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A Novel IDS-based Control Design for Tire Blowout

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Abstract: Tire blowout, as a hazardous and inevitable event, strongly affects vehicle driving stability. In such a critical situation, improper interventions from panicked human drivers may cause severer accidents. Therefore, a dedicated automatic control design needs to be properly developed for ground vehicles to ensure vehicle stability and path following performance after a tire blowout. In this paper, a novel automatic control design of tire blowout is proposed by using the Lyapunov stability theory for impulsive differential systems (IDS). Different from the existing control designs, the proposed controller can generate not only continuous (smooth) lateral force and yaw moment control efforts, but also impulsive yaw moment control efforts to overcome impulsive disturbances from tire blowout. Results of Matlab/Simulink and CarSim® co-simulation on a C-class vehicle indicate that, with the impulsive yaw moment control efforts, the vehicle direction can be quickly corrected and the path following performance can be largely improved after tire blowout.

Keywords: Tire blowout, impulsive differential system, vehicle stability control

1. INTRODUCTION

Tire blowout of ground vehicles has significantly challenged vehicle directional stability and handling performance (Choi, 2012), since tire properties and structure would instantaneously change due to the explosive and rapid loss of inflation pressure (Clark, 1981). When tire blowout occurs, additional lateral force and yaw moment are introduced, which is caused by the significant changes of tire friction forces between the normal tires and the blown-out tire(s). Without appropriate controls, the vehicle would rapidly deviate from the driving lane and may collide with other vehicles. Even worse, improper operations (e.g., excessive steering and/or massive braking) from panicked drivers, may lead to tire-rim separations or even vehicle rollovers (Tandy et al., 2013). Tire blowout cannot be completely prevented nowadays (Wang et al., 2016). A report from National Highway Traffic Safety Administration (NHTSA) estimated that tire blowout can cause more than 400 deaths and over 78,000 crashes every year (Davis, 2014). Given the adverse effects of tire blowout and mental and/or physical limits of human drivers in critical situations (Huang et al., 2019), it is crucial and necessary to propose dedicated automatic control designs for tire blowout.

To design an automatic controller for tire blowout, some simplified control-oriented vehicle models for tire blowout were adopted in the literature. A control-oriented model was considered as a two-degree-of-freedom (2-DoF) vehicle model perturbed by additional lateral force and yaw moment due to tire blowout (Patwardhan et al., 1997; Wang et al., 2016). Namely, the lumped effect of tire blowout was modeled to introduce additional lateral force and yaw moment on the vehicle center of gravity (CG), which deviated the vehicle from its driving lane, as shown in Figure 1. Moreover, a 2-DoF linear vehicle model perturbed by both parameter uncertainties (lateral stiffness) and moment disturbance (from increased rolling resistance coefficient) was also applied (Wang et al., 2015; Jing and Liu, 2019). Another 2-DoF linear vehicle model was adopted with only perturbed disturbances (no parameter uncertainties), including an additional steering angle generated by a varied but simplified self-alignment torque and a moment disturbance associated with the increased rolling resistance coefficient (Meng et al., 2019). By approximating the varied parameters, force, and/or moment disturbances as smooth and differentiable signals, the aforementioned control-oriented vehicle models consist of normal nonlinear or linear ordinary differential equations (ODEs).

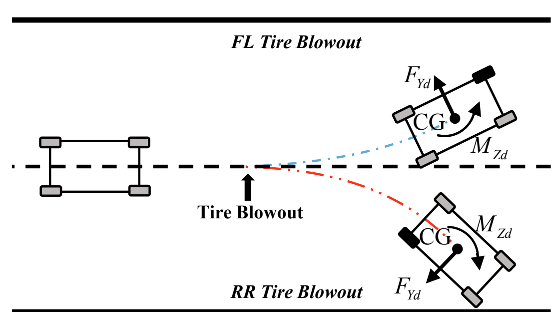

Figure 1. Vehicle deviation due to additional lateral force and yaw moment introduced by tire blowout.

