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OPTIMAL MECHATRONIC DESIGN OF A QUADRUPED ROBOT WITH COMPLIANT LEGS

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ABSTRACT

The objective of this paper is to design a quadruped robot with compliant legs. Compliant legs are developed using flexible joints, which allow the robot to attenuate the effect of support-leg exchange. A model of the legs is generated in SimMechanics and optimized to minimize the maximum torque required by the actuators. The control of the robot is divided into two stages: (i) a gait central pattern generator, and (ii) a control system with feedback linearization. The gait pattern generator is developed based on optimal inverse kinematics with the use of Bezier polynomials. The resulting gait is used as a set-point in a closed-loop feedback control, which achieves a stable gait locomotion over rough, uncertain terrain. The uncertainty on the terrain causes unknown impacts in the robot. These impacts are absorbed by the compliance of the leg mechanisms. The proposed leg mechanisms are tested in a 3D-printed quadruped robot with fifteen degrees of freedom. Since, the robot is designed with eight actuators, the robot has seven degrees of under-actuation. The lack of actuators in this robot is overcame through the proposed gait pattern generator.

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1 INTRODUCTION

Legged systems such as quadruped robots can maneuver better than wheeled vehicles in non-smooth terrains such as the ones found after devastation (e.g., natural disaster or infrastructure demolition) or in wild, uncertain terrains. Quadruped robots have been also used to assist human activities such as transportation of heavy loads and mobility [1, 2]. Despite their intrinsic stability, the coordination of their legs and the corresponding trajectory generation are not a trivial tasks.

Bioinspired approaches have been proposed to mimic animal motion using the theory of central pattern generators (CPG) [3, 4] and, particularly, extracting kinematic Motion Primitives (kMPs) to create a natural (animal/human-like) locomotion profile [5, 6]. Trajectory generation strategies based on online computation of the zero-moment point (ZMP) have been used to produce stable walking patterns that allows the robot pace walking, jumping, running, and trot [7]. Despite the effectiveness of the current approaches, the minimization of energy consumption is an unsolved issue. The trajectory generator proposed in this paper uses an off-line optimization strategy that guarantees a periodic walking with an optimal energy use.

Due to the dynamic complexity of the quadruped robot dynamics, a simplified model based on a Spring Loaded Inverted Pendulum (SLIP) [8] is used in this work to analyze the stability of the passive dynamics of a quadruped robot. These applications have shown the importance of the compliant mechanisms in the robots. Therefore, compliant joints in quadruped robots have been used to attenuate the effect of the support leg exchange by isolating the gearboxes from the impact at the landing phase [9, 10]. The flexible elements of the compliant mechanisms also store energy at the landing phase, which is released as a propulsion source of energy in the next phase of the walking [11].

The paper is organized as follows. Sec. 2 presents the hybrid dynamic model for quadruped robots. Sec. 3 shows the optimization strategy to generate the periodic leg trajectories. Sec. 5 describes the physical prototype of the quadruped robot designed to test the proposed trajectory generator. Sec. 6 shows the preliminary simulations and tests.

2 HYBRID DYNAMIC MODEL OF THE WALKING ROBOT

This section presents a mathematical model that describes the dynamics of a quadruped robot walking in the saggital plane. The gait of the quadruped has a forelegs support phase and a hind legs support phase. These two phases are connected through instantaneous support-leg exchanges. Thus, the model of the robot is divided into two continuous domains and two discrete reset functions that represent the support leg exchanges. Figure 2 shows the two continuous domains where the robot is in forelegs support and hind legs support. Each leg is formed by a compliant delta mechanism. The kinematic and kinetic model are described in the remaining of this section.

