}, \mathbf{H}) = \begin{cases} \frac{\tilde{Q}\gamma_i}{\sigma^2}, & g_1 \le \gamma_i \\ \frac{\tilde{Q}g_1}{\sigma^2}, & g_1 > \gamma_i \end{cases} \tag{41}$$

where the threshold $\gamma_i = \mathbb{E}_{\mathbf{H}}\{\tilde{g}_{i+1}\}$. Therefore, at the i-th time block, if the current channel gain g_1 is larger than the threshold γ_i, then the device should turn on and transmit.

Otherwise, it should stay silent. The effective channel gain is

$$\tilde{g}_i = \begin{cases} \gamma_i, & g_1 \leq \gamma_i \\ g_1, & g_1 > \gamma_i. \end{cases} \tag{42}$$

Furthermore, the thresholds can be calculated recursively as

$$\begin{aligned} \gamma_i &= \int_0^{\gamma_{i+1}} \gamma_{i+1} f_{g_{\max}}(x)dx + \int_{\gamma_{i+1}}^{\infty} x f_{g_{\max}}(x)dx \\ &= \gamma_{i+1} F_{g_{\max}}(\gamma_{i+1}) + \int_{\gamma_{i+1}}^{\infty} x f_{g_{\max}}(x)dx, \end{aligned} \tag{43}$$

where $f_{g_{\max}}(x)$ denotes the probability density function (PDF) of the maximum singular value of the channel matrix $\mathbf{H}$, and $F_{g_{\max}}(x)$ is the cumulative density function (CDF). If the entries of $\mathbf{H}$ are distributed as i.i.d. zero-mean Gaussian random variables $\mathcal{CN}(0,1)$, the maximum singular value has the CDF [28]

$$\begin{aligned} F_{g_{\max}}(g \leq x) = \frac{C\Gamma_M(M)}{C\Gamma_M(N+M)} \frac{x^{MN}}{M^n} {}_1 \\ \times F_1(N, N+M, -x\mathbf{I}), \end{aligned} \tag{44}$$

where ${}_1F_1$ is the hypergeometric function with a matrix input and $C\Gamma_m(n)$ is the complex multivariate Gamma function, given as

$$C\Gamma_m(n) = \pi^{m(m-1)/2} \prod_{k=1}^{m} \Gamma(n-k+1), \; n > m-1. \tag{45}$$

Clearly, the thresholds are in a decreasing order, $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_{T-1} \geq \gamma_T = 0$.

If the device has access to the SAR matrices, then DP "on-off" policy works in a very similar way. For the i-th time block, we have

$$\begin{aligned} J_i(\mathbf{q}, \mathbf{H}) &= \max \left\{ s(\mathbf{q}, g_1), \mathbb{E}_{\mathbf{H}} \left\{ J_T(\mathbf{q}, \mathbf{H}) \right\} \right\} \\ &= \max \left\{ \frac{g_1}{\sigma^2} - 1, \frac{\mathbb{E}_{\mathbf{H}}\{g_1\}}{\sigma^2} - 1 \right\} \\ &= \begin{cases} \frac{\mathbb{E}_{\mathbf{H}}\{g_1\} \tilde{Q}}{\sigma^2}, & g_1 \leq \gamma_i \\ \frac{g_1 \tilde{Q}}{\sigma^2}, & g_1 > \gamma_i. \end{cases} \end{aligned} \tag{46}$$

The device compares the current beamforming gain g_1 of the effective channel $\mathbf{H}_i \mathbf{K}_i^{1/2}$ with a threshold, and it only turns on and transmits if the gain is higher than the threshold. Thresholds are also recursively defined as in (43). However, unlike the case without SAR information, there is no closed form expression for the PDF and CDF of the largest singular values of the effective channel $\mathbf{H}\mathbf{K}^{1/2}$. Empirical CDF and PDF functions can be used to find proper threshold values for the SAR-aware "on-off" algorithm.

