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Modeling the Role of Efficiency for the Equitable and Effective Distribution of Donated Food

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Abstract Food banks operate with an objective to serve as many of food insecure people as possible with the limited supply available to them. This paper presents a mixed integer programming model to identify the efficient assignment of demand zones (counties) to distribution centers (branches) and equitable allocation of donated food from the food bank branches to the demand zones. The model objective function minimizes the total cost of branch operation, the cost of receiving and distributing food, the cost of undistributed food while maintaining the maximum allowed deviation from perfect equity. Data from the Food Bank of Central and Eastern North Carolina (FBCENC) is used to characterize the major attributes controlling the food distribution system of a food bank. Results from the optimization model using FBCENC data show that the optimal allocation under perfect equity follows a particular structure depending on the shipping cost and the cost of undistributed supply. Sensitivity analyses exploring the trade-offs between efficiency and effectiveness as a function of the cost of shipping, truck capacity, and a user-specified maximum inequity cap show that marginal sacrifice in equity can significantly improve effectiveness. The corresponding improvement in effectiveness is greater when comparatively larger trucks are used and the cost of shipping is relatively higher. The analyses also suggest that while efficiency is less sensitive to the allowable limit on the deviation from perfect equity it is sensitive to truck size. A comparison of direct shipping to branches to operating a local hub suggests the former option to be more cost efficient.

Keywords Food bank · Food insecurity · Donation distribution · Equity · Efficiency · Effectiveness

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1 Introduction

The United States Department of Agriculture (USDA) defines food insecurity as a household-level economic and social condition of limited or uncertain access to adequate food [1]. Feeding America (FA), the largest national nonprofit hunger-relief organization in the United States (U.S.), distributes food to the 15.6 million (12.3%) food insecure U.S. households through a network of more than 200 food banks [2]. The food banks are each a microcosm of FA distributing food to people with hunger need through a network of autonomous partners. The primary goals of a food bank are three-fold: 1) to serve as many people as possible by maximizing food distribution, 2) to distribute food at the lowest possible cost, and 3) to maintain fairness in the distribution of donated food within its service area.

The third goal, equity, mentioned is regularly monitored by FA. Although the food banks within the FA network are autonomous, each food bank reports its monthly food distribution and the corresponding equity level in terms of the proportion of demand served within its service region. In this study, we model this goal/requirement as a constraint in the food distribution model. Similar to Sengul Orgut et al., we define equity as equal food distribution per demand within the service region, while effectiveness is the degree to which food waste is minimized, i.e., maximizing the total food distribution [3]. We define efficiency by the resources required for food distribution with the goal of minimizing the associated cost [4, 5, 6]. The food distribution process of a food bank is complicated. It is even more complicated when food banks try to balance the effective, efficient and equitable distribution of donated food. In this article, we focus on the food distribution system of a food bank in order to design an efficient and effective distribution policy while maintaining equity at a certain level.

The prevalence of hunger in North Carolina is one of the most alarming in the U.S. North Carolina was ranked the 10$^{\text{th}}$ food insecure state in 2018 with 14.0% (1,456,200 people) of the population struggling with hunger. It would require a budget of $719.9 million more per year to meet the food needs of the people facing hunger in North Carolina [7]. The county-level food insecurity rate varies between 9.8% and 25.3% in the 100 counties in North Carolina [8]. The Food Bank of Central and Eastern North Carolina (FBCENC) is one of seven FA food banks in North Carolina, serving 34 counties[9]. FBCENC serves more than 630,000 people at hunger risk annually through a network of more than 900 autonomous partner agencies including soup kitchens, food pantries, shelters, and programs for children and adults[10]. These partner agencies are nonprofit organizations, local to the counties they serve, with the mission of fighting hunger in their respective counties. FBCENC operates through six branches located in different counties across their service territory. These branches are warehouses that store food donations received from various donors before the final distribution to agencies. The branch directors also solicit local donations and work with local partner agencies for food distribution. The branch located in Raleigh serves as the hub in FBCENC's distribution network. This facility is owned by FBCENC whereas the other five facilities are rented.

FBCENC receives different types of food donations broadly categorized as dry goods, produce, refrigerated food and frozen food. Each FBCENC branch is responsible for inspecting these food donations as per the food safety guidelines set by FA, storing the food and distributing it to the agencies that distribute to people in need. Each branch has dedicated permanent staff to perform these operations. FBCENC also owns delivery vehicles, which are used to collect donations from donors and to send donations to agencies. Although some agencies pick up their donations from their respective FBCENC branch, approximately 50% of the agencies have food delivered by FBCENC. The inspection, storage and distribution activities require investment from FBCENC. As a nonprofit organization, FBCENC tries to optimize these expenses while focusing on distributing food donations effectively and equitably. Although each food bank's operations may differ depending on their location and their priorities, the investment requirements for FBCENC are representative of other food banks within the FA network. For example, compared to the 34 counties FBCENC serves, Los Angeles Regional Food Bank (LARFB) only serves Los Angeles County, which they divide into eight Service Planning Areas (SPAs). LARFB has two warehouses located centrally in Los Angeles to serve the eight SPAs. LARFB serves 300,000 food insecure people through a network of more than 650 partner agencies, which is comparable to FBCENC[11]. Similar to FBCENC, LARFB invests in maintaining warehouses and delivery vehicles, inspecting and storing food donations.

FBCENC estimates the need in each of the 34 counties it serves using the US census poverty population estimates. FBCENC tries to allocate food to each county as per their share of the total estimated need within the entire service region. To ensure equitable distribution, on a rolling basis, FBCENC monitors the difference between the actual share of the need of a county and the share of donation allocated to that county. Based on this metric, FBCENC may adjust its allocation of donations to the counties. In this article, we capture the inequity in the system by measuring the differences between the demand served for any county over its total demand and the same for any other county. Demand for any county is derived from the government estimated poverty population and the Map the Meal Gap defined by FA [12].

A significant challenge for a food bank is to distribute as much of the food donations received as possible while maintaining equity[3]. A perfectly equitable solution for a food bank could always be to distribute no food to any of the service regions in their territory. But that will waste all the food, which contradicts with the goal of a food bank being effective in distributing food donations and may also incur a huge wastage cost depending on the waste management policy of the local authority the food bank operates within. In addition, food banks need to be efficient in minimizing operational expenses while attaining the best equitable and effective distribution of food donations. This means a food bank needs to distribute food within its service region efficiently, i.e., assign counties to branches to minimize the shipping cost. Similarly, food banks have to collect donations from local and federal donors efficiently, as the food bank is typically responsible for picking up and transporting donations to their site. These potentially contradicting goals complicate the process of finding an optimal distribution policy for donated food.

In this paper, we present a mixed integer program assignment and distribution model that minimizes the total cost of distributing food donations and the wastage cost while maintaining a user-specified cap on maximum inequitable distribution. We use data from FBCENC to explore the relationship between the maximum allowed inequity cap, effectiveness (waste reduction) and distribution costs. Results from this analysis demonstrate that a slight sacrifice in equity can positively impact effectiveness and reduce cost.

The remainder of this paper is organized as follows. In Section 2, we review the relevant studies in the area of donation distribution in emergency and long-term humanitarian relief efforts. Section 3 presents the mathematical model formulation. In Section 4, we summarize the experimental results. Section 5 discusses the research findings and future research scope.

2 Related Literature

The scope of humanitarian logistics has been defined as a "special branch of logistics managing the response supply chain of critical supplies and services with challenges such as demand surges, uncertain supplies, critical time windows and the vast scope of its operations" [13] including "the process of planning, implementing and controlling the efficient, cost-effective flow and storage of goods and materials, as well as related information, from point of origin to point of consumption for the purpose of meeting the end beneficiary's requirements"[14]. Humanitarian logistics issues can be categorized as - (i) disasters and (ii) long-term humanitarian development issues [15]. A significant focus of the literature on humanitarian logistics addresses disaster management and last mile distribution. The importance and the significance of long-term humanitarian development, especially with a focus on food banks operations, is still limited [3, 16, 17, 18]. This study concentrates on tactical decision making under constrained resources to optimize the performance of a long-term humanitarian development system.

Studies in long-term humanitarian development address different issues. Celik et al. categorize the literature on long-term humanitarian development as food and supply distribution, infrastructure network planning in healthcare, and supply chain optimization [15]. While many of these studies use lead time and cost as the performance measures of the system [19], equity and coverage have also been considered as performance measures [19, 20, 21]. Gutjahr et al. survey the literature on quantitative decision making approaches to humanitarian aid that use multicriteria optimization methods [22]. The authors find cost to be the most common attribute and identify reliability, security and equity to be potential areas of focus for future research. Balcik et al. introduce a multi-vehicle sequential resource allocation problem for food redistribution from donors, e.g., restaurants and grocery stores, to agencies, e.g., soup kitchens and homeless shelters, considering two critical objectives: providing equitable service and minimizing unused donations [23]. The authors focus on assigning donors and agencies to multiple vehicle routes while primarily maximizing equity, where equity is defined as the expected minimum fill rate among all agencies. Davis et al. focus on identifying a more cost-effective food delivery and collection strategy developing transportation schedules to collect food donations from local sources and to deliver food to charitable agencies [17].Fianu and Davis present a discrete-time, discrete-space MDP model that assists food banks in equitable distribution of uncertain donation supplies [18]. The authors define equity as a function of the pounds distributed per person in poverty. Sengul Orgut et al. propose a network flow model to identify the equitable and effective distribution of food donations under constrained capacity [3, 16, 24]. Our work focuses on optimizing a humanitarian food and supply distribution network considering efficiency, effectiveness, and equity as the performance measures. The inclusion of efficiency distinguishes our work from the problem studied in Sengul Orgut et al. [3, 16, 24].

Disaster-related studies often differ in problem definition, type of resource constraints and periodic consideration from long-term development studies. However, these two types of studies share similar objectives or performance measures, e.g., equity, efficiency, and effectiveness. These performance measures are defined differently in these studies according to the context of the problems and the standpoint of the decision makers. For example, Balcik et al. define equity as maximizing the minimum fill rate (the ratio of allocated amount to observed demand) and effectiveness as minimizing waste in their formulation of a multi-vehicle sequential resource allocation problem [23]. The paper focuses on providing equitable service and minimizing unused donations for nonprofit operations. Krejci defines efficiency as fulfilling the demand for aid using minimal resources (i.e., money and time) in designing a coordination mechanism [25].

Efficiency, equity, and effectiveness have been the primary objectives for various studies focusing on last mile relief distribution. Huang et al. present an analysis of the last mile distribution problem focusing on efficacy (i.e., the extent to which the goals of quick and sufficient distribution are met), and equity (i.e., the extent to which all recipients receive comparable service) but do not consider effectiveness (maximizing total distribution) [5]. The authors analyze the impact of different objectives on route structures and the performance of aid distribution in terms of efficiency (transportation costs), efficacy and equity. Balcik et al. propose a two-phase modeling approach to enable relief practitioners to make efficient and effective last mile distribution decisions [26]. Ekici et al. emphasize minimizing total cost while satisfying demand in their models for planning a food distribution network considering facility location and resource allocation decisions during an influenza pandemic [6].

Equity and efficiency in resource allocation have also been addressed in the context of public health. Kong et al. focus on maximizing the efficiency of intra-regional liver transplants considering the levels of geographical proximity as local, regional and national [27]. Demirci et al. develop an optimization model to explore the effect of efficiency and regional equity on the organ allocation process [28]. The authors define equity as the per-patient rate of likelihood-adjusted intraregional transplants for each organ procurement organization in the respective region. Heier Stamm et al. investigate the problem of potential spatial accessibility, or the opportunity to receive care as moderated by geographic factors, with horizontal equity, or fairness across populations regardless of need [29]. The paper defines equity as the absence of systematic disparities between different groups of people, distinguished by location or socioeconomic variables.

Several studies also operationalize equity in humanitarian logistics using economic inequity measures, e.g., Gini index. For example, Gutjahr and Fischer use of the Gini index to penalize inequity while modeling the frequency of relief distribution in the context of post-disaster response [30]. Eisenhandler and Tzur also use the Gini coefficient to address equity in their pickup and distribution model that focuses on a food bank's logistical challenges [31]. Enayati et al. use the Gini index as one of the equity measures in their multicriteria optimization model for location and dispatching policy decisions for emergency medical service systems [32]. Carlson et al. develop a Gini-like index to address equity in the context of post-disaster resource distribution [33].

While equity, effectiveness, and efficiency in distributing donations under a food bank supply chain network have been considered in different studies, few studies have combined these three attributes in a single study. Gralla et al. develop a piecewise linear utility function to evaluate the trade-offs between efficiency, equity and effectiveness based on the responses from expert humanitarian logisticians identifying their preferences among these objectives for the particular case of a natural disaster – earthquake [34]. The authors considered the amount of aid delivered and the speed of delivery as measures of effectiveness, the prioritization of aid by type and location as the measure of equity and the operational cost as the efficiency. Nair et al. examine the food recovery and redistribution logistics of food rescue organizations that collect and deliver perishable surplus food without storage [35]. The authors formulate the problem as a food rescue allocation and routing problem with an objective of identifying the cost effective routing and efficient and fair food allocation. The paper defines efficiency as maximizing the utility of the delivery system and fair allocation as maximizing the utility of the worst off delivery location or minimizing the deviation of the utilities among the delivery locations. Eisenhandler and Tzur consider the distribution of perishable products (e.g., dairy products, fruits, vegetables, meat), which require rapid distribution directly to agencies [31]. The authors refer to the problem as the humanitarian pickup and delivery problem, where the core decisions consist of selecting a subset of "suppliers" and "customers", assigning them to vehicle routes that are subject to a time constraint. The authors model this problem with the aim of maintaining equity (i.e., the extent to which it maintains fair allocation among the different agencies) while delivering as much food as possible (effectiveness).

