Tracking Equilibrium Point Under Real-Time Price-Based Residential Demand Response

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***Abstract*—This letter proposes a model for tracking the equilibrium point of the real-time locational marginal price (LMP) based residential demand response program, where elastic demand is modeled as a monotonously decreasing linear function of the LMP. The resulting bi-level model contains both primary and dual variables, making it difficult to solve. Using duality, the dual model is formulated as a convex quadratic problem which is tractable to solve and find the global optimum. Furthermore, the condition for the existence of the equilibrium point is given. Numerical results on the IEEE 30-bus system verifies the effectiveness of the demand response model.**

***Index Terms*—Security constrained economic dispatch (SCED), demand response, locational marginal price (LMP), equilibrium point, bi-level optimization.**

I. Introduction

RECENTLY, residential demand response (RDR) programs have gained sustainable popularity in the power industry [1]. In the transmission system, with the development of control technologies like load aggregators (LAs), demand can rapidly respond to the locational marginal price (LMP) variations to reduce the total generation cost and mitigate the LMP spikes at peak load periods. Since independent system operators (ISOs) cannot directly control thousands of consumers, the load aggregator can be served as an agent consumer. However, unlike commercial and industrial counterparts, participation of RDR by LAs in the wholesale electricity market requires upgrades of market design, system operation, and metering technology. In most ISOs in the U.S., RDR is only allowed to participate in the capacity market instead of energy market and they serve as voluntary reliability resources during emergency situations [2]–[3]. Even though RDR is unable to participate in the wholesale market based on today's ISO market rules, the RDR can respond to market prices as a market price-taker.

This letter aims to bridge the gap between the wholesale market and RDR programs. Herein, LAs act as intermediators between RDR programs and ISOs to receive the real-time LMP from ISOs and control the load demand of RDR programs with load elasticity. Furthermore, ISOs update the LMP based on a new load demand value and send a new LMP to LAs, which in turn will control the demand accordingly. Mathematically, it can be represented as a bi-level iterative process to optimize the coordination between RDR programs and ISO's wholesale electricity market. The process stops when it converges to an equilibrium point. This letter proposes an optimization model to track and quantify this equilibrium point, which can be used, for instance, by ISOs to predict negotiation results in the context of improving day-ahead dispatch schedules and market clearing. To realize the proposed concept, we assume that the LA can collect the required amount of exact information from all participants, i.e., information symmetry is achieved. In particular, if the LA enrolls RDR participants, the original inelastic demand needs to be precisely estimated by using advanced techniques, e.g., big data, etc.

II. Proposed Method for Tracking Equilibrium Point

The interaction between the ISO/RTO and LAs can be characterized as follows: at h^{th} iteration, the LMP $\boldsymbol{\lambda}_j^h$ at bus j is computed by a security-constrained economic dispatch (SCED) model with the current load demand value D_j^h. Then, the load demand is updated to a new value D_j^{h+1}. Furthermore, the new LMP at $(h+1)^{\text{th}}$ iteration can be obtained by the SCED model. This interaction is characterized as a bi-level iterative process, which essentially constructs a bi-level optimization model.

Upper Level (for ISO/RTO): The upper level models the ISO/RTO side as a SCED problem at each time period to minimize the generation cost while guaranteeing the physical constraints of generators and transmission lines. This SCED

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problem is as follows [4]:

$$\left(\pi^h, \mu_l^{h+}, \mu_l^{h-}\right) = \arg\min \sum_{i=1}^{N}\left(a_iP_i^2 + b_iP_i + c_i\right) \tag{1a}$$

$$s.t. \quad \sum_{i=1}^{N} P_i - \sum_{j=1}^{M} D_j^h = 0 : \pi^h \tag{1b}$$

$$P_i^{\min} \le P_i \le P_i^{\max}, \quad i = 1, \ldots, N \tag{1c}$$

$$-F_l^{\max} \le \sum_{i=1}^{N} G_{li}P_i - \sum_{j=1}^{M} H_{lj}D_j^h \le F_l^{\max} : \left(\mu_l^{h+}, \mu_l^{h-}\right), \quad l = 1, \ldots, L \tag{1d}$$