2.1 Kinematic model

With respect to the body (chassis) of the robot, the position r_Q of a foot can be expressed in closure equation form as

$$R_Q = R_1 + R_2 + R_5 + R_6 = R_1 + R_3 + R_4 + R_6 \tag{1}$$

where $R_j = r_j[\cos\theta_j, \sin\theta_j]^{\mathrm{T}}$. Since the leg utilizes a (compliant) delta connection as illustrated the figure, one observes that $r_2 = r_4$ and $r_3 = r_5$, which results in $\theta_1 = \theta_3 = \theta_5 = \theta_6$ and $\theta_2 = \theta_4$. For a given a target point $R_Q^t = [r_{Qx}^t, r_{Qy}^t]^{\mathrm{T}}$, the unknowns θ_1 and θ_2 can be solved using inverse kinematics [12]. Isolating the unknown R_2, one can express the closure equation (1) as

$$R_2 = R_Q^t - R_1 - R_5 - R_6, \tag{2}$$

as shown in Fig. 1. Simplifying, this can be expressed as

$$\begin{aligned} r_2^2 = &\left[r_{Qx}^t - (r_1 + r_5 + r_6)\cos\theta_1\right]^2 + \\ &\left[r_{Qy}^t - (r_1 + r_5 + r_6)\sin\theta_1\right]^2, \end{aligned}$$

or

$$A\cos\theta_1 + B\sin\theta_1 + C = 0 \tag{3}$$

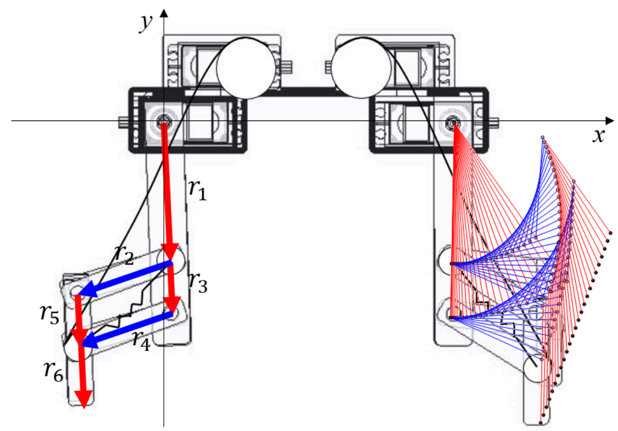

FIGURE 1. Closure equation and inverse kinematics solving for θ_1 and θ_2 for a given (linear) target foot trajectory. In this design, $r_1 =$ 65 mm, $r_2 = r_4 = 46$ mm, $r_3 = r_5 = 25$ mm, $r_6 = 28$ mm.

where $t = \tan\theta_1/2$ and

$$A = (r_1 + r_5 + r_6)r_{Qx}^t \tag{4}$$

$$B = (r_1 + r_5 + r_6)r_{Qy}^t \tag{5}$$

$$C = \frac{1}{2}\left[(r_1 + r_5 + r_6)^2 - r_2^2 + (r_{Qx}^t)^2 + (r_{Qy}^t)^2\right]. \tag{6}$$

Using trigonometric identities, this expression can be written as

$$(C - A)t^2 + 2Bt + (A + C) = 0, \tag{7}$$

where $t = \tan\theta_1/2$, which leads to

$$t = \frac{-B + \sigma\sqrt{B^2 - C^2 + A^2}}{(C - A)}, \tag{8}$$

and $\theta_1 = 2\tan^{-1} t$. The value of θ_2 can be derived from the closure equation (2). In (8), $\sigma = \pm 1$ represents the assembly mode, which is opposite for the fore and the hind legs.

