Environmental Factors in Indoor Navigation Based on Real-World Trajectories of Blind Users

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ABSTRACT
Indoor localization technologies can enhance quality of life for blind people by enabling them to independently explore and navigate indoor environments. Researchers typically evaluate their systems in terms of localization accuracy and user behavior along planned routes. We propose two measures of path-following behavior: deviation from optimal route and trajectory variability. Through regression analysis of real-world trajectories from blind users, we identify relationships between a) these measures and b) elements of the environment, route characteristics, localization error, and instructional cues that users receive. Our results provide insights into path-following behavior for turn-by-turn indoor navigation and have implications for the design of future interactions. Moreover, our findings highlight the importance of reporting these environmental factors and route properties in similar studies. We present automated and scalable methods for their calculation and to encourage their reporting for better interpretation and comparison of results across future studies.

ACM Classification Keywords
H.5.m Information interfaces and presentation (e.g., HCI): Miscellaneous.

Author Keywords
blind; accessibility; trajectory; indoor navigation; turn-by-turn navigation.

INTRODUCTION
When designing an indoor navigation system for blind users, the development process requires careful analysis of user motion within the context of the real world environment. Consider a navigation system that gives a verbal instruction to 'walk straight' in a very long narrow corridor where the blind user can use the acoustic properties of the environment to walk straight down the corridor. From our analysis we conclude that the command 'walk straight' is a sufficient command for very long straight paths. Then we discover to our dismay that giving the instruction 'walk straight' in a very open room with very different acoustic properties, results in the user constantly veering from the straight path. Though this is a contrived example, it illustrates how the lack of modeling of the physical environment (*i.e.*, whether there are walls nearby or not) leads to an incorrect interpretation of the performance of the navigation system. The motion of the user must also be interpreted in the context of the real world environment.

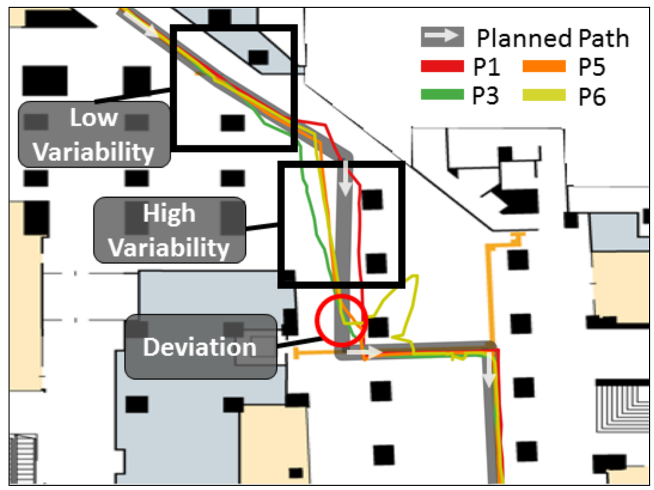

Figure 1. Given the trajectories of blind participants (*e.g.*, P1, P3, P5, and P6) along a planned path and a floor plan of the indoor environment, we estimate variability and deviation, two quantitative measures of a user's path-following behavior, and examine how they relate to the environment.

Traditionally, to assess the performance of a given navigation approach, findings are often reported in terms of localization accuracy, participant feedback, and observations from videos. Aware that the geometry of the real world environment may have an effect on the findings, researchers often report, in

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passing, floor plan layouts of the route stimuli to provide a level of qualitative commentary. Surprisingly, much of the prior work on navigation for people with visual impairments fails to perform a *quantitative* analysis of a proposed approach with respect to the features of the physical environment.

We believe that lack of analysis in the context of the physical environment is due in part to a lack of a quantitative measure of a user's behavior with respect to the features of the environment. In response to this observation, we propose a rigorous analysis framework that fits a linear regression model to a wide range of environmental and experimental factors to path-following behavior of blind users (Fig. 1). In particular, we propose two measures of path-following behavior: (1) variability - how much a user adheres to the recommended path and (2) deviation - points of anomalous behavior such as walking in the opposite direction of the recommended path or clear instances of wandering. We then define a wide range of environmental and experimental features (we propose 6 classes of features in this paper) over each segment of the recommended path. We treat these proposed features as a feature vector in a large linear regression model and attempt to model the 'variability' and 'deviation' (our two proposed measures of behavior). As a result of learning the regression model, we are able to discover which features contribute the most to path-following behavior. We show that variations in user trajectories are best explained by both the design of the system (*e.g.*, localization error and instructions), and the characteristics of the physical environment (*e.g.*, obstacles, pathways, doors, and stairs). Thus, we argue that the reporting of these factors, along with our data and code, enables comparisons of results across studies, which is not currently possible.

RELATED WORK

First, we examine how prior research on assistive indoor navigation has considered environmental and planned route characteristics when conducting their studies. We focus primarily on studies that have been conducted with blind participants. Second, we provide background information on trajectory analysis, both for indoor navigation and broader real-world applications.

Environmental Factors in Indoor Navigation Studies

The primary focus of this paper is to examine how the environmental and experimental characteristics in a study relate to path-following behavior of the participants. The following is a survey of prior indoor navigation studies with blind participants to identify the types of planned routes and the information on their floor layout that researchers reported. The goal of this literature survey was to understand the diversity of environmental setups in prior studies and the types of route stimuli that researchers commonly engineer. While there are a few examples of published results where only little information about the indoor environment is reported, *e.g.* [12], in general the trend in the field is to include more details about the geometry and other characteristics of the route(s) participants are called to navigate.

Table 1 presents examples of representative papers in the filed from 2008-2017, though similar patterns may be found when examining larger surveys of prior evaluation studies, *e.g.*, [13, 10, 35]. With few exceptions, common characteristics reported include the number of blind participants, their familiarity with the indoor environment where the data were collected, and images of the floor plan. The length and complexity of the planned routes vary in these studies. As shown in Table 1, we also see variability in the reported presence or absence of other environmental characteristics, such as tactile paving or floor texture, open entrances or doors, stairs, elevators, and obstacles or point of interests. Turns at corridor junctions are found to be the most commonly reported characteristic of planned routes. The number of turns in a route and turn types (*e.g.* 45, 90, and 180 degrees) seem to be indicative of the route complexity [21, 11]. In many cases, obstacles are either not mentioned (*e.g.* [27]) or cleared from the routes (*e.g.* [9]). Some researchers also consider open doors along a planned route as side obstacles (e.g. [7]) and others report closed doors along the planned routes (*e.g.* [9]). While stairs are often present in the floor maps, they tend to be absent along the routes. For example, researchers in [11] state: "We decided against including staircases in the paths, due to safety concerns." Thus, there is little consensus on the complexity of routes and which environmental characteristics should be included. Further, most studies tend to favor simple routes.