Studies with simultaneous consideration of equity, efficiency and effectiveness are limited even in areas other than food bank operation or hunger relief distribution. Adday et al. develop a framework for assessing the effectiveness, efficiency,and equity of behavioral healthcare. The paper defines effectiveness as the improvement in health conditions, efficiency as minimizing cost or achieving highest performance of the inputs, and equity as fairness in care delivery [36]. Davis et al. focus on evaluating hospital performance in three dimensions - efficiency, effectiveness, and equity [37]. The paper measures efficiency as a function of the length of stay at the hospital, effectiveness as the chance of readmission or death within 30 days of a hospital visit, and equity is assessed as the level of efficiency and effectiveness among different ethnic and socioeconomic groups. Sparrow et al. discuss the trade-offs between equity, efficiency and effectiveness in the context of Demand Side Management (DSM) rate design [38]. The authors define effectiveness as recovering program costs and lost revenues from all utility customers, efficiency and equity as covering all program costs from participants, and setting their bill reductions equal to avoided costs, leaving all non-participants unaffected. Young and Tilley articulate the value and importance of moving the sustainable business agenda for any business beyond the notion of eco-efficiency and socio-efficiency [39]. The authors present a new theoretical model for corporate sustainability. The model presented in this paper links six criteria that a sustainable business will need to satisfy, namely eco-efficiency, socio-efficiency, eco-effectiveness, socio-effectiveness, sufficiency and ecological equity. While these studies consider equity, efficiency and effectiveness, simultaneously, their definitions of these terms differ from our study. In addition, the papers propose frameworks for exploring the qualitative and quantitative relationships between equity, efficiency, and effectiveness. Our work introduces an analytic decision model for food distribution that seeks to identify optimal distribution and allocation policies that simultaneously satisfy each of these criteria.

In this study, we focus on developing tactical plans for a food bank for receiving and distributing donated food in an efficient and effective manner while maintaining a certain level of equity among the service regions with constrained capacity on different levels, e.g., capacity of the delivery vehicle, the capacity of the food bank's branches with respect to receiving and processing food donations, and the combined capacity of agencies working in a service region. Unlike Balcik et al., Nair et al. and Eisenhandler et al., who study a food collection and distribution problem without storage at the food bank level, we address a food collection and distribution problem with storage at the food bank level [23, 35, 31]. We also consider the trade-off between equity, efficiency and effectiveness in receiving and distributing donated food similar to Gralla et al., not in the context of short-term disaster relief distribution, but in the context of a long-term food bank operational network. However, while Gralla et al. elicit decision maker preferences regarding these criteria, we integrate these criteria to identify optimal tactical strategies for the food bank network [34]. We address equity as the deviation between proportional demand served for any two demand zones similar to Sengul Orgut et al. (which is one of the measures of equity mentioned in Nair et al.) [3, 35]. Effectiveness in our study is defined as maximizing the amount of donations distributed, similar to Gralla et al., Eisenhandler and Tzur [34, 31], which translates to minimizing the food waste as used in Balcik et al. and Sengul Orgut et al. [23, 3]. Efficiency in this context, like Stones , Gralla et al., and Krejci, is defined as minimizing the operational cost [4, 34, 25]. The contribution of our paper is to characterize the trade-offs between efficiency, effectiveness and equity in tactical distribution and allocation decision making for long-term hunger relief organizations.

3 Model Formulation

3.1 Problem Statement and Assumptions

Our model formulation is inspired by the supply chain network of FBCENC. In its service region, FBCENC operates six branches located in Durham, Greenville, New Bern, Raleigh, Sandhills (Southern Pines) and Wilmington as shown in Figure 1, which are the distribution centers in its supply chain network. Each branch distributes donations to partner agencies working in specific counties, that eventually distribute the donations to food-insecure people in their counties. FBCENC receives donations from several sources, the contribution of donations from various sources is dynamic changing each year. In 2016, around 73% of its total supply came from local donors, 3.3% from state and federal government sources, 13% from USDA, 5.4% from other food banks

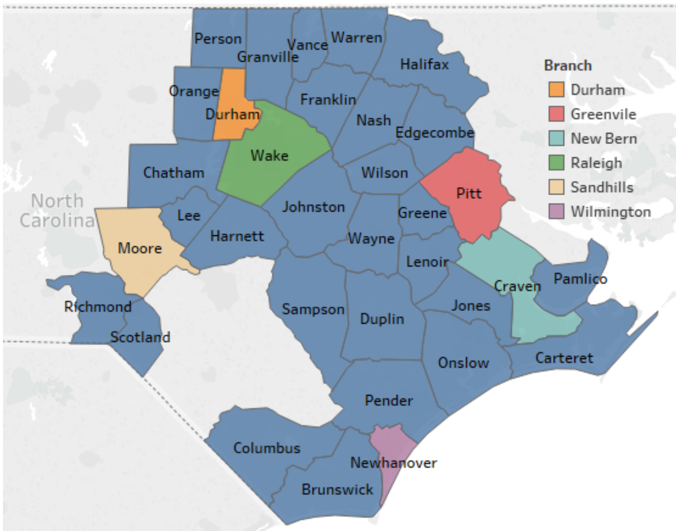


Fig. 1: FBCENC's service region and branch locations

and 5.3% from other food drives. Most of the USDA donations mainly come from Florida whereas the other donors are mostly local in the service area of

FBCENC. In our model, considering the location of the donor, we define the USDA donations as the national source and consider Florida as the location of the national source, whereas all other donations are considered local and coming from different counties within North Carolina. In the current practice of FBCENC, all of the USDA donations, and a significant portion of the local donations from different counties go to the Raleigh branch, which serves as the hub for FBCENC. Donations are processed and stored in the Raleigh branch and then redistributed to different branches as required to serve the food insecure people within its service region. In order to reduce the shipping costs, we evaluate the effect of sending the donations from the national source and local sources directly to the regional branch and then redistribute them among the partner agencies working in the counties served by that branch.

For shipping donations, FBCENC uses both rented and owned trucks of the following sizes - 10/12-pallet box trucks, 36' trailers, 53' trailers, which have capacities of 11,000 lbs, 26,000 lbs and 40,000 lbs, respectively. Ninety percent of the shipments involve the 10/12-pallet box trucks. More recently, FBCENC has introduced the "middle mile" system for collecting and distributing food donations. The "middle mile" uses volunteers' vehicles, which have considerably lower capacity, for collecting and distributing food donations similar to Uber. Except for the volunteer donated transportation via middle mile, FBCENC is responsible for the shipping cost associated with collecting the donations. In case of the distribution of donations, currently, around 50% of the donations are picked up by the agencies, whereas FBCENC delivers the remaining 50%.

With respect to the collection and distribution of donations FBCENC generally adapts to the needs of the donor or the agency. For example, for some local donations, donations are available for a relatively short time window and require special arrangements for pick up. To the extent possible, FBCENC tries to schedule a pickup to coincide with a delivery in close proximity to the donation point. However, if there is a risk of losing a donation, an empty truck is sent for pick up. In case of distribution to some agencies, FBCENC identifies a common location and ships food to that particular location where the agencies can collect their donations.

Given the complex supply chain network of FBCENC, the problem is to determine the amount of donations from national and local sources to be collected by each of the distribution centers and redistributed to the partner agencies working in each county. The goal of this assignment is to serve as many food-insecure people as possible, at the minimum cost, while maintaining a desired level of fairness among the counties FBCENC serves. To ensure that the primary objectives and challenges of FBCENC are addressed, we formulate a mixed integer programming assignment and distribution model that identifies counties' efficient allocation to branches and an equitable and effective food distribution policy. Our model extends the model proposed in Islam and Ivy in several ways [40]. First, we introduce the requirement for an integer number of trucks for shipping donations to calculate the transportation costs more accurately instead of considering a per pound shipping cost. Second, we relax the assumption of unique assignment of counties to branches for receiving and distributing local donations, which allows the system to identify the efficient assignment of counties to a single branch or multiple branches. Third, we study the impact of the food categories on different attributes of the problem. Fourth, we prove the optimal allocation policy's structural properties. Finally, we explore the trade-offs between the three attributes as a function of model parameters.

Our modeling assumptions are as follows.

- Existing branches of the food bank are in operation with known capacity and maintenance costs and are not considered for relocation or closure to avoid disruption in FBCENC's distribution network.
- As food banks may deliver food to a common location for multiple agencies, agencies working in a county are aggregated into a single agency considered to be located at the centroid of the county in order to simplify the outbound transportation cost estimation.
- Based on the location of the donation, all the sources of food donations other than the USDA sources are considered local supply. The USDA donations are considered national supply. To simplify the model complexity and inbound transportation costs, we also consider all local food donations to be collected at the centroid of the donor's county.
- No branch can receive food donations in excess of its capacity and the amount of food distributed to each county also cannot exceed its capacity.
- Trucks are used to ship food to one location on each trip.
- The majority of FBCENC's donation collection and distribution uses vehicles of a similar size. The agencies pick up 50% of the donations using their own vehicles, the capacity of which may vary from agency to agency. For simplicity, we assume that 100% of the collection and distributions is performed using a similar truck size.

We model effective and efficient distribution based on monthly aggregated demand, supply, allocation, and the level of equity maintained at the county level in a given month. One of the model assumptions is to aggregate all the agencies in a county as one agency located at the centroid of the county. This assumption reduces the number of decision variables significantly (from an average of 24 to 1 per county) as we consider 34 distribution points in 34 counties instead of more than 900 partner agency locations. While one of the reasons we aggregate is for model and computational simplicity, this approach also best matches FBCENC's representation and evaluation of their distribution decisions, particularly regarding equity. FBCENC aggregates the distribution to the county level and considers the county level poverty population to assess the equity level associated with distribution. Hence, our primary focus is on monthly distribution decisions at the county-level where we estimate monthly demand from the poverty population estimates. Moreover, data available from FBCENC at the time of this study did not provide the agency level demand information, which is necessary to allocate the donations among the agencies equitably.

We also evaluate efficiency by estimating the monthly cost of distributing donations to all the counties. Our focus is the monthly distribution cost to mirror FBCENC's current practice rather than optimizing a vehicle routing problem for daily distribution. We introduce the model notation and formulation in the next section, where the above-mentioned assumptions are translated into appropriate model constraints.

3.2 Notation and Formulation

The description of the index sets, parameters and variables used in the model are as follows. All the parameters and variables associated with demand, capacity or food distribution are measured in pounds, whereas costs are in dollars.

Index Sets

I	Set of potential branches
J	Set of counties
n	National source for food supply
M	Set of existing branches

Parameters

D_i	Demand in county i
C_i	Capacity of county i
κ_i	Capacity of branch i
d_{ij}	Centroidal distance in miles between branch i and county j
d_{ni}	Centroidal distance in miles between the national source n and branch i
c_s	Per mile shipping cost
S	Amount of supply available from the national source
ξ_i	Amount of supply available locally at county i
c_w	Cost of discarding one pound of food as waste
c_{oi}	Cost of operating a branch at county i
τ	Capacity of a truck
K	Maximum allowable deviation from equitable distribution

Decision Variables

f_i	$\begin{cases} 1, & \text{if county } i \text{ is selected as a branch location} \\ 0, & \text{otherwise} \end{cases}$
p_{ni}	number of truck shipments from the national source n to branch i
q_{ji}	number of truck shipments from county j to branch i
r_{ij}	number of truck shipments from branch i to county j
X_{ni}	Amount of food shipped from the national source n to branch i
W	Total amount of wasted food
u_{ij}	Amount of food distributed to county j from branch i
v_{ji}	Amount of donations distributed to branch i from local sources at county j

Model

$$\text{minimize} \quad \sum_{i\in I} c_{oi} f_i + 2c_s\left(\sum_{i\in I} d_{ni}p_{ni} + \sum_{i\in I}\sum_{j\in J} d_{ji}q_{ji} + \sum_{i\in I}\sum_{j\in J} d_{ij}r_{ij}\right) + c_w W \tag{1}$$

subject to:
Assignment:

$$\sum_{m\in M} f_m = |M| \tag{2}$$

Equity:

$$\left|\frac{\sum_{i\in I} u_{ig}}{D_g} - \frac{\sum_{i\in I} u_{ih}}{D_h}\right| \leq K, \qquad \forall g, h \in J, g < h \tag{3}$$

Flow Conservation:

$$\sum_{i\in I} X_{ni} = S \tag{4}$$

$$\sum_{i\in I} v_{ji} = \xi_j \qquad \forall j \in J \tag{5}$$

$$W = S + \sum_{j\in J} \xi_j - \sum_{i\in I}\sum_{j\in J} u_{ij} \tag{6}$$

Shipping:

$$\sum_{j\in J} u_{ij} \leq X_{oi} + \sum_{j\in J} v_{ji}, \qquad \forall i \in I \tag{7}$$

$$\frac{X_{ni}}{\tau} \leq p_{ni}, \qquad \forall i \in I \tag{8}$$

$$\frac{v_{ji}}{\tau} \leq q_{ji}, \qquad \forall i \in I, j \in J \tag{9}$$

$$\frac{u_{ij}}{\tau} \leq r_{ij}, \qquad \forall i \in I, j \in J \tag{10}$$

Capacity:

$$X_{ni} + \sum_{j \in J} v_{ji} \leq \kappa_i f_i, \qquad \forall i \in I \tag{11}$$

$$\sum_{i \in I} u_{ij} \leq C_j, \qquad \forall j \in J \tag{12}$$

Integrality and Nonnegativity:

$$f_i \in \{0, 1\}, \qquad \forall i \in I \tag{13}$$

$$p_{ni}, q_{ji}, r_{ij} \in \mathbb{Z}, \qquad \forall i \in I, j \in J \tag{14}$$

$$X_{ni}, u_{ij}, v_{ji}, W \geq 0, \qquad \forall i \in I, j \in J \tag{15}$$

The objective function (1) minimizes the total cost of maintaining the branches, shipping donations from various sources to the branches, distributing donations to agencies located in the counties (including the cost of shipping an empty truck for one way), and undistributed food. Constraint (2) ensures the continuation of the existing branches. Constraints (3) limit the absolute difference in the proportion of demand fulfilled between any two counties (i.e., equity constraints). Constraint (4) ensures that all donations available at the national source are shipped to the branches. Constraints (5) assure that available local donations are shipped to the branches. Constraint (6) preserves the flow conservation of the system. Constraints (7) ensure total distribution from a branch is less than the donations received by that branch. Constraints (8), (9) and (10) capture the number of truck shipments required to ship the food donations from the source to destination. Constraints (11) and (12) ensure that food shipped to a branch and a county are below its respective capacity. And finally, Constraints (13), (14) and (15) impose the binary, integer and non-negativity restrictions on f_i, p_i, q_j, r_{ij}, X_{oi}, u_{ij}, v_{ij},and W, respectively. Constraints (3) are equivalent to

$$-K \leq \frac{\sum_{i \in I} u_{ig}}{D_g} - \frac{\sum_{i \in I} u_{ih}}{D_h} \leq K, \qquad \forall g, h \in J, g < h \tag{16}$$

which can further be simplified as

$$\frac{\sum_{i \in I} u_{ig}}{D_g} - \frac{\sum_{i \in I} u_{ih}}{D_h} \leq K, \qquad \forall g, h \in J, g < h \tag{16a}$$

$$\frac{\sum_{i \in I} u_{ih}}{D_h} - \frac{\sum_{i \in I} u_{ig}}{D_g} \leq K, \qquad \forall g, h \in J, g < h \tag{16b}$$

Replacing (3) with (16a) and (16b) make the constraint sets of the model formulation linear and convex.