2.2 Equation of motion

The equation of motion of a leg with a compliant delta mechanism can be derived from the Lagrangian

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{L}{\partial q_j} = \Gamma_j \tag{9}$$

where q_j are the generalized coordinates and Γ_j are the generalized forces not derivable from a potential function such as friction forces and time-varying forcing functions. The Lagrangian function is $L = T - V$, where T is the kinetic energy of the system and V is the potential energy of the system. For a compliant leg, the kinetic energy is

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i v_i^2 + \frac{1}{2}\sum_{i=1}^{N} I_i \dot{q}_i^2 \tag{10}$$

where m_i is the mass of each link, v_i is the velocity of the center of mass, and I_i is the moment of inertia of the link about a transverse axis through its center of mass. In this design, $v_i = (l_i \dot{q}_i)/2$. The potential energy is given by

$$V = \sum_{i=1}^{N} m_i g h_i + \frac{1}{2} k \delta^2, \tag{11}$$

where $m_i g$ is the weight force of the link, h_i is the height of the center of mass, and k is spring constant of the delta mechanism. The deformation of the δ corresponds to $\delta = \delta_0 + |R_2 + R_5| = \delta_0 + |R3 + R_4|$, where δ_0 is the free length of the extension spring, where $\delta_0 > r_2 - r_5$. Finally, given the generalized coordinates (angles) q_i, the generalized forces (torques) Γ_i are derived from the equation of motion (9). To determine such torques for the whole robot, a kinetic hybrid model is required.

2.3 Kinetic hybrid model

Their dynamic kinetic model is defined with the Euler-Lagrange equation as

$$D_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + G_s(q_s) = \Gamma_s \tag{12}$$

where q_s is the vector of generalized coordinates, the sub-index s represents the forelegs support and the sub-index p represents the hind legs support. $D_s(q_s)$ is the inertia matrix, $C_s(q_s, \dot{q}_s)$ is represents the matrix of centripetal and Coriolis effects, $G_s(q_s)$ represents the gravity action, and Γ_s is the vector of generalized forces, which includes the torque generate by the actuators and also the effect the springs in the compliant joints. The model of the continuous dynamics can be presented in a general form by rewriting (12) as

$$\dot{x}_s = f_s(x_s) + g_s u_s \tag{13}$$

where $x_s : [q_s \; \dot{q}_s]^{\mathrm{T}}$ is the state space vector, u_s is the vector of control inputs,

$$f_s(x_s) := \begin{bmatrix} \dot{q}_s \\ D_s^{-1}(q_s)\left[-C_s(q_s, \dot{q}_s)\dot{q}_s - G_s(q_s)\right] \end{bmatrix},$$
$$g_s := \begin{bmatrix} 0 \\ D_s^{-1}(q_s)B_s \end{bmatrix}, \quad \text{and} \quad B_s := \begin{bmatrix} I_{4\times 4} \\ 0 \end{bmatrix}.$$

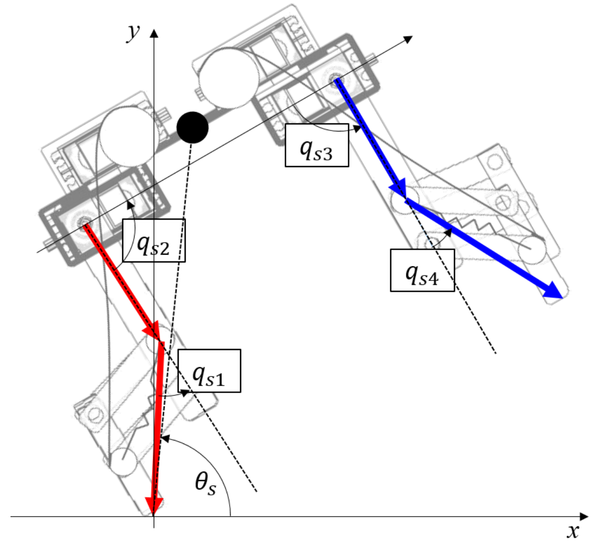

FIGURE 2. Support leg domains.