Reported	[21]	[15]	[9]	[31]	[11]	[8]	[2]	[30]	[27]
# participants	9	8	6	8	8		6	8	3
Familiarity	•	•	•	•		•	•	•	
Floor layout	•	•	•	•	•	•	•		•
Floor texture					•			•	
Entrance/door	•	•	•		•		•	•	•
Stair	•	•	•		•		•	•	•
Elevator	•		•		•		•	•	•
Obstacle/POI	•		•	•				•	
Turn	•	•	•	•	•	•	•	•	•

Table 1. Users, floor, and route characteristics in example user studies.

We observe that some studies include anecdotal evidence of relationships between the floor plan and the path-following behavior of blind participants: *e.g.*, veering in open spaces [9, 11], challenges with open doors [7, 9], and non clear turns [11]. However, we are not aware of any prior study that has explored how environmental characteristics, localization error, and instructional cues may relate to the quality of path-following behavior in indoor navigation.

Granularity and Metrics in Trajectory Analysis

We consider how path-following behavior has been studied in prior work and the broader field of trajectory analysis.

Path-following behavior in indoor navigation. Depending on the objective of their study (*e.g.*, feasibility, user interactions, and localization) researchers in this field have employed different methods for assessing path-following behaviors. More often this involves subjective feedback from blind participants *e.g.*, [9, 30, 2, 27, 3]). When an objective method is deployed, it typically focuses on analysis of high-level measures, such as task completion time (*e.g.*, [9, 31, 27, 22]) and success rate (*e.g.*, [15, 9, 8, 27]). Very few researchers base their analysis on deeper analysis of participant trajectories. While location estimates of trajectories can be obtained from logs of the localization systems, accurate trajectories require tedious annotation of sub-meter location data and are very difficult to get, especially for blind participants. We observe two ways adopted in the literature to workaround the lack of accurate data. First, some researchers opt for high level descriptors on blind user trajectories such as number of missed turns [2], extensive veering [1], and correction areas [27]. Usually these descriptors are obtained by visually inspecting videos of the study and comparing them to an ideal path-following behavior. These methods yield subjective estimates. Second, accurate trajectories are obtained with few sighted participants walking along shorter predefined paths. Sighted participants provide time-stamped input on specified points such as turning points [8] or sub-meter markers along the path [2]. The localization accuracy is then calculated as the difference between the estimated and ground-truth trajectories. However, these methods are biased toward path-following behavior of sighted people. To our knowledge, our work is the first to analyze and compare both sub-meter annotated and estimated indoor trajectories from blind participants in a real-world setting.

Data-driven trajectory analysis. Trajectory analysis is commonly adopted in many real-world applications where the goal is to detect (*e.g.*, [20, 4, 5, 26]), predict (*e.g.*, [36, 18]), or better understand (*e.g.*, [25, 23]) movement behavior of users. Most often user trajectories are obtained in outdoor environments where ground-truth location information is automatically estimated with GPS technology. The analyses vary based on study objectives and user population, where researchers define domain appropriate trajectory measures. For example, to detect wandering and disorientation in elderly and people with dementia, researchers (*e.g.*, [20, 4, 26]) look for outliers, cycles, and random patterns in user trajectories or compare them to trajectories of other users not exhibiting the same behavior. To detect real-time deviation from a routine path for people with cognitive impairment, researchers (*e.g.*, [5]) compare a trajectory's similarity to prior trajectories of the same user. In this paper, we are interested in predicting the quality of path-following behavior such as veering and deviation that blind participants exhibit when interacting with an indoor-navigation system. Since the planned route is known a priori, user trajectories are compared against a reference trajectory, which represents an ideal path-following behavior.

USER STUDY AND DATA COLLECTION

We employ trajectory and indoor environment data [1] from a large-scale real-world deployment of a real-time turn-by-turn navigation system tested with blind participants.

Environment. Participants are asked to navigate three planned routes in a shopping mall that consist of three towers of five story buildings (basement through fourth floor, see Fig. 2). The mall covers an area of $21,000m^2$ and includes more than 100 points of interest such as shops and restaurants. People can enter the mall from street level or through the metro station in the basement, and can move between floors by using elevators, escalators, or stairs. The open area between the towers in the basement includes tactile pavings.

[1]Our data and code available at https://envfactors.github.io/.

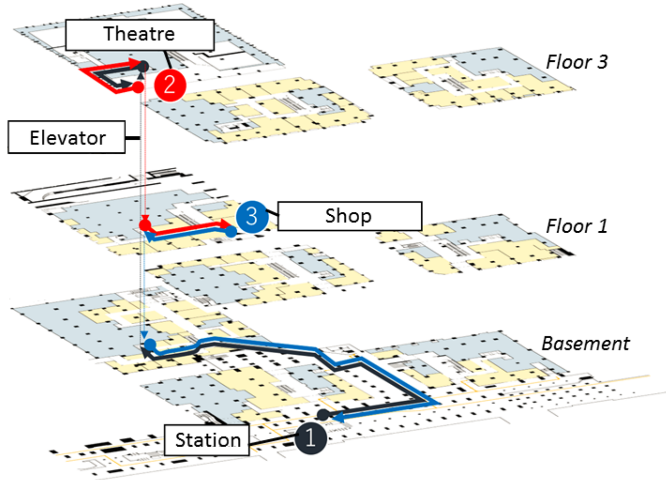


Figure 2. The three multi-floor routes in the study. Participants start at the subway station on the basement floor to movie theater on the third floor in route 1. They then continue to a candy shop on the first floor in route 2, and finally back to the train station in route 3. [Tokyo, Japan]

As illustrated in Fig. 2, participants are asked to navigate across three floors. In the first route they navigate from a subway to an elevator located in the basement floor, then to a movie theatre on the third floor. In the second route, they return to the elevator and head towards a shop on the first floor. Finally, in the third route, participants traverse back to the elevator, and to the study starting point.

With an overall length of 407 meters, our route stimuli were engineered to include diverse layout characteristics exceeding those present in the related work. Routes included open space, varying width and junction complexity halls, tactile paving, obstacles (signage, chairs, and racks), automatic doors, and elevators. Moreover, some points along the routes were in proximity to stairs, escalators, and shop entrances.