Based on the aforementioned vehicle models for tire blowout, different model-based control designs were proposed to control vehicle motions after tire blowout. For example, a coordinated active front steering (AFS) and differential braking control scheme was proposed to enhance vehicle directional stability by using a triple-step nonlinear control

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method (Wang et al., 2016). Considering the parameter uncertainties and moment disturbance introduced by tire blowout, a coordinated control approach with AFS and differential braking was developed based on a constrained H∞ method to ensure path following performance after tire blowout (Wang et al., 2015). A speed-dependent gain-scheduling H∞ control strategy was designed to maintain driving stability through both AFS and differential braking (Jing and Liu, 2019). A finite-time output feedback controller was designed to promptly stabilize an electric vehicle after tire blowout (Meng et al., 2019). A directional stability control was proposed by using a robust tube-based model predictive control (MPC) approach for an over-actuated electric vehicle (Yang et al., 2019). Since all of the tire blowout controllers (though different control methods) were designed based on vehicle models consisting of normal ODEs, the obtained control efforts were all smooth and differentiable.

Considering that a typical tire blowout process is intensive and extremely short (e.g., 0.1 seconds (Blythe, et al., 1998)), both additional lateral force (F_{Yd}) and yaw moment (M_{Zd}) will first have impulsive and non-differentiable variations and then change smoothly. Correspondingly, the vehicle states (lateral velocity v_y and yaw rate r) would suddenly jump (impulsive and non-differentiable) and then vary smoothly, as shown in Figure 2. For the same physical processes, the impulsive variations of the force, moment, and vehicle states can also be observed for blowouts at other tire locations and during different maneuvers (e.g., cornering), different from the cases in Figure 2. The aforementioned impulsive phenomenon due to tire blowout cannot be described by normal ODEs with continuous and differentiable solutions. Therefore, in the authors' recent work (Li et al., 2021), a new control-oriented vehicle model was proposed through an impulsive differential system (IDS) approach, in which an impulsive equation was developed to effectively depict the abrupt changes of vehicle states when tire blowout happens. Through both simulation and experimental validations, the proposed IDS-based control-oriented model can more accurately describe tire blowout impacts on vehicle dynamics, compared with existing ODE models in the literature.

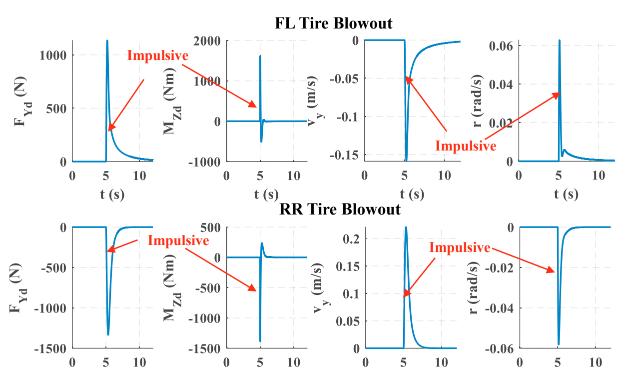

Figure 2. Vehicle lateral force, yaw moment, and state variations for a front left (FL) and rear right (RR) tire blowout in a straight-line driving maneuver at 100 km/h.

In this paper, based on the developed IDS-based control-oriented vehicle model, the Lyapunov stability theory for IDS (Lakshmikantham et al., 1989; Yang, 2001) is utilized to design a novel automatic controller for tire blowout. Although the Lyapunov stability theory was extended to IDS in the applications of other research areas, such as swing-up control of a pendubot (Jafari et al., 2015), chemistry (Bainov and Simeonoy, 1989), and medicine (Choisy et al., 2006), the application to ground vehicles for a specific physical phenomenon (e.g., tire blowout) is never discussed in the literature. The proposed controller can generate not only continuous and smooth lateral force and yaw moment control efforts, but also impulsive yaw moment control efforts at certain instants to timely correct the vehicle driving direction after tire blowout. Matlab/Simulink and CarSim® co-simulation results are shown to validate that the proposed controller can achieve better directional stability and path following performance after tire blowout, compared with the one that only generates continuous and smooth control efforts.

The rest of this paper is organized as follows. In Section 1, the models for controller design, including the IDS-based control-oriented vehicle model and a path following reference model, are described. The novel IDS-based automatic control design of tire blowout is developed based on Lyapunov theory in Section 3. In Section 4, simulation results and discussions are presented to demonstrate the enhanced directional stability and path following performance in a tire blowout situation. Conclusions and future work are described in Section 5.

2. MODELS FOR CONTROLLER DESIGN

In this section, the IDS-based control-oriented vehicle model and path following reference model for controller design are described in subsections 2.1 and 2.2, respectively.