The complete walking model of the quadruped is a hybrid dynamic system with two domains, one for forelegs support and other for hind legs support, and two reset transition maps presented in the Figure 3. Thus, the hybrid dynamics of the robot is defined as

$$\Sigma_s := \begin{cases} x_s \in \mathbb{R}^n \\ \dot{x}_s = f_s(x_s) + g_s u_s \\ x_p^+ := \Delta_{s\to p}(x_s^-) \end{cases} \tag{14}$$

$$\Sigma_p := \begin{cases} x_p \in \mathbb{R}^n \\ \dot{x}_p = f_p(x_p) + g_p u_p \\ x_s^+ := \Delta_{p\to s}(x_p^-) \end{cases} \tag{15}$$

where Σ_p and Σ_s are the hybrid dynamics for forelegs support and hind legs support, respectively. The support-leg exchanges are modeled as inelastic impacts that map the states of the robot from the pre-impact state to the post-impact state, which are defined by negative and positive super-index, respectively. These impacts are described by the functions $\Delta_{s\to p}(x_s^-)$ and $\Delta_{p\to s}(x_p^-)$.

3 CONTROL STRATEGY FOR WALK TRAJECTORY TRACKING

In order to design the gait pattern of the quadruped robot, a tracking trajectory control law with a feedback linearization is proposed. The tracking trajectory controller produces a stable closed-loop behavior, which is used to define a trajectory that

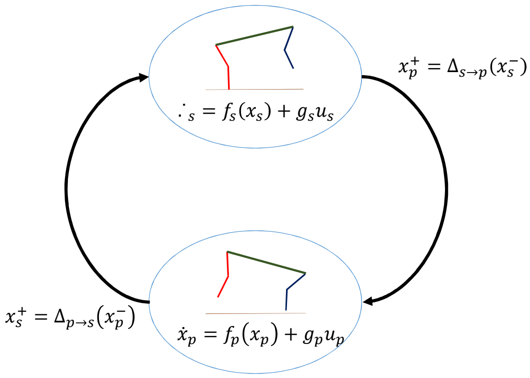

FIGURE 3. Hybrid dynamics.

minimizes the torque control required to perform a periodic stable walking.

In order to track the trajectories in each continuous part of the model, two continuous control laws based on feedback linearization with proportional derivative (PD) control are proposed.

In order to design the control strategy, let us define the target trajectories $q_s^*(t)$ and $q_p^*(t)$ for forelegs and hind legs support, respectively. These target trajectories are used as references in the tracking trajectory control task. Then, the control law for the forelegs support (equivalent for hind legs support) is defined as

$$\Gamma_s = g_s^{-1}\left(-f_s(x_s) + \ddot{q}_s^* - Kd_s\left(\dot{q}_s - \dot{q}_s^*\right) - Kp_s\left(q_s - q_s^*\right)\right), \quad (16)$$

where Kp_s and Kd_s are the proportional and derivative control gains. Note, that since g_s is a function of the inertial matrix, g_s is invertible around the trajectory of q_s^* (equivalent for q_p^*). Applying the control (16) to the continuous part of the model (12), the closed-loop dynamics is described as

$$\ddot{e}_s(t) + Kd_s\dot{e}_s(t) + Kp_s e_s(t) = 0, \quad (17)$$

where $e_s(t) := q_s - q_s^*$ is the tracking error, and the appropriated selection of the control gains Kd_s and Kp_s guarantees an asymptotic convergence of the tracking error to zero.

4 OPTIMAL WALK TRAJECTORY GENERATION

An optimization problem is formulated to define the walking target trajectories, $q_s^*(t)$ and $q_p^*(t)$, that ensure a periodic behavior in the quadruped robot and that minimize the torque required during a step. Since the reference trajectories are functions of the time, they are evaluated during a full step that starts at the beginning of the forelegs support phase and finalizes at the end of the hind legs support phase. Thus, the evaluation of the target trajectories is performed by integrating the hybrid dynamic model of the robot in (14) and (15) and computing the control signal required to allow the robot's joints follow the path proposed by the target trajectories.