Participants. A total of 9 blind participants (ages 38–65) are recruited for the study with their detailed information presented in Table 2. Five participants are totally blind and the remaining four have category 3 blindness [2]. All participants used a white cane as their mobility aid, except P4 who used a guide dog. Five participants reported owing a smartphone and only P3, P5, and P7 have had experience with turn-by-turn navigation tools (*e.g.*, Google Maps). While P4 and P9 had previously visited the shopping mall (over a year ago), they confirmed not remembering or having visited the planned routes. All participants were compensated for their time.

[2]http://apps.who.int/classifications/icd10/browse/2015/en#/H54

ID	Gen.	Age	Visual Acuity	Since	Mobility	SMA
P1	M	65	totally blind	12y	white cane	
P2	F	42	20/2000 both eyes		white cane	4y
P3	M	54	totally blind	2y	white cane	4y
P4	F	44	totally blind	10y	guide dog	1w
P5	M	48	20/2000 both eyes	28y	white cane	3y
P6	M	38	totally blind	5y	white cane	
P7	F	40	totally blind	20y	white cane	
P8	M	42	20/500	1y	white cane	
P9	F	46	20/500 right, blind left	0y	white cane	1.5y

Table 2. Participants' demographics and smartphone use period.

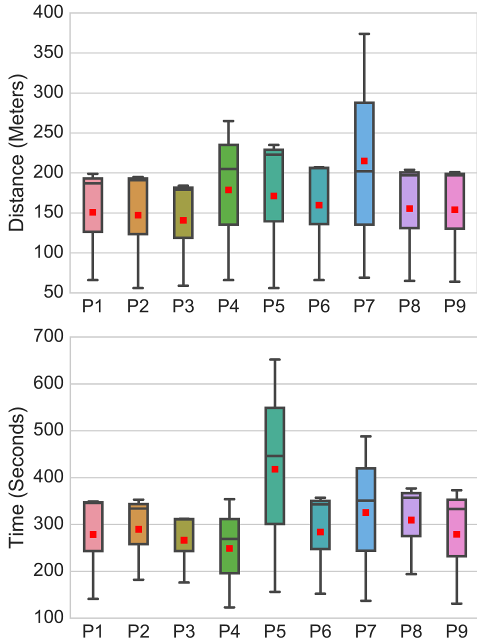


Figure 3. Distance covered and time spent per participant across the three routes in the dataset. Route lengths in meters were 177, 54, and 176, respectively.

Fig. 3 depicts the distance traveled by each participant across the multiple routes in meters and the time taken in seconds. Overall, participants traversed 491 meters on average for about 15 minutes. While this participant pool size is representative of other studies in the field (see related work section), to our knowledge, this is the first study examining sub-meter ground-truth trajectory data for so many blind participants over such long routes in indoor navigation.

Apparatus and Procedure. Participants were all equipped with an iPhone 6 smartphone and bone conduction headphones that allow for situational awareness. They traversed the planned routes based on turn-by-turn instructions (see next section) from NavCog 3 [33], a real-time smartphone-based indoor navigation system. The app requires installation of bluetooth low energy beacons in the environment. In our study, the environment was instrumented with a total of 218 beacons positioned at 5-10 meter intervals. Prior to the study, participants were informed on the scope of the experiment, signed a consent form, and were familiarized with the NavCog system and the headset through a training session. During the study, a 360° camera and three researchers followed each of the participants to record the session and ensure participants' safety. Subjective feedback on the system is reported in [30].

Trajectories. Fine-grained estimates of user trajectories were extracted form NavCog logs of user location. Furthermore, ground-truth trajectories were obtained from frame-by-frame hand-annotations of user locations by visually inspecting 360° video recordings from the study. For example, Fig. 7 visualizes both the ground-truth and estimated participant trajectories from a portion of the first and third route in the study. The localization error of the system is 1.97 meters on average (min-max: 0.02-11.2, sd:1.29) across all participants and routes.

MEASURES OF PATH-FOLLOWING BEHAVIOR

As discussed in related work, there is no consensus in the field on how to objectively assess blind users' path-following behavior in indoor navigation. For our analysis, we define two quantitative measures, "variability" and "deviation." The first can be automatically calculated, whereas the second requires visual inspection of trajectories. For meaningful results, user trajectories in both measures are compared against the planned path.

Variability

Trajectory variability measures how well all user trajectories adhere to a planned path. By taking the planned path to be the ideal reference trajectory, we quantify how well that path is followed by comparing it with participants' trajectories. For this measure, we use a *path-normalized chi-squared distance*.

We denote a trajectory T as a sequence of points, $(t_p)_{p=1}^{n}$, the reference trajectory as T^R, and participant i's trajectory as T^i. Trajectory points $t_p^i \in T^i$ are indexed by time and sampled every 1 second. Variability is calculated over an area around a reference point along the planned path, $t_p^R = (x_p^R, y_p^R)$. Let $B_r(t_p^R)$ be a neighborhood around t_p^R defined by a radius, r, such that $B_r(t_p^R) = \{t_j \mid d(t_j, t_p^R) < r\}$. Here, d is a distance function (Euclidean in our case). Trajectory points from each participant that fall within $B_r(t_p^R)$ are used to compute the variability value. The points are first normalized by the reference point, $t_p^i - t_p^R$, and then used to construct a histogram count of vector orientations, H_p^i. The corresponding orientation count vector is also constructed for the reference path, H_p^R. This process results in a fixed size movement distribution for each participant, P_i. An average distribution $\bar{H}_p$ is computed as $\bar{H}_p = (1/N)\sum_i H_p^i$, and the **variability score** V for reference point p, illustrated in Fig. 4, is defined as:

$$V_p = d_{\chi^2}(\bar{H}_p, H_p^R), \text{ where } d_{\chi^2}\text{: chi-squared distance} \quad (1)$$

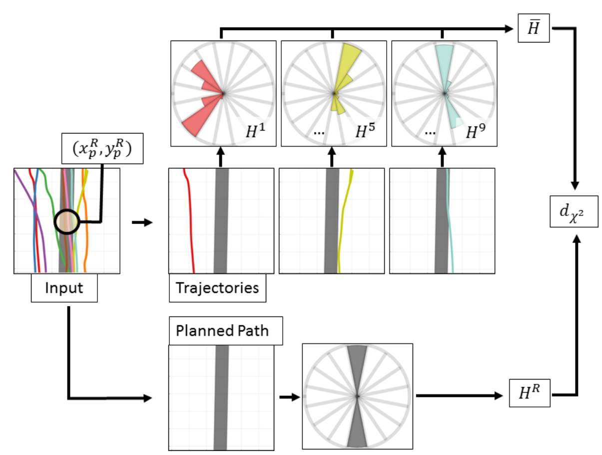


Figure 4. Variability in participant trajectories is defined over a neighborhood around points along the planned path. A path-normalized chi-squared variability is computed by: (i) representing each participant trajectory with a distribution of movement orientations relative to the reference point (*e.g.* H^1, H^5, and H^9); and (ii) comparing the average distribution with the distribution along the planned path (H^R).

