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Development of A Novel Control-Oriented Vehicle Model for Tire Blowout: An Impulsive Differential System Approach

Ao Li, Yan Chen*, Wen-Chiao Lin, and Xinyu Du

Abstract—**Hazardous and inevitable tire blowout accidents significantly threaten vehicle stability and road safety, and need to be safely controlled. An authentic model to describe tire blowout impacts on vehicle dynamics is crucial for model-based control design. However, existing vehicle models typically simplify the forces and/or moments caused by tire blowout as continuous and smooth (differentiable) disturbances, and thus consist of normal linear or nonlinear ordinary differential equations (ODEs). To accurately describe tire blowout impacts that correspond to an intensive and quick physical process, this paper proposes a new control-oriented vehicle model through an impulsive differential system (IDS) approach. In the IDS-based vehicle model, the lateral force and moment caused by tire blowout, are described by impulsive inputs that are not differentiable. Consequently, vehicle states are modeled by impulsive differential equations instead of ODEs. Through both simulation and experimental results, the proposed IDS-based control-oriented model is more accurate than existing models in describing tire blowout impacts on vehicle dynamics. The developed model will benefit the control design of tire blowout to ensure vehicle stability and safety on road.**

I. Introduction

Tires of ground vehicles are the most essential components because external and controllable friction forces are solely generated from direct interactions between the road and tires. Vehicle motions, transient behaviors, fuel consumption, and riding comforts all strongly depend on the working conditions of tires [1][2]. Operating in complex and open environments [3] under different workloads [4], tires may fail in different ways due to various reasons. The most hazardous tire failure is tire blowout during driving, which introduces suddenly changed forces/moments to deviate vehicle motions and endanger traffic safety. Even worse, improper reactions (e.g., excessive steering and/or braking) from panicked drivers, could cause severer road accidents, such as vehicle rollovers [5]-[7]. In 2015, the National Highway Traffic Safety Administration (NHTSA) estimated that tire blowouts caused more than 400 deaths and over 78,000 crashes every year [8].

To understand the impacts of tire blowout, much research work has been conducted in the modeling of tire blowouts. Variations of some key tire parameters due to tire blowout (e.g., longitudinal stiffness C_x, cornering stiffness C_y, tire effective radius R_e, and rolling resistance coefficient K_r) were analyzed and assumed to linearly vary over an extremely short tire blowout duration Δt [3][5][9]-[11]. Other important factors, such as self-alignment torque (SAT) variations and a two-stage vertical load redistribution, were considered in an enhanced tire blowout model in the authors' recent work [12]. By integrating the changed parameters and factors into a normal tire model, sophisticated and nonlinear tire blowout models were developed [9][12]. It is important to note that these physical tire blowout models were mainly utilized to study and simulate the tire blowout impacts. The developed tire blowout models, based on but more complex than normal tire models, are not suitable for model-based control design.

To utilize existing model-based control design techniques, some simplified control-oriented vehicle models for tire blowout were adopted in the literature. In [11][13], a control-oriented model was considered as a two-degree-of-freedom (2-DoF) linear vehicle model perturbed by both parameter uncertainties (C_y) and moment disturbances (from an increased K_r) due to tire blowout. However, other important parameters and the redistribution of vertical forces were ignored. Another 2-DoF linear vehicle model in [14] was applied with only perturbed disturbances (no parameter uncertainties), including an additional steering angle generated by a varied (but simplified) SAT and a moment disturbance associated with an increased K_r. Another common vehicle model is a 2-DoF system perturbed by the additional lateral force and moment due to tire blowout [3][5][9]. Namely, the resultant effect of a complexly pneumatic and mechanical process of tire blowout is to introduce the additional lateral force and moment on the vehicle center of gravity (CG), which can deviate the vehicle from its driving lane.