4.1 Objective function

In such a way, objective function is defined with the square of the control signal integrated during a complete step. This is,

$$J_s\left(q_s^*(t)\right) := \int_{t=t_0}^{t=t_s} \left({u_s}^{\mathrm{T}} u_s\right) dt, \quad (18)$$

$$J_p\left(q_p^*(t)\right) := \int_{t=t_s}^{t=t_p} \left({u_p}^{\mathrm{T}} u_p\right) dt, \quad (19)$$

$$J\left(q_s^*(t),\, q_p^*(t)\right) := J_s\left(q_s^*(t)\right) + J_p\left(q_p^*(t)\right). \quad (20)$$

where $t = t_0$ is the time at the beginning of the forelegs support phase, $t = t_s$ is the time at the end of the forelegs support phase, which coincides with the time at the beginning of the hind legs support phase, and $t = t_p$ is the time at the end of the hind legs support phase.

The optimization problem can be stated as

$$\begin{array}{ll} \min\limits_{q_s^*(t),\, q_p^*(t)} & J\left(q_s^*(t),\, q_p^*(t)\right), \\ \text{subject to:} & \text{Nonlinear equality constraints,} \\ & \text{Nonlinear inequality constraints.} \end{array} \quad (21)$$

4.2 Nonlinear equality constraints

A set of nonlinear equality constraints are imposed into the optimization problem to guarantee continuity in the join of the target trajectories for each phase of the walking. Thus, a relation between the pre-impact and post-impact states variables in both support-leg exchanges are enforced. This constraints also ensures that the feasible solution produces a periodic walking. Thus the equality constraints are defined as

$$x_p^{*+} = \Delta_{s\to p}(x_s^{*-}), \quad (22)$$

$$x_s^{*+} = \Delta_{p\to s}(x_p^{*-}), \quad (23)$$

where x_s^{*+} and x_s^{*-} are the target state vectors at the beginning and ending of the forelegs support phase, respectively. x_p^{*+} and x_p^{*-} are the target state vectors at the beginning and ending of the hind legs support phase, respectively.

An additional nonlinear equality constraint is imposed to define a target step length in the walking of the robot. Thus, the position of the swing legs at the beginning and ending of each walking phase is compared with the target step length. Such constraint is defined as

$$P_1^h(t=t_0) = cl, \tag{24}$$

$$P_1^h(t=t_s) = cl + sl, \tag{25}$$

$$P_2^h(t=t_s) = -cl - sl, \tag{26}$$

$$P_2^h(t=t_p) = -cl, \tag{27}$$

where P_1^h and P_2^h represent the horizontal position of legs-end, cl is the minimum distance between the legs-end, and sl is the target step length.

4.3 Nonlinear inequality constraints

Nonlinear inequality constraints are imposed to the reaction forces in the support-legs during the step. In order to avoid the robot take off the ground unilateral constraints are imposed into the normal reaction forces F_s^N and F_p^N. In such a way, the values of the reaction forces during the step must be positive as

$$F_s^N > 0, \tag{28}$$

$$F_p^N > 0. \tag{29}$$

Finally, constraints in the ratio between the tangential reaction forces, F_s^T and F_p^T, and normal ones are imposed to ensure that the robot do not slip. Then, such ratio must be less than the static friction coefficient μ between the legs-en and the ground. This is,

$$F_s^T / F_s^N < \mu, \tag{30}$$

$$F_p^T / F_p^N < \mu. \tag{31}$$

5 DESIGN OF THE PHYSICAL PROTOTYPE

In order to maintain balance, the robots must be slow and often employ the use of four or more legs, increasing the mechanical complexity. To overcome that, the robots can be made to move dynamically where the legs act like pendulum and can be controlled to perform a specific gait.

