Source independent velocity recovery using imaginary FWI

Introduction

Full Waveform Inversion (FWI) has been widely used for seismic velocity inversion under active seismic data. The usage of FWI under passive seismic data is much more challenging, since aside from the velocity, the source location and activation time are also unknown. Passive seismic data refers to earthquake data collected in global seismic survey and microseismic data collected during hydraulic fracturing of unconventional reservoirs. The former is used for earthquake detection as well as subsurface imaging, and the latter is used for crack monitoring and hazard control in hydraulic fracturing.

A rich literature studied the fix-one-recover-the-other situation. Assuming source is known, velocity can be recovered by FWI; assuming velocity is known, source can be recovered by back-propagation (Ishii et al. 2005), time-reversal (Larmat et al. 2006), or sparsity-promoting based least-squares fitting (Sharan et al. 2019). However, with passive data where the accurate knowledge of both are absent, such reconstructions become unstable.

Recently, joint recovery approaches have been proposed (Wang et al. 2018; Sun et al. 2016), where source and velocity are alternatively updated. In each iteration, the velocity is updated via FWI based on the currently estimated source and then the source's timing and location are updated based on the currently estimated velocity using back-propagation. As shown in (Wang et al. 2018), provided that the sources are point sources that are separable in the seismic trace, the alternating procedure can greatly improve the source estimation upon the initial one, even when the initial velocity is rough.

The assumption of separable point sources made in the existing paper is somewhat strong, as both earthquake and micro-seismic events are triggered by cracks, which are more like line singularities or small regions consisting of lots of point sources very close to each other in time and in space. In this paper, we propose a source-independent velocity reconstruction method that imposes no requirement on the source spatial distribution, but we do require the source activation time to be brief. More precisely, the method recovers the source activation time along with the velocity using a modified FWI approach that only matches the imaginary part of a phase shifted version of the Fourier data. In the following, we focus our illustration on the velocity recovery, as once the velocity is recovered, one can recover the source using standard approaches. The acquisition requirement of our approach is full boundary data and its normal derivative, under which theoretical uniqueness for the joint recovery problem has been proved for radially symmetric medium (Finch and Hickmann 2013) and medium with certain orthogonality relation (Liu and Uhlmann 2015; Knox and Amir 2020). It is still unknown whether or not the joint source-velocity recovery problem has a unique solution in general media or with partial data. Thus here we assume the availability of full boundary data and defer the partial data case to future study.

Method and Theory

The proposed method. Consider the following wave equation

$$\begin{cases} m(x)\partial_t^2 u(x,t) - \Delta u(x,t) = m(x) f(x)\delta(t-t_0), & (x,t) \in \mathbb{R}^d \times (t_0,\infty), \\ u(x,t) = 0, & t < t_0,\ x \in \mathbb{R}^d, \end{cases} \tag{1}$$

where the real-valued spatial source distribution $f(x)$, the source activation time $t_0 > 0$ and the slowness square $m(x)$ are all unknown quantities. Our goal is to recover $m(x)$ using the wavefield and its normal derivative measured on the boundary $\partial\Omega$ of the domain of interest Ω,

$$u(x_r,t) = d_{obs}(x_r,t), \quad \partial_n u(x_r,t) = g_{obs}(x_r,t), \quad \text{for } x_r \in \partial\Omega,$$

where n is the unit outer normal vector field. During discretization, this boundary data has to be interpolated on the boundary grid. By taking the temporal Fourier-Laplace transform $\hat{u}(x,k) = \frac{1}{2\pi}\int_0^\infty u(x,t)e^{ikt}\,dt$, the wave equation becomes the Helmholtz equation:

$$\Delta\hat{u}(x,k) + k^2 m(x)\hat{u}(x,k) = -\frac{e^{ikt_0}}{2\pi} f(x)m(x) \qquad \text{in } \mathbb{R}^d, \qquad k \in (0,\infty) \tag{2}$$

