A Differentially Private Incentive Design for Traffic Offload to Public Transportation

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Increasingly large trip demands have strained urban transportation capacity, which consequently leads to traffic congestion and rapid growth of greenhouse gas emissions. In this work, we focus on achieving sustainable transportation by incentivizing passengers to switch from private cars to public transport. We address the following challenges. First, the passengers incur inconvenience costs when changing their transit behaviors due to delay and discomfort, and thus need to be reimbursed. Second, the inconvenience cost, however, is unknown to the government when choosing the incentives. Furthermore, changing transit behaviors raises privacy concerns from passengers. An adversary could infer personal information (e.g., daily routine, region of interest, and wealth) by observing the decisions made by the government, which are known to the public. We adopt the concept of differential privacy and propose privacy-preserving incentive designs under two settings, denoted as two-way communication and one-way communication. Under two-way communication, passengers submit bids and then the government determines the incentives, whereas in one-way communication, the government simply sets a price without acquiring information from the passengers. We formulate the problem under two-way communication as a mixed integer linear program and propose a polynomial-time approximation algorithm. We show the proposed approach achieves truthfulness, individual rationality, social optimality, and differential privacy. Under one-way communication, we focus on how the government should design the incentives without revealing passengers' inconvenience costs while still preserving differential privacy. We formulate the problem as a convex program and propose a differentially private and near-optimal solution algorithm. A numerical case study using the Caltrans Performance Measurement System (PeMS) data source is presented as evaluation. The results show that the proposed approaches achieve a win-win situation in which both the government and passengers obtain non-negative utilities.

CCS Concepts: • **Computer systems organization → Embedded and cyber-physical systems**; • **Security and privacy → Human and societal aspects of security and privacy**;

Additional Key Words and Phrases: Intelligent transportation system, incentive design, differential privacy

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1 INTRODUCTION

Rapid urbanization is a global trend [13]. Compared to public and non-motorized transport modes, private vehicles are an increasingly popular transport choice to meet the huge traffic demands associated with the fast-growing urban population [15]. It has been shown that around 47% of daily trips in cities are made by private motorized vehicles [46]. If such trends continue, it is predicted that there will be 6.2 billion daily trips made by private vehicles in 2025 [46].

Several challenges are raised due to the fast-growing trip demands and increasingly pervasive uses of private vehicles. First, road transport is overly consumed, resulting in traffic congestion, which leads to economic losses. For example, the cost caused by congestion in urban areas in the US in 2010 is approximately $101 billion [50]. The European Union (EU) estimates the cost incurred due to congestion to be 1% of its annual gross domestic product (GDP) [8]. Second, environmental concerns are raised due to the growth of private car use. The Greenhouse Gas (GHG) emissions due to the transportation sector will reach 40% by 2050 [30].

Reducing the dependence on private cars has been identified as one of the objectives of governments to achieve sustainability [14]. One approach is to promote public transportation, which is shown to be more sustainable compared to private cares [7, 26], as an alternative [24].

In this work, we investigate the problem of how the government could incentivize the passengers to use public transport instead of private cars. There are several challenges faced by the government to encourage passengers to change transit behaviors—from private cars to public transport. First, although the government discourages the use of private cars, passengers' trip demands still need to be satisfied. Moreover, the passengers incur inconvenience costs when switching from private cars to public transport. The inconvenience cost is due to several factors including reduced quality of service (QoS) and delay of arrival time. The passengers need to be reimbursed for these costs. Furthermore, the inconvenience cost, which varies from passenger to passenger, is unknown to the government. The passengers might be unwilling to reveal the inconvenience costs or lie about their inconvenience costs to earn benefits during the interaction with the government. Although existing literature has experimentally identified the factors that prevent passengers from changing transit behavior [5, 31, 47, 54], a theoretical analysis on how to incentivize the passengers to change their transit behaviors with privacy guarantee has received little research att ention.

In this article, we model and analyze how the government could incentivize the passengers to satisfy their traffic demands via public transport instead of private cars under two settings, named two-way communication and one-way communication. Under two-way communication, the government and passengers can communicate with each other. Under one-way communication, the government can send information to the passengers but not vice versa. We not only address the challenges faced by the government but also address the passengers' privacy concerns when shifting from private cars to public transport. The privacy concerns are raised since an untruthful party can observe how the passengers respond to incentives and learn the passengers' private information including region of interest and daily routine. Such privacy concerns, which have been reported in transportation system design [2, 25], discourage privacy-sensitive passengers to switch from private cars to public transport. We make the following contributions:

- We model the interaction between the government and passenger under two-way communication using a reverse auction model. We formulate the problem of incentivizing the passengers as a mixed integer linear program. We propose an efficient algorithm to reduce the computation complexity for computing the passengers selected by the government and the associated incentives.
- We prove that the proposed mechanism design under two-way communication achieves approximate optimal social welfare, truthfulness, individual rationality, and differential privacy.

- For the one-way communication setting, we formulate the problem as an online convex program. We give a polynomial-time algorithm to solve the problem. We prove that the proposed mechanism is differentially private and provides the same asymptotic utility as the best fixed price, that is, achieves Hannan consistency.
- We present a numerical case study with real-world trace data as evaluation. The results show that the proposed approach achieves individual rationality and non-negative social welfare, and is privacy preserving.

The remainder of this article is organized as follows. We discuss the related works in Section 2. In Section 3, we present the problem formulation under two-way and one-way communication settings, respectively. We present the proposed incentive mechanism design in Section 4 for the two-way communication setting. Section 3.3 gives the proposed solution for the one-way communication setting. The proposed approaches are demonstrated using a numerical case study in Section 6. We conclude the article in Section 7.

2 RELATED WORK

In this section, we present literature review on intelligent transportation systems and differential privacy. Significant research effort has been devoted to achieving intelligent and sustainable transportation systems. Planning and routing navigation problems have been investigated by transportation and control communities [9, 10, 34, 45, 52, 55]. Various approaches have been proposed to improve operation efficiency of existing transportation infrastructure, among which vehicle balancing [38] has been extensively studied for bike sharing [51] and taxis [39]. Metering strategies have also been investigated [11, 20]. Traffic signal scheduling has been extensively studied [19, 28, 40, 49] to mitigate urban traffic congestion. Different from the works mentioned above, this article focuses on the demand-side management, with particular interest on how to encourage passengers to change their transit behaviors via incentive design.

We next discuss related works on demand-side management. Alternative travel infrastructures such as a bike-sharing system [12] have been implemented all over the world. Moreover, the ride-sharing system and the associated ride-sharing match system have been investigated [33, 36], which group passengers with similar itineraries and time schedules together to reduce the number of operating vehicles. Most of these works focus on taxis and ride-hailing services such as Uber and Lyft and ignore the potential from public transport. Pricing schemes have been proposed to reduce the number of operating vehicles at peak hours [21, 32]. These works focus on private cars and ignore public transport. Researchers have identified the factors (e.g., passengers' attitude and government's policy) that prevent passengers from taking public transport [4, 5, 31, 44, 47, 54]. However, to the best of our knowledge, there has been little research attention on how to design incentives to encourage passengers to switch from private to public transit services.

Mechanism design has recently been used in engineering applications such as cloud computing. In particular, the Vickrey-Clarke-Groves (VCG) mechanism [53] is widely used to preserve truthfulness. However, truthful communication raises privacy concerns. To address the privacy issue, we adopt the concept of differential privacy [16–18]. Mechanism designs with differential privacy, such as exponential mechanism, have been proposed [18, 23, 29, 37]. However, they are not readily applicable to the problem investigated in this article because the presence of inconvenience cost functions leads to violations of individual rationality. Moreover, the exponential mechanism is computationally complex, and hence in this article we propose efficient approximation algorithms.

Trial-and-error implementation for toll pricing has been proposed in [56]. Different from [56], we consider a closed-loop Stackelberg information pattern and compute the optimal incentive price. To solve the problem under the one-way communication setting, we adopt the Laplace mechanism to preserve differential privacy [18]. This article extends our preliminary conference version [42], in which the two-way communication setting is studied. We extend the preliminary work by also investigating the one-way communication setting.

3 PROBLEM FORMULATION

In this section, we first give the problem overview. Then we present the problem formulations under two settings, denoted as two-way communication and one-way communication. We finally discuss the privacy model.

3.1 Problem Overview

Let $\mathcal{S} = \{1, \dots, S\}$ denote the set of origin-destination (OD) pairs that will require passengers to switch to public transport over time horizon $t = 1, \dots, T$. When passengers switch from private cars to public transport, they can provide some amount of traffic offload.[1] We assume each OD pair $s \in \mathcal{S}$ requires $Q_{s,t}$ amount of traffic offload at time t to achieve sustainability. Let $\mathcal{N} = \{1, \dots, N\}$ be the set of passengers. At each time t, any passenger $i \in \mathcal{N}$ that switches to public transport for any OD pair $s \in \mathcal{S}$ receives revenue $r_{i,s,t}(q_{i,s,t})$ issued by the government, where $q_{i,s,t} \geq 0$ is the amount of traffic offload that passenger i can provide for OD pair s at time t and $r_{i,s,t}(0) = 0$. Passenger i also incurs inconvenience cost $C_{i,s}(q_{i,s,t})$ if it switches from private to public transit service due to discomfort and time-of-arrival delays. We remark that each passenger is physically located close to some OD pair s at each time t. Hence, each passenger is only willing to switch to public transport for one OD pair s that is physically close to its current location. For other OD pairs $s' \neq s$, we can regard the associated inconvenience cost as infinity. We assume that the inconvenience cost function $C_{i,s}(q_{i,s,t})$ is continuously differentiable, strictly increasing with respect to $q_{i,s,t}$ for all $s \in \mathcal{S}$, and convex with $C_{i,s}(0) = 0$ for all i and s. The utility of the passenger at each time step t is given by

$$U_{i,t} = \sum_{s} [r_{i,s,t}(q_{i,s,t}) - C_{i,s}(q_{i,s,t})]. \tag{1}$$

In this work, we assume the passengers are *selfish* and *rational*; that is, the passengers selfishly maximize their utilities.

3.2 Case 1: Interaction with Two-Way Communication

In this subsection, we present the problem formulation under two-way communication. In this case, the interaction between the government and the set of passengers is captured by a reverse auction model.

The passengers act as the bidders. Each passenger can submit a bid $\mathbf{b}_{i,t} = [b_{i,1,t}, \dots, b_{i,S,t}]$ to the government at each time t, where element $b_{i,s,t} = (\zeta q_{i,s,t}, \bar{C}_{i,s}(q_{i,s,t}))$ contains the amount of traffic offload that passenger i can provide and the associated inconvenience cost. Here ζ converts the amount of traffic offload from utilities in dollars. Without loss of generality, we assume $\zeta = 1$ in the remainder of this article. Note that $\bar{C}_{i,s}(q_{i,s,t})$ is the inconvenience cost claimed by passenger i, which does not necessarily equal the true cost $C_{i,s}(q_{i,s,t})$.

The government is the auctioneer. It collects the bids from all passengers and then selects a set of passengers that should participate in traffic offload. In particular, the government computes a selection profile $X \in \{0, 1\}^{N \times S \times T}$, with each element $x_{i,s,t} = 1$ if passenger i is selected and 0 otherwise. If a passenger i is selected by the government for OD pair s, an associated incentive $r_{i,s,t}(q_{i,s,t})$ is issued to passenger i.