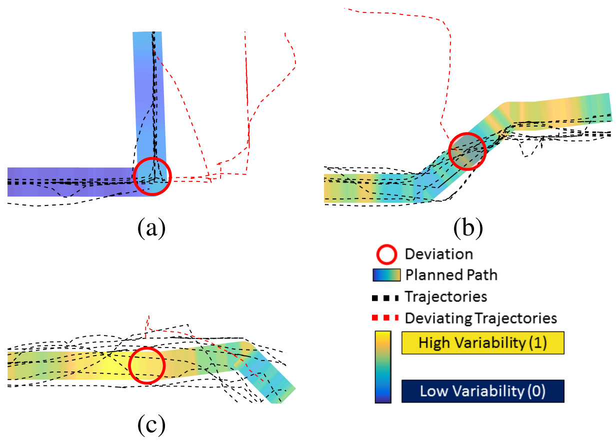


Figure 5. Examples of variability and deviation in our data. Deviation can be subtle, and may occur in areas of both high and low variability.

Although alternative approaches for trajectory analysis exist (*e.g.*, dynamic-time warping [32, 24]), the chi-squared distance is a common approach for comparing distributions [29, 34] which has also been previously adopted in user trajectory analysis [23, 25].

Variability provides an automated objective measure that is especially useful for capturing overall trends across all participants, such as extensive veering and path correction areas. However, as shown in Fig. 5(a), it is less sensitive to outlier participant trajectories. Moreover, it is meant to reflect the maximum points of divergence and not the onset points. This motivates the introduction of "deviation", a complimentary trajectory measure for path-following behavior.

Deviation

In addition to variability, which is a local phenomenon, we are interested in analyzing onset of deviation. These are cases where a user might wander away from the planned route, perhaps due to confusion over instructions or the environment. Identifying such cases requires knowledge of the overall trend (*e.g.*, a participant's trajectory may vary from the planned route but still in the right general direction), the history of motion, and the global context of the participant's goals. We annotate such instances by visually inspecting participant trajectories and video data. To mark a deviation onset we place a 2D Gaussian distribution on top of each annotated deviation point with a radius of 1 meter. Deviation, as will be shown, is significantly more challenging to predict just by looking at the environment, localization error, or instructional cues. In calculating deviation, we consider any deviation event, including those performed by a single participant. In other words, deviation points are annotated for each participant trajectory, and are aggregated to produce the deviation values for the entire set of participant trajectories, which we employ in our analysis. Deviation examples from our study are shown in Fig. 1 and Fig. 5.

ENVIRONMENTAL AND EXPERIMENTAL FACTORS

The goal of our work is to examine whether environmental factors related to floor plan layout (*e.g.*, open spaces, escalators) or route characteristics can explain some of the path-following behavior observed in indoor navigation studies with blind participants. To put our results into perspective, the effect of environmental factors is compared to those of localization error and instructional cues, which are considered to be the main factors impacting participants' behavior in studies. This section discusses concrete variables for these factors, which are employed in our multiple regression models in the following section.

Walls and Tactile Pavings

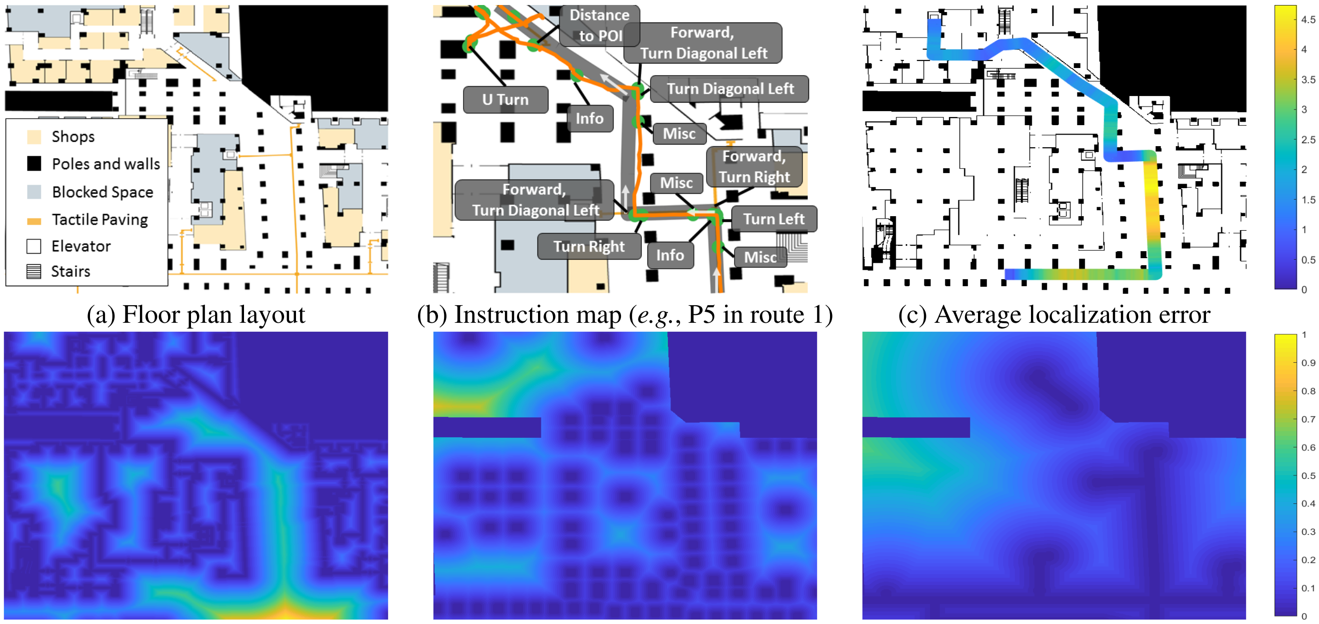
We extract wall and tactile paving characteristics automatically from the floor plan layout, such as the one in Fig. 6)(a) with encoded information on walls, pillars (black rectangles), stairs, tactile pavings (orange lines - only available on this one floor), shops (orange areas), and non-walkable space (grey). We use color thresholding and connected component analysis to obtain distance maps for all the walls (6(b)), pillars (6(c)), and tactile pavings (6)d)) in the image. Distance values in these maps are color coded from dark to light for lower to higher values.