where π^h is the shadow price corresponding to (1b) at the h^{th} iteration; μ^{h+} and μ^{h-} are the shadow prices corresponding to $-F_l^{\max} \le \sum_{i=1}^{N} G_{li}P_i - \sum_{j=1}^{M} H_{lj}D_j^h$ and $\sum_{i=1}^{N} G_{li}P_i - \sum_{j=1}^{M} H_{lj}D_j^h \le F_l^{\max}$ at h^{th} iteration, respectively; N is the number of generators; M refers to the number of LAs; L represents the number of transmission lines; P_i is the power output of dispatchable generator i; (a_i, b_i, c_i) are the triplet coefficients of the convex quadratic cost function of generator i with $a_i > 0$; F_l^{max} is the transmission capacity limit of line l; D_j^h is the load at bus j at h^{th} iteration; P_i^{min} and P_i^{max} are the maximum and minimum generation output of generator i; H_{lj} denotes the distribution factor of power injection at load bus j on line l and G_{li} denotes the distribution factor of power injection at generator bus i on line l. In this model, (1b), (1c) and (1d) represent the energy balance, generator limit, and transmission capacity limit, respectively. By solving the above SCED model, we can obtain the locational marginal price (LMP) using the Karush–Kuhn–Tucker (KKT) conditions as:

$$\lambda_j^h = \pi^h + \sum_{l=1}^{L} H_{jl}\left(\mu_l^{h+} - \mu_l^{h-}\right), \quad j = 1, \ldots, M \tag{2}$$

where λ_j^h is the LMP at bus j at h^{th} iteration.

Lower Level (for LAs): The lower level models the LA side, where the load demand changes once each LA receives the real-time LMP signal, leading to a real-time RDR framework. Consider the load at the bus j is elastic to its LMP λ_j^h, which is in fact a monotonously decreasing function with respect to the LMP λ_j^h, denoted as $D_j(\lambda_j^h)$. This implies that $\partial D_j(\boldsymbol{\lambda}_j)/\partial \boldsymbol{\lambda}_j \le 0$. Moreover, the price-elasticity in many papers, such as [5]–[6], is modeled as a linear function, which can be written as:

$$\begin{aligned} D_j^{h+1} &= D_j\left(\lambda_j^h\right) = D_j^0 - k_j\left(\lambda_j^h - \lambda_j^0\right) \\ &= a_j - k_j\lambda_j^h, \quad j = 1, \ldots, M \end{aligned} \tag{3}$$

where D_j^0 is the normal inelastic electricity demand at bus j; λ_j^0 is the given j^{th} load side price (e.g., retail price); $a_j = D_j^0 + k_j\lambda_j^0$; k_j denotes the inherent demand elasticity of load j in the linear function, which is a nonnegative value (i.e., $k_j \ge 0$); the RDR at j^{th} bus is considered/not considered when $k_j > 0/k_j = 0$; $D_j(\lambda_j^h)$ is the elastic load at bus j which is a function of LMP λ_j^h; D_j^{h+1} is the elastic load demand at bus j at $(h+1)^{\text{th}}$ iteration, which has been updated based on LMP λ_j^h.

When the LMP increases, the load demand decreases by k_j according to the RDR function (3). Then, by decreasing the load demand, the LMP decreases or remains constant, since the LMP response matrix is positive semi-definite as given in **Proposition 1**. Thus, the RDR prevents the LMP from rising. Once the LMP doesn't change, this bi-level iterative process stops and converges to an equilibrium point which mathematically satisfies $\lambda_j^h = \lambda_j^{h+1} = \lim_{h\to\infty}\lambda_j^h = \lambda_j$ and $D_j^h = D_j^{h+1} = \lim_{h\to\infty}D_j^h = D_j(\lambda_j)$, leading to the following compact form:

$$\boldsymbol{\lambda}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}) = \arg\min_{\boldsymbol{P}, \boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}} \; 1/2\boldsymbol{P}^T\boldsymbol{A}\boldsymbol{P} + \boldsymbol{b}^T\boldsymbol{P} + \text{sum(c)} \tag{4a}$$

$$s.t. \quad \boldsymbol{A}_1\boldsymbol{P} + \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda}) = \boldsymbol{b}_1 : \boldsymbol{\pi} \tag{4b}$$