One special case is when there is only one active SAR constraint for the device. Then the DP algorithm compares the largest singular value of $\mathbf{H}\mathbf{R}^{-1/2}$ to the threshold, and in (43) the CDF is given by [28]

$$\begin{aligned} F_{g_{\max}}(g \leq x) = \frac{C\Gamma_M(M)}{C\Gamma_M(N+M)} \frac{x^{MN}}{|\mathbf{R}^{-1}|^n} {}_1 \\ \times F_1(N, N+M, -x\mathbf{R}). \end{aligned} \tag{47}$$

V. Simulation Results

In this section, we present Monte-Carlo simulation results for the proposed user exposure allocation algorithms. We assume that the device is constrained by a power constraint and by SAR limitations characterized by the matrices obtained from [6]

$$\mathbf{R}_1 = \begin{bmatrix} 8 & -6j \\ 6j & 8 \end{bmatrix}, \tag{48}$$

$$\mathbf{R}_2 = \begin{bmatrix} 3.94 & -2.65 - 2.53j \\ -2.65 + 2.53j & 4.57 \end{bmatrix}, \tag{49}$$

where both SAR matrices are subject to a single constraint Q. All simulations use $M = 2$ transmit antennas, $N = 4$ receive antennas, and i.i.d Rayleigh fading channel realizations.

Simulation results show that in most cases, uniform allocation and asymptotic allocation perform almost optimally, and there is almost no advantage to performing asymptotic allocation over uniform allocation. In this section, we showcase two practical settings in which the asymptotic allocation scheme performs significantly better than the uniform allocation scheme. In the first scenario, we assume that both the power constraint P and SAR constraint Q are small. This case models body-area devices which may have strict limits on power consumption and tight exposure limits due to their close proximity to users. In the second scenario, we assume the SAR constraint Q is small and the noise variance is large, which models devices in noisy environments with relatively small SAR thresholds. The final simulations demonstrate the effectiveness of the DP allocation method.

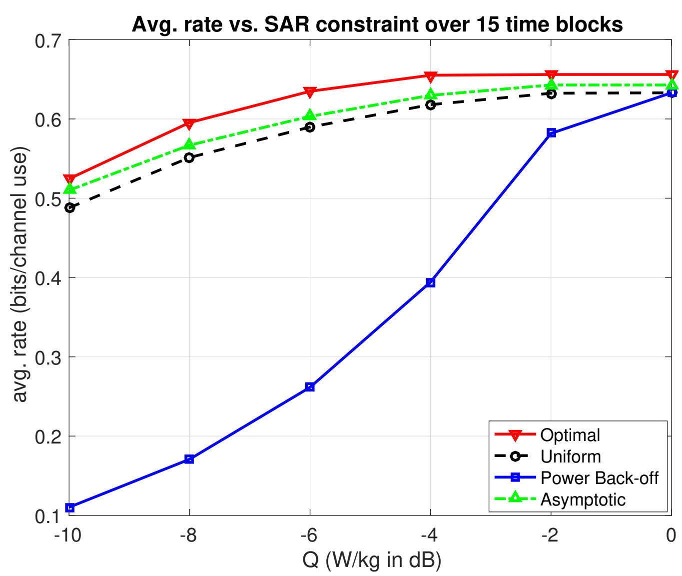

Fig. 2: Average rate comparison with $T = 15$ and $P = 0.05$ W. The SAR-aware allocation methods (uniform and asymptotic) have substantial performance improvement over the power back-off approach.

In Fig. 2, we demonstrate that the SAR information substantially helps in the exposure allocation decision, improving the average rate over 15 coherence time blocks. The power constraint and noise variance are fixed at $P = 0.05$ W and $\sigma^2 = 1$, respectively. Both uniform and asymptotic exposure allocation perform close to optimal performance and greatly