2.1 IDS-based Control-Oriented Vehicle Model

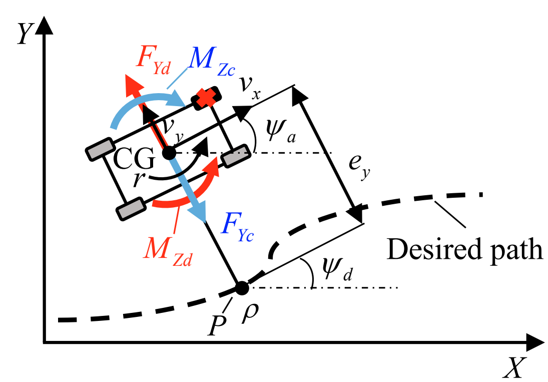

Figure 3. IDS-based control-oriented model and path following reference model.

As shown in Figure 3, the IDS-based control-oriented vehicle model, which consists of continuous differential equation and impulsive equations, is formulated as follows,

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -v_x r \\ 0 \end{bmatrix} + \begin{bmatrix} F_{Yc}/m \\ M_{Zc-c}/I_z \end{bmatrix} + \begin{bmatrix} F_{Yd-c}/m \\ M_{Zd-c}/I_z \end{bmatrix}, \; t \neq t_b, t_c, \tag{1}$$

$$\begin{cases} \Delta v_y = v_y(t_b^+) - v_y(t_b) = -v_x \Delta r \Delta t + \dfrac{F_{Yd-i}}{m}\Delta t \\ \Delta r = r(t_b^+) - r(t_b) = \dfrac{M_{Zd-i}}{I_z}\Delta t \end{cases}, \; t = t_b, \tag{2}$$

$$\begin{cases} \Delta v_y = v_y(t_c^+) - v_y(t_c) = -v_x \Delta r \Delta t \\ \Delta r = r(t_c^+) - r(t_c) = \dfrac{M_{Zc-i}}{I_z} \Delta t \end{cases}, \ t = t_c, \qquad (3)$$

where m is the total mass of the vehicle and I_z is the moment inertia about the center of gravity (CG). v_x and v_y denote longitudinal and lateral velocities, respectively. r represents the yaw rate. F_{Yc} is the continuous lateral force control effort applied on the CG. M_{Zc-c} and M_{Zc-i} denote the continuous and impulsive yaw moment control efforts applied on the CG, respectively. F_{Yd-c} / F_{Yd-i} and M_{Zd-c} / M_{Zd-i} represent the continuous/impulsive phase of the additional lateral force and yaw moment (disturbances) introduced by tire blowout, respectively. Tire blowout is assumed to be detectable (e.g., by tire pressure sensors) in this work, which happens at a certain time instant t_b and finishes at t_b^+ with an extremely short duration Δt ($t_b^+ = t_b + \Delta t$). t_c is the time instant(s) when M_{Zc-i} is applied. The applied duration for M_{Zc-i} is chosen as the same as Δt .

As shown in the impulsive equation (2), when tire blowout occurs at t_b, the impulsive phase of the disturbances, F_{Yd-i} and M_{Zd-i}, will cause sudden jumps in vehicle states v_y and r. After t_b, as shown in the normal ODE (1), F_{Yc} and M_{Zc-c} are applied to not only overcome the continuous phase of the disturbances F_{Yd-c} and M_{Zd-c}, but also ensure the vehicle can follow a desired path. At certain time instant(s) t_c, as shown in the impulsive equation (3), M_{Zc-i} is applied to impulsively change r and thus the vehicle driving direction can be quickly corrected. Note that the IDS-based control-oriented vehicle model in the authors' recent work (Li et al., 2021) mainly focused on accurately describing the tire blowout effects (especially the impulsive effect) on vehicle dynamics. The corresponding control design was not presented and control efforts were not included in the model. On that basis, control efforts (F_{Yc} , M_{Zc-c} , and M_{Zc-i}) are considered in this work for control design, as shown in (1)-(3).