We formulate the model to minimize the total cost of the system obtaining the efficient assignment of counties to branches, while the effective solution is ensured by the penalty on undistributed food. We control the level of inequitable distribution of food donations to each county by modeling equity as a constraint, where the maximum allowable deviation from a perfectly equitable distribution is an input parameter. This aligns with the actual practice of FBCENC to maintain the fair-share allocation. Moreover, it maintains the convexity of the formulation.

Constraints (4) and (5) ensure that all food donations available locally and at the national source get shipped to the branches of the food bank for inspection, processing and further distribution. This is a reasonable assumption because as a nonprofitable organization running on donations, food banks may be reluctant to decline available donations (e.g., FBCENC does not refuse donations). From the formulation it is noticeable that the cost parameters in the objective function, e.g., c_{oi}, c_s, c_w, along with the distances d_{ni} and d_{ij}, the receiving capacities C_i and κ_i, demands D_i, and allowable deviation limit from perfect equity K are the key factors in identifying the optimal solution structure for this problem. While c_{oi}, d_{oi}, d_{ij}, and κ_i affect the efficient assignment of counties to different branches, C_i, D_j, and c_w determine the optimal food distribution to the counties. This paper focuses on characterizing the potentially conflicting roles of efficiency and effectiveness in determining the optimal equitable distribution of food donation to people with food insecurity. To explore the structure of the optimal distribution policy of food donation to different counties, in the next section, we study the donation distribution problem under a given optimal assignment of branches to counties.

3.3 Perfectly Equitable Distribution

At perfect equity (K=0), constraints (16a) and (16b) become binding constraints equivalent to

$$\frac{\sum_{i\in I} u_{ih}}{D_h} = \frac{\sum_{i\in I} u_{ig}}{D_g}, \qquad \forall g, h \in J, g < h.$$

As each county must receive the same fraction of its demand, the maximum fraction of demand that could be satisfied is restricted by county j with $\min(\frac{C_j}{D_j})$, [3]. To characterize the optimal distribution policy further, we study the problem of allocating food donations given an assignment of counties to branches with sufficient supply available at the branch level to satisfy the maximum possible demand ($\min(\frac{C_j}{D_j})D_j$) fraction of each county. We aggregate the local supplies and the supplies from the national source to represent the total available food donation and denote it by TS, where $TS = S + \sum_{j\in J} \xi_j$. The index set for the optimal branch locations is denoted as I', where I' is a subset of I and the set J_i defines the counties receiving food donations from branch $i \in I'$. Incorporating these assumptions, the model formulation can be rewritten as follows.

Model P2

$$\text{minimize} \quad 2c_s \sum_{i \in I'} \sum_{j \in J_i} d_{ij} r_{ij} + c_w W \tag{17}$$

subject to:

$$\frac{\sum_{i \in I'} u_{ig}}{D_g} - \frac{\sum_{i \in I'} u_{ih}}{D_h} = 0, \qquad \forall g, h \in J, g < h \tag{18}$$

$$W = TS - \sum_{i \in I'} \sum_{j \in J_i} u_{ij} \tag{19}$$

$$\frac{u_{ij}}{\tau} \leq r_{ij}, \qquad \forall i \in I', j \in J_i \tag{20}$$

$$\sum_{i \in I'} u_{ij} \leq C_j, \qquad \forall j \in J \tag{21}$$

$$r_{ij} \in \mathbb{Z}, \qquad \forall i \in I', j \in J_i \tag{22}$$

$$u_{ij}, W \geq 0, \qquad \forall i \in I', j \in J_i \tag{23}$$

Model $P2$ is a mixed integer linear model that minimizes the cost of shipping food to the counties from branches and the cost of undistributed food. It explores the trade-offs between efficiency associated with donation distribution and effectiveness under a requirement of perfect equity. We consider two cases of Model $P2$ - (i) constraint (22) relaxed, i.e., $r_{ij} \in \mathbb{R}$, and (ii) constraint (22) holds.

(i) Constraint (22) relaxed, i.e., $r_{ij} \in \mathbb{R}$

Relaxing the integrality restrictions on the variable r_{ij}, constraints (20) in Model $P2$ can be written as equality constraints and substituted into the objective function to revise the objective function as (17a).

$$\text{minimize} \quad 2c_s \sum_{i \in I'} \sum_{j \in J_i} d_{ij} \frac{u_{ij}}{\tau} + c_w W \tag{17a}$$

Interaction between the cost parameters, c_s and c_w in the objective function determines the structure of the optimal food distribution to the counties. Proposition 1 summarizes the optimal food distribution policy for cost parameter values.

Proposition 1: Under perfect equity, optimal food distribution to different counties given sufficient supply available ($TS > min(\frac{C_j}{D_j}) \sum_{j \in J} D_j$) at the branch level is as following -

Case 1: If $c_s = 0$ and $c_w = 0$, i.e., food can be distributed to the counties from the branches or can be wasted without incurring any cost, the objective function value is 0 and the optimal food allocation to the counties ($\sum_{i \in I'} u_{ij}$) has multiple optimal solutions.

Case 2: If $c_s = 0$ and $c_w > 0$, i.e., there is no shipping cost involved in distributing food to counties from branches but any undistributed food will be penalized with a wastage cost. The optimal objective function value is

$$c_w(TS - \frac{C_{j*}}{D_{j*}} \sum_{j \in J} D_j), \qquad where \qquad j* = argmin(\frac{C_j}{D_j})$$

with the optimal amount of food distributed to each county

$$\sum_{i \in I'} u_{ij} = \frac{C_{j*}}{D_{j*}} D_j, \qquad \forall j \in J.$$

This corresponds to similar results presented in Sengul Orgut et al. under the consideration of equity and effectiveness [3].

Case 3: If $c_s > 0$, and $c_w = 0$, i.e., there is a cost to distribute food to the counties from the branches but there is no cost for wasting food. The optimal objective function value is 0, and the optimal food allocation to the counties is

$$\sum_{i \in I'} u_{ij} = 0 \qquad \forall j \in J.$$

Case 4: If $c_s > 0$ and $c_w > 0$, i.e., there is a cost for distributing food to the counties from the branches as well as a cost for wasting food. Denoting the fraction of demand satisfied for any county j by z_j is equivalent to the following expression.

$$\frac{\sum_{i \in I'} u_{ij}}{D_j} = z_j, \qquad \forall j \in J \tag{24}$$

As constraints (18) ensure that each county must receive the same proportion of its demand, we can represent z_j, $\forall j \in J$ by z, where z may take on any value within the following range.

$$0 \leq z \leq \frac{C_{j*}}{D_{j*}} \tag{25}$$

So, (24) can be written as follows.

$$\sum_{i \in I'} u_{ij} = zD_j, \qquad \forall j \in J \tag{26}$$

Substituting (26) into (17a), the objective function is revised as (17b).

$$\text{minimize} \qquad \frac{2zc_s}{\tau} \sum_{i \in I'} \sum_{j \in J_i} d_{ij} D_j + c_w W \tag{17b}$$

As per (19), W is equivalent to,

$$W = TS - z\sum_{j\in J} D_j.$$

Hence, (17b) can be simplified as (17c).

$$\text{minimize} \quad z\left(\frac{2c_s}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j - c_w\sum_{j\in J} D_j\right) + c_w TS \tag{17c}$$

If $c_s > \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j}$, the optimal objective function value is $c_w TS$ and the optimal fraction of demand served for each county $z = 0$, i.e., it is optimal to distribute nothing. Thus,

$$\sum_{i\in I'} u_{ij} = 0 \qquad \forall j \in J.$$

If $c_s = \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j}$, the objective function value is $c_w TS$ and the fraction of demand served for each county z has multiple solutions, i.e., the food allocations to the counties $(\sum_{i\in I'} u_{ij})$ have multiple optimal solutions.

If $c_s < \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j}$, the optimal objective function value is

$$\frac{C_{j*}}{D_{j*}}\left(2c_s\sum_{i\in I'}\sum_{j\in J_i} d_{ij}\frac{D_j}{\tau} - c_w\sum_{j\in J} D_j\right) + c_w TS,$$

and the optimal fraction of demand served for each county is $z = \frac{C_{j*}}{D_{j*}}$, i.e., the optimal food allocation to the counties are

$$\sum_{i\in I'} u_{ij} = \frac{C_{j*}}{D_{j*}} D_j \qquad \forall j \in J.$$

Proof of Proposition 1: The proof is provided in the Appendix.

(ii) Constraint (22) holds

Constraints (20) in Model $P2$ capture the integer number of trucks required to transport the allocation from branch i to county j, which is equivalent to $\lceil \frac{u_{ij}}{\tau} \rceil$. Because of the integer requirement of variable r_{ij}, without loss of generality, we can say, $r_{ij} = \lceil \frac{u_{ij}}{\tau} \rceil$. Substituting this into the objective function of Model $P2$, we can revise the objective function as (17d).

$$\text{minimize} \quad 2c_s\sum_{i\in I'}\sum_{j\in J_i} d_{ij}\left\lceil\frac{u_{ij}}{\tau}\right\rceil + c_w W \tag{17d}$$

For this form of the objective function, it is not possible to separate u_{ij} to study the closed form structure of the solution. We can still compare the solutions obtained in Proposition 1. Cases 1, 2 and 3 presented in Proposition 1 would also be directly applicable for this form of the problem. Cases 1 and 2 are true as with $c_s = 0$, the first term in (17d) becomes 0 and hence, the problems are respectively equivalent to the problems discussed under the Cases 1 and 2 in Proposition 1. Case 3 of Proposition 1 also holds for this form of the problem as with $c_w = 0$, the optimal solution is always distribute nothing to all the counties. Under Case 4 of Proposition 1, c_s and c_w, both are strictly positive and hence, the optimal solutions are not trivial to find. Now, substituting (19) into objective function (17d) we can rewrite it as follows.

$$\text{minimize} \quad 2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij} \lceil \frac{u_{ij}}{\tau} \rceil + c_w(TS - \sum_{i\in I'} \sum_{j\in J_i} u_{ij}) \tag{17e}$$

It is noticeable that, as one county can receive donations from multiple branches each requiring a different number of trucks, $\sum_{i\in I'} \lceil \frac{u_{ij}}{\tau} \rceil \neq \lceil \frac{\sum_{i\in I'} u_{ij}}{\tau} \rceil$. Thus, we cannot substitute (26) into objective function (17e). Hence, we cannot establish the scenarios discussed in Case 4 of Proposition 1 to explore the closed form structure of the solutions under this case. However, we can evaluate the objective function value as per (17e) for the solutions discussed in Case 4 of Proposition 1 and compare the optimality gap for each scenario.

As per Proposition 1, for the relaxed problem, if $c_s > \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau} \sum_{i\in I'} \sum_{j\in J_i} d_{ij} D_j}$, the optimal distribution policy is,

$$\sum_{i\in I'} u_{ij} = 0 \qquad \forall j \in J.$$

Hence, the objective function value is $c_w TS$. Substituting these solutions into (17e) yields an objective function value of $c_w TS$, which is the same as the relaxed problem with zero optimality gap.

If $c_s = \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau} \sum_{i\in I'} \sum_{j\in J_i} d_{ij} D_j}$, the objective function value is $c_w TS$ and there are multiple optimal food allocations to the counties, $(\sum_{i\in I'} u_{ij})$. In this case the optimality gap is

$$2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij} \lceil \frac{u_{ij}}{\tau} \rceil - c_w \sum_{i\in I'} \sum_{j\in J_i} u_{ij}.$$

Now, from (17a) and (19), we know that $c_s = \frac{c_w \sum_{j\in J} D_j}{\frac{2}{\tau} \sum_{i\in I'} \sum_{j\in J_i} d_{ij} D_j}$ can also be written as $2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij} \frac{u_{ij}}{\tau} = c_w \sum_{i\in I'} \sum_{j\in J_i} u_{ij}$. By definition, $\lceil \frac{u_{ij}}{\tau} \rceil \geq \frac{u_{ij}}{\tau}$.
So, $2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij} \lceil \frac{u_{ij}}{\tau} \rceil \geq 2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij} \frac{u_{ij}}{\tau} = c_w \sum_{i\in I'} \sum_{j\in J_i} u_{ij}$. Thus, it can be

said that,

$$2c_s \sum_{i \in I'} \sum_{j \in J_i} d_{ij} \lceil \frac{u_{ij}}{\tau} \rceil - c_w \sum_{i \in I'} \sum_{j \in J_i} u_{ij} \geq 0,$$

i.e., the optimality gap in this case is nonnegative.