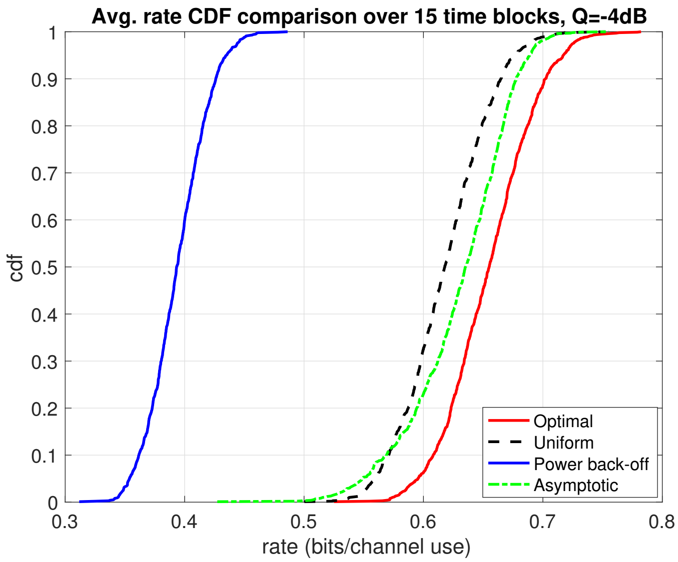

Fig. 3: Average rate CDF with $T = 15$, $P = 0.05$ W and $Q = -4$ dB. The asymptotic algorithm has higher 50-percentile rate and lower 10-percentile rate compared to the uniform allocation.

outperform the allocation algorithm without knowledge of the SAR matrices. For large values of Q, the average rate of all methods reaches a ceiling since the power constraint becomes the only active constraint. The asymptotic allocation achieves better performance than the uniform allocation due to the relatively small SAR and power budgets.

In Fig. 3, we plot the rate CDF with the SAR constraint $Q = -4$ dB, and power constraint $P = 0.05$ W over 15 time blocks. The asymptotic algorithm has a higher 95-percentile and 50-percentile rate than the uniform allocation, but it also exhibits a slightly lower 10-percentile rate. Therefore, in most cases, the asymptotic SAR allocation outperforms the uniform SAR allocation method.

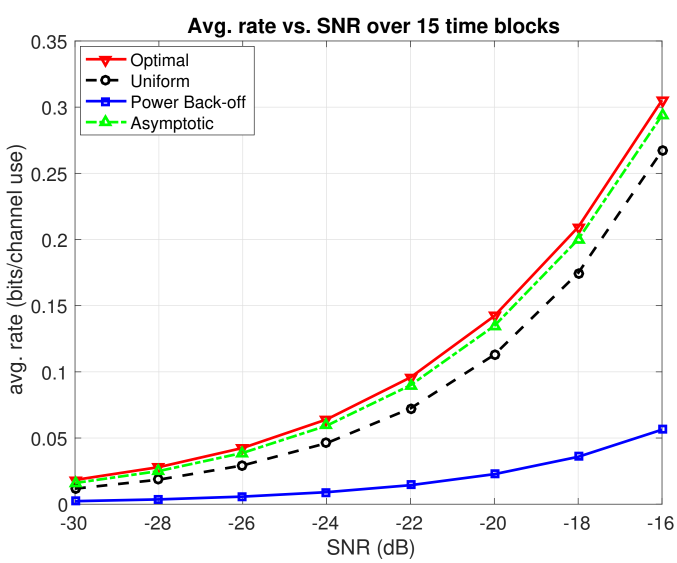

Fig. 4: Average rate comparison with $T = 15$, $P = 0.2$ W and $Q = -4$ dB. In the low SNR regime, the asymptotic allocation algorithm has close performance compared to the optimal solution.

In Fig. 4, the average rate of the proposed methods in the low SNR regime is plotted. It is common for a single SAR constraint to dominate system performance, so from here on we assume that the device has a single SAR limitation, which is characterized by $\mathbf{R}_1$. The power and SAR constraints are fixed at $P = 0.2$ W and $Q = -4$ dB, respectively. As above, the proposed SAR-aware methods show higher average rates than the back-off approach, with substantial gains as the transmit SNR increases. The asymptotic algorithm performs close to the optimal allocation.