5.1 Mechanical Design

Our robot features a lightweight 3D-printed leg design which takes its inspiration from the mammal gait. This section explains the concept of our system and subsystems, attempting to give some insight into the reasoning behind its design. The spacing of rear and front legs is mirrored across the plane bisecting the trunk of our robot for symmetry purposes. The knee joint is made to be compliant using a crank slider mechanism with an integrated spring for the retraction effect. For actuation of joints (4 hip, 4 knee), a HITEC servo with a torque range of 3.3-4.1 kg/cm was used, since it came as a complete unit; with motor, a potentiometer for position control and gearhead. The easy interface of the servo motors with the microcontroller at hand also contributed to the process of picking out the right model. The lengths of the links were chosen to minimize the torque of the chosen servos and hence current using optimization algorithm(s), and to enable the robot to possibly acquire different dynamic gaits.

Our leg design is a single link/segment, which has been found to be energetically advantageous since it absorbs shock activating its sliding mechanism in the face of uneven terrain. It is directly bolted to the servo arm of the motors. To facilitate rapid prototyping, hot glue and bolts were typically used to secure the motors and the legs in place. The fact that the robot parts were 3d printed and lightweight limited the load carrying capacity of the robot, making buckling/yielding of ABS a consequence. However, this resulted in advantages as well, low cost, easy and safe manufacture.

The leg has two DOFs, which influenced the leg length. Having previously implemented a construction variant, it was decided to implement compliance by the crank-slider-spring mechanism. The compression and retraction of the leg is guided by the slider surrounded by 3D-printed spring-like mechanism on either side. During stance phase, the spring is compressed by the external forces acting on the leg, thus absorbing energy parts of the impact energy from touchdown. At the end of the stance phase, before takeoff, it restitutes its energy, thus contributing to the forward motion. The leg retracts with the help of a string on a pulley mounted on hip-servo. It always has two legs touching the ground at the same time during each support phase. Therefore, the walking gait of the robot can be approximated as two-dimensional pendulums movement in the x-y plane. The objective here was to determine the length of the pendulum with slider/spring and plan the trajectory of it.

To get an approximation of the required spring properties, a spring-mass model of single leg was used; and the stresses, deformation and strain were analyzed on ANSYS for better modelling parameters. When a robot is supporting its body on one xleg, its movement can be simplified as a linear pendulum. In our case, we assumed the hip joint as the supporting point, the trunk of the robot which contains the center of mass connects the supporting point to the foot(point-contact) through the leg design. The coordinate system is located on the supporting point of the linear pendulum. The self-stabilizing property has been adopted, which would allow stable locomotion without neuronal feedback [1]. A genuine model of running, Spring Loaded Pendulum, proves that self-stability may rest on properly adjusted

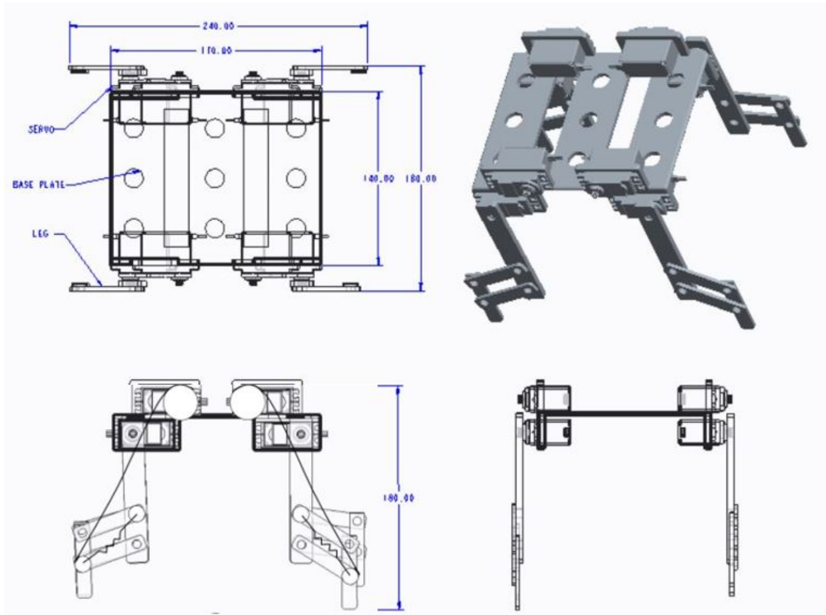

FIGURE 4. Mechanical design of the quadruped walking robot depicting the front, top, and isometric views.