For each point in the planned path we calculate its minimum distance to a pillar (**PillarMinDist**) and the standard deviation of its distance to a pillar in a 6 meter radius (**PillarDistStd**). The latter allows us to measure properties of surrounding placement of multiple pillars. Similar variables are computed for the tactile pavings (**TactileMinDist** and **TactileDistStd**).

In addition to the minimum distance to any wall (**WallMinDist**), we also encode the distance to a wall in four directions: north (**NWallDist**), east (**EWallDist**), south (**SWallDist**), and west (**WWallDist**). Moreover, we encode the orientation of the nearest wall in these four directions. These orientation variables are computed by (i) extracting a 2D gradient at each pixel from the wall-only floor plan and (ii) finding the nearest wall pixel in each of the four directions and recording its gradient orientation value. The intuition behind these orientation variables is to capture the shape of the scene, such as narrow hallway, complex junctions, or open spaces. In order to use an orientation value in the regression (two angles may be close to each other modulo 2π but far away in numerical value), it is transformed into two variables: the x and y axis value along a unit circle. These variables are referred to as **NWallOriX** and **NWallOriY** for the north wall, and similarly to east, south, and west. These variables are motivated by observations from the data. For instance, we suspect that the presence of an open space in route 1 leads to a confusion across several participants. Furthermore, it is useful to separate walls from pillars, as the latter can be bypassed from two directions, leading to differences in path-following behavior and deviation from the path.

Scene

In addition to automatic visual extraction of wall, tactile, and pillar features from the floor map, we also employ annotations of obstacles and other static landmarks in the scene that are made available to the navigation system a priori. These include locations of elevators, doors, stairs, chairs, signage, and other obstacles. These type of variables are referred to with a prefix 'Dist', such as **DoorDist** or **StairsDist** for minimum distance



(a) Floor plan layout (b) Instruction map (*e.g.*, P5 in route 1) (c) Average localization error

(d) Wall distance map (e) Pillar distance map (f) Tactile distance map

Figure 6. Given a floor plan (a), we use the instruction locations (b) and average localization error along the planned path (c). Environmental factors such as distance from walls (d), pillars (e), and tactile pavings (f) are automatically extracted and used to analyze trajectory behavior.

to a door or stairs, respectively. Dynamic scene information such as moving obstacles and pedestrians are beyond the scope of this study since it would require automatic crowd and scene detection from videos, which is beyond state of the art in indoor navigation technology.

Path

In order to fully study the interaction between the planned path and the scene, features from the planned path itself are also extracted. We compute a spatial numerical derivative of the planned path trajectory. In the analysis, we refer to this variable as **derivativeX** and **derivativeY** since it is applied separately to each coordinate of the trajectory. Because the path **curvature** value may also relate with trajectory behavior, it is also used as a feature.

Instruction Type

NavCog 3, used in the study, determines a user's current location using Bluetooth beacons and consequently provides turn-by-turn navigation and information about the surroundings. For instance, Fig. 6 shows the types of verbal instructions that P5 received during portion of route 1. The app informs users of the next turn early on, provide them with a reminder a bit before the turn, and repeat the turn instruction at the turning point. So for this portion of the route P5 heard [3]: "It is about time", "Turn left", "Tactile paving ahead", "Go ahead for 9 meters and turn right", "It is about time", "Turn right", "No tactile paving", "Go ahead for 17 meters, turn diagonally left", "It is about time", "Turn diagonally left", 'Go ahead for 26 meters, turn diagonally left", "Tactile paving ahead", "15 meters remaining", "Koledo Muromachi 2 underground front entrance", "No tactile paving", "Two automatic doors", and "It seems to be in the opposite direction".

[3]Instructions were provided originally in Japanese.

The instructions that participants receive in real time are typically considered to be one of the most important factors in defining participants experience with an indoor navigation system. In order to obtain meaningful analysis, the overall set of instructions is first clustered into 8 semantically similar groups: **sharp turn** (*e.g.*, turn left, turn right), **mild turn** (*e.g.*, turn diagonally left, turn diagonally right), U-**turn**, **forward** (*e.g.*, go ahead for x meters), **point of interest** (POI, *e.g.*, store, restroom, and restaurants), **arrived**, and **obstacle** (*e.g.*, automatic doors, signage, and chairs). The presence of each cluster type around a point on the optimal path is used as a feature. The regression analysis can help reveal how certain instructions are more likely to co-occur with certain variability and deviation scores.

Localization Error

For assistive indoor applications, trajectories are often obtained with location estimation sensors, such as an IMU, WiFi, or Bluetooth beacons. The localization quality in each time step is crucial to our application domain, as mis-localization could result in inaccurate instructions to the users and higher trajectory variability. This is why improvements in localization are emphasized as the main proxy for improved user experience. While not always available to the researchers, we include this factor in our analysis to provide better perspective but comparing its relative importance with the other factors.

The localization error features encode the localization quality of the system in a certain location along the path. First, we compute the Euclidean distance in meters between estimated and annotated locations of trajectory points. Since there are multiple trajectory points that fall within the variability neighborhood, we use the mean and standard deviation of the localization errors as our features. A visualization example can be seen in Fig. 6. The localization error is of particular

Figure 7. We explore the difference between accurate human-annotated trajectories and estimated trajectories obtained from the system and beacon sensors. The figure depicts examples obtained trajectories for P5 as he traverses the same area during route 1 (left) and route 3 (right).

concern around turns, which also happen to be difficult areas for navigation. Therefore, we expect this factor to act as an overall contextual feature for variability and deviation. The variables for this factor are referred to as **LocErrorMean** and **LocErrorStd** for the average and standard deviation of localization error in the proximity of the current point along the planned path.

ANALYSIS AND RESULTS

The goal of our analysis is to examine how environmental and experimental factors in assisted indoor navigation studies relate to participants' path-following behavior. We employ multiple regression to analyze data from 9 participants in a large-scale real-world environment, collected in the study described above. Our independent variables cover all six independent factors: walls, tactile paving, scene, path, localization error, and instruction type (see section on Environmental and Experimental Factors). Our dependent variables are the variability and deviation scores (see section on Measures of Path-Following Behavior).

