

SUPPORTING CONNECTIONS TO TEACHING IN AN UNDERGRADUATE CALCULUS COURSE

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The Mathematical Education of Teachers II report by the Conference Board of the Mathematical Sciences (2012) recommends that undergraduate programs enhance prospective secondary mathematics teachers' (PSMTs) understanding of connections between the advanced undergraduate mathematics content and the mathematics they will teach. This paper examines the connections to teaching made by one instructor and one undergraduate PSMT after implementation of two calculus lessons aimed at supporting connections to teaching. Each lesson embedded approximations of practice tasks in the learning of calculus content. Findings suggest that these lessons enabled both deepening mathematical content knowledge and insight into the work of teaching.

Calculus is a gateway course for science, technology, engineering, and mathematics majors and is part of every PSMTs' program of study. It is quite common to embed applications to physics, biology, or chemistry in calculus courses, but despite recommendations from the Conference Board of the Mathematical Sciences' (CBMS) Mathematical Education of Teachers II (MET II) report (CBMS, 2012), applications to teaching that make explicit connections between advanced content in mathematics to mathematics taught in secondary schools are much less common (e.g., Lai & Patterson, 2017). As the MET II report encourages making connections to teaching throughout a prospective teacher's mathematics program of study, the *Mathematical Education of Teachers as an Application of Undergraduate Mathematics* (META Math) project's initial focus has been on developing lessons in calculus, discrete mathematics, abstract algebra, and statistics, typical undergraduate mathematics courses in which PSMTs enroll.

For each course, META Math developed two lessons focused on building connections to teaching via applications of mathematics to teaching and collected data on implementation at over 14 universities nationwide.

As part of a larger study, this paper explores one instructor's experiences implementing two META Math lessons in calculus and one PSMT's experiences with the lessons. We focus on how the lessons support instructors' and undergraduates' awareness of connections to teaching. We explore the following research question: In what ways does the use of applications to teaching in calculus support an instructors' and an undergraduates' awareness of the role of connections to teaching in a mainstream mathematics course?

Background and Theoretical Perspective

Mathematical Knowledge for Teaching (MKT) was studied by Ball et al. (2008) as being “the mathematical knowledge needed to carry out the work of teaching mathematics” (p. 395). While the MET II (CBMS, 2012) recommends that future teachers make connections between advanced and school mathematics throughout their mathematics program, there are few studies that show that future teachers are making these connections in their undergraduate mathematics programs. Wasserman (2018) found that “teachers and their students appear to gain little from a teacher’s study of advanced mathematics” (p. 4). Other researchers have noted that PSMTs have found the advanced content to be disconnected to what they will one day teach, or even that they do not understand the foundational concepts at a deep enough level to teach them (e.g., Goulding et al., 2003; Wasserman et al., 2018).

For university mathematics instructors, identifying and using appropriate resources poses unique challenges as many are unfamiliar with MKT, how to highlight connections to teaching in a mainstream course, how to identify applications to teaching in their extant curriculum, or the role of applications to teaching for PSMTs (Álvarez & Burroughs, 2018; Lai, 2016).

Ball et al. (2008) proposed that a skill necessary for teachers was to be able to “hear and interpret students’ emerging and incomplete thinking” (p. 401). Incorporating experiences throughout teacher preparation programs that engage undergraduates in practices used in mathematics teaching can take the form of including approximations of practice tasks in mainstream courses. Grossman et al. (2009) describes these tasks as “opportunities to engage in practices that are more or less proximal to the practices of a profession” (p. 2058; see also Álvarez et al. 2020). For example, Ghousseini and Herbst (2016) use constructed dialogues and Campbell et al. (2020) employ “planted errors” to approximate the work of teaching mathematics.

Methodology

To incorporate mathematics teaching connections as a legitimate application area of undergraduate mathematics, the META Math project developed inquiry-based lessons for calculus that address MKT and the recommendations of the MET II (CBMS, 2012) report. The lessons consist of content-specific and pedagogical connections to engage undergraduates in connections to teaching. Several tasks in the lessons require students to analyze a hypothetical student’s work or choose or pose guiding questions for probing student thinking (e.g., Figure 1).

Figure 1

Assessment Example from Newton's Method

2. A student tried to use Newton's method to find the zeros of the function $f(x) = x^3 - 3x + 1$ by beginning with an initial guess of 1. She was unsuccessful and asked you for help.
- (a) Why was she unsuccessful? Explain, using tangent lines, what is causing this behavior.
 - (b) Write two questions you can ask the student to help her correct her work. Explain how your questions might help the student.