[1] In the remainder of this article, we use 'traffic offload' and 'reduce the use of private car' interchangeably.

The utility (Equation (1)) of each passenger i at time t is rewritten as

$$U_{i,t} = \sum_{s} x_{i,s,t} \left[r_{i,s,t}(q_{i,s,t}) - C_{i,s}(q_{i,s,t}) \right], \forall i, t. \tag{2}$$

The social welfare can be represented as

$$\Omega(X, B) = \sum_{t} \sum_{s} \sum_{i} \left[x_{i,s,t}(q_{i,s,t} - C_{i,s}(q_{i,s,t})) \right], \tag{3}$$

where B contains $\mathbf{b}_{i,t}$ for all i and t. The government aims at maximizing social welfare $\Omega(X, B)$. This social welfare maximization problem is given as

$$\max_{X} \quad \sum_{t} \sum_{s} \sum_{i} \left[x_{i,s,t}(q_{i,s,t} - C_{i,s}(q_{i,s,t})) \right] \tag{4a}$$

$$\text{s.t.} \quad \sum_{s \in \mathcal{S}} x_{i,s,t} \leq 1, \ \forall i, t \tag{4b}$$

$$\sum_{i \in \mathcal{N}} x_{i,s,t} q_{i,s,t} \geq Q_{s,t}, \ \forall s, t \tag{4c}$$

$$x_{i,s,t} \in \{0, 1\}, \ \forall i, s, t. \tag{4d}$$

Constraint (4b) implies that a passenger can only be selected for one OD pair at each time t. Constraint (4c) requires that the desired traffic offload $Q_{s,t}$ must be satisfied for all s and t. Constraint (4d) defines binary variable $x_{i,s,t}$.

Under the two-way communication setting, a malicious adversary aims at inferring the inconvenience cost function of each passenger by observing the selection profile X. The adversary can observe X by eavesdropping on the communication channel. Let X_t be the selection profile at time t. Then the information perceived by the adversary up to time t is $\mathcal{I}_t^{two} = \{X_{t'} | t' \leq t\}$. Therefore, the government needs to compute a privacy-preserving incentive mechanism so that the social welfare is (approximately) optimal. Compared to encryption, which successfully prevents an eavesdropper, we will propose a mechanism that ensures a desirable trade-off between social welfare and privacy guarantee and requires no computation from any passenger. Moreover, our proposed mechanism is not only resilient to a third-party adversary but also an honest-but-curious passenger.

Besides the privacy guarantees, we state some additional desired properties that the government needs to achieve under this two-way communication setting. First, individual rationality for each passenger should be achieved; that is, each passenger must obtain non-negative utility when being selected by the government. Second, the government wishes to reveal the true inconvenience cost functions from the passengers to seek the optimal solution to Equation (4). Therefore, the government needs to ensure that the passengers bid truthfully. Truthfulness is defined as follows.

Definition 3.1 (Truthfulness). An auction is truthful if and only if bidding the true inconvenience cost function, that is, $\bar{C}_{i,s}(q_{i,s,t}) = C_{i,s}(q_{i,s,t})$ for all $q_{i,s,t}$, is the dominant strategy for any passenger i regardless of the bids from the other passengers. In other words, bidding $\bar{C}_{i,s}(q_{i,s,t}) = C_{i,s}(q_{i,s,t})$ maximizes the utility (Equation (2)) of passenger i for all i.

3.3 Case 2: Interaction with One-Way Communication

In this subsection, we present a problem formulation when two-way communication is infeasible while one-way communication from the government to the passengers is enabled. Under this setting, the passengers cannot report any information to the government. The government hence broadcasts an incentive price $p_{s,t}$ for each OD pair s at each time step t, and then observes the responses from the passengers to design the incentive price for the next time step $(t + 1)$. Different from two-way communication, the passengers respond to the incentive price rather than bidding a fixed amount of traffic offload. Hence, the amount of traffic offload provided by each passenger i for OD pair s at time t is defined as a function of incentive price $p_{s,t}$, denoted as $q_{i,s}(p_{s,t})$. We assume that the traffic offload $q_{i,s}(p_{s,t})$ provided by each passenger i is strictly increasing with respect to $p_{s,t}$.

The government predicts the traffic condition for the set of OD pairs $\mathcal{S} = \{1, 2, \dots, S\}$ in the near-future time horizon $t = 1, \dots, T$ based on the historical traffic information (e.g., traffic conditions during rush hours). Suppose the government requires $Q_{s,t} \geq 0$ amount of traffic offload on OD pair s at each time index t. To satisfy $Q_{s,t}$ amount of traffic offload, the government designs a unit incentive price $p_{s,t}$ for each time index t to incentivize individual passengers to participate in the traffic offload program. The information perceived by the government $\mathcal{I}_t^{gov}$ up to time t includes the following: (1) the historical incentives $\{p_{s,t'} | t' = 1, \dots, t-1, s \in \mathcal{S}\}$ and (2) the historical traffic offload offered by the passengers $\{q_{i,s}(p_{s,t'}) | i \in \mathcal{N}, s \in \mathcal{S}, t' = 1, \dots, t-1\}$. Thus, the government's decision on $p_{s,t}$ for each time t and OD pair s can be interpreted as a policy mapping from the information set to the set of non-negative real numbers $p_{s,t} : \mathcal{I}_t^{gov} \mapsto \mathbb{R}_{\geq 0}$.

At each time step t, the passengers observe the incentives $p_{s,t}$ and then decide whether to participate in traffic offload and earn the incentive $p_{s,t} q_{i,s}(p_{s,t})$ based on their own utility functions. Passengers that participate in traffic offload incur inconvenience cost $C_{i,s}(q_{i,s}(p_{s,t}))$. The inconvenience cost function $C_{i,s}(q_{i,s}(p_{s,t}))$ is private to each passenger i. The information $\mathcal{I}_t^i$ available to passenger i up to time t includes the following: (1) the historical incentives $\{p_{s,t'} | t' = 1, \dots, t, s \in \mathcal{S}\}$, (2) the traffic offload function $\{q_{i,s}(\cdot) | s \in \mathcal{S}\}$, and (3) its inconvenience cost function $\{C_{i,s}(\cdot) | s \in \mathcal{S}\}$.

Let $\mathbf{p}_t = [p_{1,t}, \dots, p_{S,t}]$ be the incentive prices for all OD pairs $s \in \mathcal{S}$ at time t. The utility of each passenger i at time step t can be represented as

$$U_{i,t}(\mathbf{p}_t) = \sum_{s \in \mathcal{S}} \{p_{s,t} q_{i,s}(p_{s,t}) - C_{i,s}(q_{i,s}(p_{s,t}))\}, \ \forall i, t. \tag{5}$$

The social cost is given by

$$\Lambda(\mathbf{p}) = \sum_t \sum_{s \in \mathcal{S}} \left\{ \sum_{i \in \mathcal{N}} C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p_{s,t}) \right]^+ \right\}, \tag{6}$$

where $\mathbf{p} = [\mathbf{p}_1, \dots, \mathbf{p}_T]^T$ contains the incentive prices for all s and t, $[\cdot]^+$ represents $\max\{\cdot, 0\}$, and β_s represents the penalty due to deficit of traffic offload. The social cost minimization problem is formulated as $\min_{\mathbf{p}} \Lambda(\mathbf{p})$.

Under the one-way communication setting, the malicious party could not observe the participation of each passenger directly as in a two-way communication setting. We focus on a malicious party that can observe the incentive prices issued by the government up to time t and then infer the amount of traffic offload offered by each passenger i, which might be further used to infer private information of the passengers [2, 25]. Denote the information obtained by the government up to time t as $\mathcal{I}_t^{gov}$. Then we have $\mathcal{I}_t^{gov} = \{p_{s,t'}, q_{i,s}(p_{s,t'}) | \forall s, \forall t' \leq t\}$. The objective of a malicious party is to compute $q_{i,s}(p_{s,t'})$ given $\mathcal{I}_t^{mal}$. In this case, the government's objective is to compute a privacy-preserving incentive design such that the social welfare is (approximately) maximized.

Besides the privacy guarantee, we briefly discuss the game-theoretic properties under the one-way communication setting. Since the government broadcasts the incentive price while the passengers decide if they will participate or not, individual rationality is automatically guaranteed for rational passengers. Truthfulness is not required under the one-way communication setting since the passengers cannot send messages to the government under this setting.

3.4 Notion of Privacy

In this subsection, we give the notion of privacy adopted in this article. We focus on differential privacy [16, 17], which is defined as follows.

Definition 3.2 (ϵ-Differential Privacy). Given $\epsilon \geq 0$, a computation procedure M is said to be ϵ-differentially private if for any two inputs C_1 and C_2 that differ in a single element and for any set of outcomes $L \subseteq \text{Range}(M)$, the relationship $Pr(M(C_1) \in L) \leq \exp(\epsilon) \cdot Pr(M(C_2) \in L)$ holds, where $\text{Range}(M)$ is the set of all outcomes of M.

Definition 3.2 requires computation procedure M to behave similarly given similar inputs, where parameter ϵ models how similarly the procedure should behave. The choice of ϵ is application dependent and is not considered in this work. A risk-analysis-based approach is given in [35], while a budget-constraint-based approach is presented in [27]. In this article, we provide an analysis on the trade-off between performance and privacy guarantee. A more relaxed and general definition of differential privacy is as follows.

Definition 3.3 ((ϵ, δ)-Differential Privacy). Given $\epsilon \geq 0$ and $\delta \geq 0$, a computation procedure M is said to be (ϵ, δ)-differentially private if for any two inputs C_1 and C_2 that differ in a single element and for any set of outcomes $L \subseteq \text{Range}(M)$, inequality $Pr(M(C_1) \in L) \leq \exp(\epsilon) \cdot Pr(M(C_2) \in L) + \delta$ holds.

To quantify the privacy leakage using the proposed incentive designs, we adopt the concept of min-entropy leakage [3]. We first introduce the concepts of min-entropy and conditional min-entropy [48], and then define the min-entropy leakage. Let V and Y be random variables. The min-entropy of V is defined as $H_\infty(V) = \lim_{\alpha \to \infty} \frac{1}{1-\alpha} \log_2 \sum_v Pr(V = v)^\alpha$, where $Pr(V = v)$ represents the probability of $V = v$. The conditional min-entropy is defined as $H_\infty(V|Y) = -\log_2 \sum_y Pr(Y = y) \max_v Pr(v|y)$, where $Pr(v|y)$ is the probability that $V = v$ given that $Y = y$. Then the min-entropy leakage [3] is defined as $L = H_\infty(V) - H_\infty(V|Y)$.

Under the two-way communication setting, the min-entropy leakage is computed as

$$L = \lim_{\alpha \to \infty} \frac{1}{1-\alpha} \log_2 \sum_B Pr(B)^\alpha - \left(-\log_2 \sum_X Pr(X) \max_B Pr(B|X) \right),$$

where $Pr(B)$ is the probability that a bidding profile B is submitted, and $Pr(B|X)$ is the probability that the bidding profile B is submitted given the selection profile X is observed. Under the one-way communication setting, the min-entropy leakage is computed as

$$L = \lim_{\alpha \to \infty} \frac{1}{1-\alpha} \log_2 \sum_C Pr(C)^\alpha - \left(-\log_2 \sum_{\mathbf{p}} Pr(\mathbf{p}) \max_C Pr(C|\mathbf{p}) \right),$$

where $Pr(C)$ is the probability that the collection of passengers' inconvenience cost functions is C, and $Pr(C|\mathbf{p})$ is the probability that the collection of inconvenience costs is C given the historical incentive $\mathbf{p}$ is observed.