Remark 1: Lateral force and yaw moment control efforts applied on the CG, F_{Yc} , M_{Zc-c} and M_{Zc-i}, can be generated from Active Front Steering (AFS) and Direct Yaw Moment Control (DYC), respectively. The M_{Zc-c} and M_{Zc-i} can be realized by applying traction or braking torques to each wheel. Considering the quick response and powerful capability of the braking system, the impulsive yaw moment control effort M_{Zc-i} is assumed to be achievable. Note that this work mainly focuses on the high-level control design. The control efforts (high-level) are directly applied to a high-fidelity tire blowout model integrated with CarSim®, which will be described in the simulation section, to preliminarily demonstrate the enhanced control performance from the impulsive yaw moment control effort. The real-time realization of the control efforts and the practical implementation of the impulsive yaw moment control effort will be considered in the future work.

Moreover, the impulsive and continuous disturbances due to tire blowout are assumed to be known in this work. The introduced disturbances from one tire blowout location in a straight-line driving maneuver should have similar vehicle impacts for different driving speeds. If the magnitudes and patterns of the disturbances for one driving speed are calibrated and stored offline, the disturbances for other driving speeds can be similarly obtained. The same principle can be applied to one tire blowout location in the cornering maneuver with different steering angles (scaling with steering angle).

2.2 Path Following Reference Model

The automatic controller for tire blowout aims to overcome the disturbances introduced by tire blowout and ensure a vehicle to follow the desired path, as shown in Figure 3. In Figure 3, ρ is denoted as the curvature at the closest point *P* on the desired path. e_y represents the lateral offset, which is the distance from vehicle CG to point *P*. e_ψ is the heading error, which is defined as the difference between the actual vehicle heading ψ_a and the desired heading ψ_d (i.e., $e_\psi = \psi_a - \psi_d$). ψ_d is along the tangential direction of the desired path at point *P*. A path following reference model (Wang et al., 2019; Huang et al., 2020) is adopted to generate the desired lateral velocity v_{yd} and yaw rate r_d , as described in (4),

$$\begin{cases} v_{yd} = 0 \\ r_d = \rho v_x - k_2 \left(e_\psi + k_1 e_y \right) \end{cases}, \qquad (4)$$

where k_1 and k_2 are two positive constants and $k_2 > k_1 v_x$.

By tracking the desired values of v_y and r in (4), e_y and e_ψ can be asymptotically stabilized to follow the desired path (Wang et al., 2019; Huang et al., 2020).

3. IDS-BASED CONTROL DESIGN FOR TIRE BLOWOUT

Based on the IDS-based control-oriented vehicle model in (1)-(3) and the path following reference model in (4), the novel IDS-based control design of tire blowout is developed using the Lyapunov stability theory for IDS (Lakshmikantham et al., 1989; Yang, 2001).

Proposition: The continuous control efforts F_{Yc} and M_{Zc-c} in (5), together with impulsive moment control effort M_{Zc-i} in (6) can asymptotically stabilize the tracking errors of two vehicle states in (1).

$$\begin{cases} F_{Yc} = m\left[\dot{v}_{yd} + v_{yd} - v_y + v_x r - F_{Yd-c} \right] \\ M_{Zc-c} = I_z \left[\dot{r}_d + r_d - r - M_{Zd-c} \right] \end{cases}, \ t \neq t_c. \qquad (5)$$

$$M_{Zc-i} = \frac{-2I_z \left[\left(r - r_d \right) + p\left(v_y - v_{yd} \right) \right]}{\left(1 + p^2 \right) \Delta t}, \ t = t_c, \qquad (6)$$

where $p = -v_x \Delta t$.

Proof: The tracking errors are defined in (7),

$$e = x - x_d = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} v_y - v_{yd} \\ r - r_d \end{bmatrix}. \quad (7)$$

A (positive definite) candidate Lyapunov function is defined as,

$$V = \frac{1}{2}(e_1^2 + e_2^2) > 0. \quad (8)$$

The following proof will be categorized into two parts. One is for the continuous system at $t \neq t_c$ and the other is for the impulsive part at $t = t_c$. When $t \neq t_c$, the derivative of (8) is derived as,

$$\dot{V} = \dot{e}_1 e_1 + \dot{e}_2 e_2. \quad (9)$$

By selecting

$$\dot{e}_1 = -e_1,\ \dot{e}_2 = -e_2, \quad (10)$$

Equation (9) will become negative definite,

$$\dot{V} = -(e_1^2 + e_2^2) < 0, \quad (11)$$

which guarantees that e_1 and e_2 will approach to zero asymptotically.