The calculus lessons used by the instructor and PSMT in this study focused on inverse functions and Newton's method. *Inverse Functions* reviews commonly taught methods of finding an inverse function and explores how these methods influence the formulation of derivatives of inverse functions. *Newton's Method* introduces undergraduates to an iterative method of approximating the zeros of a function by looking at hypothetical students' work of using tangent lines at points on the function to see where the line intersects the x-axis. Both of these lessons incorporate opportunities to analyze another student's work.

The META Math lessons incorporate five types of connections to teaching between college-level mathematics and knowledge for teaching school mathematics (see Table 1). Arnold et al. derived these connections from Ball et al.'s (2008) six categories of MKT (as cited in Álvarez et al., 2020). Both lessons consist of an activity-based lesson (separated into pre- and class-activities), homework questions, and assessment items. Instructors receive a detailed annotated lesson plan (ALP), which serves as a guide for implementing the lessons effectively.

Table 1

Five Types of Connections to Teaching (Álvarez et al., 2020)

Connection	Description
<i>Content Knowledge (CK)</i>	Undergraduates use course content in applications or to answer mathematical questions in the course.
<i>Explaining Mathematical Content (EC)</i>	Undergraduates justify mathematical procedures or theorems and use of related mathematical concepts.
<i>Looking Back / Looking Forward (FB)</i>	Undergraduates explain how mathematics topics are related over a span of K-12 curriculum through undergraduate mathematics.
<i>School Student Thinking (ST)</i>	Undergraduates evaluate the mathematics underlying a student's work and explain what that student may understand.
<i>Guiding School Students' Understanding (GSU)</i>	Undergraduates pose or evaluate guiding questions to help a hypothetical student understand a mathematical concept and explain how the questions may guide the student's learning.

Setting and Participants

In the Fall 2019 semester two instructors at two different universities implemented both first-semester calculus lessons in their calculus courses. The universities are both large public research universities in the Midwest and Southeast United States, respectively. As part of their participation in the project, instructors were also invited to three interviews that occurred during the semester. All undergraduates in these courses, as part of their regular coursework, completed all parts of each lesson. We invited a subset of consenting undergraduates to participate in an hour-long semi-structured interview at the end of the semester. From a total of 63 consenting undergraduates, six undergraduates from each site consented to participate in interviews. For this paper we will focus on one instructor, Bruce, and one of his undergraduate students, Kayla.

Bruce has taught calculus for the last 15 years. At his institution, calculus is not a coordinated course. His department has 174 mathematics majors, about 20% are PSMTs. Kayla intends to teach middle school mathematics. She is an interdisciplinary liberal studies major with a minor in mathematics. We chose Bruce as a representative case as he is an experienced calculus instructor with no prior experience using these pedagogical ideas. Kayla was chosen as a critical case as the only self-identifying PSMT in the course. Participants were given pseudonyms.

Data Collection and Analysis

Both instructor and undergraduate interviews lasted between 45-60 minutes and were audio-recorded and transcribed. During the undergraduate interviews, students re-examined their work on the assessment items from each interview (e.g., Figure 1). While reconsidering their work, they provided explanations of their thought processes where appropriate, considered alternative approaches, and discussed the potential connections to previous math content. Interview questions were often posed through the lens of connections for teachers, but interviewees discussed their own perceptions of the assessment items and connections to teaching emphasized in the lesson regardless of their intent to formally teach in a classroom environment.

There were three instructor interviews, two occurred shortly after each lesson was taught, which focused on how the implementation went and how prepared instructors felt to teach the lesson based on the provided resources. At the end of the semester, the third interview reviewed both lessons, eliciting instructors' views of the project and the five types of connections.

We used thematic analysis (e.g., Braun & Clarke, 2006; Nowell et al., 2017) to qualitatively analyze the interview transcriptions. Each interview was first coded for the five connections to teaching. These codes were then expanded inductively with any emergent thematic ideas.

These additional codes tended to relate to teaching, implementation of the lesson, or the format of the activities. Once each lesson was coded independently, we compared codes until we were in agreement. Less pervasive codes, such as those that did not relate to the types of connections or were only present in one interview, were eliminated or integrated into broader categories.