$$\boldsymbol{A}_2\boldsymbol{P} + \boldsymbol{B}_2\boldsymbol{D}(\boldsymbol{\lambda}) \le \boldsymbol{b}_2 : \boldsymbol{\mu} \tag{4c}$$

$$\boldsymbol{A}_3\boldsymbol{P} \le \boldsymbol{b}_3 : \boldsymbol{\nu} \tag{4d}$$

where $\boldsymbol{A} = \text{diag}\{2a_i\}$; $\boldsymbol{\pi}$, $\boldsymbol{\mu}$, and $\boldsymbol{\nu}$ are the Lagrange multipliers corresponding to (4b)-(4d); $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{A}_3$, $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ are coefficient matrices and right-hand sides corresponding to constraints (1b)-(1d). The Lagrange function is given by:

$$\begin{aligned} L_{\boldsymbol{D}(\boldsymbol{\lambda})}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}) &= 1/2\boldsymbol{P}^T\boldsymbol{A}\boldsymbol{P} + \boldsymbol{b}^T\boldsymbol{P} + \boldsymbol{\pi}^T(\boldsymbol{b}_1 - \boldsymbol{A}_1\boldsymbol{P} - \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda})) \\ &+ \boldsymbol{\mu}^T(\boldsymbol{b}_2 - \boldsymbol{B}_2\boldsymbol{D}(\boldsymbol{\lambda}) - \boldsymbol{A}_2\boldsymbol{P}) + \boldsymbol{\nu}^T(\boldsymbol{b}_3 - \boldsymbol{A}_3\boldsymbol{P}) + \text{sum(c)} \end{aligned} \tag{5}$$

According to the definition of the LMP, we have:

$$\boldsymbol{\lambda} = \partial L_{\boldsymbol{D}(\boldsymbol{\lambda})}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu})/\partial \boldsymbol{D}(\boldsymbol{\lambda}) = -\boldsymbol{B}_1^T\boldsymbol{\pi} - \boldsymbol{B}_2^T\boldsymbol{\mu} \tag{6}$$

However, model (4) contains primal and dual variables simultaneously, which cannot be solved directly. To solve it, we first take dual of (4), which leads to:

$$\max_{\boldsymbol{\eta}, \boldsymbol{\mu}, \boldsymbol{\nu}, \boldsymbol{y}} \; Z(\boldsymbol{\lambda}) = (\boldsymbol{b}_1 - \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda}))^T\boldsymbol{\pi} + (\boldsymbol{b}_2 - \boldsymbol{B}_2\boldsymbol{D}(\boldsymbol{\lambda}))^T\boldsymbol{\mu} + \boldsymbol{b}_3^T\boldsymbol{\nu} + \text{sum}(\boldsymbol{c}) - 1/2\boldsymbol{y}^T\boldsymbol{A}\boldsymbol{y} \tag{7a}$$

$$s.t. \quad \boldsymbol{A}_1^T\boldsymbol{\pi} + \boldsymbol{A}_2^T\boldsymbol{\mu} + \boldsymbol{A}_3^T\boldsymbol{\nu} - \boldsymbol{A}\boldsymbol{y} = \boldsymbol{b} \tag{7b}$$

$$\boldsymbol{\mu} \le \boldsymbol{0}, \boldsymbol{\nu} \le \boldsymbol{0} \tag{7c}$$

where there are only dual variables in (7). Furthermore, using (6), (7a) can be written as:

$$Z(\boldsymbol{\lambda}) = \boldsymbol{D}^T(\boldsymbol{\lambda})\boldsymbol{\lambda} + \boldsymbol{b}_1^T\boldsymbol{\pi} + \boldsymbol{b}_2^T\boldsymbol{\mu} + \boldsymbol{b}_3^T\boldsymbol{\nu} + \text{sum(c)} - 1/2\boldsymbol{y}^T\boldsymbol{A}\boldsymbol{y} \tag{8}$$