Factor-Level Analysis

As shown in Fig. 8, we trained ten separate models: wall (w), scene (s), tactile (t), path (p), localization error (l), instruction (i), floor (w+s+t), environment (w+s+t+p), environment with localization error (w+s+t+p+l), and environment with localization error and instruction variables (w+s+t+p+l+i). Each model performed regression of a dependent variable against the independent variables within the factors. For example, model (w+s+t+p+l+i) uses all 47 independent variables (from each of the six factors). We do this for each dependent variable (variability of estimated, variability of annotated, deviation of estimated, and deviation of annotated trajectory data) resulting in a total of forty models. The rationale for this choice is to investigate at a high level how each of the independent factors and their combinations relate with variability and deviation for both estimated and annotated trajectories.

Variability & Deviation on Annotated Data. Fig. 8 illustrates how the variability model based upon the wall variables accounts for more variance than all the other models trained on the other independent factors. Loosely speaking, this indicates that you can more accurately predict trajectory variability by considering distances and directionality of walls and pillars, rather than relying on any of the other variables. However, including the other variables to the model can improve the prediction, with the scene variables such as obstacles, elevator, stairs, and doors being the second best.

Fig. 8 also illustrates how the deviation model based upon verbal instructions accounts for similar variance to the wall variables and for more variance than the rest of the models trained solely on another independent factor. More important this figure depicts the difference between the two measures of trajectory behavior. Comparing the two, explaining or predicting deviation is more challenging, as it involves reasoning over long-term movement and navigation goals, rather than just instantaneous variability. Another observation is that the combination of different factors produces a larger increase in adjusted R^2 for variability, while for the deviation, the same combination leads to minor improvement. This can be seen by the grater uptick in the lines for variability relative to deviation in Fig. 8 when factors are combined. This observation validates the complexity of automatically identifying deviation.

Estimated vs Annotated Trajectories: Our overall research objective involves quantifying the explanaibility of trajectory behavior based on different environmental and experimental factors. Hence, the most revealing experiments involve analysis of variability and deviation calculated on accurate annotated trajectories of participants. As, discussed in section User Study and Data Collection, ground truth data are obtained from human annotators and thus less noisy than the estimated trajectories due to localization error (see Fig. 7).

However, it is difficult to extract accurate trajectory data for large-scale deployments of indoor navigation systems especially with multiple users and environments. Given the challenges in obtaining trajectories of human-annotated location information, many researchers may perform their analysis over estimated trajectories which are inherently noisy. We examine whether the effect of factors on variability and deviation, calculated over this noisy estimated location, are similar to factors revealed by the accurate annotated data.

Fig. 8 shows that the explainability afforded by the different models follows a similar trend in each case (for estimated and annotated trajectories). There are two differences: the scene factor model alone shows a smaller contribution to the estimated trajectory variability, and the localization error factor model alone has a higher contribution. This is a promising result. It implies that that we can use estimated trajectories as proxies for annotated location data to reveal effects of the environment on user behavior. Furthermore, as localization technology improves, we expect these differences to become even smaller.

Variable-Level Analysis

Henceforth, our discussion will focus only on the best performing models combining all the independent factors over the accurate annotated data. We have engineered many independent variables for each of the high level factors (section Environmental and Experimental Factors). This leads to the concern that variables may be correlated and exhibit multicollinearity, thus preventing the model from finding an optimal set of explanatory variables. We overcome analytic limitations related

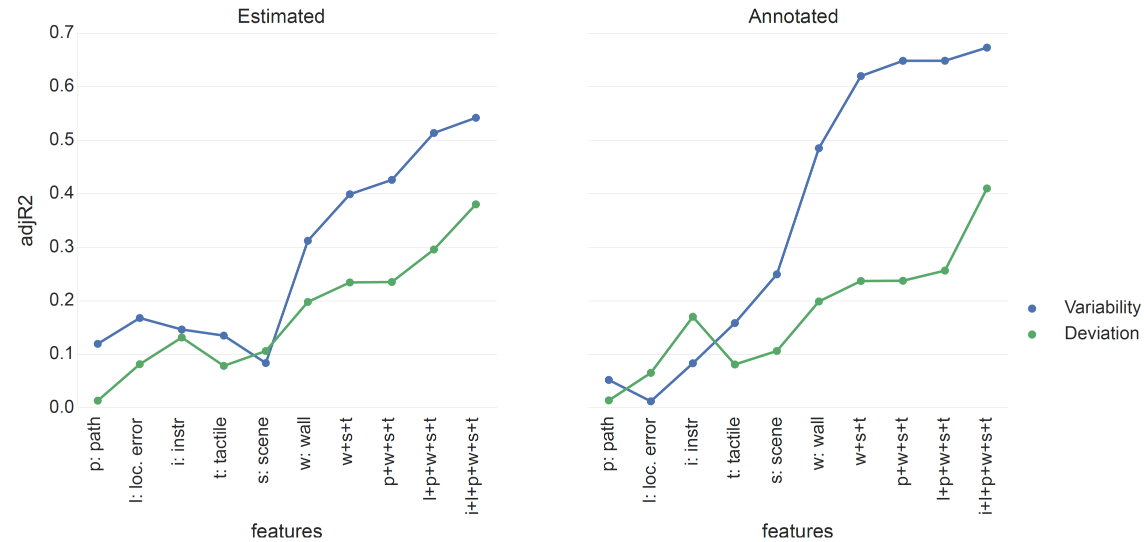


Figure 8. Variability and deviation explainability on different independent factors and their combinations for estimated and annotated trajectories.

Table 3. Multiple Regression Model.