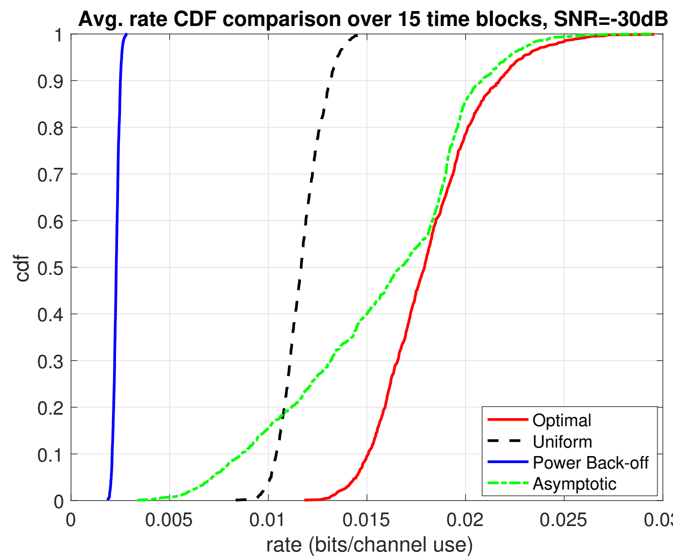

Fig. 5: Average rate CDF with $T = 15$, $P = 0.2$ W and $Q = -4$ dB. The asymptotic algorithm has significantly higher 50-percentile rate and 95-percentile rate compared to uniform allocation.

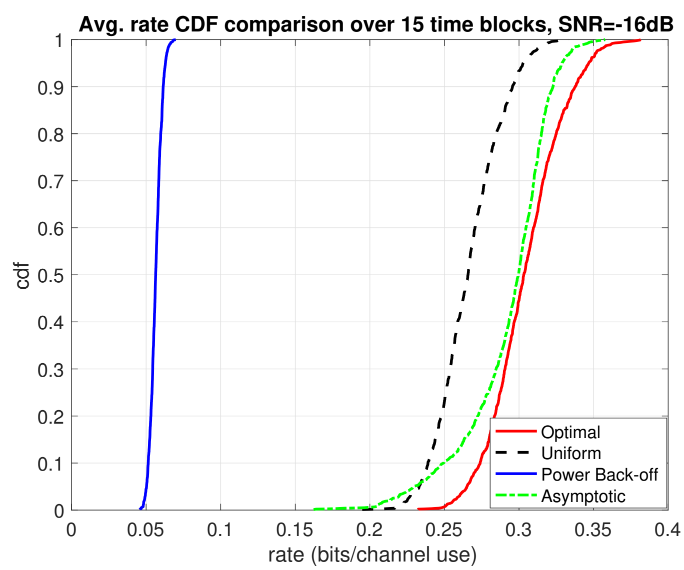

Fig. 6: Average rate CDF with $T = 15$, $P = 0.2$ W and $Q = -4$ dB. The asymptotic algorithm performs better than the uniform allocation in the 50-percentile and 95-percentile.

In Fig. 5, we present the rate CDF with SNR $= -30$ dB, SAR constraint $Q = -4$ dB, and power constraint $P = 0.2$ W over 15 time blocks. The asymptotic allocation approach performs worse than uniform allocation up to the 20-percentile, but also has a significantly higher 95-percentile and 50-percentile rate. In Fig. 6, we present the rate CDF with SNR $= -16$ dB and the same SAR and power constraints as in Fig. 5. Similarly to Fig. 3, the asymptotic algorithm outperforms uniform allocation in the 95-percentile and 50-percentile. These results demonstrate that while the advantage of the asymptotic allocation scheme is more pronounced at extremely low SNR, fewer users will reap the full benefits of asymptotic allocation in this regime compared to the moderately low SNR regime.

For the following two simulations, we assume that the device is limited by a single SAR constraint and has no power limitation since the SAR constraint governs performance in the low SAR constraint regime. In Fig. 7, we demonstrate the performance of the DP allocation method with and without access to the SAR matrices. We assume the random selection algorithm arbitrarily selects one out of 5 time blocks to transmit. The DP algorithm provides around 0.8dB gain over random selection in terms of receive SNR. The receive SNR of SAR-aware method is almost 7 times higher than the SNR without SAR knowledge. In Fig. 8, the receive SNR CDF with $Q = -10$ dB is presented. The DP algorithm has 0.8 dB higher in 50-percentile SNR than the random selection. The benefits of applying a SAR-aware solution are clear.