4 SOLUTION FOR TWO-WAY COMMUNICATION SETTING

Motivated by the exponential mechanism [29, 37], we present an incentive design for the two-way communication setting in this section. We propose a payment scheme that achieves individual rationality. We mitigate the computation complexity incurred by the exponential mechanism using an iterative algorithm. We prove that the desired properties are achieved using the proposed incentive design.

4.1 Solution Approach

In this subsection, we give an exact solution under two-way communication. We formally prove that truthfulness, approximate social welfare maximizing, and differential privacy are achieved using the proposed mechanism.

The mechanism is presented in Algorithm 1. The algorithm takes the bid profile from the passengers as input and gives the selection profile X and the incentives issued to each selected passenger. The algorithm proceeds as follows. At each time $t \leq T$, the government selects a feasible solution to the social welfare maximization problem (Equation (4)). The probability of selecting each feasible X is proportional to the exponential function evaluated at the associated social welfare $\Omega(X, B)$ with scale $\frac{\epsilon}{2\Delta}$, where Δ is the difference between the upper and lower bound of social welfare $\Omega(X, B)$.

Although the computation of selection profile X is motivated by the exponential mechanism [29, 37], the VCG-like payment scheme adopted by the exponential mechanism is not applicable to the problem investigated in this work. The reason is that the VCG-like payment scheme violates individual rationality and truthfulness in our case, due to the fact that the passengers not only have valuations over the incentives but also inconvenience costs during traffic offload. To this end, the payment scheme (Equation (11)) is proposed for the problem of interest, in which the incentive issued to each passenger is determined by the social cost introduced by each passenger. In the following, we characterize the mechanism presented in Algorithm 1.

THEOREM 4.1. *The mechanism described in Algorithm 1 achieves truthfulness, individual rationality, near-optimal social welfare, and ϵ-differential privacy.*

PROOF. We first show that the mechanism achieves near-optimal social welfare. Suppose the selection profile is subject to some probability distribution $\tilde{D}$. Then the expected social welfare can be rewritten as

$$\begin{aligned}
&\mathop{\mathbb{E}}_{X\sim\tilde{D}}\{\Omega(X,B)\} \\
&= \sum_X Pr_{X\sim\tilde{D}}(X) \sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s}) \\
&= \frac{2\Delta}{\epsilon}\sum_X Pr_{X\sim\tilde{D}}(X) \sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} \frac{\epsilon}{2\Delta} x_{i,s}(h_{i,s}-c_{i,s}) \\
&= \frac{2\Delta}{\epsilon}\sum_X Pr_{X\sim\tilde{D}}(X) \ln\left(\exp\left(\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} \frac{\epsilon}{2\Delta} x_{i,s}(h_{i,s}-c_{i,s})\right)\right) \\
&= \frac{2\Delta}{\epsilon}\sum_X Pr_{X\sim\tilde{D}}(X) \ln\left(\frac{\exp\left(\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} \frac{\epsilon}{2\Delta} x_{i,s}(h_{i,s}-c_{i,s})\right)}{\sum_X \exp\left(\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} \frac{\epsilon}{2\Delta} x_{i,s}(h_{i,s}-c_{i,s})\right)}\right) \\
&\qquad + \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s})\right)\right) \\
&= \frac{2\Delta}{\epsilon}\sum_X Pr_{X\sim\tilde{D}}(X) \ln\left(Pr_{X\sim D}(X)\right) + \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s})\right)\right),
\end{aligned} \tag{7}$$

where the last equality follows from Equation (10). Following [41], we introduce the concept of free social welfare defined as

$$\tilde{\Omega}(X,B) = \mathop{\mathbb{E}}_{X\sim\tilde{D}}\{\Omega(X,B)\} + \frac{2}{\epsilon}E(\tilde{D}), \tag{8}$$

where $E(\cdot)$ is the Shannon entropy. Substituting Equation (7) into Equation (8), the free social welfare can be rewritten as

$$\begin{aligned}\tilde{\Omega}(X,B) &= \frac{2\Delta}{\epsilon}\sum_X Pr_{X\sim\tilde{D}}(X)\ln\left(\frac{Pr_{X\sim D}(X)}{Pr_{X\sim\tilde{D}}(X)}\right) + \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s})\right)\right)\\ &= \frac{2\Delta}{\epsilon}D_{KL}(D||\tilde{D}) + \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s})\right)\right),\end{aligned} \tag{9}$$

where $D_{KL}(D||\tilde{D})$ is the KL-divergence. Observing that the second term is independent of $\tilde{D}$, by the property of KL-divergence, we have that Equation (9) is maximized if $\tilde{D}$ is computed following Equation (10).

ALGORITHM 1: Mechanism design for the government

1: **procedure** MECHANISM(B)
2: **Input**: Bid profile B
3: **Output:** Selection profile X, incentives R
4: **while** $t \leq T$ **do**
5: Choose a selection profile X that is feasible for social welfare maximization problem (Equation (4)) with probability

$$Pr(X) \propto \exp\left(\frac{\epsilon}{2\Delta}\Omega(X,B)\right). \tag{10}$$

6: For each passenger that is selected, issue incentive r_i as

$$\begin{aligned} r_{i,s,t} = \mathop{\mathbb{E}}_{X\sim D(\mathbf{b}_{i,t},B_{-i,t})}\left\{\sum_j\sum_s x_{j,s,t}q_{j,s,t} - \sum_{j'\neq i}\sum_s x_{j',s,t}C_{j',s}(q_{j',s,t})\right\} + \frac{2\Delta}{\epsilon}E(D(\mathbf{b}_{i,t},B_{-i,t}))\\ -\frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i},B_{-i})\right)\right),\end{aligned} \tag{11}$$

where Δ is the difference between the upper and lower bound of social welfare $\Omega(X,B)$, $E(\cdot)$ is the Shannon entropy, $D(\cdot)$ is the probability distribution over selection profile B, and $X_{-i,t}$ and $B_{-i,t}$ are the matrix obtained by removing the ith row and ith column in the selection profile and bid profile, respectively.
7: $t \leftarrow t+1$
8: **end while**
9: **end procedure**

By the definition of free social welfare (Equation (8)), we have that the free social welfare is obtained by adding a term into the social welfare $\Omega(X,B)$. Hence, we have that the mechanism described in Algorithm 1 gives near-optimal social welfare.

We then show truthfulness. We prove truthfulness by showing that for each player i, truth-telling is the dominant strategy. Denote the truthful and non-truthful bid from player i as $\mathbf{b}_i$ and $\hat{\mathbf{b}}_i$, respectively. By definition, the truth-telling bid $\mathbf{b}_i$ contains the real inconvenience cost of passenger i, that is, $\bar{c}_{i,s} = c_{i,s}$, $\forall s$, while bid $\hat{\mathbf{b}}_i$ is the bid in which the passenger lies about its inconvenience cost, that is, $\bar{c}_{i,s} \neq c_{i,s}$, $\forall s$. Let $r_{i,s}$ be the incentive associated with bid $\mathbf{b}_i$ and $\hat{r}_{i,s}$ be the incentive associated with bid $\hat{\mathbf{b}}_i$. Then we prove truthfulness by showing that for each player i, truth-telling is the dominant strategy. The difference between the utility of passenger i by bidding

$\mathbf{b}_i$ and $\hat{\mathbf{b}}_i$ is represented as

$$
\begin{aligned}
&\left(r_i - \sum_s x_{i,s} c_{i,s}\right) - \left(\hat{r}_i - \sum_s \hat{x}_{i,s} c_{i,s}\right) \\
&= \mathop{\mathbb{E}}_{X \sim D(\mathbf{b}_i, B_{-i})} \{\Omega(X, (\mathbf{b}_i, B_{-i}))\} + \frac{2\Delta}{\epsilon} E(D(\mathbf{b}_i, B_{-i})) - \frac{2\Delta}{\epsilon} \ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i}, B_{-i})\right)\right) \\
&\quad - \mathop{\mathbb{E}}_{X \sim D(\hat{\mathbf{b}}_i, B_{-i})} \left\{\Omega\left(X, (\hat{\mathbf{b}}_i, B_{-i})\right)\right\} - \frac{2\Delta}{\epsilon} E(D(\hat{\mathbf{b}}_i, B_{-i})) + \frac{2\Delta}{\epsilon} \ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i}, B_{-i}\right)\right) \\
&= \mathop{\mathbb{E}}_{X \sim D(\mathbf{b}_i, B_{-i})} \{\Omega(X, (\mathbf{b}_i, B_{-i}))\} + \frac{2\Delta}{\epsilon} E(D(\mathbf{b}_i, B_{-i})) - \mathop{\mathbb{E}}_{X \sim D(\hat{\mathbf{b}}_i, B_{-i})} \left\{\Omega\left(X, (\hat{\mathbf{b}}_i, B_{-i})\right)\right\} - \frac{2\Delta}{\epsilon} E(D(\hat{\mathbf{b}}_i, B_{-i})) \\
&= \tilde{\Omega}\left(X, (\mathbf{b}_i, B_{-i})\right) - \tilde{\Omega}\left(X, (\hat{\mathbf{b}}_i, B_{-i})\right),
\end{aligned}
\tag{12}
$$

where the last equality holds by definition of free social welfare (Equation (8)). Since near-optimal social welfare is achieved, the free social welfare is maximized when $X \sim D(\mathbf{b}_i, B_{-i})$. Therefore, we have

$$
r_i - \sum_s x_{i,s} c_{i,s} - \left(\hat{r}_i - \sum_s \hat{x}_{i,s} c_{i,s}\right) = \tilde{\Omega}\left(X, (\mathbf{b}_i, B_{-i})\right) - \tilde{\Omega}\left(X, (\hat{\mathbf{b}}_i, B_{-i})\right) \geq 0.
$$

Hence, bidding truthfully is the dominate strategy for each passenger i.

Next, we show individual rationality. By truthfulness, we have that the passengers always bid truthfully. In the following, we show that the passengers obtain non-negative utilities when bidding truthfully.