	Trajectory Variability (Adj.R^2: 0.4436)				Path Deviation (Adj. R^2: 0.3637)			
Factor	**Variable**	**Estimate**	**Std. Error**	**t score**	**Variable**	**Estimate**	**Std. Error**	**t score**
	(Intercept)	0.6178	0.0023	270.624***	(Intercept)	0.0941	0.0014	68.976***
Path	Curvature	0.0119	0.0023	5.188***	Curvature	0.0022	0.0014	1.603
	derivativeX	-0.0076	0.0025	-2.997**	derivativeX	0.0123	0.0015	8.135***
	derivativeY	0.0062	0.0029	2.150*	derivativeY	0.0161	0.0017	9.281***
Wall	WallMinDist	-0.0153	0.0028	-5.459***	WallMinDist	0.0025	0.0017	1.516
	NWallDist	0.0136	0.0033	4.172***	NWallDist	-0.0123	0.0019	-6.322***
	WWallOriX	-0.0003	0.0030	-0.111	WWallOriX	0.0064	0.0018	3.557***
	WWallOriY	0.0266	0.0026	10.246***	WWallOriY	-0.0033	0.0016	-2.153*
	EWallOriX	-0.0454	0.0034	-13.214***	EWallOriX	0.0065	0.0021	3.165**
	EWallOriY	-0.0026	0.0025	-1.016	EWallOriY	0.0033	0.0015	2.188*
	NWallOriX	-0.0554	0.0035	-15.692***	NWallOriX	-0.0010	0.0021	-0.460
	NWallOriY	-0.0124	0.0044	-2.840**	NWallOriY	-0.0041	0.0026	-1.581
	SWallOriX	-0.0026	0.0029	-0.899	SWallOriX	-0.0071	0.0018	-4.074***
	SWallOriY	-0.0058	0.0049	-1.191	SWallOriY	-0.0111	0.0029	-3.797***
	PillarMinDist	0.0270	0.0039	6.987***	PillarMinDist	-0.0141	0.0023	-6.126***
	PillarDistStd	-0.0542	0.0038	-14.408***	PillarDistStd	0.0457	0.0022	20.372***
Tactile	TactileDistStd	-0.0143	0.0036	-4.011***	TactileDistStd	0.0035	0.0021	1.665.
Scene	DoorDist	0.0128	0.0042	3.054**	DoorDist	-0.0031	0.0025	-1.218
	StairsDist	-0.0893	0.0032	-28.243***	StairsDist	0.0025	0.0019	1.296
Loc. Error	LocErrorMean	-0.0029	0.0039	-0.736	LocErrorMean	-0.0111	0.0023	-4.761***
	LocErrorStd	0.0080	0.0044	1.834.	LocErrorStd	0.0161	0.0026	6.151***
Instruction	SharpTurn	0.0076	0.0034	2.235*	SharpTurn	-0.0056	0.0020	-2.750**
	UTurn	-0.0088	0.0027	-3.229**	UTurn	0.0320	0.0016	19.661***
	MildTurn	-0.0058	0.0032	-1.831.	MildTurn	-0.0085	0.0019	-4.446***
	Obstacle	0.0473	0.0035	13.716***	Obstacle	-0.0250	0.0021	-12.133***
	Arrived	0.0123	0.0025	4.894***	Arrived	0.0092	0.0015	6.096***
	POI	-0.0261	0.0026	-9.900***	POI	0.0146	0.0016	9.219***
	Forward	-0.0079	0.0035	-2.220*	Forward	0.0381	0.0021	18.037***

Trajectory Variability: Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.1448 on 4295 degrees of freedom
F-statistic: 128.6 on 27 and 4295 DF, p-value: < 2.2e-16

Path Deviation: Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.08629 on 4273 degrees of freedom
F-statistic: 92.02 on 27 and 4273 DF, p-value: < 2.2e-16

to collinearity by applying a step-wise selection of variables until all variance-inflation (VIF) values are below a threshold (with a VIF threshold value set to 10 in our analysis) [6]. From 47 variables, only 27 had VIF < 10 in our analysis.

Variability. Table 3 summarizes the regression analysis for the variability model based on 27 selected independent variables (from a total of 47). In consistency with the factor-level analysis, the top four variables (largest coefficients) lie in the wall, scene, and instruction factor groups (see "StairDist", "PillarDistStd", "EWallOriX", and "Obstacles" rows). The first variable,"StairDist", indicates that the variability among trajectories will be lower when stairs are not very close to the route. The second variable, "PillarDistStd", captures the deviation of distances from pillars within a 6 meter radius. Lower variability seems to be related with higher values of this variable. Intuitively, the third variable, "EWallOriX", captures diagonal paths. From our data, we observe that participants adhere to the path when entering this diagonal part of the route. As expected, instructions informing users about surrounding "Obstacles", the fourth variable, are related to reduced variability. Variables from the tactile and path were also key components. Surprisingly, localization error variables are not significant to trajectory variability, implying that further improvements in localization accuracy may not prevent behaviors such as veering off the prescribed path.

Deviation. Table 3 summarizes the regression analysis for the deviation model based on 27 selected independent variables (identical to the variability model above). Consistent with the factor-level analysis, the top four variables lie in the wall and instructions factor groups (see "PillarDistStd", "Forward", "UTurn", and "Obstacles" rows). Only the "Obstacles" instruction cue has a downward effect on deviation. This makes sense since users are less prone to deviations when warned about obstacles in their path. The "Forward" and "UTurn" variables are positively related to deviation, suggesting that instructions were challenging for participants. Variables from the localization error and path were also key components. Interestingly, none of the tactile and scene variables were significant to deviation. One would expect tactile indication from the ground to reduce deviation. We have two possible explations. First, tacticle paths were only available on the first floor and perhaps limited data resulted in insignificance. Second, participants may go off a tactile path if they follow false instructions due to localization error.

Variability vs Deviation. The regression models also highlight differences between variability and deviation. "PillarDistStd" coefficient is negative for the variability but positive for deviation. While localization error is not significant to variability, it is significant to deviation. On the other hand, none of the tactile and scene variables were significant to deviation, but were significant to variability. This suggests that the two metrics capture complimentary aspects of trajectory behaviors for people with visual impairment and should be reported together.

DISCUSSION

In the Variable-Level Analysis section, we used the regression coefficients ("Estimate" columns in Table 3) to identify the most influential variables. For more meaningful interpretation, we calculated the relative importance of each variable in the variability and deviation models, using the squared standardized coefficient calculated using the 'relaimpo' package [14]. This analysis assigns an R-squared percent contribution to each correlated variable obtained from all possible orderings of the variables in the regression model. Higher bars in Fig. 9 indicate variables with greater importance in the model. For better readability, we pruned the figure by keeping only the top-10 variables. We estimated the bootstrap variance of the relative importance value to determine 95% confidence intervals (shown as whiskers in Fig. 9). Importance values are significant when a bar's whiskers do not cross the zero line in the graph. We consider variables with significant and higher relative importance values to be more important.

We observe that the most important variables for explaining path-following behavior come from both environmental and experimental factors. The presence of stairs in the scene contributes most toward explaining how closely participants adhere to a planned route – trajectory variability is lower when stairs are not very close to the route. As discussed in the related work, many researchers restrain from including staircases in proximity to the evaluation routes thus limiting the applicability of their results to real-world scenarios. Also, the layout of walls and pillars around the planned route is significant in explaining both trajectory variability and deviation, with pillars and open spaces presenting a real challenge for indoor navigation systems. Our results confirm that informing participants about surrounding obstacles improves path-following behavior. However, instructions about forward movements (distance to advance forward) and U-turns seem to pose challenges. We suspect that participants poorly estimate the distance they have traversed, as shown in prior work [19], and deviate from the planned route. An interesting result is that the direction in which participants approach the same static scene also contributes to the deviation. This indicates the importance of taking both directions (forward and backward along the same path) into account when designing route stimuli. We confirm that localization error can explain some of the path following behavior. In particular, we observe that large variation in localization error within a short segment can lead participants off the path.