Although different vehicle models for tire blowout were discussed in the literature, one common feature or approximation is that the varied parameters, forces, and/or moment are all modeled as smooth and differentiable signals. Consequently, normal ordinary differential equations (ODEs) were used to describe vehicle states [3][5][9][11][13][14]. However, a typical tire blowout process is very short (e.g., 0.1 seconds [10]) and intensive due to a high-pressure tire air leakage. In this process, both tire friction force and moment

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* Corresponding Author. Ao Li is with School for Engineering of Matter, Transport and Energy and Yan Chen is with The Polytechnic School, Arizona State University, AZ, USA (e-mail: yanchen@asu.edu). Wen-Chiao Lin and Xinyu Du are with General Motors Global R&D, Warren, MI, USA.

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will first have impulsive and non-differentiable variations and then change smoothly, as shown in Figure 1. The open-loop simulation results without any control in Figure 1 are obtained from a high-fidelity vehicle dynamics simulation software, CarSim®, integrated with the enhanced tire blowout model developed in the authors' previous work [12]. The impulsive friction force (F_{Yd}) and moment (M_{Zd}), shown in Figure 1, consequently cause impulsive and non-differentiable lateral velocity (v_y) and yaw rate (r) [15]. Although these impulsive changes of vehicle inputs and states due to tire blowout are physically comprehensible and verified through simulation results, unfortunately, a corresponding control-oriented model has never been developed in the literature.

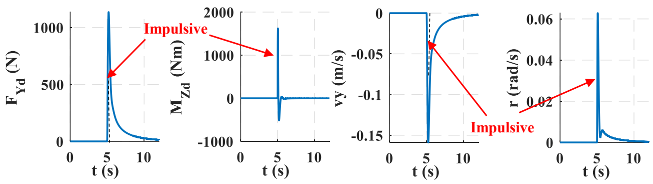

Figure 1. Vehicle lateral force, yaw moment, and state variations for a front left tire blowout in straight-line driving at 100 km/h.

Mathematically, the impulsive force and moment should be described as Dirac delta functions [15]-[18], which may not exist in real physical systems. However, if the magnitudes of the force and moment variations are much larger than the corresponding state variations in an extremely short time duration (e.g., a tire blowout), the impulsive variations can be represented by impulsive differential systems (IDS) [16][17] through differential and difference equations [19].

In this paper, the IDS theory is utilized to develop a new control-oriented vehicle model for tire blowout. Since the impulsive force and moment variations depend on the essential tire blowout process, the proposed IDS-based model will work for different tire blowout locations and maneuvers, not limited to the example of the front left tire blowout during a straight-line driving maneuver shown in Figure 1. Simulation results of the proposed control-oriented model are compared with those of the existing models in the literature to show the improved model accuracy. Furthermore, the experimental results of a scaled test vehicle are demonstrated to validate the effectiveness and accuracy of the proposed model.

The rest of this paper is organized as follows. In Section II, the new control-oriented vehicle model for tire blowout based on the IDS approach is described. Simulation results and discussions are presented in Section III to compare the proposed model with the existing models in the literature. In Section IV, the proposed control-oriented model is further validated through experimental results and analyses. Conclusions are described in Section V.

II. Development of IDS-Based Control-Oriented Model

In this section, the vehicle lateral dynamic model considering tire blowout is reviewed first in subsection *A*. In subsection *B*, the development of the IDS-based control-oriented model is described.

A. Vehicle Lateral Dynamic Model

The vehicle lateral dynamic model is shown in Figure 2. The front steering angle is represented by δ_f . Indexes, $i = 1,2,3,4$, denote the subscripts for the front left, front right, rear left, and rear right tires, respectively.

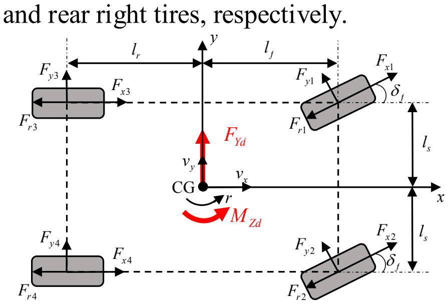

Figure 2. Vehicle lateral dynamic model.