subject to the outgoing Sommerfeld radiation condition $|x|^{\frac{d-1}{2}}(\frac{\partial}{\partial|x|}-ik)\hat{u}\to 0$ as $|x|\to\infty$. Solving (2) directly is challenging due to the presence of the unknown spatial source distribution $f(x)$. The key observation is that if we define $\hat{U} := \Im(\hat{u})e^{-ikt_0}$ where $\Im$ takes out the imaginary part of the input, then $\hat{U}$ satisfies the source-independent Helmholtz equation

$$\begin{aligned}&\Delta\hat{U}(x,k)+k^2m(x)\hat{U}(x,k)=0. \quad (x,k)\in\Omega\times(0,\infty),\\&\hat{U}(x_r,t)=\Im(\hat{d}_{obs}(x_r,k)e^{-ikt_0}), \quad \partial_n\hat{U}(x_r,t)=\Im(\hat{g}_{obs}(x_r,k)e^{-ikt_0}), \quad x_r\in\partial\Omega.\end{aligned} \tag{3}$$

This system now only involves the unknown slowness $m(x)$ as well as the unknown source activation time t_0 through the term e^{-ikt_0}, which is only a complex scalar. Therefore, we can form an FWI-like optimization on the concatenated variable $[m(x),e^{-ikt_0}]$ to look for the minimizer.

Optimization. Let $H(m)$ be the discrete Helmholtz operator applied to the interior points of Ω, $P_{\partial\Omega}$ be the restriction operator to the boundary grids, V, D_{obs}, G_{obs} be the discretized $\hat{U}$, d_{obs} and g_{obs}, respectively. In addition, let L be the discrete normal derivative operator on the boundary. Specifically, if M is the number of boundary grid points and N is the total number of grid points within Ω, then we formulate the normal derivative operator L as an $N\times N$ matrix with only M nonzero rows, each of which takes the normal derivative of the wavefield at the boundary grid. Then equation (3) is discretized to

$$H(m)V=0, \quad P_{\partial\Omega}V=\Im(e^{-ikt_0}D_{obs}), \quad LV=P^*_{\partial\Omega}\Im(e^{-ikt_0}G_{obs}) \tag{4}$$

where $P^*_{\partial\Omega}$ is the transpose of $P_{\partial\Omega}$. We add the first to the third equation in (4) to get

$$(H(m)+L)V=P^*_{\partial\Omega}\Im(e^{-ikt_0}G_{obs}). \tag{5}$$

Since $H(m)$ applies to the interior points and L applies to the boundary points, $H(m)+L$ is an invertible operator, and we denote it as $\tilde{H}(m):=H(m)+L$. Solving for the wavefield V from (5) and matching the resulting $P_{\partial\Omega}V$ to the boundary data (i.e., second equation in (4)), we obtain the least squares fitting problem

$$\min_{m,t_0}\|P_{\partial\Omega}V-\Im(e^{-ikt_0}D_{obs})\|_2^2=\min_{m,t_0}\|P_{\partial\Omega}\tilde{H}^{-1}(m)P^*_{\partial\Omega}\Im(e^{-ikt_0}G_{obs})-\Im(e^{-ikt_0}D_{obs})\|_2^2\equiv F(m,t_0) \tag{6}$$

The slowness square $m(x)$ can then be reconstructed by solving (6) using LBFGS. The variable is the concatenated $[m,t_0]$ and the inversion can be performed sequentially on incremental frequency batches. The derivative of the objective function with respect to m is the same as that in the normal FWI and that with respect to t_0 is

$$\frac{\partial F}{\partial t_0}=k\left\langle (R^2),\sin(2kt_0-2\Theta)\right\rangle,$$

where the vectors R and Θ are consisted of the element-wise absolute value and the element-wise phase of the vector $P_{\partial\Omega}\tilde{H}^{-1}(m)P^*_{\partial\Omega}G_{obs}-D_{obs}$, respectively.