Note that equation (10) can be rewritten as,

$$\begin{bmatrix} \dot{v}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dot{v}_{yd} + v_{yd} - v_y \\ \dot{r}_d + r_d - r \end{bmatrix}. \quad (12)$$

Substitute (12) into (1), we have,

$$\begin{bmatrix} \dot{v}_{yd} + v_{yd} - v_y \\ \dot{r}_d + r_d - r \end{bmatrix} = \begin{bmatrix} -v_x r \\ 0 \end{bmatrix} + \begin{bmatrix} F_{Yc} / m \\ M_{Zc-c} / I_z \end{bmatrix} + \begin{bmatrix} F_{Yd-c} / m \\ M_{Zd-c} / I_z \end{bmatrix}. \quad (13)$$

By rearranging both sides of (13), the continuous control efforts F_{Yc} and M_{Zc-c} in (5) can be obtained.

When $t = t_c$, by denoting $U = (M_{Zc-i}\Delta t) / I_z$, equation (3) can be rewritten as,

$$\begin{cases} \Delta r = U \\ \Delta v_y = -v_x \Delta r \Delta t = pU \end{cases},\ t = t_c. \quad (14)$$

Based on Corollary 4.1.1 (Yang, 2001), the asymptotic stability requires $V(e(t_c^+)) \le V(e(t_c))$ for the impulsive part. $V(e(t_c^+))$ is written as,

$$\begin{aligned} V(e(t_c^+)) &= \frac{1}{2}(v_y(t_c^+) - v_{yd})^2 + \frac{1}{2}(r(t_c^+) - r_d)^2 \\ &= \frac{1}{2}(v_y + \Delta v_y - v_{yd})^2 + \frac{1}{2}(r + \Delta r - r_d)^2 \\ &= \frac{1}{2}(v_y + pU - v_{yd})^2 + \frac{1}{2}(r + U - r_d)^2 \\ &= \frac{1}{2}(v_y - v_{yd})^2 + \frac{1}{2}(r - r_d)^2 \\ &\quad + pU(v_y - v_{yd}) + \frac{1}{2}p^2U^2 + U(r - r_d) + \frac{1}{2}U^2 \\ &= \frac{1}{2}e_1^2(t_c) + \frac{1}{2}e_2^2(t_c) \\ &\quad + pU(v_y - v_{yd}) + \frac{1}{2}p^2U^2 + U(r - r_d) + \frac{1}{2}U^2. \end{aligned} \quad (15)$$

Comparing (8) with (15), to satisfy $V(e(t_c^+)) \le V(e(t_c))$, we have,

$$pU(v_y - v_{yd}) + \frac{1}{2}p^2U^2 + U(r - r_d) + \frac{1}{2}U^2 \le 0. \quad (16)$$

Therefore, if $U \le 0$, we have,

$$\frac{-2\left[(r - r_d) + p(v_y - v_{yd})\right]}{1 + p^2} \le U \le 0. \quad (17)$$

If $U \ge 0$, we have,

$$0 < U \le \frac{-2\left[(r - r_d) + p(v_y - v_{yd})\right]}{1 + p^2}. \quad (18)$$

By selecting $U = -2\left[(r - r_d) + p(v_y - v_{yd})\right] / (1 + p^2)$, the impulsive yaw moment control effort in (6) can be obtained. ■

Since the IDS-based vehicle model consists of a continuous differential equation and impulsive equations, it is important to note that two criteria are applied to ensure the asymptotical stability of the IDS, as described in (11) and (16), respectively. Specifically, the continuous control efforts F_{Yc} and M_{Zc-c} in (5) ensure two tracking errors can be asymptotically stabilized by making the Lyapunov function (8) decreasing. At the time instants (t_c) when the impulsive control effort M_{Zc-i} is applied, the Lyapunov function (8) subject to the impulsive states due to M_{Zc-i} is obtained, as shown in (15). By making such a Lyapunov function non-increasing (as shown in (16)) at the impulsive control instants t_c, the asymptotical stability of two tracking errors can also be guaranteed (Yang, 2001).

Remark 2: As discussed in Section 1, the existing control-oriented vehicle model (Patwardhan et al., 1997; Wang et al., 2016) perturbed by smooth (differentiable) lateral force and yaw moment from tire blowout can be presented in (1). The corresponding control design equivalently follows the derivation in (7)-(13) to only generate the smooth and differentiable control efforts as shown in (5). In contrast, the control design in this work is based on the IDS model in (1)-(3) and Lyapunov stability theory for IDS, which can generate not only continuous (smooth) control efforts but also impulsive yaw moment control efforts (in (6)) at certain time instants. In the simulation section, the path following performance of the aforementioned two control designs will be compared to demonstrate the benefits from the impulsive yaw moment control effort.