Results

Bruce

While preparing for both lessons, Bruce reported carefully reading through the ALP a week before implementation. Bruce had students complete the pre-activity in class to gauge, based upon their work, the undergraduates' prior knowledge of the topics. During the next class, he began with a brief discussion of the pre-activity before undergraduates worked on the class activity in groups. Bruce's class took two class periods to complete the activity. He found that the materials, especially the ALP, prepared him to implement the lesson, although he was unprepared for how little his students remembered inverse functions from past math courses.

While discussing his views on the five types of connections and how they influenced his teaching, Bruce points out that *ST* and *GSU* were connections he had not previously considered using in his mathematics teaching. He said that he does not often consider questions like that in Figure 1 because they are not "really relevant to [him] directly." After seeing how his undergraduates had responded to these questions, he added that if he had even one future teacher in his class he would "be thinking more clearly about [those connections] and trying to make ties [to school mathematics]." When discussing his experiences implementing *Inverse Functions*, he explained that he often hesitates to show others' work because of time constraints and privacy issues, but by looking at hypothetical students' work, he found it "valuable and worth [his] time" to see students interacting with others' work and that "it can be really useful for students to observe other students work. As opposed to just my pre-scripted, professor style writing."

Bruce also discussed how he sees a difference in the way he might present content with these connections in mind, especially *CK* and *FB*. He mentions that normally, as a mathematics professor, he wants "to emphasize [the] fancy ways of solving hard things" but that is not always "what's most useful for the students." He then explains that for future high school teachers having a strong background in these connections to then be able to teach the content to school students would be most helpful. He learned better scaffolding techniques for tasks when implementing the lessons and realized the importance of scaffolding material, especially for

future teachers. Building connections to teaching resonates for Bruce as it is coupled with the rigor of an undergraduate course. For *Inverse Functions*, he acknowledges the appropriateness of the content for all of his undergraduate students, specifically that this lesson is “crucial for anyone whose major requires [calculus].” As mentioned, he also realized “how little about inverse functions [his students] remember.” This allowed him to provide support from which to build the class activities. For *Newton’s Method* he said that he was “going to entirely use the *Newton’s Method* worksheet... [it] was a great take-away, content-wise.” Overall, Bruce recognized that these lessons were both appropriate for future teachers and other students.

Most of Bruce’s undergraduates were not PSMTs, but he indicated that he wanted to motivate these connections for all of his students by emphasizing that many fields require explaining content or analyzing someone’s thinking; so, for non-PSMTs, Bruce highlighted communication skills as important outside of a classroom environment. He found the lessons easily adaptable and suitable for all students while maintaining fidelity to the connections to teaching.

Kayla

Kayla plans to teach middle school mathematics after getting her master’s degree in education. We focus on Kayla since she was the only PSMT in Bruce’s class. Kayla participated in both class activities for the lessons and then consented to participate in an end of the semester interview about her experiences during those days where the lessons were implemented.

Kayla said that she “loved” having the experience with student work and posing guiding questions. She saw a direct connection to teaching since she expects to see children’s work that displays unfamiliar strategies or methods or that may not be thorough enough to readily determine the strategies or methods used. She also indicated that she had fun looking at student work and decoding “what they have done in their thought processes” when it was not possible to speak with them directly. Moreover, Kayla saw the depth of the mathematical thinking required in the lessons as an important feature for her as a prospective teacher since “there’s always going to be the kid who’s curious in your class as to why things work, and I think it’s really important to ... have some understanding and then be able to be like well I, maybe I don’t know why, but why don’t we figure it out together.” Kayla also commented, “I think a lot of people who maybe don’t have upper level math understanding will shut that down out of fear of like, not knowing the answers themselves.” This was linked to stunting school students’ interest in math and contrasted

with “being able to like explore stuff like this and be comfortable with exploring it ... really, it's gonna benefit students in the future, being able to help them explore math too.”

Kayla discussed that explaining mathematical content in the lessons provided insight into connections to teaching. When engaging in the *Inverse Functions* activity she recognized the value in explaining her work as it mirrors what she would want her future students to do. Looking forward she adds that, “as a future teacher you want your kids to know the meaning of the work that’s behind what they’re doing, you don’t just want to memorize steps.” She then explains that this lesson reinforced ideas that she would want her future students to have, such as a good grasp of definition usage and conceptual understanding versus overreliance on formulas.