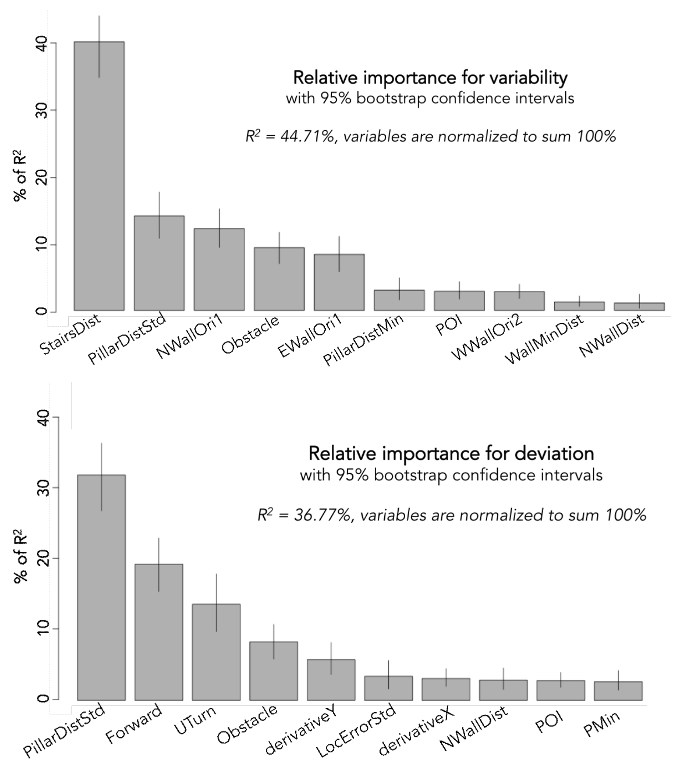


Figure 9. Relative importance of top-10 variables for variability and deviation for annotated trajectories.

Generalizability and Predictability

To avoid overfitting and increase explainability, our analysis opts for a simple linear model. This provides an interpretable model which may be adopted in multiple studies with different locations and diverse blind user groups. However, the extensive features capturing the characteristics of the physical environment used in our work are also designed to give high prediction accuracy model, and hence generalize to wider settings. As user studies are expensive and must be constrained in time, an application of our work would be to inform system deployment by identifying the most challenging areas. To demonstrate this, we explore the predictive power of our model using leave-one-floor-out cross validation (CV). We find that deviation and variability have a CV-estimated MSE of 0.106 and 0.268 after feature selection. Both measures are in the [0, 1] interval, thus these predictions errors demonstrate potential for predictability given more data. We hope to motivate future researchers to pursue this research direction of scalability and generalizability.

Implications

Our findings and insights can contribute to the design and evaluation of future indoor navigation systems for blind users in the following ways.

User interactions. Our analysis provides a better understanding of challenges arising from environmental factors for path-following. For example, it highlights how proximity of stairs or pillar layout can impact a participant's performance independently of the localization error. This calls for novel interactions that can inform and guide users through paths with such structural elements that have the potential to mislead.

Comparison across studies. The preceding highlights how environments rich in physical characteristics, such as open spaces and stairs, can affect the ability of users to adhere to a planned route. In table 1 we assembled all the physical characteristics we had seen in the literature and engineered rich routes which included *all* of these elements. We further quantified their relationship with the path-following behavior of users. We recommend that future researchers incorporate all of these elements in to their stimuli design (*i.e.*, routes) when evaluating their systems. Further, we encourage that researchers report specific characteristics of their routes for better interpretation and comparison of results across future studies.

New directions. Our analysis shows that relationships between path-following behavior and environmental or experimental factors can be observed even with noisy estimates of trajectories. We release code [4] to calculate trajectory variability, extract environmental features from floor plan layout, and quantify the effects of route characteristics on path-following behavior. Researchers may use their own stimuli and data to arrive at estimates for their proposed systems. This opens up new directions for data-driven methods in indoor navigation. First, these estimates may be used in a predictive model to forecast performance of an indoor navigation system deployed in new environments prior to user studies. Second, our measures and features can contribute to future work that can uncover navigation strategies from interaction patterns in a similar analysis to [17] for outdoor environments or user expertise [28]. Third, our analysis can be applied to single users with the goal of personalization [16]. For example, data from a user may be used to determine challenging aspects of the environment specific to the user, and adapt the verbosity of the instructions accordingly.

Limitations

Our study was conducted with nine participants on a large-scale, real-world environment which exceeds characteristics of previous studies (in terms of route scale and richness of environment). While the participant size is representative of the upper limit of previous studies in this field, we can see results benefiting from a larger participant pool. Furthermore, such analysis would benefit from additional data from diverse locations to account for culture- or locale-specific behaviors and architecture. Our measure of path deviation is human-coded and while it aligns with other observation based metrics in the field, it is susceptible to subjective interpretation of the annotator. As shown in the results, it is more challenging to predict due to the global context required to judge deviation onset.

CONCLUSIONS AND FUTURE WORK

This paper presents an analysis of factors contributing to trajectory variability and deviation in assistive indoor navigation for blind people. We provide evidence that characteristics of the physical environment affect successful navigation, in addition to generally acknowledged variables such as localization error of the system and instructional cues that users receive. For example, we show that users tend to adhere more to a path where scene and layout elements such stairs and pillars are not present (likely due to varying acoustic properties of the path). Moreover, we show that factors contributing to path-following behavior calculated over noisy location estimates are well-aligned with factors revealed by annotated, ground-truth location data. Given the challenges in annotating blind users trajectories with sub-meter accuracy, we provide a scalable solution for data-driven analysis and evaluation within the field. Our findings highlight the importance of including and reporting these environmental factors in future studies.

In future work, we plan to replicate this analysis with larger datasets from estimated trajectories of blind users across multiple environments. A larger dataset would also allow us to investigate how dynamic scene elements, *e.g.*, presence of people, affect the performance of indoor navigation systems for blind users.

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[4]Our data and code available at https://envfactors.github.io/.

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