Besides, using (6), we can formulate (3) in a matrix form as:

$$\boldsymbol{D}(\boldsymbol{\lambda}) = \boldsymbol{a} - \boldsymbol{K}\boldsymbol{\lambda} = \boldsymbol{a} + \boldsymbol{K}\left(\boldsymbol{B}_1^T\boldsymbol{\pi} + \boldsymbol{B}_2^T\boldsymbol{\mu}\right) \tag{9}$$

where $\boldsymbol{K}$ is a diagonal matrix with the j^h element being k_j and $\boldsymbol{a}$ is a vector consisting of a_j elements, for $\forall j$. Since $k_j \ge 0$ for $\forall j$, $\boldsymbol{K}$ is a positive semi-definite matrix. Then, by substituting $\boldsymbol{\lambda}$ from (6) and $\boldsymbol{D}(\boldsymbol{\lambda})$ in (9), we obtain:

$$Z(\lambda) = \left(\boldsymbol{b}_1^T - \boldsymbol{a}^T\boldsymbol{B}_1^T\right)\boldsymbol{\pi} + \left(\boldsymbol{b}_2^T - \boldsymbol{a}^T\boldsymbol{B}_2^T\right)\boldsymbol{\mu} + \boldsymbol{b}_3^T\boldsymbol{\nu} - \left(\boldsymbol{\eta}^T, \boldsymbol{\mu}^T\right)\boldsymbol{Q}\begin{pmatrix}\boldsymbol{\pi} \\ \boldsymbol{\mu}\end{pmatrix} + \text{sum(c)} \tag{10}$$

where

$$\boldsymbol{Q} = \begin{pmatrix} \boldsymbol{B}_1\boldsymbol{K}\boldsymbol{B}_1^T & \boldsymbol{B}_1\boldsymbol{K}\boldsymbol{B}_2^T \\ \boldsymbol{B}_2\boldsymbol{K}\boldsymbol{B}_1^T & \boldsymbol{B}_2\boldsymbol{K}\boldsymbol{B}_2^T \end{pmatrix} = \begin{pmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{pmatrix}\boldsymbol{K}\begin{pmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{pmatrix}^T \tag{11}$$

It is easy to show that $\boldsymbol{Q}$ is positive semi-definite, since $\boldsymbol{K}$ is positive semi-definite. Therefore, (7) becomes a convex quadratic problem, which can be tractably solved to obtain the global optimal solution.

To find the equilibrium point, we introduce the LMP response matrix representing the sensitivity of the LMP at a load bus with respect to the infinitesimal increment of load demand.

Proposition 1: LMP response matrix, denoted by $\boldsymbol{O}$, is symmetric and positive semi-definite.

Proof: The SCED model (1) and (4) with only marginal units and binding constraints is written as

$$\min_{\overline{\boldsymbol{P}}} \ 1/2\overline{\boldsymbol{P}}^T\overline{\boldsymbol{A}\boldsymbol{P}} + \boldsymbol{b}^T\overline{\boldsymbol{P}} + \mathrm{sum}(\overline{\mathrm{c}}) \tag{12a}$$

$$s.t. \ \ \boldsymbol{A}_1\overline{\boldsymbol{P}} + \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda}) = \boldsymbol{b}_1 : \boldsymbol{\pi} \tag{12b}$$

$$\overline{\boldsymbol{A}_2\boldsymbol{P}} + \overline{\boldsymbol{B}_2}\boldsymbol{D}(\boldsymbol{\lambda}) = \overline{\boldsymbol{b}_2} : \overline{\boldsymbol{\mu}} \tag{12c}$$

$$\boldsymbol{A}_3\overline{\boldsymbol{P}} < \overline{\boldsymbol{b}_3} : \overline{\boldsymbol{\nu}} \tag{12d}$$

where $\overline{\boldsymbol{P}}$ is the decision variables $\boldsymbol{P}$ of only marginal units; $\overline{\boldsymbol{A}_2}$, $\overline{\boldsymbol{A}_3}$, $\overline{\boldsymbol{B}_2}$ are coefficient matrices; $\overline{\boldsymbol{b}_2}$ and $\overline{\boldsymbol{b}_3}$ are right-hand side vectors; $\overline{\boldsymbol{\mu}}$ and $\overline{\boldsymbol{\nu}}$ are Lagrange multipliers corresponding to the binding constraints (12c) and (12d); $\overline{\boldsymbol{c}}$ is the constant cost plus the cost of non-marginal units. The Lagrange function and KKT conditions are given by (13) and (14).