If $c_s < \frac{c_w \sum_{j \in J} D_j}{\frac{2}{\tau} \sum_{i \in I'} \sum_{j \in J_i} d_{ij} D_j}$, the optimal objective function value for the relaxed problem is,

$$\frac{C_{j*}}{D_{j*}} (2c_s \sum_{i \in I'} \sum_{j \in J_i} d_{ij} \frac{D_j}{\tau} - c_w \sum_{j \in J} D_j) + c_w TS,$$

and the optimal food allocation to the counties are,

$$\sum_{i \in I'} u_{ij} = \frac{C_{j*}}{D_{j*}} D_j \qquad \forall j \in J, \qquad \text{where}, j* = argmin(\frac{C_j}{D_j}).$$

Direct substitution of these solutions into the objective function (17e) is not always feasible as $\sum_{i \in I'} \lceil \frac{u_{ij}}{\tau} \rceil \neq \left\lceil \sum_{i \in I'} \frac{u_{ij}}{\tau} \right\rceil$, unless the branch to county distribution has a unique assignment, i.e., each county receives donations from only one branch. In case of a unique assignment, plugging these solutions into (17e) obtains an objective function value of

$$2c_s \sum_{i \in I'} \sum_{j \in J_i} d_{ij} \lceil \frac{\frac{C_{j*}}{D_{j*}} D_j}{\tau} \rceil + c_w (TS - \frac{C_{j*}}{D_{j*}} \sum_{j \in J_i} D_j),$$

which has an optimality gap with the relaxed problem equivalent to,

$$2c_s (\sum_{i \in I'} \sum_{j \in J_i} d_{ij} \lceil \frac{\frac{C_{j*}}{D_{j*}} D_j}{\tau} \rceil - \frac{C_{j*}}{D_{j*}} \sum_{i \in I'} \sum_{j \in J_i} d_{ij} \frac{D_j}{\tau}).$$

It is evident that, this will also be nonnegative as $\lceil \frac{\frac{C_{j*}}{D_{j*}} D_j}{\tau} \rceil \geq \frac{\frac{C_{j*}}{D_{j*}} D_j}{\tau}$.

If the optimal distribution does not have a unique assignment, i.e., at least one county receives donations from multiple branches, we cannot mathematically evaluate the value of objective function (17e) and its optimality gap with the objective function value of the relaxed problem.

4 Computational Study

4.1 Data and Experimental Settings

We obtained daily transaction data from FBCENC for a 10-year period from 2007 to 2016. Table 1 presents a summary of the annual number of transactions and volume of food donations distributed by FBCENC. We aggregate the daily food distribution on a monthly basis to identify the food donations received by each county in a month. Table 2 provides a glimpse of the monthly average distributions received by 10 counties in the service region of FBCENC. Donations received by FBCENC can be classified into four broad categories: dry goods, produce, refrigerated food, and frozen food. The majority of the food donations received by FBCENC were dry goods, e.g., in 2016 about 52.8% of food donations distributed by FBCNEC. In 2016, refrigerated, frozen, and produce were 32.7%, 12.6% and 1.9% of the food donations distributed by FBCENC, respectively. The overall distribution of dry goods and frozen food by FBCENC over the period 2007-2016 are summarized in Table 3. Table 4 displays the monthly average food distributed to 10 counties in the service region of FBCENC in 2016. In this work, a major portion of our analysis focuses on dry goods distribution data. However, we also evaluate our model's performance with frozen food distribution data to understand the impact of the capacity of the counties and the branches' ability to process different types of food on the distribution decision.

Table 1: FBCENC's total number of transactions and distribution in pounds

Year	No. of Transactions	Distribution (lbs)
2007	165,856	32,103,696
2008	173,670	33,706,911
2009	186,769	39,718,212
2010	182,121	41,886,278
2011	183,321	42,846,031
2012	201,047	49,411,833
2013	201,241	52,874,347
2014	203,373	56,790,411
2015	194,260	58,959,669
2016	199,077	65,752,967

Table 2: Average pounds of food received per month by 10 counties from 2014 to 2016

County	2014	2015	2016
BRUNSWICK	140,257	152,518	169,516
CARTERET	73,692	98,242	139,033
CHATHAM	54,182	55,219	57,722
COLUMBUS	74,305	87,277	122,945
CRAVEN	133,695	108,570	163,332
DUPLIN	97,137	78,413	78,605
DURHAM	240,073	291,191	327,298
EDGECOMBE	141,843	132,823	138,227
FRANKLIN	101,788	94,308	108,178
GRANVILLE	42,230	56,511	61,166

Table 3: Number of transactions and distribution for dry goods and frozen food

	Dry Goods		Frozen	
Year	No. of Transactions	Distribution (lbs)	No. of Transactions	Distribution (lbs)
2007	108,976	15,883,958	21,883	4,482,503
2008	109,053	16,397,033	26,346	4,722,709
2009	123,549	21,342,047	30,478	5,559,677
2010	121,058	23,803,116	29,024	5,438,342
2011	119,594	24,181,482	30,927	5,630,817
2012	131,438	23,522,108	34,175	6,094,249
2013	126,828	24,889,452	39,392	6,776,095
2014	128,536	30,322,435	39,362	6,471,103
2015	128,761	33,352,202	33,361	7,144,943
2016	130,360	34,733,415	32,783	8,256,645

Table 4: Average dry goods and frozen food received per month by 10 counties from 2014 to 2016

	Dry Food			Frozen		
County	2014	2015	2016	2014	2015	2016
BRUNSWICK	82,410	83,902	82,828	24,652	24,751	27,177
CARTERET	42,301	45,385	57,242	8,858	13,505	22,388
CHATHAM	32,637	29,743	30,233	7,106	7,700	8,111
COLUMBUS	45,234	46,417	65,496	15,934	21,018	21,766
CRAVEN	59,606	60,408	75,117	16,943	14,804	15,422
DUPLIN	52,489	42,849	42,826	7,004	7,343	7,860
DURHAM	145,052	181,627	193,420	30,331	35,593	47,268
EDGECOMBE	73,883	74,847	74,707	16,119	14,899	17,643
FRANKLIN	47,275	47,711	51,461	13,100	13,317	15,130

4.1.1 Demand characterization

Actual demand in a county is difficult to characterize as FBCENC did not maintain a record of demand in each county at the time of this study. A reasonable assumption is to consider the demand in each county to be correlated with the estimated poverty population of that county [41]. We obtain the 2016 poverty population estimation of each county in the service region of FBCENC from the census data [42]. We use the "Map the Meal Gap" program by FA to transform the estimated poverty population of a county into demand in pounds of food [12]. We use the per person weekly food budget shortfall (\$16.9) and the average meal cost (\$3.0) in 2016 as specified by the FA Map the Meal Gap program to estimate the weekly meal demand [7]. As shown in (27), the estimated monthly demand per person in pounds can be obtained using the weekly meal demand, the number of weeks in a month and the pounds per meal (approximately 1.2 lb/meal) reported by FBCENC [43]. The steps to obtain the monthly demand estimate per person are shown in Table 5.

$$\text{Estimated monthly demand per person (EMDPP)} = \frac{\text{avg. budget shortfall}}{\text{avg. meal cost}} \times \text{lbs/meal} \times \frac{\text{\# of weeks/year}}{\text{\# of months/year}} \tag{27}$$

The total monthly demand in pounds for a county is calculated using the estimated monthly demand per person and the poverty population estimates (PPE) in that county as shown in (28).

$$\text{Estimated Monthly demand} = \text{EMDPP} \times \text{PPE} \tag{28}$$

As we study the food distribution problem for different types of food, we characterize the monthly demand by food type from the total monthly demand obtained from (28) using the USDA Center for Nutrition Policy and Promotion recommended intake amount by food group for 2000 calorie level [44]. Table 6 presents the approximate demand proportion by food type. We use these proportions to estimate the monthly food demand for each county for each type of food. Table 7 presents the estimated monthly demand for dry goods and frozen food for the counties within the service region of FBCENC.

Table 5: Estimation of monthly demand in pounds per person

Weekly budget shortfall (\$)	Cost/meal (\$)	Weekly demand (meal) (rounded up)	lbs/meal	Yearly demand/person (lb)	Monthly demand/person (lb)
16.9	3.0	6.0	1.2	374.4	31.2

Table 6: Demand proportion by types of food

Type of Food	Dry Goods	Frozen	Refrigerated	Produce
Share (%)	41.41	18.69	26.60	13.30

Table 7: Monthly demand, capacity, supply in pounds for dry goods and frozen food by counties

County (j)	Dry goods			Frozen food		
	Demand (D_j)	Capacity (C_j)	Local supply (ξ_j)	Demand (D_j)	Capacity (C_j)	Local supply (ξ_j)
Brunswick	276,513	130,665	85,328	101,499	35,962	29,985
Carteret	132,188	83,030	54,171	48,522	33,716	31,260
Chatham	133,189	44,694	41,842	48,889	21,276	5,707
Columbus	208,267	121,000	70,270	76,448	31,119	28,699
Craven	239,038	111,571	47,880	87,743	23,871	22,922
Duplin	197,273	75,364	30,133	72,413	13,245	6,505
Durham	750,919	265,356	210,550	275,639	70,530	44,704
Edgecombe	198,798	98,672	75,043	72,973	28,490	16,260
Franklin	152,888	72,433	36,647	56,120	18,053	15,017
Granville	133,491	81,627	62,719	49,000	12,367	10,309
Greene	70,248	37,626	9,276	25,786	10,065	7,964
Halifax	215,971	74,208	54,588	79,276	17,564	10,943
Harnett	324,076	84,707	57,970	118,958	19,532	17,122
Johnston	398,185	137,877	89,926	146,161	39,002	31,884
Jones	33,170	43,748	21,038	12,176	4,097	869
Lee	156,780	69,849	62,470	57,549	16,721	15,108
Lenoir	184,866	182,612	101,012	67,859	35,108	33,250
Moore	171,030	142,862	50,932	62,780	22,787	19,304
Nash	240,690	108,675	87,655	88,350	28,784	19,864
Newhanover	595,283	159,012	134,887	218,510	49,927	45,524
Onslow	376,024	142,186	127,693	138,026	40,752	37,292
Orange	270,842	103,319	86,463	99,417	12,972	9,783
Pamlico	35,426	64,678	12,130	13,004	6,816	4,502
Pender	137,939	91,577	63,472	50,633	21,191	17,959
Person	94,713	94,822	22,044	34,766	9,149	5,211
Pitt	581,335	277,049	116,378	213,390	59,287	52,343
Richmond	173,158	87,660	37,772	63,561	27,317	26,431
Sampson	192,444	60,726	36,080	70,640	10,234	10,169
Scotland	143,499	100,269	62,337	52,674	33,484	18,206
Vance	167,630	142,662	39,893	61,532	18,482	13,197
Wake	1,493,308	512,798	355,865	548,146	143,551	126,140
Warren	79,415	23,125	14,544	29,151	5,601	2,742
Wayne	395,389	166,064	68,627	145,135	38,448	29,472
Wilson	282,741	98,922	55,503	103,785	25,349	23,200

4.1.2 Supply data

To specify the supply for a particular type of food we use actual donations received by FBCENC by food type in December 2016. FBCENC reports donations totaling 2,854,182 pounds and 907,868 pounds received in December 2016 for dry goods and frozen food, respectively. As mentioned in Subsection 3.1, 87% of these supplies come from local sources and the rest from the national source. Food donations received from local sources are listed in Table 7. Food donations received from the national source are listed in Table 10 for dry goods and Table 11 for frozen food.

4.1.3 Capacity estimation

Capacity is defined as the ability to receive and distribute food. For FBCENC, we consider capacity from the perspective of branches and agencies' ability to distribute food donations to food insecure households. This refers to branches' and agencies' physical storage capacity, staffing to process donations, and financial ability to receive and distribute food within the counties they serve. We use the 90^{th} percentile of the empirical distribution of the amount of food shipped to a county in each month during the fiscal year 2016 to estimate each county's capacity which is adapted from Sengut Orgut et al. 2016 [3]. We use a similar approach to identify the capacity of a branch. The capacities of the branches are characterized by type of food. The capacities of the counties by dry goods and frozen food are listed in Table 7. Table 8 shows the capacities of the branches to process dry goods and frozen food in 2016.

4.1.4 Transportation cost characterization

To identify the cost of shipping food from the local and national sources to FBCENC's branches and from the branches to the counties, we use the centroidal distance between the two locations, the estimated transportation cost per mile and the number of truckloads required to ship the given amount of food. FBCENC does not report the transportation expenses per mile in their financial data. Thus, for a standard transportation cost per mile we use the total variable costs and drivers' salaries per mile ($0.7235) for trucking reported by RTSFinancial [45]. As per our assumption that trucks ship food to one location for each trip mentioned in Section 3.1, the shipping cost for each trip is multiplied by two to represent the round trip cost including the cost of shipping an empty truck one way. As mentioned in Section 3.1, although FBCENC uses trucks of different capacities, most of their shipments use trucks with a capacity of 11,000 pounds so we use this as our default truck size to study the model. The other cost components such as the branch operating cost and the cost of undistributed food were acquired from FBCENC. The monthly average branch operating cost for FBCENC in 2016 are listed in Table 8 for the existing branches. FBCENC considers the worth of any undistributed food as the value of the food and the inspection and handling cost incurred to process the food, which is about $1.85 per pound as reported by FBCENC. This constitutes the cost of undistributed food for our analysis.