Kayla found the structure of both lessons to be good models for an inquiry-oriented class environment. She commented that she could apply the scaffolded approach in her future lessons. The chance for students to “explore the meaning [of these topics] for themselves” tied into her belief that the concepts will “stick with kids more” when actively engaged in learning. Having students actively work through the steps aligned with her belief in having students “understand something for themselves and make connections.” Recognizing the connections to high school, Kayla explains that understanding “basic algebra and ... graphing and lines” would prepare a student for investigating Newton’s method. She expressed that revisiting and applying these connections would better prepare her for explaining elementary ideas to her future students.

Discussion and Conclusion

Both Bruce and Kayla reported that concepts in the lesson helped them see connections to teaching, especially the problems that presented a hypothetical student’s thinking which then had to be analyzed. Kayla expresses not only her appreciation for the student thinking questions addressing the needs of future teachers’ development of communication skills, but also addresses the connection *Looking Back/ Looking Forward* discussing how knowing advanced content can help a teacher explain concepts to curious students that may require more justification.

Kayla found that the lesson structure helped her see the need to scaffold concepts to students, and Bruce said that the use of scaffolding benefited all students, especially PSMTs. Each reported that the approximation of practice tasks supported their awareness of connections to teaching. Bruce recognized the importance of including these tasks in his course and how they address the needs of all students. Using the materials raised his awareness of the needs of PSMTs in his courses and the importance of integrating these ideas throughout the curriculum.

Acknowledgements

This research is based upon work partially supported by the National Science Foundation (NSF) under grant number DUE-1726624. Any opinions, findings, conclusions or recommendations are those of the authors and do not necessarily reflect the views of the NSF.

References

- Álvarez, J. A. M., & Burroughs, E. (2018). Faculty and undergraduate students' challenges when connecting advanced undergraduate mathematics to school mathematics. In A. Weinberg, D. Moore-Russo, H. Soto, & M. Wawro (Eds.), *Proceedings of the 22nd Annual Conference on Research in Undergraduate Mathematics Education* (pp. 1069-1070). RUME.
- Álvarez, J. A. M., Arnold, E. G., Burroughs, E. A., Fulton, E. W., & Kercher, A. (2020). The design of tasks that address applications to teaching secondary mathematics for use in undergraduate mathematics courses. *Journal of Mathematical Behavior*, 60, <https://doi.org/10.1016/j.jmathb.2020.100814>
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3, 77-101.
- Campbell, M. P., Baldinger, E. E., & Graif, F. (2020) Representing student voice in an approximation of practice: Using planted errors in coached rehearsals to support teacher candidate learning. *National Council of Teachers of Mathematics*. 9(1), 23-49.
- Conference Board of the Mathematical Sciences (2012). *Mathematical Education of Teachers II*. American Mathematical Society and Mathematical Association of America.
- Ghousseini, H., & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education*. 19, 79–103.
- Goulding, M., Hatch, G., & Rodd, M. (2003). Undergraduate mathematics experience: Its significance in secondary mathematics teacher preparation. *Journal of Mathematics Teacher Education*, 6(4), 361-393.
- Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P.W. (2009). Teaching practice: A cross-professional perspective. *Teachers Coll. Rec.*, 111(9), 2055–2100.
- Lai, Y. (2016). Lacking confidence and resources despite having value: Mathematicians teaching prospective secondary teachers. In T. Fukawa-Connelly, N. Infante, M. Wawro, & S. Brown (Eds.), *Proceedings of the 19th Annual Conference on RUME* (pp. 281-295). RUME.
- Lai, Y., & Patterson, C. (2017). Opportunities presented by mathematics textbooks for prospective teachers to learn to use mathematics in teaching. In W. M. Smith, B. R. Lawler, J. Bowers, and L. Augustyn (Eds.). *Proceedings of the sixth annual mathematics teacher education partnership conference*. Association of Public and Land-grant Universities.
- Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic analysis: Striving to meet the trustworthiness criteria. *International Journal of Qualitative Methods*, 16, 1-13.
- Wasserman, N. H. (2018). Exploring advanced mathematics courses and content for secondary mathematics teachers. In N. H. Wasserman (Ed.), *Connecting abstract algebra to secondary mathematics, for secondary mathematics teachers* (pp. 1-15). Springer.
- Wasserman, N., Weber, K., Villanueva, M., & Mejía-Ramos, J. P. (2018). Mathematics teachers' views about the limited utility of real analysis: A transport model hypothesis. *Journal of Mathematical Behavior*, 50(1), 74-89.