Numerical Simulation

For validation, the proposed method is applied to the Marmousi model and the Camembert model. All simulations are performed in the frequency domain. In the first experiment, we show that the proposed method works for super-positions of point sources with small separations. For that, we perform velocity inversion on the Marmousi model with small within-group source spacing and large inter-group source spacing. Explicitly, three groups of point sources (denoted by different colors in Fig. 1(a)) are placed at the depth of 1.12km. Sources within each group are close to each other (with a spacing of 50m) and are activated simultaneously. The corresponding three data records for the three simultaneous sources are used together for the velocity inversion. Specifically, we first estimate the timing t_0 of each simultaneous source by solving the optimization (6) using data at the lowest 2 frequencies. Then we fix the t_0s and only update the velocity using data at the rest of the frequencies. The computational domain is 2.2 km $\times$ 6.6 km with a grid spacing of 10 m in both directions. Receivers are densely located surrounding the computational domain, as is shown in Fig. 1(a). We use the Gaussian-filter-smoothed Marmousi model for velocity initialization, as is shown in Fig. 1(b). Each frequency sweep starts from 0.5 Hz to 5 Hz and two sweeps are implemented. Each frequency batch contains two frequencies that differ by 0.25 Hz. We see that the final velocity reconstruction by the proposed method (Fig. 1(c)) with unknown source information and only imaginary data is almost as good as the ordinary FWI reconstruction (Fig. 1(d)) with known source and full (i.e., real and imaginary) data under the same setup.

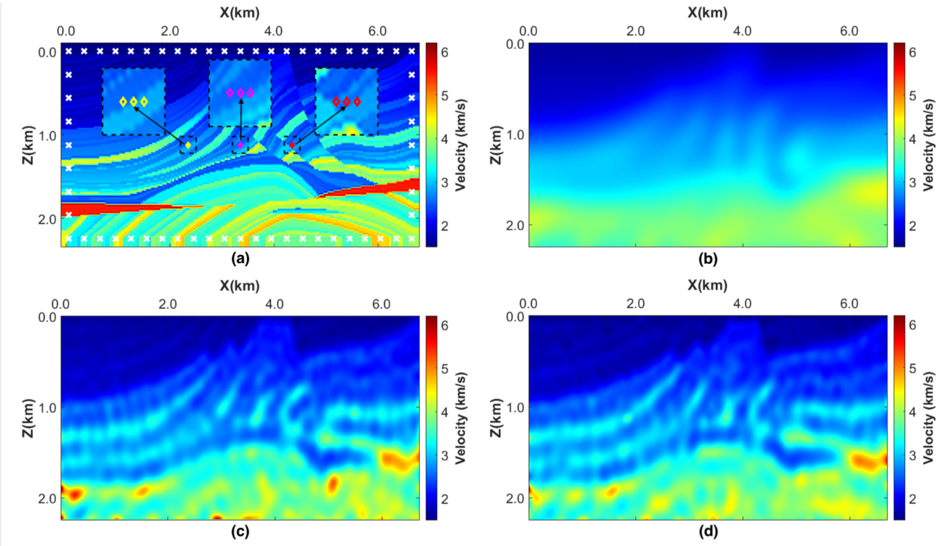

Figure 1: (a) True velocity and sources (in yellow, pink, and red diamonds with a 1:4 zoomed in version)/receivers (white cross) distribution; (b) Initial velocity; (c) Reconstructed velocity via the proposed imaginary FWI with unknown sources; (d) Reconstructed velocity via the ordinary FWI with known sources.