Based on Figure 2, the vehicle lateral dynamic model is formulated as follows,

$$\begin{cases} m\left(\dot{v}_y + v_x r\right) = \sum_{i=1}^{2}\left[F_{yi}\cos\delta_f + \left(F_{xi} - F_{ri}\right)\sin\delta_f\right] + \sum_{i=3}^{4} F_{yi} \\ \quad = F_{Yd}\left(F_{xi}, F_{yi}, F_{ri}, \delta_f\right) \\ I_z\dot{r} = l_s\left[\left(F_{x2} - F_{r2}\right)\cos\delta_f - F_{y2}\sin\delta_f + \left(F_{x4} - F_{r4}\right)\right] \\ \quad -l_s\left[\left(F_{x1} - F_{r1}\right)\cos\delta_f - F_{y1}\sin\delta_f + \left(F_{x3} - F_{r3}\right)\right] \\ \quad + l_f\sum_{i=1}^{2}\left[F_{yi}\cos\delta_f + \left(F_{xi} - F_{ri}\right)\sin\delta_f\right] - l_r\sum_{i=3}^{4} F_{yi} \\ \quad = M_{Zd}\left(F_{xi}, F_{yi}, F_{ri}, \delta_f\right) \end{cases}, \tag{1}$$

where m is the vehicle total mass and I_z is the yaw moment of inertia. The front and rear axles have l_f and l_r distances from the CG, respectively. The length, l_s, is half of the wheel track. Variables v_x and v_y denote the longitudinal and lateral velocities, respectively. r represents the yaw rate. F_{xi}, F_{yi}, and F_{ri} are the longitudinal friction forces, lateral friction forces, and rolling resistance forces, respectively. The tire friction forces of four wheels and resulted moments are lumped together as the total lateral force F_{Yd} and yaw moment M_{Zd} in Eqn. (1).

B. Development of IDS-based Control-Oriented Model

With varied tire parameters over the extremely short duration Δt, friction forces of the blown-out tire and the normal tires will change rapidly and significantly. Assuming that the blown-out tire location is known (e.g., from tire pressure sensors), the impacts of tire blowout on vehicle dynamics can be described through the variations of F_{Yd} and M_{Zd} (with both impulsive and differentiable phases) as external disturbances. Hence, the IDS-based control-oriented model can be developed as follows.

Eqn. (1) is rewritten as,

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -v_x r \\ 0 \end{bmatrix} + \begin{bmatrix} F_{Yd}/m \\ M_{Zd}/I_z \end{bmatrix}. \tag{2}$$

Without loss of generality, tire blowout is assumed to happen at a certain time instant t_b. The phase of impulsive force and moment disturbances can be approximated by difference equations over Δt, which will generate state jumps. Taking the time integral over Δt of Eqn. (2) at the tire blowout moment t_b, we have

$$\begin{cases} \int_{t_b}^{t_b+\Delta t} \dot{v}_y \, dt = \int_{t_b}^{t_b+\Delta t} -v_x r \, dt + \int_{t_b}^{t_b+\Delta t} \frac{1}{m} F_{Yd-d} \, dt \\ \int_{t_b}^{t_b+\Delta t} \dot{r} \, dt = \int_{t_b}^{t_b+\Delta t} \frac{1}{I_z} M_{Zd-d} \, dt \end{cases}, \tag{3}$$

where F_{Yd-d} and M_{Zd-d} represent the impulsive phase of the disturbances over Δt. The impulses of the disturbances, $\int_{t_b}^{t_b+\Delta t} \frac{1}{m} F_{Yd-d} \, dt$ and $\int_{t_b}^{t_b+\Delta t} \frac{1}{I_z} M_{Zd-d} \, dt$ (approximated by differences $\frac{F_{Yd-d}}{m} \Delta t$ and $\frac{M_{Zd-d}}{I_z} \Delta t$, respectively) cause the state jumps at the time instant t_b, denoted as $\Delta v_y \triangleq \int_{t_b}^{t_b+\Delta t} \dot{v}_y \, dt$ and $\Delta r \triangleq \int_{t_b}^{t_b+\Delta t} \dot{r} \, dt$ after a tire blowout finishes at $t_b^+ = t_b + \Delta t$. Considering the coupling effect between v_y and r, the substitution of Δr into the first equation of Eqn. (3) will give the impulsive equation,