Bill of Materials				
Tag	**Qty**	**Part Nr**	**Manufacturer**	**Description**
Parts				
Microcontroller	1	RM001	Arduino UNO	Based on ATmega328, 14 DI/DO pins and 6 AI
ServoMotors	8	HS-422	HITEC	Dual iron-oilite bushings and high impact resin gear train
Robot body parts	17	RM002-005	-	3D Printed Parts in PLA
Wiring	1	-	-	Jumper wires
Springs	4	RM006	-	Rubber bands with known damping coefficeint

FIGURE 5. Bill of Materials

leg compliance. Which is what we tried to simulate. One of the problems encountered by a robot moving on a slope is how to reduce a joint applying excessive torque.

Curbing joint torque is an important matter because excessive torque output results in high energy consumption and requires the adoption of high spec actuators. In this project, we focus on limb lengths for the reduction of joint torques.

5.2 Electrical components

The servos used for the design were controlled by an Arduino Uno. The Arduino is powered by a 9V battery pack which has a common ground with the Arduino. The Arduino provides PWM to all 8 servos which are powered by the battery pack too. The PWM of each servo differs from the other. The functioning of the front half was mirrored in the rear half. Hip servos in the rear receive the signals first so as to simulate the natural gait of a quad-legged animal. The front ones receive PWM with a little delay among themself as well to make it as realistic as possible. Each servo having a stall torque of 4.1kg.cm

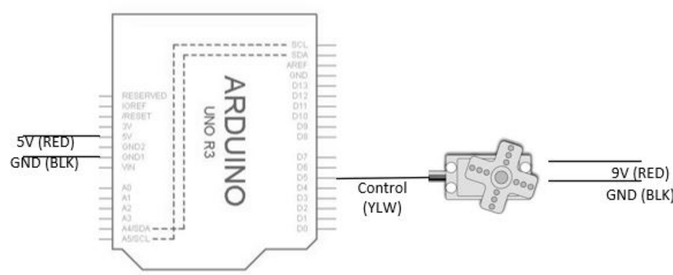

FIGURE 6. Wiring Diagram

could very easily pull on the leg with the spring mechanism incorporated in it. To code in Arduino, eight different arrays of angles were generated using a MATLAB code that used inverse kinematics for angle generation given the position of both links. This MATLAB code was fed with the path of the point contact to the ground, and configuration of both links. It in turn generated angles for the hip servos given an origin, and for the leg servos, given the hip angle as reference. Function used for control was `servo.write()` and to keep track of the movement, `servo.read()`. Each servo powers up by a current of 180 mA which is why powering it up using just Arduino was unsuitable. The 9 V battery pack being rated at 400–600 mAh is used for powering up all servos in parallel.

The choice to use a fixed pulley at the hip was made because it is the only pulley that when used individually, uses more effort than the load to lift the load. It is fixed to the shaft of the servo. When the shaft moves, so does the pulley either pulling on the leg or releasing it when the motor de-energizes after it has already pulled on the leg. The equations of motion derived using Lagrangian took care of the selection of the stiffness constant of the spring used in the leg.