4. SIMULATION RESULTS AND ANALYSES

In this section, co-simulation between Matlab/Simulink and CarSim® is conducted to validate the effectiveness of the proposed controller for tire blowout. The parameters of the adopted C-Class hatchback vehicle in CarSim® are shown in TABLE 1. The final values of tire parameters in TABLE 1 are utilized to characterize tire properties after tire blowout. In the authors' previous work (Li et al., 2020), an enhanced tire blowout model considering self-alignment torque (SAT) variations and a two-stage vertical load redistribution was proposed. With high model accuracy in describing tire blowout impacts on vehicle dynamics, the tire blowout model is used together with CarSim® for control performance evaluation. The simulation framework is shown in Figure 4. By implementing the control efforts defined in (5) and (6), the directional stability and path following performance of the proposed controller can be investigated.

TABLE 1. Parameters of the C-class hatchback vehicle

Parameter	Symbol	Value	Final value
Vehicle Parameters			
Vehicle total mass	m	1412 kg	-
Moment inertia around CG	I_z	1536.7 kg-m²	-
Distance from front axle to CG	l_f	1.105 m	-
Distance from front axle to CG	l_r	1.895 m	-
Wheel track	l_t	1.675 m	-
Front suspension stiffness	K_{sf}	27000 N/m	-
Rear suspension stiffness	K_{sr}	30000 N/m	-
Tire Parameters			
Tire effective radius	R_e	0.325 m	To 2/3
Longitudinal stiffness of each tire	C_x	47000 N/slip	To 1/10
Lateral stiffness of each tire	C_y	-55000 N/rad	To 1/10
Wheel moment inertia	I_w	0.9 kg-m²	-
Rolling resistance coefficient	K_r	0.018	To 30 times

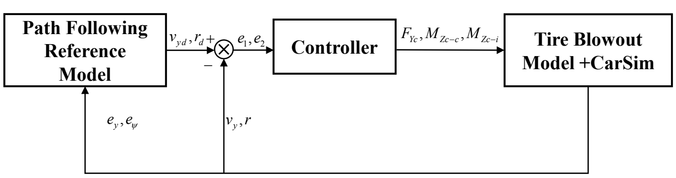

Figure 4. Simulation framework.

In the simulation, the vehicle is driven in a straight line at a constant driving speed of 100 km/h (62 mph) and the front left tire blowout occurs at the 5th second of the total 12-second simulation time. The parameters in the path following reference model (4) are selected as $k_1 = 3/v_x$ and $k_2 = 30k_1$ (Wang et al., 2019; Huang et al., 2020). The aforementioned two control designs are denoted as Controller 1 (with impulsive control, (5) and (6)) and Controller 2 (without impulsive control, only (5)), respectively. The disturbances introduced by tire blowout in (1) and (2) are assumed to be known, which are obtained through open-loop simulation without any control by using the tire blowout model. The simulation results are shown in Figure 5-Figure 7.

As shown in Figure 5, the path following performance of the proposed Controller 1 is significantly improved given the largely reduced lateral offset e_y and heading error e_ψ, compared with those of Controller 2. Considering the lane boundary and the vehicle width, the proposed Controller 1 can effectively stabilize the vehicle within the driving lane by the applied impulsive yaw moment control efforts. However, without impulsive control, the vehicle body deviates out of the driving lane with Controller 2. The control efforts are shown in Figure 6. The proposed Controller 1 can generate not only continuous (smooth) control efforts, but also impulsive yaw moment control effort, which is applied 5 times at equidistant instants in a short period when the vehicle is about to reach the maximum heading error e_ψ. In contrast, Controller 2 can only generate continuous (smooth) control efforts. Since the impulsive yaw moment control effort in Figure 6 (b) is directly and frequently applied, the chattering phenomenon is observed, which can be handled by considering the real-time realization and practical implementation of the control efforts in the future work. The tracking errors are shown in Figure 7. When the impulsive yaw moment control effort is applied, the vehicle heading can be corrected faster with a large negative yaw rate. Correspondingly, the yaw rate tracking error e_2 of the proposed Controller 1 is reduced sharply and more quickly as shown in Figure 7 (b). With such a yaw rate, the lateral velocity tracking error e_1 of the proposed Controller 1 is slightly larger than that of Controller 2 as shown in Figure 7 (a). In sum, by producing both continuous and impulsive control efforts, the proposed controller can largely improve path following performance and ensure vehicle directional stability after tire blowout.