The utility of passenger i can be rewritten as

$$
\begin{aligned}
U_i &= r_i - \sum_s x_{i,s} c_{i,s} \\
&= \mathop{\mathbb{E}}_{X \sim D(\mathbf{b}_i, B_{-i})} \left\{\sum_j \sum_s x_{j,s} h_{j,s} - \sum_{j' \neq i} \sum_s x_{j',s} c_{j',s}\right\} + \frac{2\Delta}{\epsilon} E(D(\mathbf{b}_i, B_{-i})) \\
&\qquad - \frac{2\Delta}{\epsilon} \ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i}, B_{-i})\right)\right) - \sum_s x_{i,s} c_{i,s} \\
&= \Omega(\tilde{X}, B) - \frac{2\Delta}{\epsilon} \ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i}, B_{-i})\right)\right).
\end{aligned}
\tag{13}
$$

By Equation (9), we have that the maximum free social welfare can be represented as

$$
\tilde{\Omega}(X, B) = \frac{2\Delta}{\epsilon} \ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{N}} x_{i,s}(h_{i,s} - c_{i,s})\right)\right).
$$

Hence, we have that Equation (13) can be rewritten as

$$\begin{aligned} U_i &= \tilde{\Omega}(X,B) - \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i},B_{-i})\right)\right) \\ &= \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\sum_{s\in\mathcal{S}}\sum_{i\in\mathcal{N}} x_{i,s}(h_{i,s}-c_{i,s})\right)\right) - \frac{2\Delta}{\epsilon}\ln\left(\sum_X \exp\left(\frac{\epsilon}{2\Delta}\Omega(X_{-i},B_{-i})\right)\right) \\ &\geq 0, \end{aligned}$$

where the inequality holds by the fact that the free social welfare is maximized.

We finally show that differential privacy holds. The proof follows from the analysis on the exponential mechanism [37]. We consider two bid profiles B and $\hat{B}$ that differ in a single entry. Denote the selection profiles associated with B and $\hat{B}$ as X and $\hat{X}$, respectively. Then the ratio of the probability of obtaining selection profile X and $\hat{X}$ given bid profiles B and $\hat{B}$ is

$$\begin{aligned} \frac{Pr(X)}{Pr(\hat{X})} &= \frac{\exp\left(\frac{\epsilon}{2\Delta}\Omega(X,B)\right) / \sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\Omega(X',B)\right)}{\exp\left(\frac{\epsilon}{2\Delta}\Omega(\hat{X},\hat{B})\right) / \sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\Omega(X',\hat{B})\right)} \\ &= \frac{\exp\left(\frac{\epsilon}{2\Delta}\Omega(X,B)\right)\sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\Omega(X',\hat{B})\right)}{\exp\left(\frac{\epsilon}{2\Delta}\Omega(\hat{X},\hat{B})\right)\sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\Omega(X',B)\right)} \\ &\leq \frac{\exp\left(\frac{\epsilon}{2\Delta}\left(\Omega(\hat{X},\hat{B})+\Delta\right)\right)\sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\left(\Omega(X',B)\right)+\Delta\right)}{\exp\left(\frac{\epsilon}{2\Delta}\Omega(\hat{X},\hat{B})\right)\sum_{X'}\exp\left(\frac{\epsilon}{2\Delta}\Omega(X',B)\right)} \\ &= \exp\left(\frac{\epsilon}{2}\right)\cdot\exp\left(\frac{\epsilon}{2}\right) \\ &= \exp(\epsilon). \end{aligned} \tag{14}$$

By Equation (14), we have $Pr(X) \leq \exp(\epsilon)Pr(\hat{X})$, satisfying Definition 3.2. □

The mechanism proposed in Algorithm 1 is computationally expensive. The payment scheme (Equation (11)) is intractable when the passenger set is large since Equation (11) needs to compute the social welfare associated with X and X_{-i} for all i. Therefore, a computationally efficient algorithm is desired.

4.2 Approximation Algorithm

In this subsection, we give a mechanism that achieves the desired game-theoretic properties and privacy guarantees and runs in polynomial time.

In real-world implementation, since the passengers are geographically distributed, the government can decompose the social welfare maximization problem (Equation (4)) with respect to OD pair s. Then Equation (4) becomes a set of optimization problems associated with each OD pair s as follows:

$$\begin{aligned} \max_{\mathbf{x}_s} \quad & \sum_{i\in\mathcal{N}} x_{i,s,t}\left(q_{i,s} - C_{i,s}(q_{i,s})\right) \\ \text{s.t.} \quad & \sum_{i\in\mathcal{N}} x_{i,s,t}q_{i,s,t} \geq Q_{s,t},\ \forall s,t \\ & x_{i,s,t} \in \{0,1\},\ \forall i,s,t. \end{aligned} \tag{15}$$

Given the set of decomposed problems, if we can achieve the optimal solution to each decomposed problem using an incentive design, then we can obtain a social optimal solution. Thus, our objective is to design a mechanism that achieves the (approximate) optimal solution of each decomposed problem, individual rationality, truthfulness, and differential privacy.

The proposed efficient algorithm for each decomposed problem is presented in Algorithm 2. The algorithm iteratively computes the set of passengers $\mathcal{W}_{s,t}$ selected by the government for OD pair s at time t. First, the set $\mathcal{W}_{s,t}$ is initialized as an empty set. Then at each iteration k, the probability of selecting a passenger i that has not been selected at time t is proportional to the exponential function $\exp\left(\epsilon'(q_{i,s,t} - \bar{C}_{i,s}(q_{i,s,t}))\right)$, that is,

$$Pr\left(\mathcal{W}_{s,t} \leftarrow \mathcal{W}_{s,t} \cup \{i\}\right) \propto \begin{cases} \exp\left(\epsilon'(q_{i,s,t} - \bar{C}_{i,s}(q_{i,s,t}))\right), & \text{if } i \text{ has not been selected;} \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $\epsilon' = \frac{\epsilon}{e\ln(e/\delta)}$. Then the set of selected passengers $\mathcal{W}_{s,t}$ is removed from the passenger set $\mathcal{N}$. For each $i \in \mathcal{W}_{s,t}$, the government issues incentive $r_{i,s,t}$ computed as

$$r_{i,s,t} = (q_{i,s,t} + z)\exp\left(\epsilon'(q_{i,s,t} - \bar{C}_{i,s}(q_{i,s,t}))\right) - \int_0^{q_{i,s,t}+z} \exp(\epsilon' y)\mathrm{d}y, \tag{17}$$

where $z = \frac{\bar{C}_{i,s}(q_{i,s,t})}{\exp(\epsilon'(q_{i,s,t} - \bar{C}_{i,s}(q_{i,s,t})))}$. We characterize the solution presented in Algorithm 2 as follows.

Lemma 4.2. *Algorithm 2 achieves truthfulness, individual rationality, and $(\frac{\epsilon\Delta}{e(e-1)}, \delta)$-differential privacy. Moreover, Algorithm 2 achieves near-optimal social welfare $\Omega_s^* - \frac{O(\ln Q_s)}{\epsilon'}$ with probability at least $1 - \frac{1}{Q_s{}^{O(1)}}$, where Ω_s^* is the maximum social welfare for OD pair s and $\epsilon' = \frac{\epsilon}{e\ln(e/\delta)}$.*

Proof. First, we give a lower bound of the social welfare by using Algorithm 2. By the property of differential privacy, we have

$$Pr\left(\sum_i (h_{i,s} - c_{i,s}) < \Omega_s^* - \frac{\ln|L|}{\epsilon} - \frac{t}{\epsilon}\right) \le \exp(-t),$$

where Ω_s^* is the optimal social welfare for sub-problem indexed s. Ignore the term $\frac{\ln|L|}{\epsilon}$ and let $t = \ln(Q_s)$. We have $Pr(\sum_i (h_{i,s} - c_{i,s}) < \Omega_s^* - \frac{O(\ln Q_s)}{\epsilon'}) \le \frac{1}{Q_s{}^{O(1)}}$. Reversing the inequality, we then have that with probability of at least $1 - \frac{1}{Q_s{}^{O(1)}}$,

$$\sum_i (h_{i,s} - c_{i,s}) > \Omega_s^* - \frac{O(\ln Q_s)}{\epsilon'}. \tag{18}$$

Before analyzing the truthfulness property, we define a concept named virtual bid $\mathbf{b}_i^v$ for each passenger as $\mathbf{b}_i^v = [b_{i,1}^v, b_{i,2}^v, \ldots, b_{i,S}^v]$, where each entry $b_{i,s}^v = h_{i,s} - \bar{c}_{i,s}$. Then we characterize how the truthfulness property is preserved when using Algorithm 2 in the following theorem.

Denote the set of passengers that are selected by the government as $\mathcal{W}_s$. Assume that $i \notin \mathcal{W}_s$. Then the probability that $\mathcal{W}_s$ is selected by the government is represented as

$$Pr(i \notin \mathcal{W}_s) = \left(1 - \exp(\epsilon'(h_{i,s} - \bar{c}_{i,s}))\right)^{|\mathcal{W}_s|}.$$

We observe that the probability of not selecting i is monotone decreasing with respect to the virtual bid $b_{i,s}^v$. As a consequence, the probability of selecting passenger i is monotone non-decreasing with respect to virtual bid $b_{i,s}^v$. By [1], we have that the solution proposed in Algorithm 2 is truthful in expectation.

Next, we consider the individual rationality property. By Algorithm 3, we have that only the passengers that can provide the government non-negative social welfare can be selected. Moreover, we have shown that truthfulness is preserved using the proposed algorithm. Therefore, we have $h_{i,s} - \bar{c}_{i,s} = h_{i,s} - c_{i,s} \ge 0$. By observing Equation (17), we have that the first term models the size of a rectangle whose length is $h_{i,s} - \bar{c}_{i,s}$ and width is $\exp\left(\epsilon'(h_{i,s} - \bar{c}_{i,s})\right)$, while the second term models the size of the area below the curve $\exp\left(\epsilon'(h_{i,s} - \bar{c}_{i,s})\right)$. By the convexity of the exponential function, we have that the payment scheme in Equation (17) is always non-negative.

We finally prove that Algorithm 2 achieves differential privacy with respect to passengers' bids. Consider two bid profiles B and $\hat{B}$ that differ in a single entry for some road segment s. Denote the sets of passengers that are selected associated with B and $\hat{B}$ as $\mathcal{W}_s$ and $\hat{\mathcal{W}}_s$, respectively, where $\mathcal{W}_s = \hat{\mathcal{W}}_s = \{1, 2, \ldots, W\}$. Then the ratio of the probability of obtaining selection profile $\mathcal{W}_s$ and $\hat{\mathcal{W}}_s$ given bid profiles B and $\hat{B}$ is represented as

$$
\begin{aligned}
&\frac{Pr(\mathcal{W}_s)}{Pr(\hat{\mathcal{W}}_s)} \\
&= \prod_{i=1}^{W} \frac{\exp\left(\epsilon'(h_{i,s} - c_{i,s})\right) / \sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)}{\exp\left(\epsilon'(\hat{h}_{i,s} - \hat{c}_{i,s})\right) / \sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(\hat{h}_{j,s} - \hat{c}_{j,s})\right)} \\
&= \prod_{i=1}^{W} \frac{\exp\left(\epsilon'(h_{i,s} - c_{i,s})\right)}{\exp\left(\epsilon'(\hat{h}_{i,s} - \hat{c}_{i,s})\right)} \cdot \prod_{i=1}^{W} \frac{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(\hat{h}_{j,s} - \hat{c}_{j,s})\right)}{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)},
\end{aligned}
\tag{19}
$$

where N_s^i is the set of passengers that have not been selected at iteration i.