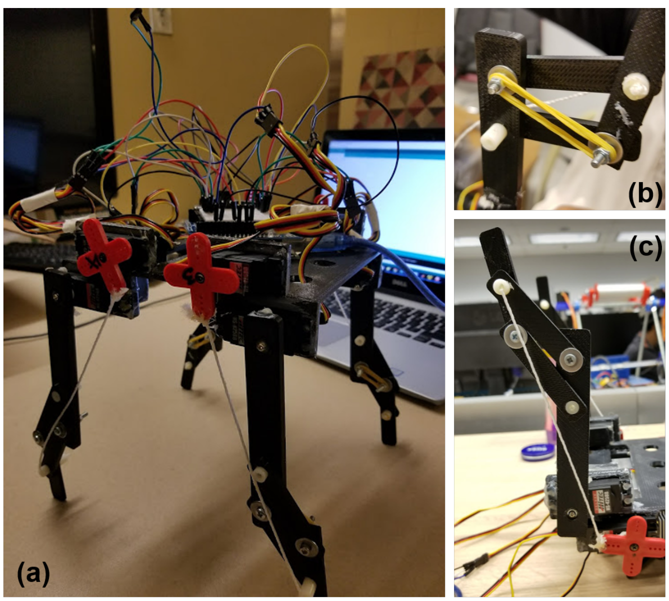

FIGURE 7. (a) Physical prototype of the quadruped walking robot. (b) Spring on legs. (c) Detail of the string pulling on the leg.

6 SIMULATIONS AND EXPERIMENTS

Although the arrays were generated, a DOE was hypothesized to obtain the required gait. The purpose of it was to form an algorithm with appropriate time delays and shifting of weight among legs. The CAD assembly file was then made into a Simscape Multibody XML file which could be simulated in SIMULINK. Its translation into an XML file generates the revolute joints for simulation of actuation through servos. The revolute joints consisting of stiffness constants and damping coefficients take care of the modelling of spring and compliance. The transformation from base to frame signifies transformation between the modelled world, mechanism configuration where gravity is set for the entire mechanism, and the linearization delta; and the entire model. The base plate further connects to eight transformation blocks in order to simulate all eight servos. Similarly, the servos connect to the leg design using transformation blocks as well. Sensing of position, velocity and torque could be done as well in order to have controlled motion of the legs by forming a closed loop. It was much easier to generate an XML file and tweak the model than to have started from scratch because the architecture that XML file picks out of CAD assembly, generally makes sense and is accurate when it comes to connections. The gait generated in MATLAB is fed to the hip and knee joints in SimMechanics in order to obtain controlled motion of the robot. Since MATLAB is easily interfaced with SimMechanics, the angles for separate joints are fed easily. The gait is smoothened using spline fitting or bezier curve. The method we used for actuation was Torque/Force actuation because we optimized torque, for that we needed position control to follow trajectory. SimMechanics also incorporates PID control and its tuning for proper optimization. Figure 8 shows trajectory of the robot in foreleg support phase.

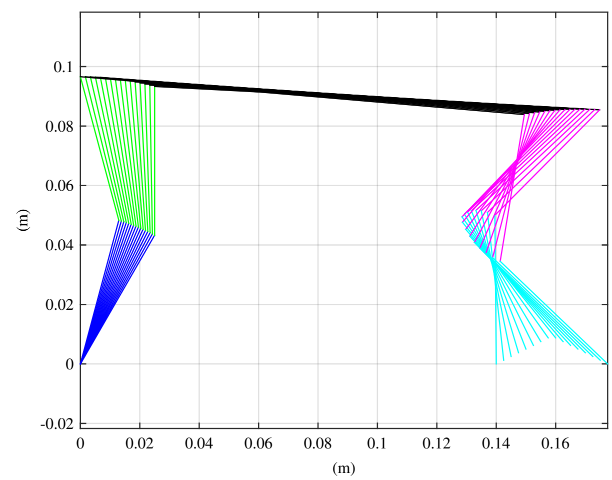

FIGURE 8. Quadruped trajectory generation.

7 CONCLUSIONS

A hybrid dynamic model for quadruped robots has been developed in this work. This model is used to design a feedback controller based on feedback linearization to track trajectory references. The effect of compliant joints on the robot is included as a model perturbation. The trajectory references are generated to provide a periodic walking pattern and minimize the robot's energy consumption. A physical prototype with compliant joints was designed and fabricated to experimentally test the proposed trajectory generation strategy. Ongoing work includes the generation of different gait patterns such as trotting, running, and jumping.

8 ACKNOWLEDGMENTS

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