$$\begin{cases} \Delta v_y = v_y(t_b^+) - v_y(t_b) = -v_x \Delta r \Delta t + \frac{F_{Yd-d}}{m} \Delta t \\ \Delta r = r(t_b^+) - r(t_b) = \frac{M_{Zd-d}}{I_z} \Delta t \end{cases}, \; t = t_b. \tag{4}$$

Except the impulsive phase from t_b to t_b^+, then the vehicle states are governed by continuous and differentiable disturbances F_{Yd-c} and M_{Zd-c}, which are described in a normal ordinary differential equation (ODE),

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -v_x r \\ 0 \end{bmatrix} + \begin{bmatrix} F_{Yd-c} / m \\ M_{Zd-c} / I_z \end{bmatrix}, \; t \neq t_b. \tag{5}$$

Note that although the lumped inputs, F_{Yd-c} and M_{Zd-c}, are different before and after tire blowout (t_b) corresponding to health and fault tire inputs, the same Eqn. (5) is applied to describe the continuous and differentiable states, with different initial conditions for two continuous phases.

Consisting of an impulsive Eqn. (4), a continuous differential Eqn. (5), and the states jump criterion $t = t_b$, the control-oriented vehicle model for tire blowout is described as a specific IDS [16][17]. The IDS theory is specially developed to describe the abrupt changes of system states at certain instant(s). The corresponding system solution has *impulsive and non-differentiable* point(s) at the instant(s) [20], which cannot be handled by normal ODEs with a *continuous and differentiable* solution. With such a unique feature, the IDS theory has been widely applied in many research areas over the past decades, such as swing-up control of a pendubot [19] and chemistry [21]. However, the application to describe a specific physical phenomenon (e.g., tire blowout) for ground vehicles is discussed for the first time in this paper.

It is worth noting that the novel IDS-based control-oriented model is specifically developed for model-based vehicle control design for tire blowout, by accurately depicting the impulsive phenomenon associated with the extremely fast tire blowout process. On the other hand, the sophisticated and nonlinear tire blowout models (e.g., the authors' recent work [12]), which is too complicated to be used for control design, were created to evaluate tire blowout impacts on vehicle dynamics in simulation.

***Remark*:** As long as the location of the blown-out tire is known, different tire blowout locations with different driving speeds have similar vehicle impacts for a straight-line driving maneuver. If the impulsive and continuous disturbances for one driving speed are calibrated and stored offline, the developed IDS model for other driving speeds can be developed through scaling. The same conclusion can be drawn for one tire blowout location in the cornering maneuver with different steering angles (scaling with steering angle). Therefore, the developed tire blowout model is generalizable for different tire blowout situations and driving maneuvers with a unique mathematical framework to describe the impulsive phenomenon in tire blowout.

III. Simulation Results and Analyses

In this section, simulation results of the proposed IDS-based control-oriented model and existing vehicle models for tire blowout are compared. Validated through experimental results, the enhanced tire blowout model considering self-alignment torque (SAT) variations and a two-stage vertical load redistribution in the authors' previous work [12] is used together with CarSim® as the baseline for comparisons. The existing vehicle models for tire blowout in the literature are first described in subsection *A*. In subsection *B*, comparison results and discussions between the existing models and the proposed model in Section II. *B* are presented.