In the following, we consider the following two cases. Suppose $h_{i,s} - c_{i,s} > \hat{h}_{i,s} - \hat{c}_{i,s}$. Then Equation (19) can be rewritten as

$$
\begin{aligned}
&\prod_{i=1}^{W} \frac{\exp\left(\epsilon'(h_{i,s} - c_{i,s})\right)}{\exp\left(\epsilon'(\hat{h}_{i,s} - \hat{c}_{i,s})\right)} \cdot \prod_{i=1}^{W} \frac{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(\hat{h}_{j,s} - \hat{c}_{j,s})\right)}{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)} \\
&\leq \prod_{i=1}^{W} \left(\exp\left(\epsilon'\left(h_{i,s} - c_{i,s} - \left(\hat{h}_{i,s} - \hat{c}_{i,s}\right)\right)\right)\right) \\
&= \exp\left(\epsilon' \sum_{i=1}^{W} \left(h_{i,s} - c_{i,s} - \left(\hat{h}_{i,s} - \hat{c}_{i,s}\right)\right)\right) \\
&= \exp(\epsilon' \Delta_s),
\end{aligned}
$$

where Δ_s is the difference between the social welfare associated with B and $\hat{B}$ for s. The first inequality holds by the fact that the second term in Equation (19) is upper bounded by one.

Next, we suppose that $h_{i,s} - c_{i,s} < \hat{h}_{i,s} - \hat{c}_{i,s}$. Then Equation (19) can be rewritten as

$$
\begin{aligned}
&\prod_{i=1}^{W} \frac{\exp\left(\epsilon'(h_{i,s} - c_{i,s})\right)}{\exp\left(\epsilon'(\hat{h}_{i,s} - \hat{c}_{i,s})\right)} \cdot \prod_{i=1}^{W} \frac{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(\hat{h}_{j,s} - \hat{c}_{j,s})\right)}{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)} \\
&\leq \prod_{i=1}^{W} \frac{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(\hat{h}_{j,s} - \hat{c}_{j,s})\right)}{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)} \\
&= \prod_{i=1}^{W} \frac{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'\left(\hat{h}_{j,s} - \hat{c}_{j,s} - (h_{j,s} - c_{j,s})\right)\right) \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)}{\sum_{j \in \mathcal{N}_s^i} \exp\left(\epsilon'(h_{j,s} - c_{j,s})\right)} \\
&= \prod_{i=1}^{W} \mathbb{E}_{j \in \mathcal{N}_s^i} \left\{\exp(\epsilon' \beta_{j,s})\right\},
\end{aligned}
$$

where $\beta_{j,s} = \hat{h}_{j,s} - \hat{c}_{j,s} - (h_{j,s} - c_{j,s})$. The first inequality holds because the first term in Equation (19) is upper bounded by one. For all $\epsilon' \leq 1$ and $\beta_{j,s} \leq 1$ (which can be achieved by normalizing the social welfare), we have

$$\prod_{i=1}^{W} \mathbb{E}_{j\in\mathcal{N}_s^i}\left\{\exp(\epsilon'\beta_{j,s})\right\} \leq \prod_{i=1}^{W} \mathbb{E}_{j\in\mathcal{N}_s^i}\left\{1+(e-1)\epsilon'\beta_{j,s}\right\}$$
$$\leq \exp\left((e-1)\epsilon'\sum_{i=1}^{W}\mathbb{E}_{j\in\mathcal{N}_s^i}\{\beta_{j,s}\}\right),$$

where the first inequality holds because for all $\beta \leq 1$, $\exp(\beta) \leq 1+\beta(e-1)$. When $\mathbb{E}_{j\in\mathcal{N}_s^i}\{\beta_{j,s}\} \leq \Delta\ln(e/\delta)$, we have

$$\exp\left((e-1)\epsilon'\sum_{i=1}^{W}\mathbb{E}_{j\in\mathcal{N}_s^i}\{\beta_{j,s}\}\right) \leq \exp\left((e-1)\epsilon'\Delta\ln(e/\delta)\right) = \exp\left(\frac{\epsilon\Delta}{e(e-1)}\right).$$

By [22], the probability that $\mathbb{E}_{j\in\mathcal{N}_s^i}\{\beta_{j,s}\} > \Delta\ln(e/\delta)$ is at most δ. Hence, we have

$$Pr(\mathcal{W}_s) \leq \exp\left(\frac{\epsilon\Delta}{e(e-1)}\right)Pr(\hat{\mathcal{W}}_s)+\delta. \qquad \square$$

Given Algorithm 2 for each decomposed problem, we present Algorithm 3, which utilizes Algorithm 2 as a subroutine, to solve for the selection profile X for Equation (4). Algorithm 3 works as follows. It first makes S copies of the passenger set $\mathcal{N}$, with each denoted as $\mathcal{N}_s$ for all $s \in \mathcal{S}$. Then Algorithm 2 is invoked iteratively to compute the selected passengers for each OD pair s. The selection profile X for time t is finally returned as the union $\cup_s \mathcal{W}_{s,t}$.

ALGORITHM 2: Solution algorithm for decomposed Equation (15)

procedure DECOMPOSE(B)
  Input: Bid profile B, current time t
  Output: Selection profile $\mathcal{W}_{s,t}$
  Initialization: Selected passenger set $\mathcal{W}_{s,t} \leftarrow \emptyset$, $\epsilon' \leftarrow \frac{\epsilon}{e\ln(e/\delta)}$
  while $|\mathcal{W}_{s,t}| \leq Q_s \wedge \mathcal{N} \neq \emptyset$ **do**
    for $i \in \mathcal{N}$ **do**
      Compute the probability of selecting passenger i as Equation (16).
    end for
    if passenger i is chosen **then**
      $\mathcal{N} \leftarrow \mathcal{N} \setminus \{i\}$
    end if
  end while
  return $\mathcal{W}_{s,t}$
end procedure

We conclude this section by characterizing the properties achieved by Algorithm 3.

THEOREM 4.3. *Algorithm 3 achieves truthfulness, individual rationality, and* $(\frac{\epsilon\Delta S}{e(e-1)}, \delta S)$*-differential privacy. Moreover, Algorithm 3 achieves near-optimal social welfare* $\Omega^* - \frac{SO(\ln Q_s)}{\epsilon'}$ *with at least probability* $1-\frac{1}{Q^{*O(1)}}$*, where* Ω^* *is the maximum social welfare,* $Q^* = \max_s Q_s$*, and* $\epsilon' = \frac{\epsilon}{e\ln(e/\delta)}$.

PROOF. The properties of truthfulness, individual rationality, and achieving near-optimal social welfare can be shown following a similar approach used in the proof of Lemma 4.2. Differential privacy can be shown by applying the composability rule of differential privacy over all OD pairs. $\square$

ALGORITHM 3: Solution algorithm for Equation (4)

procedure Social_Max(B)
  Input: Bid profile B
  Output: Selection profile X
  while $t \leq T$ **do**
    Initialization: $\mathcal{N}_s = \mathcal{N}$ for all s
    Remove all passengers that provide negative social welfare $B \leftarrow [(q_{i,s}, \bar{C}_{i,s}) : q_{i,s,t}, \bar{C}_{i,s}(q_{i,s,t}) \geq 0]$
    for $s \in \mathcal{S}$ **do**
      Decompose(B)
      $\mathcal{N}_s = \mathcal{N}_s \setminus \cup_{s'=1}^{s-1} \mathcal{W}_{s'}$
    end for
    return $X = \cup_{s \in \mathcal{S}} \mathcal{W}_{s,t}$
    $t \leftarrow t + 1$
  end while
end procedure

5 SOLUTION FOR ONE-WAY COMMUNICATION SETTING

In this section, we analyze the problem formulated in Section 3.3. We first present an incentive mechanism design without privacy guarantee. Then we give an incentive design that satisfies differential privacy.

5.1 Incentive Mechanism Design without Privacy Guarantee

Different from the two-way communication scenario, the passengers observe the incentive price signal sent by the government and respond to it by maximizing their own utility. In the following, we first analyze passengers' best responses to the price signal. Then we analyze how the government should design the incentive price to achieve optimal social welfare.

Lemma 5.1. *Given an incentive price $p_{s,t}$, a selfish and rational passenger would contribute $q_{i,s}(p_{s,t}) = [C'^{-1}_{i,s}(q_{i,s}(p_{s,t}))]^+$ amount of traffic offload to maximize its utility $U_{i,t}(\mathbf{p}_t)$.*

Proof. Let $s \in \mathcal{S}$ be the OD pair that passenger i can contribute to. Then for any $s' \neq s$, we have $q_{i,s'}(p_{s',t}) = 0$. Given an incentive price $p_{s,t}$, the maximizer of $U_{i,t}(\mathbf{p}_t)$ can be computed as the solution to $p_{s,t} = C'_{i,s}(q_{i,s}(p_{s,t}))$ due to the convexity of $C_{i,s}(\cdot)$. Therefore, we have that if the incentive price $p_{s,t}$ is no less than the marginal cost of contributing $C'_{i,s}(0)$ for each passenger i, then passenger i participates in traffic offload. By solving $p_{s,t} = C'_{i,s}(q_{i,s}(p_{s,t}))$ for $q_{i,s}(p_{s,t})$, we have $q_{i,s}(p_{s,t}) = [C'^{-1}_{i,s}(p_{s,t})]^+$, where the operator $[\cdot]^+$ is due to the fact that $q_{i,s}(p_{s,t}) \geq 0$, and the existence of the solution follows by the convexity of $C_{i,s}(\cdot)$. □

We have the following two observations by Lemma 5.1. First, a selfish and rational passenger that optimizes its utility will contribute the amount of traffic offload $C'^{-1}_{i,s}(q_{i,s}(p_{s,t}))$ if and only if it can obtain non-negative utility. Moreover, by observing the participation of each passenger, the government can infer the gradients of inconvenience cost functions.

Taking the amount of traffic offload of each participating passenger $q_{i,s}(p_{s,t})$ as feedback, the government can then use the gradient descent algorithm [57] to approximately minimize the social cost. In Algorithm 4, the government first initializes a set of learning rates $\{\eta_1, \ldots, \eta_T\}$ that adjusts the step size between two time instants. Meanwhile, Algorithm 4 initializes $\mathbf{p}_1$ of small value for time $t = 1$. Then for each time step $t = 2, \ldots, T$, the government iteratively updates the incentive price $\mathbf{p}_{t+1}$ as $\max\{p_{s,t} - \sum_i \eta_t C'_{i,s}(q^*_{i,s}), 0\}$.

ALGORITHM 4: Computation of incentive price

1: Initialize the sequence of learning rates $\eta_1, \ldots, \eta_{T-1}$
2: **while** $t \leq T$ **do**
3: Initialize incentive price $\mathbf{p}_1 > 0$ for time step $t = 1$ arbitrarily
4: Update incentive price as $p_{s,t+1} = \max\left\{p_{s,t} - \sum_i \eta_t C'_{i,s}(q^*_{i,s}), 0\right\}$ for all s
5: **end while**

In the following, we characterize Algorithm 4 by analyzing the social cost incurred using the incentive price returned by Algorithm 4. Analogous to the online convex algorithm [57], we define the regret of the government. The regret over time horizon T is defined as

$$R(T) = \Lambda(\mathbf{p}) - \Lambda^*, \tag{20}$$

where $\Lambda(\mathbf{p})$ is the social cost when selecting a sequence of incentive prices $\{p_{s,t}\}_{s=1,t=1}^{S,T}$ as defined in Equation (6), and

$$\Lambda^* = \min_p \sum_t \sum_{s \in \mathcal{S}} \left\{ \sum_{i \in \mathcal{N}} C_{i,s}(q_{i,s}(p)) + \beta_s \left[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p) \right]^+ \right\} \tag{21}$$

is the optimal social cost when using a fixed price. Then the regret (Equation (20)) models the difference between the social cost when selecting a sequence of incentive prices $\{\mathbf{p}_t\}_{t=1}^T$ and optimal social cost from using a fixed price p_s^* for each s.