Table 8: Average monthly operating cost in 2016 for existing branches and their capacity

Branch	Operating Cost ($)	Capacity (lbs)	
		Dry goods	Frozen
Durham	168,592	583,504	126,467
Greenville	156,612	506,108	106,889
New Bern	95,958	420,789	118,794
Raleigh	540,781	1,529,684	391,234
Sandhills	120,216	348,638	76,677
Wilmington	139,090	413,571	136,635

4.1.5 Experimental Design

We perform numerical studies to analyze the efficient and effective distribution of donated food under equity constraints. For a given level of supply available at the local sources and at the national source, we study the efficient shipment of the food donations from the local and national sources to different branches and the efficient distribution of donated food to different counties from the branches. We study the results both under perfect equity and an allowed maximum deviation from perfect equity. In addition, we solve the model independently for dry goods and frozen food and analyze the model solutions to explore the potential impact of the variable capacity of the branches and counties to process different types of donations. We study the impact on the optimal assignment and allocation for both shipping donations to the branches from the sources and from the branches to the counties. To study the interaction between the cost of shipping donations and the cost of undistributed food, we solve the model for different shipping costs under different levels of deviation from perfect equity, and for different truck capacities. Table 9 summarizes the parameters with their values or ranges considered in the analyses. As listed in Table 9, we consider a wide range of values (0-15) for the shipping cost, c_s, to better understand the interplay between the shipping cost, c_s, and the cost of undistributed food, c_w, and their impact on the model efficiency and effectiveness at different level of equitable distribution. The different truck sizes considered for the sensitivity analysis is motivated by the existing practice of FBCENC. As discussed in Section 3.1, FBCENC primarily uses trucks of capacity 11,000 lbs or lower for shipping donations. We study the model solutions for trucks with a capacity of 2000 lbs, 4000 lbs, 6000 lbs, and 11,000 lbs. There are significant differences in the capacity of different service regions of FBCENC for handling dry goods and frozen food. We solve the model independently for these two types of food and compare the results. We also analyze the model solutions for both the existing distribution network of FBCENC and the proposed flexible modified network to identify potential improvement in terms of efficiency. We evaluate the model performance to

Table 9: Parameter ranges for experimental design

Parameters	Value/Range
Shipping cost, c_s ($/mile)	0-15
Truck Capacity, τ (lbs)	2000, 4000, 6000, 11000
Equity Level (K)	0-0.10
Types of food	Dry goods, Frozen food

answer the following research questions:

1) How does the assignment of supply to the branches differ for different types of food?
2) How does the assignment of counties to branches for receiving food donations vary by type of food?
3) How does the optimal assignment of supply vary as a function of the equity cap?
4) How does the optimal donation distribution vary as a function of the equity cap?
5) How does the shipping cost and the cost of undistributed food impact the optimal distribution for a given equity cap?
6) How does the truck size impact the efficiency and effectiveness for a given level of equity?
7) Should donations be shipped to the branches directly to improve efficiency?

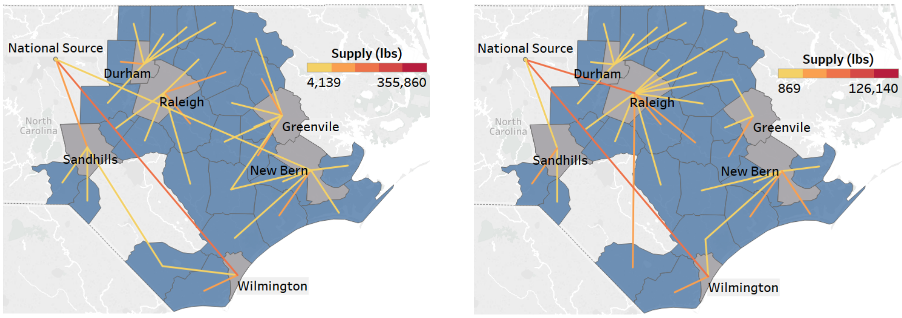
The MIP model was programmed using IBM ILOG Optimization programming language and solved using CPLEX as the underlying MIP solver. To perform the sensitivity analysis, the model was solved at the default level of tolerance (0.01%) to produce integer optimal solutions in a reasonable time.

4.2 Results and discussion

4.2.1 Perfectly equitable distribution

We study the model solutions under perfect equity (K=0) for both dry goods and frozen food for the given level of supply introduced in Subsection 4.1.2. The model determines the efficient shipment of donations from the sources to different branches of the food bank and from the branches to the counties for final distribution to people in need. The model solutions specify how much to ship from the sources to each branch and from the branches to each county. Results at perfect equity for dry goods and frozen food are summarized in Table 10 and Table 11. It is noticeable that for both types of food, the model solutions use the existing six branches as distribution centers and determine the optimal assignment and allocation of the available supplies to be shipped to one of these branches for processing, inspection, and final distribution.

Figures 2(a) and 2(b) show the relative amount of local supply shipped from each county to the optimally chosen branches under perfect equity for dry goods and frozen food, respectively. The link between a branch and any county in this figure indicates shipment of local food donations from the county to the branch connected by the link where counties (not labeled) represent the sender and the branches (labeled) represent the recipient. The darkness of the links indicates the relative amount of food shipped. In cases where the sender and the recipient are at the same location, there are no links representing those shipments as they start and end at the same point.



(a) dry goods (b) frozen food

Fig. 2: Allocation of national and local donations from the counties to the branches (labeled) under perfect equity

Fig. 2 shows that the efficient assignment of local supplies to branches for dry goods is different from that for frozen food. This happens because of differences in the food donations available in each county, capacity of the trucks used in transportation and the capacity of the branches to receive, process and store donations by food type. For example, under perfect equity for dry goods, the Sandhills branch received supply almost up to its capacity including supply from the national source and Columbus, Moore, Richmond, and Scotland counties (see Table 10). Columbus county also sends donations to the Wilmington branch. In the case of frozen food, the Sandhills branch also received supply almost up to its capacity but only received donations from the national source, and Moore, Richmond and Scotland counties (as shown in Table 11).

Fig. 2 also shows that the efficient shipment of supply may not always follow the shortest path for each county. To illustrate, refer to Columbus county in 2(b), which ships all of its donations to the Raleigh branch which is not the closest branch. The branches can be ordered by increasing distance from Columbus county where the actual distances are cited in parenthesis as follows: Wilmington (46.6 mi), Sandhills (85.8 mi), Raleigh (105.6 mi), New Bern (108.1 mi), Greenville (117.3 mi), and Durham (123.4 mi). Intuitively, in an efficient solution the Wilmington branch should receive supply from Columbus county. But, as reported in Table 11, the Wilmington branch receives supply up to its full capacity from counties comparatively closer to it, e.g., New Hanover (0 mi), Brunswick (22.7 mi), and Pender (22.9 mi), and from the national source and hence, cannot receive donations from the Columbus county.

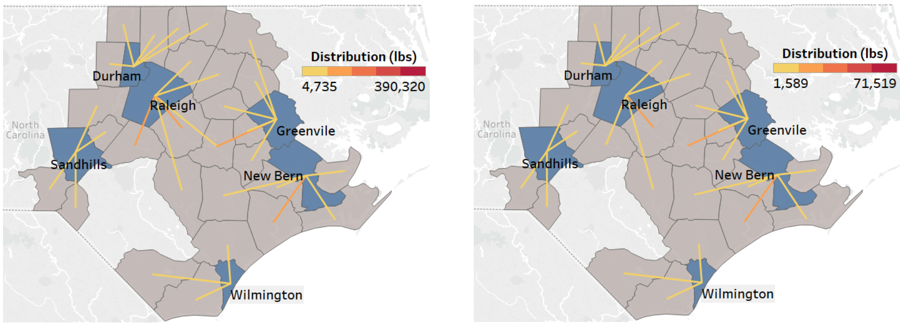
Donations available at the national sources are also not necessarily always shipped to the closest branches. For example, as shown in Table 12, Wilmington is the closest branch to the national source and the next closest branches are Sandhills, New Bern, Raleigh, Greenville, and Durham. Table 11 shows that the Wilmington branch receives donations up to its full capacity. Sandhills and New Bern have available capacity up to 1,736 lbs and 6,589 lbs, respectively, both of which are much less than the truck capacity (11,000 lbs), individually and collectively. Rather than sending two trucks to these two branches from the national source, which would require a travel distance of 941.0 miles and 959.1 miles, it is cost efficient to combine these two shipments and ship it to the next nearest branch, Raleigh, which has enough capacity to receive the entire amount. It is also noticeable that, the Greenville and Durham branches receive donations less than their capacity with no donations from the national source.

Table 10: Food donations (in pounds) received by a branch from local sources and national sources, and distributed to the counties for dry goods under perfect equity

Branch (i)	Branch Capacity (κ_i)	Assignment of donations ($X_{oi} + v_{ji}$)	Assignment of national donations (X_{oi})	Supply from county to branch			Total Food Distributed	Distribution from branch to county				
				County (j)	Amount (v_{ji})	% of local donations		County (j)	Capacity (C_j)	Amount (u_{ij})	$\frac{C_j}{D_j}$	$\frac{u_{ij}}{C_j}$
				Carteret	54,171	16.06%		Carteret	83,030	34,551	0.63	0.42
				Craven	47,880	14.19%		Craven	111,571	62,480	0.47	0.56
				Duplin	11,000	3.26%		Duplin	75,364	51,564	0.38	0.68
				Jones	21,038	6.24%		Jones	43,748	8,670	1.32	0.20
				Onslow	127,690	37.85%		Onslow	142,186	98,285	0.38	0.69
				Pamlico	12,130	3.60%		Pamlico	64,678	9,260	1.83	0.14
New Bern	420,789	389,425	52,044	Pender	63,472	18.81%	264,810					
				Total	337,381	100.00%						
				Chatham	41,842	8.95%		Durham	265,356	196,280	0.35	0.74
				Durham	210,550	45.02%		Granville	81,627	34,892	0.61	0.43
				Granville	62,719	13.41%		Orange	103,319	70,793	0.38	0.69
				Orange	86,463	18.49%		Person	94,822	24,756	1.00	0.26
				Person	22,044	4.71%		Vance	142,662	43,815	0.85	0.31
				Vance	39,893	8.53%		Warren	23,125	20,757	0.29	0.90
Durhum	583,504	467,650	0	Warren	4,139	0.89%	391,293					
				Total	467,650	100.00%						
				Columbus	63,914	29.73%		Chatham	44,694	34,813	0.34	0.78
				Moore	50,932	23.69%		Lee	69,849	40,979	0.45	0.59
				Richmond	37,772	17.57%		Moore	142,862	44,704	0.84	0.31
				Scotland	62,337	29.00%		Richmond	87,660	45,260	0.51	0.52
Sandhills	348,638	346,955	132,000	Total	214,955	100.00%	203,264	Scotland	100,269	37,508	0.70	0.37
				Brunswick	85,328	37.66%		Brunswick	130,665	72,275	0.47	0.55
				Columbus	6,356	2.81%		Columbus	121,000	54,437	0.58	0.45
				Newhanover	134,887	59.53%		Newhanover	159,012	155,600	0.27	0.98
Wilmington	413,571	413,571	187,000	Total	226,571	100.00%	318,367	Pender	91,577	36,055	0.66	0.39
				Duplin	19,133	3.83%		Edgecombe	98,672	51,962	0.50	0.53
				Edgecombe	75,043	15.02%		Greene	37,626	18,362	0.54	0.49
				Greene	9,276	1.86%		Halifax	74,208	56,451	0.34	0.76
				Halifax	54,588	10.93%		Lenoir	182,612	48,321	0.99	0.26
				Lenoir	101,010	20.22%		Pitt	277,049	151,950	0.48	0.55
				Pitt	116,380	23.30%		Wayne	166,064	98,612	0.42	0.62
				Wayne	68,627	13.74%		Wilson	98,922	73,903	0.35	0.75
Greenvile	506,108	499,560	0	Wilson	55,503	11.11%	499,560					
				Total	499,560	100.00%						
				Franklin	36,647	4.97%		Franklin	72,433	39,962	0.47	0.55
				Harnett	57,970	7.87%		Harnett	84,707	84,707	0.26	1.00
				Johnston	89,926	12.20%		Johnston	137,877	104,080	0.35	0.75
				Lee	62,470	8.48%		Nash	108,675	62,912	0.45	0.58
				Nash	87,655	11.89%		Sampson	60,726	50,301	0.32	0.83
				Sampson	36,080	4.90%		Wake	512,798	390,320	0.34	0.76
				Wake	355,860	48.28%		Wayne	166,064	4,735	0.42	0.62
Raleigh	1,529,684	737,013	0	Warren	10,405	1.41%	737,013					
				Total	737,013	100.00%						

Table 11: Food donations (in pounds) received by a branch from local and national sources, and distributed to the counties for frozen food under perfect equity

Branch (i)	Branch Capacity (κ_i)	Assignment of donations ($X_{oi}+v_{ji}$)	Assignment of national donations (X_{oi})	Supply from county to branch			Total Food Distributed	Distribution from branch to county				
				County (j)	Amount (v_{ji})	% of local donations		County (j)	Capacity (C_j)	Amount (u_{ij})	$\frac{C_j}{D_j}$	$\frac{u_{ij}}{C_j}$
New Bern	118,794	112,205	0	Carteret	31,260	27.86%	108,387	Carteret	33,716	6,331	0.55	0.19
				Craven	22,922	20.43%		Craven	23,871	11,448	0.22	0.48
				Duplin	6,505	5.80%		Duplin	13,245	9,448	0.15	0.71
				Jones	869	0.77%		Jones	4,097	1,589	0.27	0.39
				Onslow	37,292	33.24%		Onslow	40,752	18,009	0.24	0.44
				Pamlico	4,502	4.01%		Pamlico	6,816	1,697	0.42	0.25
				Pender	8,855	7.89%						
				Total	112,205	100.00%						
Durham	126,467	91,654	0	Chatham	5,707	6.23%	71,697	Durham	70,530	35,964	0.20	0.51
				Durham	44,704	48.77%		Granville	12,367	6,393	0.20	0.52
				Granville	10,309	11.25%		Orange	12,972	12,972	0.10	1.00
				Orange	9,783	10.67%		Person	9,149	4,536	0.21	0.50
				Person	5,211	5.69%		Vance	18,482	8,028	0.24	0.43
				Vance	13,197	14.40%		Warren	5,601	3,803	0.15	0.68
				Warren	2,742	2.99%						
				Total	91,654	100.00%						
Sandhills	76,677	74,941	11,000	Moore	19,304	30.19%	37,245	Chatham	21,276	6,379	0.35	0.30
				Richmond	26,431	41.34%		Lee	16,721	7,509	0.23	0.45
				Scotland	18,206	28.47%		Moore	22,787	8,191	0.29	0.36
				Total	63,941	100.00%		Richmond	27,317	8,293	0.34	0.30
								Scotland	33,484	6,873	0.51	0.21
Wilmington	136,635	136,635	52,023	Brunswick	29,985	35.44%	58,334	Brunswick	35,962	13,243	0.28	0.37
				Newhanover	45,523	53.80%		Columbus	31,119	9,975	0.32	0.32
				Pender	9,104	10.76%		Newhanover	49,927	28,510	0.18	0.57
				Total	84,613	100.00%		Pender	21,191	6,606	0.33	0.31
Greenville	106,889	101,201	0	Edgecombe	7,644	7.55%	92,402	Edgecombe	28,490	9,521	0.31	0.33
				Greene	7,964	7.87%		Greene	10,065	3,364	0.31	0.33
				Lenoir	33,250	32.86%		Halifax	17,564	10,344	0.18	0.59
				Pitt	52,343	51.72%		Lenoir	35,108	8,854	0.41	0.25
				Total	101,201	100.00%		Pitt	59,287	27,842	0.22	0.47
								Wayne	38,448	18,936	0.21	0.49
								Wilson	25,349	13,541	0.19	0.53
Raleigh	391,234	391,234	55,000	Columbus	28,699	8.54%	134,176	Franklin	72,433	7,322	0.26	0.41
				Edgecombe	8,616	2.56%		Harnett	84,707	15,521	0.13	0.79
				Franklin	15,017	4.47%		Johnston	137,877	19,070	0.21	0.49
				Halifax	10,943	3.25%		Nash	108,675	11,527	0.26	0.40
				Harnett	17,122	5.09%		Sampson	60,726	9,217	0.12	0.90
				Johnston	31,884	9.48%		Wake	512,798	71,519	0.21	0.50
				Lee	15,108	4.49%						
				Nash	19,864	5.91%						
				Sampson	10,169	3.02%						
				Wake	126,140	37.52%						
				Wayne	29,472	8.77%						
				Wilson	23,200	6.90%						
				Total	336,234	100.00%						