A. Existing Vehicle Models for Tire Blowout

Selecting vehicle states as $x \triangleq [v_y \; r]^{\mathrm{T}}$, vehicle lateral dynamics with tire blowout was modeled as a linear model perturbed by parameter uncertainties and a moment disturbance in [11][13], which is denoted as Model 1 in this paper as follows,

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)\begin{bmatrix} \delta_f \\ M_c \end{bmatrix} + B_2 M_b, \tag{6}$$

where

$$A = \begin{bmatrix} \frac{-C_f - C_r}{m v_x} & \frac{-C_f l_f + C_r l_r}{m v_x} - v_x \\ \frac{-C_f l_f + C_r l_r}{I_z v_x} & \frac{-C_f l_f^2 - C_r l_r^2}{I_z v_x} \end{bmatrix}, B = \begin{bmatrix} \frac{C_f}{m} & 0 \\ \frac{C_f l_f}{I_z} & \frac{1}{I_z} \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ \frac{1}{I_z} \end{bmatrix},$$

ΔA and ΔB consider the changes of front/rear cornering stiffness C_f / C_r, $\begin{bmatrix} \delta_f & M_c \end{bmatrix}^{\mathrm{T}}$ are the control efforts using Active Front Steering (AFS) and Direct Yaw Moment Control (DYC), respectively, and M_b is the moment disturbance from tire blowout.

In [14], vehicle dynamics with tire blowout was described as a linear model purely perturbed by disturbances (without parameter uncertainties), which is denoted as Model 2 in this paper as follows,

$$\dot{x} = Ax + B_1\left(\delta_f + \delta_{SAT}\right) + B_2\left(M_c + M_b\right), \tag{7}$$

where $B_1 = \begin{bmatrix} \dfrac{C_f}{m} & \dfrac{C_f l_f}{I_z} \end{bmatrix}^{\mathrm{T}}$, and δ_{SAT} is the additional front steering angle generated by the changed SAT when a front tire blowout happens. Note that the control efforts in Eqn. (6) and (7) are assumed zero for (open-loop) model evaluations and comparisons.

The vehicle dynamic model with tire blowout in [9] and that in [3][5] were modeled as common ODEs perturbed by continuous and differentiable external disturbances, which are denoted as Model 3 and Model 4 in this paper, respectively. The general form for both models is described as,

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -v_x r \\ 0 \end{bmatrix} + \begin{bmatrix} F_{Ydj}/m \\ M_{Zdj}/I_z \end{bmatrix}, \ (j = 3, 4). \tag{8}$$

As discussed in Section I, tire friction forces contributing to the total lateral force F_{Ydj} and moment M_{Zdj} were not fully modeled in both Model 3 ($j = 3$) and Model 4 ($j = 4$). Specifically, F_{Yd3} and M_{Zd3} in Model 3 considered the F_{xi} and F_{yi}, but ignored F_{ri} and the related variations. In Model 4, F_{Yd4} and M_{Zd4} involved F_{yi} and F_{ri}, but neglected F_{xi} and the related variations.

B. Simulation Results, Comparisons, and Discussions

A front left tire blowout in a straight-line driving maneuver at a constant speed of 100 km/h is selected as an example. In Table 1, the parameters of a C-class hatchback in CarSim® are utilized. The final values of the parameters characterize tire properties after tire blowout. At the 5th second of the total 12-second simulation, tire blowout is triggered.

By applying the changed parameters in Table 1 into the enhanced tire blowout model [12], the varied F_{xi}, F_{yi}, and F_{ri} of each tire can be obtained as inputs to the CarSim® vehicle model, whose dynamical responses will be served as the baseline for comparison. In Model 1, one cornering stiffness C_f / C_r ($C_f = C_r = 2C_y$) will change (depends on the front/rear tire blowout). In Model 2, the δ_{SAT} is simply calculated using the varied F_{xi} and F_{yi} $(i = 1, 2)$ together with unchanged moment arms. M_b is approximated using F_{ri} and l_s for both models. In Models 3 and 4, F_{Ydj} and M_{Zdj} $(j = 3, 4)$ are obtained with different selections of F_{xi}, F_{yi}, and F_{ri} as discussed in subsection *A*.