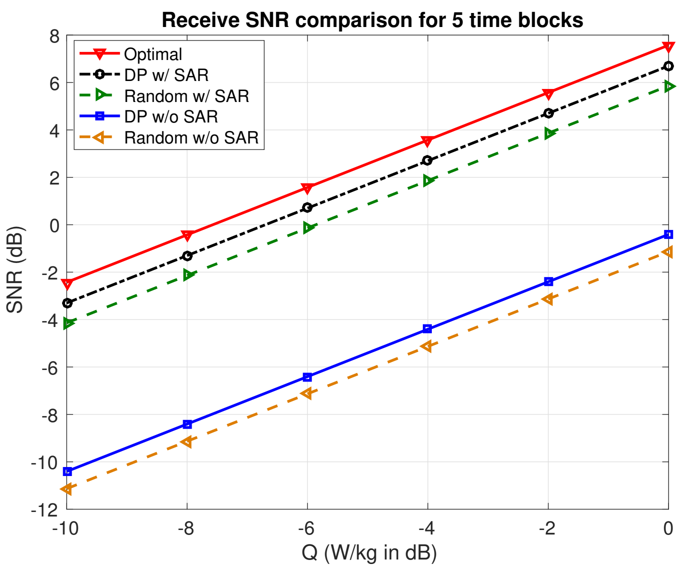

Fig. 7: Receive SNR comparison with $Q = 0.1$ W/kg. The DP algorithm gives 0.8 dB gain over random selection with and without the SAR matrices. The SAR-aware method has 7 dB improvement over algorithms without access to the SAR matrices.

We end this section with a brief comparison of the proposed SAR allocation alogorithms with other SAR-aware transmission approaches in the literature, as seen in Table I. Note that the SAR-aware precoding technique proposed in [6] is identical to the proposed uniform allocation method when $T = 1$. Power back-off approaches provide the worst rate performance but are the simplest solutions for complying with SAR regulations. The SAR code in [4], [5] achieves good performance but is restricted to the case with a single SAR constraint and two transmit antennas. The precoding method in [13] extends the results of [6] to the multi-user MIMO (MU-MIMO) case when the transmitters only have statistical CSI. The distinctive feature of this work is that we address communication over multiple time blocks. As shown in the simulations in this section, the proposed SAR allocation methods can exploit the channel diversity over time to adaptively adjust SAR levels during each time block and achieve higher rates than power back-off approaches.

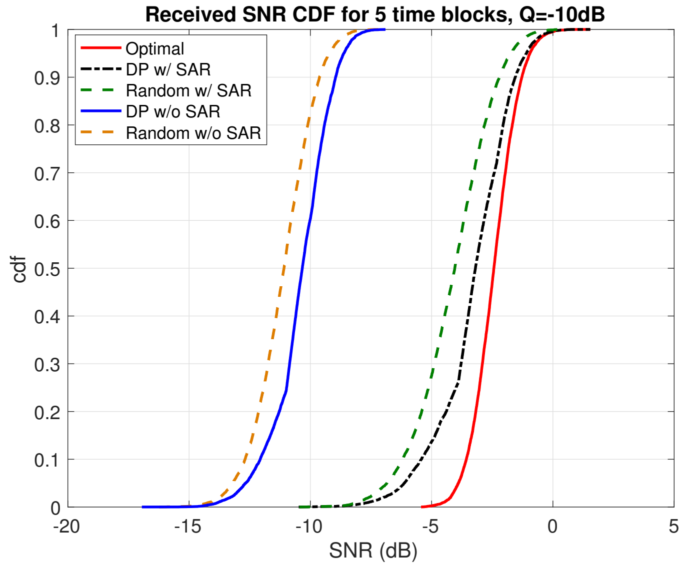

Fig. 8: Receive SNR CDF with $T = 5$ and $Q = 0.1$ W/kg. The 50-percentile rate of DP algorithm has 0.8 dB higher than the random selection.