In the following, we characterize the mechanism design proposed for one-way communication by analyzing the regret (Equation (20)). In particular, we analyze the regret (Equation (20)) by showing that it satisfies Hannan consistency, that is,

$$\limsup_{T \to \infty} \frac{R(T)}{T} \to 0. \tag{22}$$

Hannan consistency implies that the average regret (Equation (22)) vanishes when the time horizon approaches infinity. We define the following notations. Define row vectors $\mathbf{g}_{s,t} \in \mathbb{R}^N$ and $\mathbf{h}_{s,t} \in \mathbb{R}^N$ as

$$\mathbf{g}_{s,t} = \left[C'_{1,s}(q_{i,s}(p_{s,t})), \ldots, C'_{N,s}(q_{i,s}(p_{s,t})) \right] \tag{23}$$

$$\mathbf{h}_{s,t} = \left[q'_{1,s}(p_{s,t}), \ldots, q'_{N,s}(p_{s,t}) \right]. \tag{24}$$

We denote the vectors $\mathbf{g}_{s,t}$ and $\mathbf{h}_{s,t}$ that are associated with $p_{s,t} = p_s^*$ as $\mathbf{g}^*_{s,t}$ and $\mathbf{h}^*_{s,t}$, respectively. Let $\bar{g} = \max_{s,t} \mathbf{g}_{s,t}(p_{s,t})$ and $\underline{g} = \min_{s,t} g_{s,t}$. Denote the maximum incentive price the government would issue as $\bar{p}$. We also define column vectors for all s and t as $\mathbf{q}_{s,t} = [q_{1,s}(p_{s,t}), \ldots, q_{N,s}(p_{s,t})]^T$. Similarly, vector $\mathbf{q}^*_{s,t}$ represents the vector associated with $p_{s,t} = p^*_s$. We finally define $k_{s,t} = \mathbf{g}_{s,t} \cdot \mathbf{h}_{s,t} + \beta_s \mathbf{h}_{s,t} \mathbf{1}_N$, where $\mathbf{g}_{s,t} \cdot \mathbf{h}_{s,t}$ is the dot product of $\mathbf{g}_{s,t}$ and $\mathbf{h}_{s,t}$. Let $\bar{k} = \max_{s,t} k_{s,t}$ be the maximum $k_{s,t}$ for all s and t. Next we show that the regret (Equation (20)) is upper bounded.

Lemma 5.2. *The regret of Algorithm 4 is bounded as* $R(T) \leq \sum_s \left\{ \frac{\bar{p}^2 k_{s,T}}{2\eta_T \mathbf{g}_{s,T} \mathbf{1}_N} + \sum_{t=1}^T \frac{\eta_t \bar{g}^2 N^2 k_{s,t}}{2 \mathbf{g}_{s,t} \mathbf{1}_N} \right\}$.

Proof. The proof is motivated by [57]. Denote the optimal incentive price associated with optimal social cost Λ^* as p_s^* for each OD pair s. Due to the convexity of inconvenience cost functions

$C_{i,s}(\cdot)$, for any $q_{i,s}(p_{s,t})$ and $p_{s,t}$ we have

$$\sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p_{s,t})\right]^+ \\ \geq \sum_i \left\{C'_{i,s}\left(q^*_{i,s}\right)\left(q_{i,s}(p_{s,t}) - q^*_{i,s}\right) + C_{i,s}(q^*_{i,s})\right\} + \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p_{s,t})\right]^+ .$$

By definition of $\mathbf{g}_{s,t}$ (Equation (23)), we have that the optimal social cost satisfies the following inequalities:

$$\sum_i C_{i,s}(q_{i,s}(p^*_s)) + \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p^*_s)\right]^+$$

$$\geq \mathbf{g}_{s,t}\left(\mathbf{q}^*_{s,t} - \mathbf{q}_{s,t}\right) + \sum_i C_{i,s}\left(q_{i,s}(p_{s,t})\right) + \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p^*_s)\right]^+ \tag{25}$$

$$\geq \mathbf{g}_{s,t}\left(\mathbf{q}^*_{s,t} - \mathbf{q}_{s,t}\right) + \sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \mathbf{h}_{s,t}\mathbf{1}_N(p^*_s - p_{s,t}) - \sum_i q_{i,s}(p_{s,t})\right]^+ \tag{26}$$

$$\geq \mathbf{g}_{s,t}\left(\mathbf{q}^*_{s,t} - \mathbf{q}_{s,t}\right) + \sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \sum_i q_{i,s}(p_{s,t})\right]^+ - \beta_s\left[\mathbf{h}_{s,t}\mathbf{1}_N(p^*_s - p_{s,t})\right]^+, \tag{27}$$

where $\mathbf{1}_N = [1,\ldots,1]^T$ with dimension N, Inequality (25) follows by the convexity of $C_{i,s}(\cdot)$, Inequality (26) follows by the first-order Taylor expansion of concave function $q_{i,s}(\cdot)$, and Inequality (27) holds by the fact that $[a-b]^+ \geq [a]^+ - [b]^+$. Rearranging the inequality above, we have that

$$\sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p_{s,t})\right]^+ - \sum_i C_{i,s}(q_{i,s}(p^*_s)) - \beta_s \left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p^*_s)\right]^+$$
$$\leq \beta_s\left[\mathbf{h}_{s,t}\mathbf{1}_N(p^*_s - p_{s,t})\right]^+ - \mathbf{g}_{s,t}(\mathbf{q}^*_{s,t} - \mathbf{q}_{s,t})$$
$$\leq \beta_s\left[\mathbf{h}_{s,t}\mathbf{1}_N(p^*_s - p_{s,t})\right]^+ - \mathbf{g}_{s,t}\cdot\mathbf{h}_{s,t}(p^*_s - p_{s,t})$$
$$\leq k_{s,t}(p^*_s - p_{s,t}),$$

where $k_{s,t} = I_{\{p^*\geq p_{s,t}\}}\beta_s\mathbf{h}_{s,t}\mathbf{1}_N - \mathbf{g}_{s,t}\cdot\mathbf{h}_{s,t}$, $I_{\{p^*\geq p_{s,t}\}}$ is an indicator that equals to 1 if $p^* \geq p_{s,t}$ and 0 otherwise, and $\mathbf{g}_{s,t}\cdot\mathbf{h}_{s,t}$ represents the dot product of $\mathbf{g}_{s,t}$ and $\mathbf{h}_{s,t}$. At time step $t+1$, we have

$$(p_{s,t+1} - p^*_s)^2 \leq (p_{s,t} - \eta_t\mathbf{g}_{s,t}\mathbf{1}_N - p^*_s)^2 \tag{28}$$
$$\leq (p_{s,t} - p^*_s)^2 - 2\eta_t\mathbf{g}_{s,t}\mathbf{1}_N(p_{s,t} - p^*_s) + \eta_t^2\bar{g}^2N^2, \tag{29}$$

where Inequality (28) holds by the updating rule of $p_{s,t}$ and Inequality (29) holds due to $\mathbf{g}_{s,t}\mathbf{1}_N \leq \bar{g}N$. Then we obtain $\mathbf{g}_{s,t}\mathbf{1}_N(p^*_s - p_{s,t}) \leq \frac{1}{2\eta_t}[(p_{s,t} - p^*_s)^2 - (p_{s,t+1} - p^*_s)^2 + \eta_t^2\bar{g}^2N^2]$. By

Equation (20), we have

$$\begin{aligned}
R(T) &= \Lambda(\mathbf{p}) - \Lambda^* \\
&= \sum_t \sum_s \left\{ \sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \left[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p_{s,t}) \right]^+ \right. \\
&\qquad \left. - \sum_i C_{i,s}(q_{i,s}(p_s^*)) - \beta_s \left[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p_s^*) \right]^+ \right\} \\
&\leq \sum_s \frac{\bar{p}^2}{2} \left\{ \frac{k_{s,1}}{\eta_1 \mathbf{g}_{s,1} \mathbf{1}_N} + \sum_{t=2}^{T} \left(\frac{k_{s,t}}{\eta_t \mathbf{g}_{s,t} \mathbf{1}_N} - \frac{k_{s,t-1}}{\eta_{t-1} \mathbf{g}_{s,t-1} \mathbf{1}_N} \right) + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N^2 k_{s,t}}{2 \mathbf{g}_{s,t} \mathbf{1}_N} \right\} \\
&= \sum_s \left\{ \frac{\bar{p}^2 k_{s,T}}{2 \eta_T \mathbf{g}_{s,T} \mathbf{1}_N} + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N^2 k_{s,t}}{2 \mathbf{g}_{s,t} \mathbf{1}_N} \right\},
\end{aligned}$$

which completes our proof. □

Leveraging Lemma 5.2, we then show that the Hannan consistency holds for the proposed incentive mechanism design.

PROPOSITION 5.3. *Let $\eta_t = \frac{1}{\sqrt{t}}$. The regret defined in Equation (20) along with the incentive design proposed in Algorithm 4 achieves the Hannan consistency.*

PROOF. The average regret satisfies

$$\begin{aligned}
\frac{R(T)}{T} &\leq \sum_s \left\{ \frac{\bar{p}^2 k_{s,T}}{2 \eta_T \mathbf{g}_{s,T} \mathbf{1}_N T} + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N^2 k_{s,t}}{2 \mathbf{g}_{s,t} \mathbf{1}_N T} \right\} \\
&\leq \sum_s \left\{ \frac{\bar{p}^2 \bar{k}}{2 \eta_T N \underline{g} T} + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N^2 \bar{k}}{2 N \underline{g}} \right\} \\
&= \sum_s \left\{ \frac{\bar{p}^2 \bar{k}}{2 \eta_T N \underline{g} T} + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N \bar{k}}{2 \underline{g} T} \right\},
\end{aligned}$$

where the first inequality holds by Lemma 5.2, and the second inequality holds by the facts that $\bar{k} = \max_{s,t} k_{s,t}$ and $\bar{g} = \max_{s,t} \mathbf{g}_{s,t}$. Moreover, we have $\sum_{t=1}^{T} \eta_t = \sum_{t=1}^{T} \frac{1}{\sqrt{t}} \leq \int_{t=1}^{T} \frac{1}{\sqrt{t}} = 2\sqrt{T} - 1$. Thus,

$$\limsup_{T \to \infty} \frac{R(T)}{T} \leq \limsup_{T \to \infty} \left\{ \sum_s \left(\frac{\bar{p}^2 \bar{k}}{2\sqrt{T} N \underline{g} T} + \frac{\bar{g}^2 N \bar{k} \left(2\sqrt{T} - 1 \right)}{2 \underline{g} T} \right) \right\},$$

which approaches zero as $T \to \infty$. Therefore, we have that the Hannan consistency holds. □

5.2 Incentive Mechanism Design with Privacy Guarantees

In this subsection, we give the differentially private incentive price $p_{s,t}$ under the one-way communication setting.