In addition to vehicle states, two important road safety parameters, namely lateral offset e_y (deviation distance) and heading error e_ψ (deviation direction), are also selected for comparisons, which are calculated in Eqn. (9),

$$\begin{cases} \dot{e}_y = v_x \sin e_\psi + v_y \cos e_\psi \\ \dot{e}_\psi = r - \rho v_x \end{cases}. \tag{9}$$

Table 1. Parameters of the C-Class Hatchback / Scaled Test Vehicle

Symbol	Value	Final Value
	C-Class / Scaled Test Vehicle	
m	1412 / 63.2 (kg)	-
I_z	1536.7 / 6.961 (kg-m^2)	-
l_f	1.105 / 0.388 (m)	-
l_r	1.895 / 0.388 (m)	-
l_s	0.8375 / 0.311 (m)	-
C_x	47000 / 4000 (N/slip)	To 1/10
C_y	-55000 / -3000 (N/rad)	To 1/10
R_e	0.325 / 0.127 (m)	To 2/3
K_r	0.018 / 0.018	To 30 times

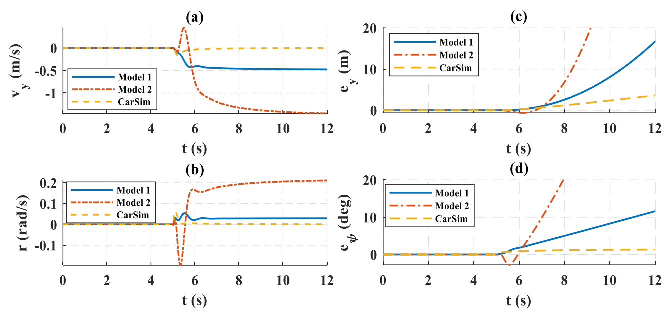

Figure 3. Simulation results and comparisons of Model 1, Model 2, and the baseline CarSim®.

Simulation results of Model 1, Model 2, and the baseline CarSim® simulation are shown in Figure 3. From Figure 3, simulation results of Models 1 and 2 largely differ from the CarSim® results. As discussed in Section I, tire blowout is a complex process with nonlinear and coupled variations of multiple tire parameters. Model 1 represents a linear tire model with only two tire parameter changes considered (C_y and K_r). Thus, large differences in vehicle states are observed in Figure 3 (a) and (b), which further influence e_y and e_ψ in Figure 3 (c) and (d).

Compared with Model 1, Model 2 only considers one tire parameter change K_r. Although a changed SAT of the front tires was considered, the generated additional steering angle δ_{SAT} is inaccurate without a comprehensive understanding of the changed SAT impacted by both longitudinal and lateral force variations. Moreover, the δ_{SAT} in Model 2 cannot represent variations of tire cornering stiffness C_y. Therefore, vehicle states have opposite trends initially as observed in Figure 3 (a) and (b), and the vehicle deviates to the opposite (right) direction initially, as shown in Figure 3 (c) and (d).

Vehicle simulation comparisons of Model 3, Model 4, and the baseline CarSim® are shown in Figure 4. In Figure 4 (a) and (b), the vehicle states of both Model 3 and Model 4 differ significantly from those of the baseline, since not all contributions to tire blowout inputs are fully considered as discussed in Eqn. (8) and shown in Figure 5. Although some forces in the longitudinal direction (e.g., F_{ri} or F_{xi}) are not considered in Model 3/Model 4, the F_{Yd} of Model 3 and Model 4 are similar to those of the baseline CarSim® and IDS model, as shown in Figure 5 (a). However, even with the similar F_{Yd}, M_{Zd} is greatly influenced by both F_{xi} and F_{ri} as illustrated in the M_{Zd} calculation in Eqn. (1). Without fully considering both F_{xi} and F_{ri}, the M_{Zd} of Model 3 and Model 4 are largely different from those of the baseline CarSim® and IDS model in Figure 5 (b). The tire blowout inputs of Model 3 and Model 4 are continuous and smooth compared with those of the baseline CarSim®. Given the similar F_{Yd} but different M_{Zd} (with large steady-state values in Figure 5 (b)), the vehicle states of Model 3 and Model 4 become unrealistically large based on Eqn. (8). With the significantly increased vehicle states, the large e_ψ (larger than 360°) and sinusoidal behaviors of e_y are observed in Figure 4 (c) and (d). The vehicle with a blown-out tire is whirling rapidly on the road, which is not realistic.