(a) dry goods (b) frozen food

Fig. 3: Allocation of food distribution from the branches (labeled) to the counties under perfect equity

With respect to distribution, the optimal assignment of counties to branches and the amount of food distributed to the counties from the branches are different for dry goods and frozen food as shown in Fig. 3. For example, as reported in Table 10, Wayne county receives food from the Greenville branch, which is the closest branch at a distance of 36.3 miles and the Raleigh branch, which is the next closest branch to Wayne county at a distance of 47.0 miles, under equitable distribution for dry goods. On the other hand, in the case of frozen food, Wayne county only receives food from the Greenville branch under perfect equity (see Table 11). This is due to the amount of food allocated to each county and the limited supply available at the branch level to support the optimal allocation for different counties. From Table 10, in the case of dry goods, the Greenville branch distributes all of the donations (499,560 lbs) it receives, which eventually limits its ability to satisfy the total allocation for Wayne county. Whereas, in case of frozen food, the Greenville branch has enough supply to satisfy the demand of Wayne county and even has some undistributed supply (Table 11). Moreover, counties sending local donations to a branch may not receive food from the same branch as shown in Figures 2 and 3. As discussed earlier, the location of the national source, the total amount of donations from the national source, and the capacity of the branches influence the distribution of local supply from counties to branches. Whereas, in distributing the donations to the agencies in different counties, the food available at different branches and the capacities of the counties to receive food play the major role in the optimal assignment and allocation decisions.

The capacity of the counties to receive a particular type of food also affects the overall effectiveness of the food distribution system. Specifically, the capacity to demand ratio (C/D) of the counties directly controls the amount of food distributed to different counties for a particular level of equitable distribution [3]. In this case, for dry goods, the county with the minimum C/D ratio is Harnett with a C/D ratio of 0.23 (please see Table 10). As a result, Harnett county receives food up to its full capacity while the others receive less than

Table 12: Truckloads of donations received from the national source

Branch	distance from national source (mi)	Dry goods		Frozen food	
		Donations received	no. of truckloads	Donations received	no. of truckloads
New Bern	959.1	52,043	5	-	-
Durham	997.0	-	-	-	-
Sandhills	941.8	132,000	12	11,000	1
Wilmington	884.7	187,000	17	52,023	5
Greenville	985.8	-	8	-	-
Raleigh	982.4	-	-	55,000	5

their capacities. Tables 10 and 11 show that the C/D ratios of the counties for frozen food are very different from those for dry goods as counties have different capacities to handle frozen and dry goods. The distribution received over demand values of the counties for frozen food is also different from the same for dry goods. The distribution policy structure is the same, the county with minimum C/D ratio, Orange county, receives frozen food up to its full capacity while the other counties receive less than their capacity.

4.2.2 Comparison of model solutions with Proposition 1

In Section 3.3, we discuss the structure of the optimal donation allocation to each county and show that it depends on the interaction between the cost parameters c_s and c_w. In Proposition 1, we establish different structures based on the relationship between c_s and c_w. As we present the results obtained from the experimental setup under perfect equity, we compare the model solutions with the structures discussed in Proposition 1. We only show the comparison for the model solutions obtained using the dry goods data. As mentioned in Section 4.1, the positive values used for $c_s = 0.7235$ and $c_w = 1.85$, which correspond to Case 4 presented in Proposition 1. As discussed in this case, we evaluate the value of $\frac{c_w \sum_{j \in J} D_j}{\frac{2}{\tau} \sum_{i \in I'} \sum_{j \in J_i} d_{ij} D_j}$ and compare it with c_s. In Table 13, we list the allocation for each county as suggested in Proposition 1 and as per the model solutions along with the distance of each county to the branch from which it is receiving donations. As shown in Table 13,

$$\sum_{j \in J} D_j = 7,512,094, \text{and}$$

$$\sum_{i \in I'} \sum_{j \in J_i} d_{ij} D_j = 155,820,062$$

With $\tau = 11000$,

$$\frac{c_w \sum_{j \in J} D_j}{\frac{2}{\tau} \sum_{i \in I'} \sum_{j \in J_i} d_{ij} D_j} = 49.05 > c_s = 0.7235.$$

Hence, the optimal donation allocation to every county as presented in Proposition 1 is,

$$\sum_{i \in I'} u_{ij} = \frac{C_{j*}}{D_{j*}} D_j \qquad \forall j \in J$$

Table 13: Comparison of food distribution (in pounds) received by each county for dry goods as suggested in Proposition 1 and as per the model solutions

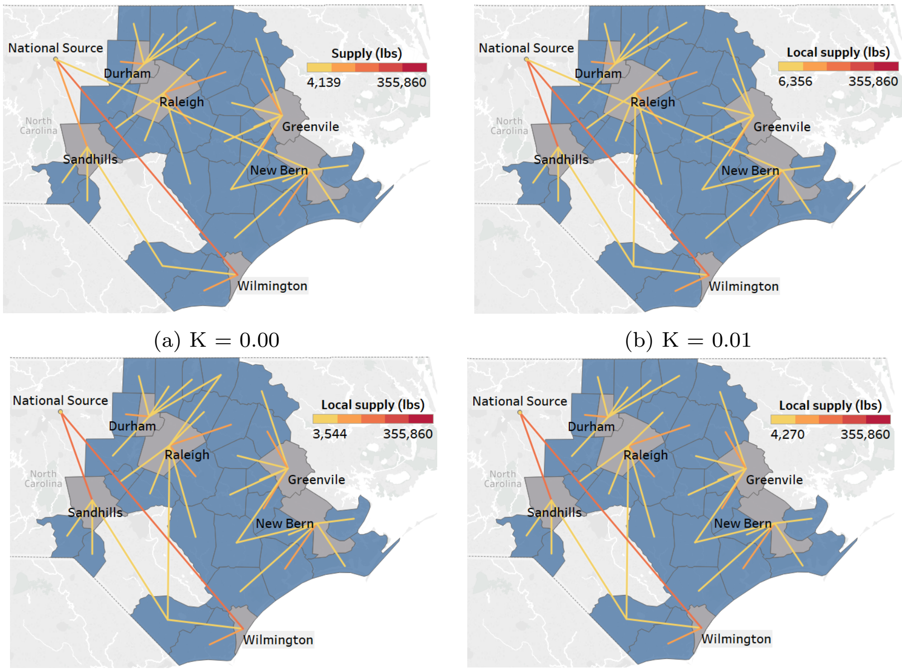
County (j)	**distance from serving branch** d_{ij}	**Demand** D_j	$d_{ij} * D_j$	**Distribution as per model solution**	**Distribution as per Proposition 1**
Brunswick	22.7	224,884	5,113,026	72,275	72,275
Carteret	31.1	107,507	3,342,243	34,551	34,551
Chatham	29.6	108,321	3,209,327	34,813	34,813
Columbus	46.6	169,380	7,888,281	54,437	54,437
Craven	0.0	194,406	-	62,480	62,480
Duplin	50.1	160,440	8,030,066	51,564	51,564
Durham	0.0	610,712	-	196,276	196,276
Edgecombe	25.3	161,680	4,090,180	51,962	51,962
Franklin	28.8	124,341	3,579,161	39,962	39,962
Granville	21.4	108,566	2,326,323	34,892	34,892
Greene	18.2	57,132	1,040,502	18,362	18,362
Halifax	48.8	175,646	8,577,050	56,451	56,451
Harnett	31.3	263,566	8,250,054	84,707	84,707
Johnston	24.4	323,838	7,893,661	104,078	104,080
Jones	18.3	26,977	494,421	8,670	8,670
Lee	21.0	127,507	2,681,041	40,979	40,979
Lenoir	28.2	150,349	4,246,850	48,321	48,321
Moore	0.0	139,096	-	44,704	44,704
Nash	39.0	195,750	7,625,129	62,912	62,912
Newhanover	0.0	484,135	-	155,596	155,600
Onslow	33.6	305,815	10,288,437	98,285	98,285
Orange	14.0	220,272	3,088,683	70,793	70,793
Pamlico	22.7	28,811	654,427	9,260	9,260
Pender	22.8	112,184	2,563,162	36,055	36,055
Person	24.9	77,029	1,920,103	24,756	24,756
Pitt	0.0	472,792	-	151,950	151,950
Richmond	26.3	140,827	3,700,091	45,260	45,260
Sampson	57.4	156,512	8,989,245	50,301	50,301
Scotland	32.4	116,706	3,786,927	37,508	37,508
Vance	33.8	136,331	4,612,597	43,815	43,815
Wake	0.0	1,214,485	-	390,323	390,323
Warren	49.6	64,587	3,201,285	20,757	20,757
Wayne	85.4	321,564	27,445,842	103,347	103,347
Wilson	31.2	229,949	7,181,948	73,903	73,903
Total		7,512,094	155,820,062		

Where, $j* = argmin(\frac{C_j}{D_j})$, which is Harnett county in this case (see Table 10) making $\frac{C_{j*}}{D_{j*}} = 0.23$. From Table 13, it is noticeable that the donation allocation per Proposition 1 is the same as obtained from the model solution for all the counties.

4.2.3 Deviation from perfect equity

Deviating from a perfectly equitable solution allows the system to distribute more food than the solution at $K = 0$. This can potentially affect the assignment of the counties to the branches for sending local donations or receiving food donations from the food bank. We explore the results obtained by solving the model for different values of K using the dry goods data to understand the impact of the relaxed equity constraint on the model solutions. Fig. 4

shows the optimal shipment decisions for local donations from different counties to the branches for different values of K within the range of 0 to 0.10. This range corresponds to the region where the optimal shipment decisions for local donations for different values of K are different. The optimal distribution of food to the counties from the branches for K within the range of 0 to 0.10 are shown in Fig. 5. This figure indicates that in distributing food donations



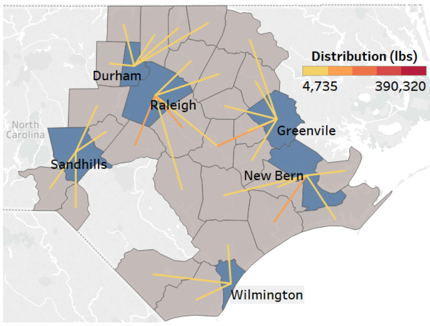
(a) K = 0.00 (b) K = 0.01

(c) K = 0.05 (d) K = 0.10

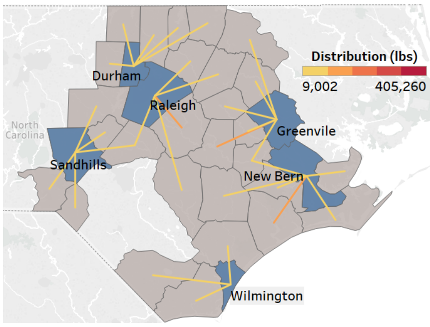
Fig. 4: Impact of deviation from perfect equity on allocation of local donations from counties to the branches (labeled)

to the counties, the optimal allocation and assignment decisions differ as a function of K. As discussed in Section 4.2.1, these changes are driven both by the distances between branches and counties, and the capacity of the branches and the counties.