Next, we demonstrate that the proposed method works for non-point sources using the Camembert model. In particular, we simulate a total of 5 groups of sources, 3 of which lie along line segments with different slopes, and another 2 lie in small disks (Fig. 2.(a)). All sources within one geometric shape (e.g.,within one group) fire simultaneously and 5 different activation times are used for the five groups. The computational domain is 2×2 km, with a grid spacing of 10 m in both directions. We use the background velocity (1 km/s everywhere) as the initial guess, which makes the inversion difficult. Nevertheless, the inversion result via the proposed method using the 0.5Hz to 5Hz data and unknown sources seems quite accurate (Fig. 2.(c)), so are the reconstructed source timings (Table. 1). To further confirm this observation, we use the reconstructed velocity and source timing to solve for the underlying sources via a simple least squares fitting (one Helmholtz solve). Fig. 2.(d) shows the reconstructed sources' spatial supports, from which we observe that the all sources are correctly detected with shapes that coincide well with the correct ones (indicated by black dashed lines).

Velocity Model	True Time t_0	Initialized Time	Estimated Time
Marmousi (3 groups of sources)	1.44/1.00/0.57	0/0/0	1.43/0.98/0.56
Camembert (5 groups of sources)	2.0/1.0/2.7/0.32/1.95	0/0/0/0/0	1.99/0.99/2.68/0.31/1.93

Table 1: Firing time reconstruction for the simultaneous sources

Conclusions and Future Directions

We introduced a source-independent velocity inversion approach, which has the same complexity as the ordinary FWI while requiring no knowledge of the source. Unlike existing methods, our method gets rid of the assumption that the source is a superposition of point sources, thus applying to more general source settings. Admittedly, the current full boundary measurements requirement is a bit strong, here we provide some thoughts on how it might be relaxed. With no essential change of the method, the full-boundary measurements assumption can be replaced by a partial boundary one except that we need several layers of measurements at different depths, and the latter may be computed from measurements

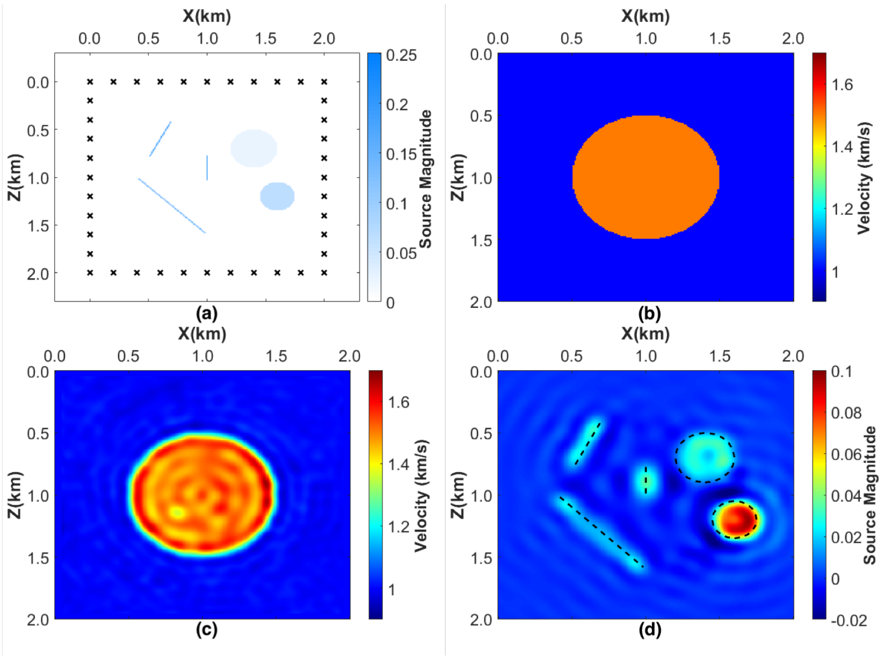

Figure 2: (a) Source (Simultaneous sources 1-3: blue lines, Simultaneous source 4-5: blue disks, darker color means larger magnitude) and receivers (black cross) distribution; (b) True velocity; (c) Reconstructed velocity from using the background velocity (1km/s) as initial guess; (d) Reconstructed source from reconstructed velocity

at a single depth by extrapolating it to other depths via wavefield extrapolation techniques.

Acknowledgements

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