To achieve the privacy guarantee, we perturb the incentive price returned by Algorithm 4 as follows:

$$p_{s,t} = p_{s,t}^* + \delta_t, \ \forall s, t, \tag{30}$$

where $\delta_t \sim \mathcal{L}(\frac{\Delta p}{\epsilon})$ is a random variable that follows Laplace distribution with scale $\Delta p / \epsilon$, and Δp is the maximum difference of the incentive price under two sets of observations that differ in one passenger, which is obtained by solving

$$\begin{aligned}\max_{s,t,\mathbf{q}_{s,t},\mathbf{q}'_{s,t}} \quad & p_{s,t+1} - p'_{s,t+1} \\ \text{s.t.} \quad & \|\mathbf{q}_{s,t} - \mathbf{q}'_{s,t}\|_1 = 1,\end{aligned}$$

where $p_{s,t+1}$ and $p'_{s,t+1}$ are incentive prices returned by Algorithm 4 given traffic offloads $\mathbf{q}_{s,t}$ and $\mathbf{q}'_{s,t}$, respectively.

In the sequel, we show that differential privacy is preserved.

THEOREM 5.4. *Incentive price design (Equation (30)) achieves $((T - \sum_{t=1}^{T-1} \eta_t)\epsilon)$-differential privacy.*

PROOF. We prove by induction. We first prove that differential privacy holds for a single time step. Then we generalize the analysis on a one-time-step scenario to a multiple-time-steps scenario. Since the passengers' utility function is deterministic, given an incentive price $p_{s,t}$, passengers' participation is deterministic. Given the initial incentive price $p_{s,1}$ at $t = 1$, the contribution of passenger i is determined as $q_{i,s}(p_{s,1})$. We compare the p.d.f.'s at $p_{s,2} = p'_{s,2}$.

$$\begin{aligned}\frac{P\left(p_{s,2}\right)}{P'\left(p'_{s,2}\right)} &= \frac{\exp\left(-\frac{\epsilon|p_{s,1}-\sum_j \eta_1 C'_{j,s}(q_{j,s}(p_{s,1}))-p_{s,2}|}{\Delta p}\right)}{\exp\left(-\frac{\epsilon|p'_{s,1}-\sum_j \eta_1 C'_{j,s}(q_{j,s}(p'_{s,1}))-p_{s,2}|}{\Delta p}\right)} \\ &\leq \exp\left(-\epsilon\left|p_{s,1} - \sum_j \eta_1 C'_{i,s}(q_{i,s}(p_{s,1})) + \left(p'_{s,1} - \sum_j \eta_1 C'_{j,s}(q_{j,s}(p'_{s,1}))\right)\right|/\Delta p\right) \\ &= \exp\left(\epsilon\left\{\left|p'_{s,1} - p_{s,1} + \eta_1\left[C'_{i,s}(q_{i,s}(p'_{s,1})) - C'_{i,s}(q_{i,s}(p_{s,1}))\right]\right|\right\}/\Delta p\right) \\ &= \exp((1-\eta_1)\epsilon),\end{aligned}$$

where the inequality follows from triangle inequality, and the last equality follows by Lemma 5.1. Thus, we have $(1 - \eta_1)\epsilon$-differential privacy.

We note that since the scheme follows Stackelberg setting, a malicious party can only infer the passengers' behavior at time $t = 1$ by observing $p_{s,2}$. Thus, the analysis on a single time step serves as our induction base.

At time t, the information perceived by the malicious party is $\mathcal{I}_t^{mal} = \{p_{s,t'} | \forall s, t = 1, \ldots, t\}$. We analyze the ratio of $\frac{P(p_{s,t})}{P'(p'_{s,t})}$ under the following scenarios. First, if $p_{s,t'} = p'_{s,t'}$ for all $t' < t$ and $p_{s,t}$ distinguishes from $p_{s,t}$, then we have $(1 - \eta_t)$-differential privacy. In the following, we focus on the general setting in which $p_{s,t'}$ differs from $p'_{s,t'}$ for all $t' < t$ such that $\mathbf{q}_{s,1:t}$ and $\mathbf{q}'_{s,1:t}$ differ in at most one entry. Then we have

$$\begin{aligned}\frac{P\left(p_{s,t}\right)}{P'\left(p'_{s,t}\right)} &= \prod_{\tau=2}^{t}\left(\frac{Pr\left(p_{s,\tau}|p_{s,\tau-1}\right)}{Pr\left(p'_{s,\tau}|p'_{s,\tau-1}\right)}\right) \\ &= \prod_{\tau=1}^{t}\left\{\exp\left[\epsilon\left[\left|p_{s,\tau} - p'_{s,\tau} + \eta_\tau\left(C'_{i,s}(q_{i,s}(p'_{s,\tau})) - C'_{i,s}(q_{i,s}(p_{s,\tau}))\right)\right]\right|/\Delta p\right]\right\} \\ &= \exp\left(\left(T - \sum_{\tau=1}^{t} \eta_\tau\right)\epsilon\right).\end{aligned}$$

Therefore, we have that the proposed approach achieves $((T - \sum_{t=1}^{T-1} \eta_t)\epsilon)$-differential privacy. □

In the remainder of this section, we characterize the social welfare using the incentive design (Equation (30)). We start with the expected regret defined as the probabilistic counter-part of Equation (20): $\mathbb{E}\{R(T)\} = \mathbb{E}_{\mathbf{p}}\{\Lambda(\mathbf{p})\} - \Lambda^*$, where $\mathbb{E}_{\mathbf{p}}\{\cdot\}$ represents expectation with respect to $\mathbf{p}$.

Lemma 5.5. *The expected regret $\mathbb{E}\{R(T)\}$ under the incentive (Equation (30)) is bounded from above as*

$$\mathbb{E}\{R(T)\} \leq \mathbb{E}\left\{\sum_s \left[\frac{\bar{p}^2 k_{s,T}}{2\eta_T \mathbf{g}_{s,T}\mathbf{1}_N} + \sum_{t=1}^{T} \frac{\eta_t \bar{g}^2 N^2 k_{s,t}}{2\mathbf{g}_{s,t}\mathbf{1}_N}\right]\right\}. \tag{31}$$

Proof. The proof is the probabilistic counter-part of that of Lemma 5.2. □

Before closing this section, we finally show that Hannan consistency holds under incentive design (Equation (30)), that is,

$$\limsup_{T\to\infty} \frac{R(T)}{T} = 0 \text{ with probability one.} \tag{32}$$

Theorem 5.6. *The Hannan consistency (Equation (32)) holds for incentive design (Equation (30)).*

To prove Theorem 5.6, we first give the following lemma.

Lemma 5.7. *Let $Pr(\cdot)$ be the probability of an event. Then the following inequality holds:*

$$Pr\left(\limsup_{T\to\infty}\left\{\sum_{t=1}^{T} S\bar{k}\max_s \|p_s^* - p_{s,t}\|_\infty/T\right\} \leq 0\right) \geq \limsup_{T\to\infty} Pr\left(\sum_{t=1}^{T} S\bar{k}\max_s \|p_s^* - p_{s,t}\|_\infty/T \leq 0\right). \tag{33}$$

Proof. Let $V = \limsup_{T\to\infty}\sum_{t=1}^{T} S\bar{k}\max_s \|p_s^* - p_{s,t}\|_\infty/T$. Then V can be interpreted as the following two statements for all for all $\phi > 0$:

$$\left\{t : \sum_{t'=1}^{t} S\bar{k}\max_s \|p_s^* - p_{s,t'}\|_\infty/t > V + \phi\right\} \text{ is finite}$$
$$\left\{t : \sum_{t'=1}^{t} S\bar{k}\max_s \|p_s^* - p_{s,t'}\|_\infty/t < V - \phi\right\} \text{ is infinite.}$$

Then, by the property of limit superior for sequence of sets, we have that Equation (33) holds. □

Now we are ready to prove Theorem 5.6.

Proof (Proof of Theorem 5.6). Let $\bar{k} = \max_{s,t} k_{s,t}$ be the maximum $k_{s,t}$ for all s and t, and $\underline{g} = \min_{s,t} g_{s,t}$. Then, following the proof of Lemma 5.2, we have

$$\sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s\left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p_{s,t})\right]^+ - \sum_i C_{i,s}(q_{i,s}(p_s^*)) - \beta_s\left[Q_{s,t} - \sum_{i\in\mathcal{N}} q_{i,s}(p_s^*)\right]^+$$
$$\leq k_{s,t}(p_s^* - p_{s,t}) \leq \bar{k}(p_s^* - p_{s,t}).$$

Summing the inequality above over t and s, we have

$$\begin{aligned}
&\Lambda(\mathbf{p}) - \Lambda^* \\
&= \sum_t \sum_s \Bigg(\sum_i C_{i,s}(q_{i,s}(p_{s,t})) + \beta_s \Big[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p_{s,t}) \Big]^+ \\
&\qquad - \sum_i C_{i,s}(q_{i,s}(p_s^*)) - \beta_s \Big[Q_{s,t} - \sum_{i \in \mathcal{N}} q_{i,s}(p_s^*) \Big]^+ \Bigg) \\
&\leq \sum_t \sum_s (k_{s,t}(p_s^* - p_{s,t})) \\
&\leq \sum_t \sum_s \left(\bar{k}(p_s^* - p_{s,t}) \right) \leq \sum_t \sum_s \left(\bar{k} \| p_s^* - p_{s,t} \|_\infty \right).
\end{aligned}$$

Let $Pr(\cdot)$ be the probability of an event. Then we have

$$\begin{aligned}
Pr\left(\limsup_{T\to\infty} \frac{R(T)}{T} \leq 0 \right) &\geq Pr\left(\limsup_{T\to\infty} \left\{ \sum_t \sum_s \left(\bar{k}\|p_s^* - p_{s,t}\|_\infty \right)/T \right\} \leq 0 \right) \\
&\geq Pr\left(\limsup_{T\to\infty} \left\{ \sum_t S\bar{k} \max_s \|p_s^* - p_{s,t}\|_\infty / T \right\} \leq 0 \right) \\
&\geq \limsup_{T\to\infty} \left\{ Pr\left(\sum_t \frac{S\bar{k} \max_s \|p_s^* - p_{s,t}\|_\infty}{T} \leq 0 \right) \right\} && (34) \\
&\geq \limsup_{T\to\infty} \left\{ 1 - Pr\left(\sum_t \frac{S\bar{k} \max_s \|p_s^* - p_{s,t}\|_\infty}{T} \geq \frac{\Delta p}{\epsilon} \right) \right\} && (35) \\
&= \limsup_{T\to\infty} \left\{ 1 - \exp\left(-\frac{T}{S\bar{k}} \right) \right\} = 1,
\end{aligned}$$

where Inequality (34) holds by Lemma 5.7, and Inequality (35) holds by Equation (30) and the definition of δ_t. Therefore, we have that Hannan consistency holds. □

6 NUMERICAL CASE STUDY

6.1 Case Study Setup

We consider a government aiming at initiating traffic offload for $S = 5$ OD pairs for the next day. Suppose the time horizon $T = 24$ and each time slot t is set as 1 hour. The desired amount of traffic offload at each OD pair is obtained from the Caltrans Performance Measurement System (PeMS) data source [6]. The five roads that we used in the dataset are county "INY" with direction S, county "LA" with direction N, county "KER" with direction W, county "FRE" with direction S, and county "IMP" with direction S[2]. If a road appears multiple times in the data source, we take the average over the peak volume as the data used in the case study. To show the performance of traffic offload, we use the ahead peak hour traffic volume in [6] as the traffic volume without traffic offload. Since the ahead hourly traffic volume data is not available, we treat the ahead traffic data at different post miles as the traffic volume data at different times.