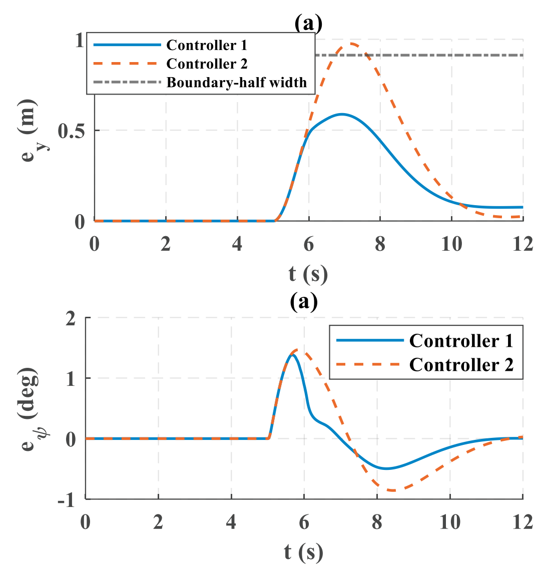

Figure 5. Lateral offsets and heading errors in the simulation.

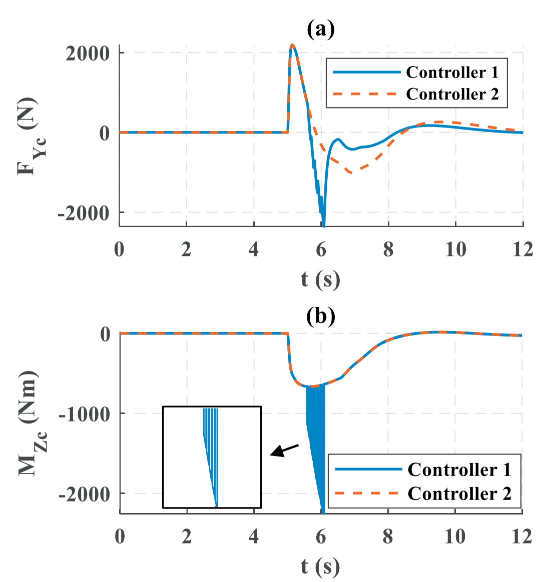

Figure 6. Control effects in the simulation.

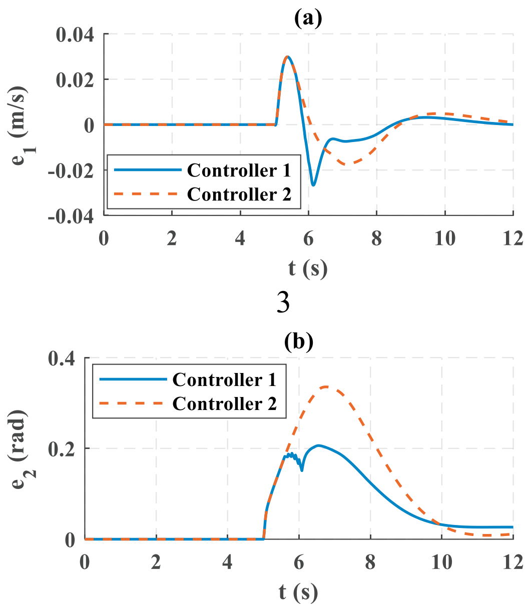

Figure 7. Tracking errors in the simulation.

5. CONCLUSIONS

In this work, a novel IDS-based automatic control design was developed to achieve vehicle path following after tire blowout. The proposed controller can generate not only continuous (smooth) lateral force and yaw moment control efforts, but also an impulsive yaw moment control effort at certain time instants. The effectiveness of the proposed controller was validated through Matlab/Simulink and CarSim® co-simulation on a C-class hatchback vehicle. With the impulsive yaw moment control, the proposed controller can effectively ensure vehicle directional stability and largely improve path following performance after tire blowout. The real-time realization and practical implementation of the control efforts, and experimental validations of the proposed controller using a scaled test vehicle will be included in the future work.

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