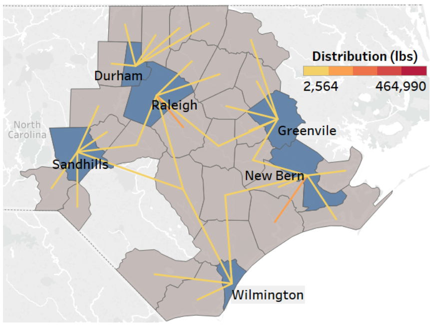
We also explore the impact of deviation from perfect equity on the effectiveness and efficiency of the system. We compare the change in total cost, cost of waste and cost of transportation along with the deviation from perfect equity. In Fig. 6, the horizontal axis shows the maximum allowable deviation from perfect equity, K, the primary vertical axis represents the cost. The undistributed supply, i.e., the waste, is represented by bars for different values of K with its value on the secondary vertical axis. The yellow line represents the wastage cost, which is proportional to the waste in lbs. The blue line shows



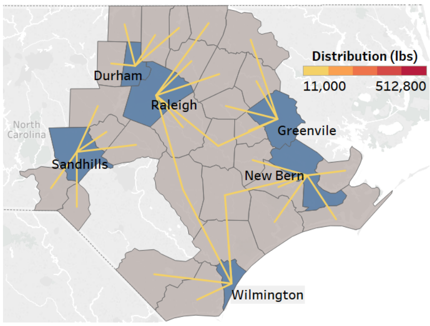
(a) K = 0.00



(b) K = 0.01



(c) K = 0.05



(d) K = 0.10

Fig. 5: Impact of deviation from perfect equity on food distribution from the branches (labeled) to the counties

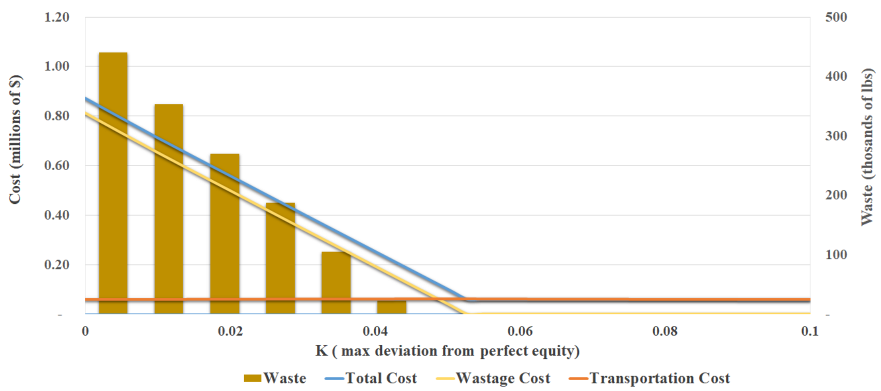


Fig. 6: Impact on efficiency and effectiveness with increasing K (total cost is shown without operating costs)

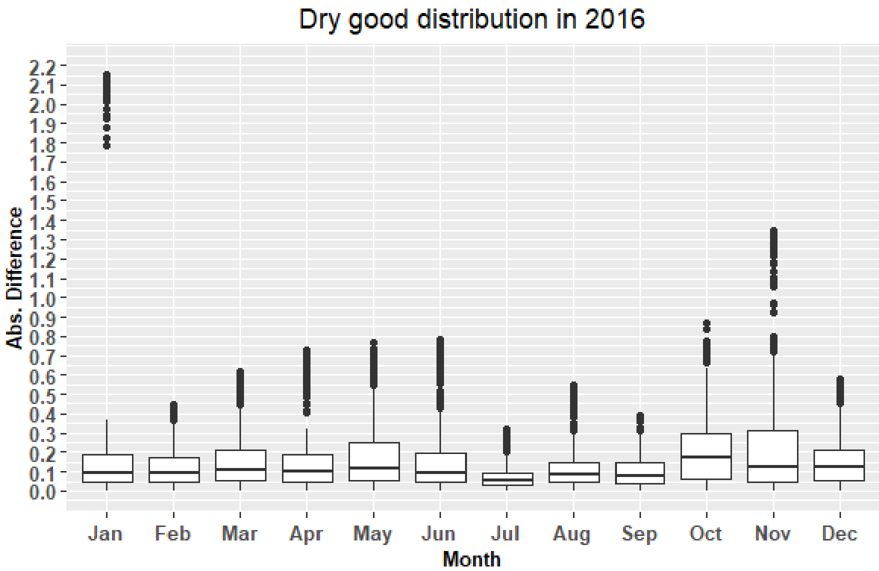
the total cost of the system excluding the branch operating costs and the orange line refers to the transportation cost for different K. As K increases, the overall cost of the distribution system and the wastage cost decreases whereas the cost of transportation initially increases slightly up to a certain limit of K until all food is distributed. This initial increase in transportation cost is because of the increase in the cost of shipping more food as the equity constraint is relaxed. As the reduction in wastage cost is much higher than the increase in transportation cost, the total cost of the system decreases with increasing K up to a certain value of K, which in this case is 0.0527. Once K is larger than this value the system become supply constrained and no supply is wasted. The total cost of the system is affected only by the transportation cost.

Figure 6 also shows a monotonic relationship between K and waste (and total cost). The waste and correspondingly the total cost monotonically decrease as K increases, i.e., as the constraint on equity is relaxed it is possible to distribute more food. The relaxation of the constraint on K essentially identifies a Pareto frontier associated with the objectives of effectiveness-efficiency and equity. An interesting direction for future work would be to formulate the problem as a bi-objective optimization problem to minimize the weighted sum of K and cost.

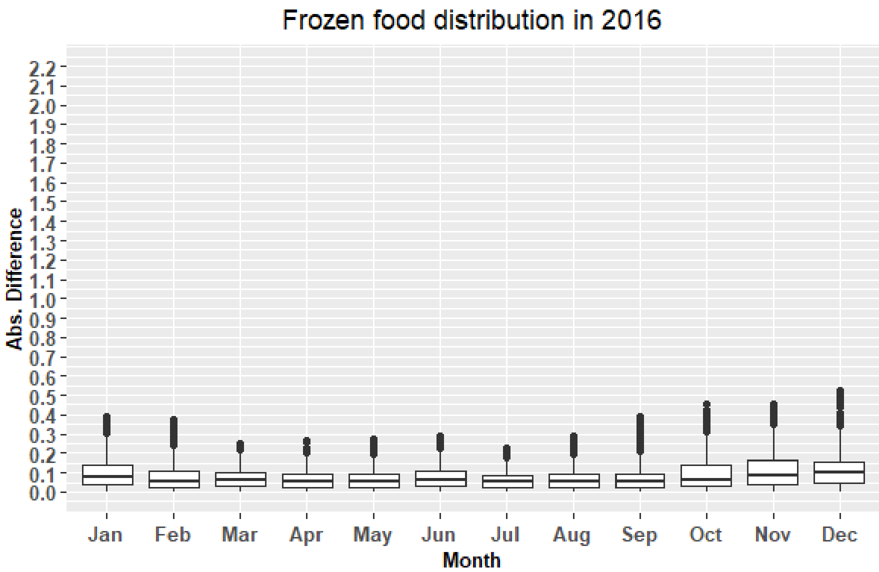
4.2.4 Deviation from equitable distribution in real data

As shown in Section 4.2.3, the model solution has a waste under perfect equity. We also show that if we allow a deviation from perfect equity with a maximum value of K =0.0527, the model solution has zero waste. In this section, we investigate the actual distribution data of FBCENC and evaluate the level of deviation from perfect equity following our equity definition mentioned in Section 2 and later on translated into constraint (3). For this purpose, we consider FBCENC's actual monthly dry goods and frozen food distribution in 2016.

We compare the proportional demand served for each county with the other counties proportional demand served to identify deviation from perfect equity. We perform this analysis for both dry goods and frozen food for each month in 2016. For this analysis, we use demand data reported in Tables 10 and 11. Figure 7 shows the boxplot of all the individual values of the absolute difference between the proportional demand served between any two counties. As shown in Figure 7(a), the maximum difference between the proportional demand served for any two counties in the monthly dry good distribution by FBCENC in 2016 varied from month to month within a range of 0.3 to 2.2. Figure 7(b) shows this maximum difference for frozen food varied between 0.25 and 0.55. Figure 3(a) shows that the maximum difference between the proportional demand served for any two counties in January 2016 was as high as 2.2. That is attributable to a high allocation to Pamlico county, which received a 2.32 times higher allocation of dry goods compared to its demand for dry goods. That high allocation created a large difference in proportional demand served compared to counties like Greene and Harnett who received an



(a) dry goods



(b) frozen

Fig. 7: The absolute difference between the proportional demand served for any two counties in the actual monthly donation distribution by FBCENC in 2016

allocation of 0.16 and 0.17 times of their demand for dry goods, respectively. It is also noticeable in Figure 7(a) that the 50$^{\text{th}}$ percentile of the differences varies from 0.05 to 0.2, the 75$^{\text{th}}$ percentile of the differences varies from 0.1 to 0.35 across the months in 2016.

4.2.5 Deviation from FBCENC's fair share allocation

FBCENC monitors the level of equity through their "Fairshare" program. According to the Fairshare program, FBCENC allocates donations to each county in its service region according to the county's estimated need relative to their proportion of the total demand in FBCENC's service region. They monitor the deviation between the proportion of a county's demand relative to the total demand and the proportion of donation allocation to each county relative to the total donation allocated to all counties. The equivalent constraint in terms of our model is as follows.

$$\left| \frac{\sum\limits_{i \in I} u_{ig}}{\sum\limits_{i \in I} \sum\limits_{j \in J} u_{ij}} - \frac{D_g}{\sum\limits_{j \in J} D_j} \right| \leq K, \qquad \forall g \in J \tag{29}$$

We insert our model solution obtained for $K = 0.06$, where the model distributes all the available donations resulting in zero waste, into (29) to assess the deviation from the fairshare for each county. Figure 8 shows the fairshare for each county, the model allocated share and the actual share allocated by FBCENC in December 2016. The horizontal axis represents the counties and the vertical axis shows the relative values.The red line shows the fairshare for each county, the green line shows the share as per the model solutions, and the blue line shows actual share as per FBCENC's allocation in December 2016. Figure 9 reports the deviation from fairshare allocation as per FBCENC's actual allocation and the model solution for the month of December 2016 for all the counties. While the horizontal axis represents the same as Figure 8, the vertical axis, in this case, represents the difference in share. The red line shows the difference between the actual share and fairshare for each county, and the blue line shows the difference between fairshare and the share allocated as per the model solution. Figure 9 shows that for most counties the model solution deviates less from the fairshare allocation than the actual allocation. This suggests that the model's allocation is more equitable than the actual distribution performed by FBCENC in December 2016.

4.2.6 Shipping cost, truck capacity and cost of undistributed food

We investigate the interplay between per mile shipping cost (c_s) of a truck and per unit cost of undistributed food (c_w) and their impact on the optimal distribution decision. We evaluate the optimal policy, total cost (excluding the operating cost), and the amount of undistributed food as a function of c_s (from 0 to 15) and K for fixed value of c_w (\$1.85) to explore how efficiency and effectiveness vary for different equity constraints. We also evaluate the effect

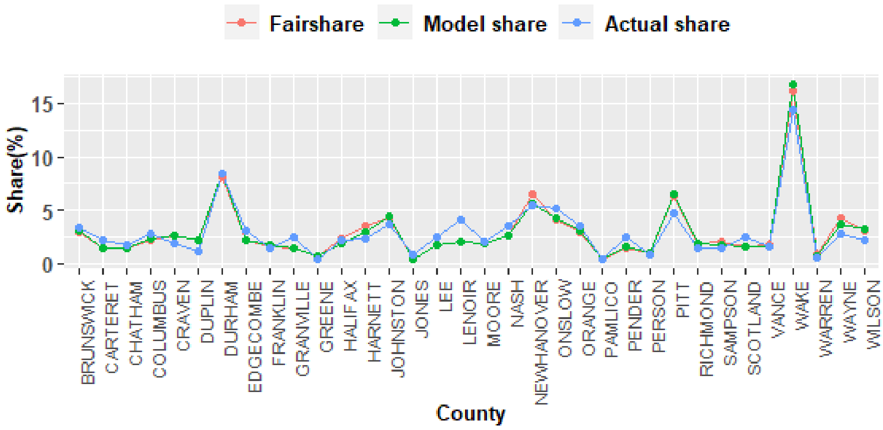


Fig. 8: Fairshare , actual share, and model share as per the model solution at K=0.06

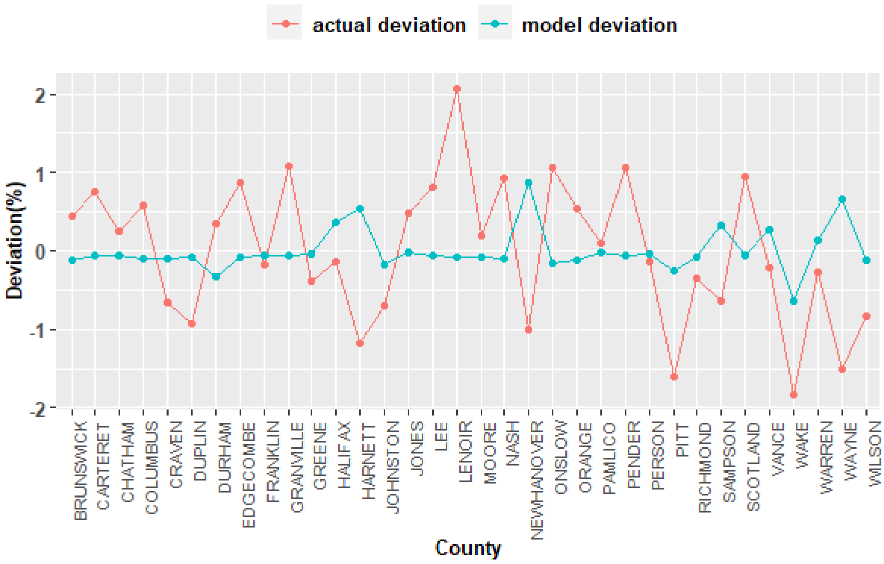


Fig. 9: Deviation from the fairshare allocation as per FBCENC's actual distribution and model solutions for K=0.06

of different truck sizes (2000, 4000, 6000 and 11000 lbs) used for shipping donations on efficiency and effectiveness.