The size of the passenger set is $N = 50{,}000$. We assume the inconvenience cost function $C_{i,s}(q_{i,s})$ of each passenger i is a linear combination of four factors denoted as comfort, reliability, delay on time of arrival, and cost [5, 31]. Different passengers assign different weights on these factors. The

[2]We omit the result of OD pair IMP due to space limits. See [43] for the result of IMP.

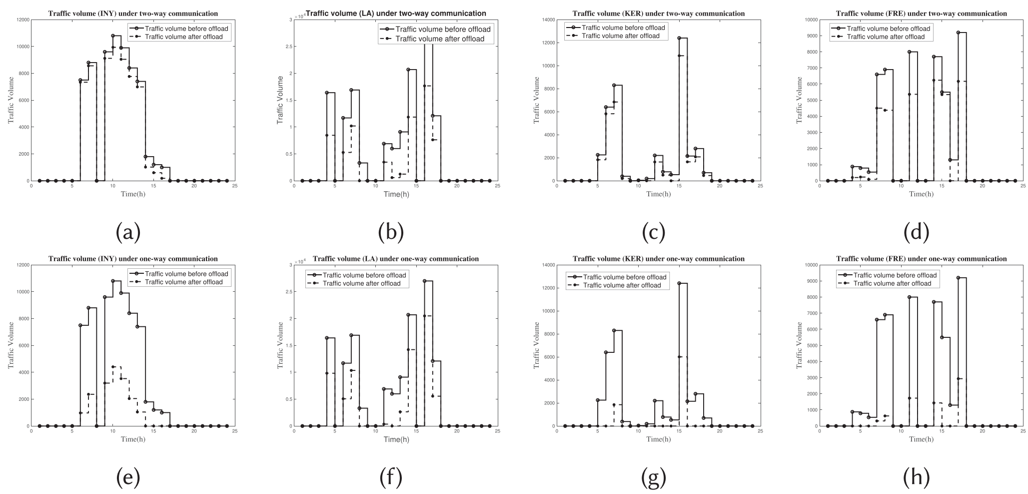


Fig. 1. In Figure 1(a) to Figure 1(d), we present the traffic volume before and after traffic offload under two-way communication. The solid curve is the traffic volume before traffic offload, whereas the dashed curve represents the traffic volume after traffic offload. In Figure 1(e) to Figure 1(h), we present the traffic volume before and after traffic offload under one-way communication. The solid curve is the traffic volume before traffic offload, whereas the dashed curve represents the traffic volume after traffic offload.

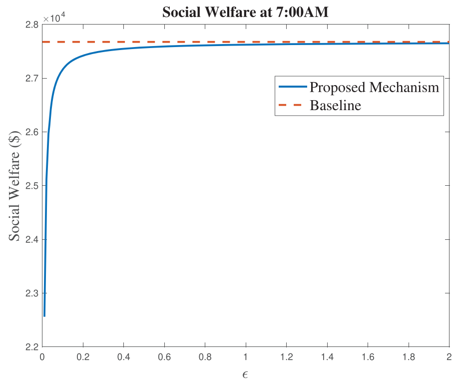
weights for each passenger are generated using a multivariate normal distribution, with mean $[0.16, 0.27, 0.36, 0.21]$ and variance $0.3I$ [31], where I is the identity matrix.

We set two baselines for comparison under two-way and one-way communication settings. In the baseline of two-way communication, the government solves the social welfare maximization problem (Equation (4)) without any privacy guarantee by adopting the VCG mechanism [53]. In the baseline of one-way communication, the government uses a fixed price p_s^* to minimize the social cost for each $s \in \mathcal{S}$.

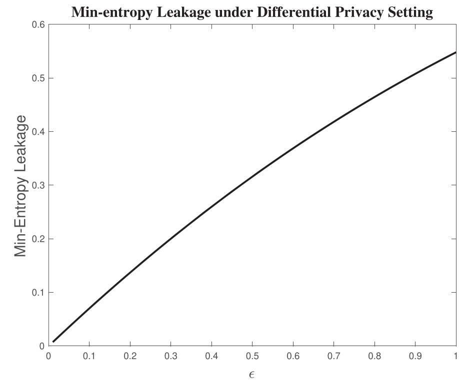
6.2 Two-Way Communication

In this section, we demonstrate the proposed approach for the two-way communication scenario. We first generate the passengers' bids. As shown in Theorem 4.3, the passengers bid truthfully to the government, and hence the government knows the inconvenience cost function of each passenger. The amount of traffic offload contributed by each passenger is generated using a normal distribution with mean 3.5 and variance 0.3. We remark that the contributions model the best effort of all passengers, that is, the capabilities of all passengers.

We compute the incentives and selection profile following Algorithm 3. First, we show the traffic volume on each OD pair before and after traffic offload in Figure 1(a) to Figure 1(d). The solid curve is the traffic volume before traffic offload, whereas the dashed curve represents the traffic volume after traffic offload. As observed in Figure 1(a) to Figure 1(d), the traffic volume decreases by incentivizing the passengers to switch from private to public transit services. Moreover, the gap between the solid curve and dashed curve gives the amount of traffic offload due to passengers switching from private to public transit services. We next compare the total social welfare achieved at all OD pairs at 7 : 00AM using our proposed approach and the baseline with respect to the privacy parameter ϵ in Figure 2(a). Since the baseline ignores the privacy guarantee, it achieves the optimal social welfare (the dotted red line in Figure 2(a)). We also observe that we can tune the



(a)



(b)

Fig. 2. In Figure 2(a), we present the total social welfare of all OD pairs at 7 : 00AM under two-way communication using the proposed approach (blue solid line) and the baseline approach (red dotted line) with respect to the privacy parameter ϵ. In Figure 2(b), we present the min-entropy leakage for OD pair INY at 12 : 00 PM when parameter ϵ varies from 0.01 to 1.

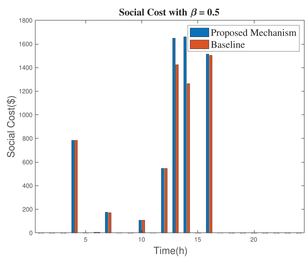
trade-off between the social welfare and privacy guarantee. In particular, by decreasing the value of parameter ϵ, we achieve a stronger privacy guarantee at the expense of social welfare loss.

We present the min-entropy leakage in Figure 2(b) to validate that the proposed incentive design in Algorithm 3 is privacy preserving. We compute the min-entropy for OD pair INY at 12 : 00PM when differential privacy parameter ϵ varies from 0.01 to 1. We observe that the min-entropy is monotone increasing with respect to parameter ϵ, which agrees with our privacy-preserving property. That is, when the mechanism is designed with a stronger privacy guarantee, there exists less min-entropy leakage for each individual passenger. We also give the min-entropy leakage of the baseline approach, which selects the passengers deterministically. The min-entropy leakage of the baseline is 2^{50000}, which is significantly higher compared to that of our proposed approach.

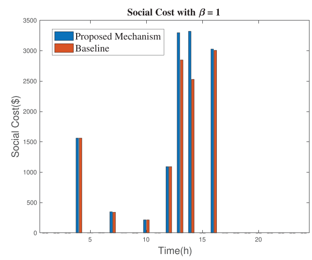
6.3 One-Way Communication

In this subsection, we demonstrate the proposed approach for the one-way communication scenario. The government initializes a first guess of incentive price 0.02. Given the incentive price $p_{s,t}$, the response from each passenger is computed by Lemma 5.1. The capability of each passenger is adopted from the setting under two-way communication.

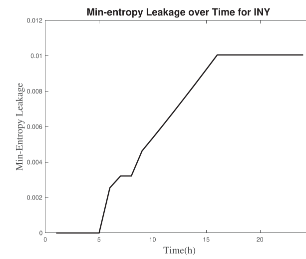
We present the traffic volume on each OD pair before and after traffic offload in Figure 1(e) to Figure 1(h). We have the following observations. First, the traffic volume decreases due to passengers switching from private to public transit services. Similar to Figure 1(a) to Figure 1(d), the gap between the curves represents the amount of traffic offload. Finally, the traffic volume after traffic offload is lower than that under the two-way communication setting for some time t; that is, the amount of traffic offload contributed by the passengers is higher than that under the two-way communication setting. The reasons are twofold. First, the government does not know the inconvenience cost function of each passenger under the one-way communication setting and has no ability to select the participating passengers. Therefore, the participating passengers could contribute more than $Q_{s,t}$ for all s and t under the one-way communication setting. However, the government selects the winners under the two-way communication setting and only $Q_{s,t}$ amount of traffic offload is realized for all s and t. Second, the passengers' inconvenience costs are modeled as a linear function. Hence, any passenger i such that $p_{s,t} \geq C'_{i,s}(q_{i,s})$ would participate in traffic



(a)



(b)



(c)

Fig. 3. In Figure 3(a) to Figure 3(b), we present the social cost incurred under one-way communication setting with different parameter β using the proposed approach and the baseline approach. The parameter β is set as 0.5 and 1 in Figure 3(a) and 3(b), respectively. In Figure 3(c), we show the min-entropy leakage L for OD pair INY over time.

offload by shedding the maximum amount of traffic offload, that is, contribute its maximum effort. We then give the social cost incurred during traffic offload under the one-way communication setting using our proposed approach and baseline in Figure 3(a) to Figure 3(b) with different parameter β. In particular, the parameter β is set as 0.5 and 1 in Figure 3(a) and 3(b), respectively. We observe that as β increases, the social cost increases. The reason is that there exist some time steps (e.g., $10:00$ AM at IMP) at which the desired traffic offload is not satisfied and hence the government incurs high penalties. Moreover, the government incurs slightly higher social cost compared to the optimal social cost incurred using the baseline approach. The average regret is 26.458 and 52.916 when $\beta = 0.5$ and $\beta = 1$, respectively.

We finally present the min-entropy leakage for OD pair INY under the one-way communication setting in Figure 3(c). In this case study, parameter ϵ is set as 0.015. We show how privacy is preserved over time. We observe that the privacy leakage increases over time. The reason is that the malicious party perceives more information over time. Hence, more information can be inferred by the adversary as time increases. Using the baseline approach, the min-entropy leakage is 2 for all time t, which is 199 times higher compared to our approach.

7 CONCLUSIONS

In this article, we investigated the problem of incentivizing passengers to switch from private to public transit service to mitigate traffic congestion and achieve sustainability. We considered two settings denoted as two-way communication and one-way communication. We modeled the interaction under the two-way communication setting using a reverse auction model and proposed a polynomial-time algorithm that achieves approximate social optimal, truthfulness, individual rationality, and differential privacy. In the one-way communication setting, we presented an online convex algorithm to solve for the incentive price. The proposed approach achieves Hannan consistency and differential privacy. We evaluated the proposed approaches using a numerical case study with the PeMS data source.

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