VI. Conclusion

In this paper, we investigated user exposure allocation algorithms over coherent time blocks. The optimal user exposure allocation method with perfect CSI is a modified waterfilling result. We showed that the uniform SAR allocation over time has performance close to the optimal solution in the medium-to-high NSarNR range, and we proposed an asymptotic method to reduce the complexity for temporal allocation. In the low NSarNR range, the receive SNR matters more than rate. We developed an "on-off" method using dynamic programming. In all cases, our simulation results demonstrated that joint SAR-aware optimization has substantial performance improvement over conventional multi-antenna transmission without SAR knowledge.

Appendix

Proof of $B(\mathbf{q})$'s concavity: First, the rate-function $\log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{HSH}^H\right|$ is a concave function of the transmit covariance matrix $\mathbf{S}$. Second, the maximum of the rate-function over the constraints, given as

$$r(\mathbf{q}, \mathbf{H}) = \max_{\substack{\mathbf{S}\succeq\mathbf{0},\\ \mathrm{tr}(\mathbf{R}_j\mathbf{S})\leq Q_j,\forall j}} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{HSH}^H\right|, \tag{50}$$

is also a concave function of the collection constraint values $\mathbf{q} = [PT, Q_1T, \ldots, Q_GT]$. This can be easily shown using the definition of concave function.

TABLE I: Comparison of SAR-Aware Transmission Schemes

Method	Complexity	Achievable Rate	SAR Diversity	Use Case
Worst case back-off [3]	very low	low	none	no SAR knowledge
Adaptive back-off [3]	low	fair	spatial	low complexity
SAR Code [4], [5]	medium	medium	spatial	MIMO, $M = 2$, $G = 1$
SAR-Aware Precoding [13]	medium	high	spatial	MU-MIMO, statistical CSI
Proposed Uniform Allocation	medium	medium	spatial	MIMO, $T \geq 1$ blocks
Proposed Asymptotic Allocation	medium	medium	spatial and temporal	MIMO, $T > 1$ blocks

For any two sets of constraints $\mathbf{q}_1$ and $\mathbf{q}_2$, we assume $\mathbf{S}_1$ is the maximizer of the rate-function constrained by $\mathbf{q}_1$ and $\mathbf{S}_2$ is the maximizer constrained by $\mathbf{q}_2$, then for any $0 \leq \gamma \leq 1$, we denote $\mathbf{S}_\gamma$ as the rate maximizer constrained by limitation set $\mathbf{q}_\gamma = \gamma\mathbf{q}_1 + (1-\gamma)\mathbf{q}_2$. Note that $\gamma\mathbf{S}_1 + (1-\gamma)\mathbf{S}_2$ is a feasible solution under the constraint set $\mathbf{q}_\gamma$.

Then,

$$\begin{aligned}
&\gamma r(\mathbf{q}_1, \mathbf{H}) + (1-\gamma) r(\mathbf{q}_2, \mathbf{H}) \\
&= \gamma \log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}_1\mathbf{H}^H \right| + (1-\gamma)\log\left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}_2\mathbf{H}^H \right| \\
&\leq \log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}(\gamma\mathbf{S}_1 + (1-\gamma)\mathbf{S}_2)\mathbf{H}^H \right| \\
&\leq \log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}_\gamma\mathbf{H}^H \right| \\
&= r(\gamma\mathbf{q}_1 + (1-\gamma)\mathbf{q}_2, \mathbf{H}). \qquad (51)
\end{aligned}$$

Finally, $B(\mathbf{q})$ is an expectation of $r(\mathbf{q}, \mathbf{H})$ over the channel $\mathbf{H}$, and it is also a concave function:

$$\begin{aligned}
B(\mathbf{q}) &= \mathbb{E}_{\mathbf{H}}\left\{ r(\mathbf{q}, \mathbf{H}) \right\} \\
&= \mathbb{E}_{\mathbf{H}} \left\{ \max_{\substack{\mathbf{S}\succeq\mathbf{0} \\ \mathrm{tr}(\mathbf{R}_j\mathbf{S}) \leq Q_j\ \forall j}} \log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{S}\mathbf{H}^H \right| \right\}. \qquad (52)
\end{aligned}$$

Therefore , $B(\mathbf{q})$ is a concave function over the constraints $\mathbf{q}$.

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