In Figure 10, the horizontal axis represents the per mile shipping cost of a truck and the vertical axis represents the amount of food undistributed and the total cost of the system for a month on two different scales for each level of equity constraint. As would be expected for fixed c_w, Figure 10 shows

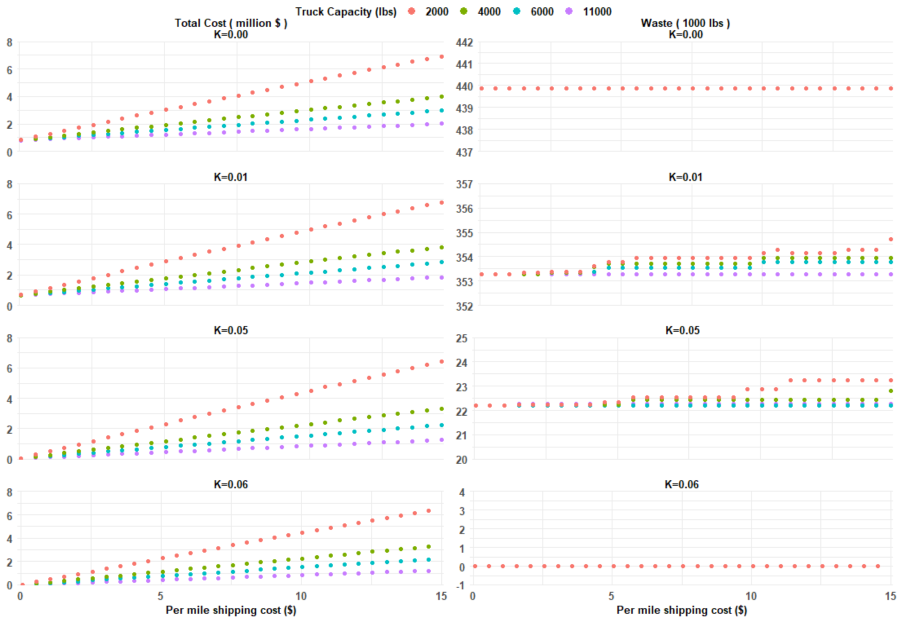


Fig. 10: Sensitivity of the model efficiency and effectiveness against varying shipping cost, truck capacity and equity (K)

that the total cost is monotonically increasing with increasing c_s and as K increases (i.e., as equity is relaxed), the total cost decreases corresponding to the reduction in waste. Note that as the equity constraint is relaxed the effectiveness increases (waste decreases to zero) across all truck sizes. However, while efficiency also improves the optimal policy is not as sensitive to changes in K. Comparing results for different truck sizes, we observe from the Fig. 10 that using larger trucks to ship donations makes the system more efficient and effective for each level of the equity constraint except for the case of perfect equity.

In case of perfect equity, the waste level remains unchanged regardless of the truck size. With the exception of the perfect equity case, the optimal policy is less sensitive to shipping cost for larger trucks in terms of effectiveness (i.e., the effect of waste). This is understandable as using smaller sized trucks requires more truckloads to ship the food and eventually costs more. Hence, the model solution chooses to waste more food than to distribute it when smaller trucks are used for shipment as the cost of shipping increases and there is no penalty to unused truck capacity.

4.2.7 Comparison of flexible network to existing network

In Section 3.1, our flexible model is introduced that allows food from local and national sources to be shipped directly to the branches of the food bank and then be redistributed to the agencies operating within the different counties. As per the current practice, FBCENC receives most of the food donations at the Raleigh branch, which serves as the hub for the network, and redistribute the food to the branches after processing. We compare the efficiency of the existing network to the efficiency of our proposed flexible network. For the flexible network, we use the model presented in Section 3.2 and the data described in Section 4.1 to obtain the total cost of the system for different levels of allowed inequity. For the existing network, we slightly modify the formulation presented in Section 3.2 to adapt the model as per the structure of the existing network. We introduce an index h to represent the Raleigh branch, the hub, and a decision variable s_h to represent the number of truckloads needed to ship all national donations to the hub h. The objective function (1) in Section 3.2 is modified as follows to incorporate the shipping cost for collecting donations from the national and local sources and redistributing them to the branches.

$$\sum_{i \in I} c_{oi} f_i + 2c_s \Big(d_{nh} s_h + \sum_{j \in J} d_{jh} q_{jh} + \sum_{i \in I} d_{hi} p_{hi} + \sum_{i \in I} \sum_{j \in J} d_{ij} r_{ij}\Big) + c_w W \quad (1a)$$

Constraints (4) and (5) in Section 3.2 are replaced by Constraint (30), which ensures that all the local and national supplies received at the hub h are shipped to the branches of the food bank.

$$\sum_{i \in I} X_{hi} = S + \sum_{j \in J} \xi_j \quad (30)$$

We add Constraint (31) to the model in Section 3.2 to capture the number of truckloads required to ship all the national donations to the hub h.

$$\frac{S}{\tau} \leq s_h \quad (31)$$

Constraints (7), (9), (11) and (14) are replaced with (7a), (9a), (11a) and (14a).

$$\sum_{j \in J} u_{ij} \leq X_{hi}, \qquad \forall i \in I \quad (7a)$$

$$\frac{v_{jh}}{\tau} \leq q_{jh}, \qquad j \in J \quad (9a)$$

$$X_{hi} \leq \kappa_i f_i, \qquad \forall i \in I \quad (11a)$$

$$p_{hi}, q_{ji}, r_{ij}, s_h \quad integer, \qquad \forall i \in I, j \in J \quad (14a)$$

All the other constraints in the model presented in Section 3.2 remain the same. We solve this model using the data presented in Section 4.1 and obtain the total monthly cost for the existing network. We calculate the percentage cost savings from the flexible network over the existing network in practice as shown in Equation 32. Under the existing cost structure, receiving food donations directly at the branches could save approximately 32% of the transportation cost for all K.

$$\text{Savings (\%)} = \frac{\text{Cost of existing network - Cost of flexible network}}{\text{Cost of existing network}} \times 100 \tag{32}$$

5 Conclusions and Future Work

We study the food donation distribution network of a food bank within the network of Feeding America. We develop an assignment and distribution model to identify the efficient and effective assignment of counties to branches and the optimal allocation of donated food to each county within an allowable deviation from perfect equity. Our objective is to maximize the distribution of food at the minimum transportation and operating cost while maintaining the FA guideline for equitable distribution. We formulate a mixed integer programming model that minimizes the total cost of operating branches, the cost of receiving and distributing food, and the cost for undistributed food. We restrict the donation distribution decision within a certain level of deviation from perfect equity through a model constraint. We analyze the case of perfect equity, which is the least effective case as it can lead to food waste due to capacity constraints at the county-level. Specifically, we study the optimal solution to understand the relationship between the shipping cost and the cost of undistributed food under perfect equity. Under the efficient assignment of counties to branches, we analyze the effect of these cost parameters on the food distribution decision under perfect equity. We prove that based on the relative values of these two cost parameters, the optimal food distribution decision can be either (1) to distribute nothing, (2) to distribute to all counties according to the minimum C/D ratio, or (3) to distribute nothing or up to the amount of food according to the minimum C/D ratio.

We perform a numerical study with data from FBCENC, categorizing the data by the type of food, e.g., dry goods, frozen. We solve the model for different values of K, the maximum allowable deviation from perfect equity, and analyze the optimal model solutions. The results show that the assignment of counties, for both the case of shipping local supplies to the branches and receiving the final distribution from the branches, differ by the type of food and by the level of equity maintained. The model solutions also suggest that a marginal increase in K can allow the system to distribute more food at a lower cost. A comparison of the inequity level of the actual distribution to the inequity level associated with the model solution shows the model solution performs better in terms of the equitable distribution of the donated food. Results from the sensitivity analysis also verify that sacrificing equity improves both efficiency and effectiveness. Moreover, the sensitivity analysis shows that using trucks with higher capacity improves efficiency and effectiveness for the same level of equity. A comparison between the proposed flexible network and FBCENC's existing network shows that FBCENC could save approximately 32% of their transportation cost by adopting a flexible network model for receiving donations at all of the food bank's branches.

The policies and results from our analyses have been shared with FBCENC's staff. Results from our analyses provide insight regarding how to improve efficiency in the collection and distribution of donations. Our results identify potential factors for reducing transportation and wastage cost while maintaining a desired level of equity. Although our study focuses on the objectives of FBCENC with the analysis of distribution data, our model and results are representative of the 200 food banks working within FA network. Our model can easily be adapted to the distribution network of other food banks.

The study of food bank operations considering equity, efficiency and effectiveness provides many potential opportunities for future research. In this work, we represent all the agencies working in a county as a single agency located at the centroid of the county. Our model could be extended to identify the efficient distribution considering the exact location of the agencies with their capacity. The equity constraint could also be studied at the agency level instead of limiting it to the county level. In addition, the model could be extended to incorporate a routing problem relaxing our assumption of shipping food to one location at a time.

In this work, we study the donation collection and distribution problem to facilitate tactical planning. Hence, optimal routing of pickups and deliveries have not been considered. While the problem may be computationally intensive considering the size of the network, it is an interesting area for future research. The model solution presented in the computational study suggests that, in many cases, donations are shipped to a county from a branch with partially full truck shipments. Instead of studying a routing problem for the entire network, which can be computationally difficult, it may be possible to disaggregate the network to reduce the computational complexity.

Another area for future study would be to incorporate non-homogeneous truck capacities to study the impact of efficiency on effective and equitable distribution. Combined equitable distribution of different types of food is another potential area for future exploration. In this model, demand, capacity, and supply are assumed to be deterministic. In practice, the demand, supply, and capacity of a food bank are uncertain. One may consider one or more of these input parameters to be uncertain to study their impact on the optimal policy. Food banks run a long term humanitarian operation. Our work analyzes the distribution problem for a single period. A next step could be to extend the model for multiple periods incorporating uncertainty in future input parameters such as supply, capacity. Food banks, as a non-profit entity, also has a limited financial support to run their operations. Adding a budgetary constraint could also be an interesting direction for further study.

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Appendix: Proof of Proposition 1

The proof addresses the cases separately.

Case 1: In this case with $c_s = 0$ and $c_w = 0$ the objective function of the model P2 is 0. So, any feasible solution to constraints sets (18) to (23) will be an optimal solution with an objective function value of 0. Constraints set (18) can be substituted by Equation (24), where z represents the fraction of demand served for each county. Thus,

$$\sum_{i \in I'} u_{ij} = zD_j, \qquad \forall j \in J$$

and

$$\sum_{i \in I'} \sum_{j \in J_i} u_{ij} = z \sum_{j \in J} D_j.$$

Constraints (21) can be written as

$$zD_j \leq C_j, \qquad \forall j \in J$$

or

$$z \leq \frac{C_j}{D_j}, \qquad \forall j \in J,$$

which is equivalent to the following inequality.

$$z \leq \min_j \left(\frac{C_j}{D_j}\right) \tag{33}$$

Constraint (19) can be written as the following equation.

$$W = TS - z \sum_{j \in J} D_j \tag{19a}$$

As there is no upper bound for r_{ij}, a feasible solution can always be obtained satisfying constraints (20). So it will suffice to prove that there exists some $z \leq \min_j(\frac{C_j}{D_j})$ that satisfies (19a). Thus $\sum_{i \in I'} u_{ij}$ have multiple optimal solutions. Given $TS > min(\frac{C_j}{D_j}) \sum_{j \in J} D_j$ and because of the non-negativity of u_{ij}, any value of z given $0 \leq z \leq min(\frac{C_j}{D_j})$ will satisfy constraint (19a) and hence provide a feasible and optimal solution to P2.

Case 2: In this case the objective function is $c_w W$ and it will be minimized when W is lowest. From (19a),

$$W = TS - z \sum_{j \in J} D_j.$$

The minimum value of W can be attained by maximizing z. With $TS > min(\frac{C_j}{D_j})$ and being constrained by (33) the maximum possible value of z is $min(\frac{C_j}{D_j})$. Hence, the optimal solution for the problem is $W = TS - \min_j(\frac{C_j}{D_j}) * \sum_{j \in J} D_j$ and $z = \min_j(\frac{C_j}{D_j})$. i.e., $\sum_{i \in I'} u_{ij} = \min_j(\frac{C_j}{D_j}) D_j, \forall j \in J$.

Case 3: In this case the objective function is $2c_s \sum_{i\in I'} \sum_{j\in J_i} d_{ij}\frac{u_{ij}}{\tau}$ and it will be minimized with $u_{ij}, \forall i \in I', j \in J_i$ being at the lowest possible values. From (26), which is obtained from (18), $z = 0$ identifies the optimal solution for $u_{ij} = 0, \forall i \in I', j \in J_i$ satisfying constraints (20) and (21). Thus the optimal solution for $W = TS$.

Case 4: From (17c) the objective function in this case is

$$z(\frac{2c_s}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j - c_w \sum_{j\in J} D_j) + c_w TS.$$

From here, we will ignore the constant part $c_w TS$ and discuss scenarios minimizing the rest of the objective function.

When $c_s > \frac{c_w \sum\limits_{j\in J} D_j}{\frac{2}{\tau}\sum\limits_{i\in I'}\sum\limits_{j\in J_i} d_{ij}D_j}$, it follows that

$$\frac{2c_s}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j - c_w \sum_{j\in J} D_j > 0.$$

Thus the optimal solution will be reached while z is minimized and from Case 3 we know that $z = 0$ is a feasible and optimal solution to this problem. Hence, the optimal solution under this condition is $\sum\limits_{i\in I'} u_{ij} = 0, \forall j \in J$ and $W = TS$.

When $c_s = \frac{c_w \sum\limits_{j\in J} D_j}{\frac{2}{\tau}\sum\limits_{i\in I'}\sum\limits_{j\in J_i} d_{ij}D_j}$, it is equivalent to

$$\frac{2c_s}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j - c_w \sum_{j\in J} D_j = 0.$$

The objective function becomes constant with a value of $c_w W$ and the problem is similar to case 1 as any feasible solution that satisfies the constraint sets (18) to (23) will provide an optimal solution here. Hence, any z within the range of $0 \leq z \leq min(\frac{C_j}{D_j})$ will provide a feasible and optimal solution to the problem proving the existence of multiple optimal solutions.

When $c_s < \frac{c_w \sum\limits_{j\in J} D_j}{\frac{2}{\tau}\sum\limits_{i\in I'}\sum\limits_{j\in J_i} d_{ij}D_j}$, it is equivalent to

$$\frac{2c_s}{\tau}\sum_{i\in I'}\sum_{j\in J_i} d_{ij}D_j - c_w \sum_{j\in J} D_j < 0.$$

Again, ignoring the constant part $c_w TS$ in the objective function, the objective function will be minimized when z is maximized. This is very similar to case 2 and hence, the optimal solution will be reached with $z = min(\frac{C_j}{D_j})$. Thus the optimal allocation $\sum_{i\in I'} u_{ij} = min(\frac{C_j}{D_j})D_j, \forall j \in J$ and $W = TS - min(\frac{C_j}{D_j}) * \sum\limits_{j\in J} D_j$.