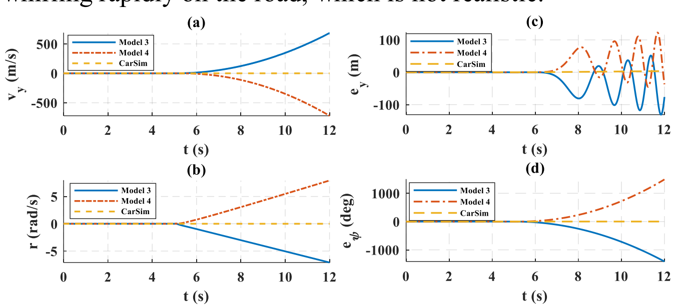

Figure 4. Simulation results and comparisons of Model 3, Model 4, and the baseline CarSim®.

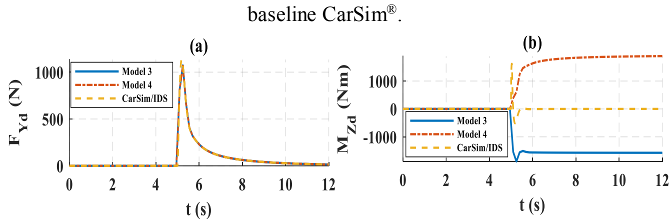

Figure 5. Tire blowout inputs comparison of Model 3, Model 4, IDS model, and the baseline CarSim®.

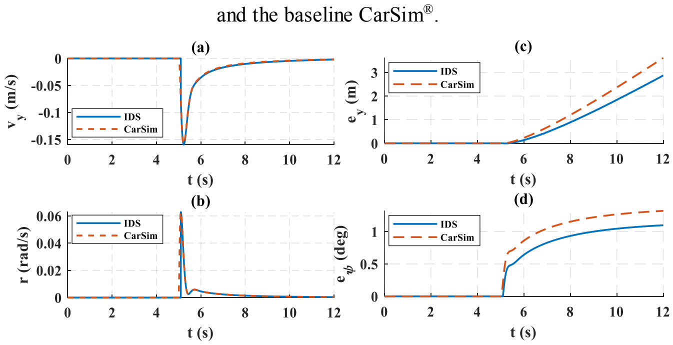

Figure 6. Simulation results of the proposed IDS model and the baseline CarSim®.

In Figure 6, simulation results of the proposed IDS-based vehicle model are compared with the baseline CarSim® results. With a full consideration of all factors contributing to the disturbances in terms of both impulsive and differentiable inputs (shown in Figure 5), the results obtained from the IDS model are close to the CarSim® results, compared with those of Models 1-4 in Figure 3 and Figure 4. The impulsive phase of the inputs causes sudden jumps in vehicle states at the tire blowout moment, as shown in Figure 6 (a) and (b). The vehicle states then change continuously with the smoothly varied inputs after tire blowout. In addition to vehicle states, e_y and e_ψ in Figure 6 (c) and (d) also closely match with those in the CarSim® simulation. In sum, compared with existing vehicle models for tire blowout, the proposed IDS control-oriented model is more accurate in representing tire blowout impacts on vehicle dynamics.

IV. Experimental Validations

The proposed IDS-based vehicle model is also validated through experimental results on a scaled test vehicle, as shown in Figure 7. The scaled test vehicle is Four-Wheel-Independently-Actuated (FWIA) with front and rear steering systems. The rear steering is locked in the experiments. To obtain precise global positions, the Swift Navigation® Piksi Multi GPS is equipped. A valve-based tire blowout device is developed with a remote controller, which can trigger tire blowout remotely. The parameters of the scaled test vehicle are also shown in Table 1, which are obtained through measurement and calculation using testing data.

