

# From Proper Scoring Rules to Max-Min Optimal Forecast Aggregation

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This paper forges a strong connection between two seemingly unrelated forecasting problems: incentive-compatible forecast elicitation and forecast aggregation. Proper scoring rules are the well-known solution to the former problem. To each such rule  $s$  we associate a corresponding method of aggregation, mapping expert forecasts and expert weights to a “consensus forecast,” which we call *quasi-arithmetic (QA) pooling* with respect to  $s$ . We justify this correspondence in several ways:

- QA pooling with respect to the two most well-studied scoring rules (quadratic and logarithmic) corresponds to the two most well-studied forecast aggregation methods (linear and logarithmic).
- Given a scoring rule  $s$  used for payment, a forecaster agent who sub-contracts several experts, paying them in proportion to their weights, is best off aggregating the experts’ reports using QA pooling with respect to  $s$ , meaning this strategy maximizes its worst-case profit (over the possible outcomes).
- The score of an aggregator who uses QA pooling is concave in the experts’ weights. As a consequence, online gradient descent can be used to learn appropriate expert weights from repeated experiments with low regret.
- The class of all QA pooling methods is characterized by a natural set of axioms (generalizing classical work by Kolmogorov on quasi-arithmetic means).

The full paper is available at <https://arxiv.org/abs/2102.07081>

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