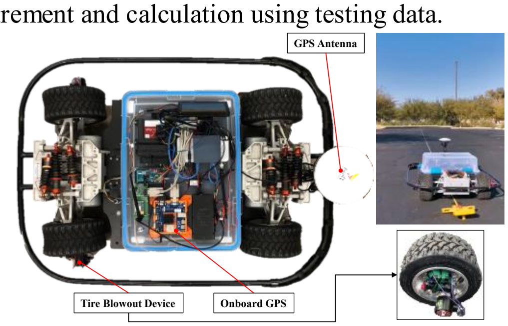

Figure 7. The scaled test vehicle with a tire blowout device in the test field.

By using the parameters of the scaled test vehicle, good accuracy of the previously proposed enhanced tire blowout model has been experimentally validated in [12]. Therefore, the tire blowout inputs in certain tire blowout experiments can be estimated offline, which are further utilized in the IDS control-oriented model. The simulation results of the IDS control-oriented model will be compared with the experimental result to validate the accuracy in describing the tire blowout impacts on vehicle dynamics.

The front left (FL) tire blowout experiment in a straight-line driving maneuver was first conducted at a constant driving speed of 5.2 m/s. The same tire blowout scenario and speed profile in the experiment was also utilized to obtain the inputs for the IDS model simulation. The vehicle trajectory comparison between the IDS model and experiment results is shown in Figure 8 (a). The baseline in Figure 8 (a) is the experimental result without tire blowout. The test vehicle deviated to the left when the front left tire blowout happened. Since the inputs for the IDS model simulation were estimated offline, the baseline of the IDS model is the straight black dot line. In this case, the final lateral offset compared with the baseline in the experiment is 0.45m and the predicted deviation from the proposed IDS control-oriented model is 0.36m. The difference is small (0.09m) along the 25-m traveling distance.

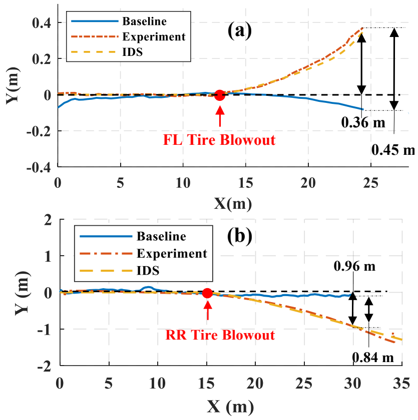

Figure 8. Validations of the proposed IDS model through tire blowout experiment results.

Similarly, the rear right (RR) tire blowout experiment was also conducted in the same driving maneuver with the same driving speed of 5.2 m/s. Simulation and experiment results were compared using the vehicle trajectories in the global coordinates, as shown in Figure 8 (b). In Figure 8 (b), the test vehicle deviated to the right side when the rear right tire blowout occurred. The final lateral offset of the test vehicle compared with the baseline (non-tire-blowout case) is 0.84m in the experiment, while the predicted deviation using the proposed IDS control-oriented model is 0.96m. The difference is still small (0.12m) along the 30-m traveling distance.

Through two different experiments, the proposed IDS control-oriented model effectively reflects the tire blowout impacts on vehicle dynamics. Since the existing vehicle models have certain fundamental issues as discussed in Sections I and III, those simulation results are not compared with the experimental result.

V. Conclusions

In this paper, an impulsive differential system (IDS) approach is applied to develop a novel control-oriented vehicle model for tire blowout, where the lateral force and moment are modeled as inputs with both impulsive and smooth variations due to tire blowout. Comparisons and validations of the proposed new model against existing vehicle models for tire blowout are conducted through both simulation and experimental results. The proposed IDS-based control-oriented model can more accurately describe tire blowout impacts on vehicle dynamics, which will benefit the model-based control design for tire blowout. The IDS-based control design and experimental evaluations will be presented in the future work.

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