$$\begin{aligned}\overline{L}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu}) = {} & 1/2\overline{\boldsymbol{P}}^T\overline{\boldsymbol{A}\boldsymbol{P}} + \boldsymbol{b}^T\overline{\boldsymbol{P}} + \boldsymbol{\pi}^T\Big(\boldsymbol{b}_1 - \boldsymbol{A}_1\overline{\boldsymbol{P}} - \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda})\Big) \\ & + \overline{\boldsymbol{\mu}}^T\Big(\overline{\boldsymbol{b}_2} - \overline{\boldsymbol{B}_2}\boldsymbol{D}(\boldsymbol{\lambda}) - \overline{\boldsymbol{A}_2\boldsymbol{P}}\Big) + \overline{\boldsymbol{\nu}}^T\Big(\overline{\boldsymbol{b}_3} - \overline{\boldsymbol{A}_3\boldsymbol{P}}\Big) \\ & + \mathrm{sum}(\overline{\mathrm{c}}) \end{aligned} \tag{13}$$

$$\partial\overline{L}(\boldsymbol{\pi}, \overline{\boldsymbol{\mu}}, \overline{\boldsymbol{\nu}})/\partial\overline{\boldsymbol{P}} = 0 \Rightarrow \overline{\boldsymbol{A}\boldsymbol{P}} + \boldsymbol{b} - \boldsymbol{A}_1^T\boldsymbol{\pi} - \overline{\boldsymbol{A}_2^T}\overline{\boldsymbol{\mu}} - \overline{\boldsymbol{A}_3^T}\overline{\boldsymbol{\nu}} = 0$$

$$\partial\overline{L}(\boldsymbol{\pi}, \overline{\boldsymbol{\mu}}, \overline{\boldsymbol{\nu}})/\partial\boldsymbol{\pi} = 0 \Rightarrow \boldsymbol{A}_1\overline{\boldsymbol{P}} + \boldsymbol{B}_1\boldsymbol{D}(\boldsymbol{\lambda}) = \boldsymbol{b}_1 \tag{14a}$$

$$\partial\overline{L}(\boldsymbol{\pi}, \overline{\boldsymbol{\mu}}, \overline{\boldsymbol{\nu}})/\partial\overline{\boldsymbol{\mu}} = 0 \Rightarrow \overline{\boldsymbol{A}_2\boldsymbol{P}} + \overline{\boldsymbol{B}_2}\boldsymbol{D}(\boldsymbol{\lambda}) = \overline{\boldsymbol{b}_2} \tag{14b}$$

$$0 < \Big(\boldsymbol{b}_3 - \boldsymbol{A}_3\overline{\boldsymbol{P}}\Big)\perp\overline{\boldsymbol{\nu}} \geq 0 = 0 \tag{14c}$$

where $\perp$ indicates the inner product of two vectors. Since $\boldsymbol{b}_3 - \boldsymbol{A}_3\overline{\boldsymbol{P}}$ is strictly positive, (14d) yields $\overline{\boldsymbol{\nu}} = 0$. Thus, (14a) can be reformulated as:

$$\begin{aligned}\overline{\boldsymbol{P}} &= -\overline{\boldsymbol{A}}^{-1}\Big(\boldsymbol{b} - \boldsymbol{A}_1^T\boldsymbol{\pi} - \overline{\boldsymbol{A}_2^T}\overline{\boldsymbol{\mu}}\Big) \\ &= -\overline{\boldsymbol{A}}^{-1}\boldsymbol{b} + \overline{\boldsymbol{A}}^{-1}\Big(\boldsymbol{A}_1^T\boldsymbol{\pi} + \overline{\boldsymbol{A}_2^T}\overline{\boldsymbol{\mu}}\Big)\end{aligned} \tag{15a}$$

Also, (14b) and (14c) can be written as:

$$\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right)\overline{\boldsymbol{P}} = \left(\frac{\boldsymbol{b}_1}{\boldsymbol{b}_2}\right) - \left(\frac{\boldsymbol{B}_1}{\boldsymbol{B}_2}\right)\boldsymbol{D}(\boldsymbol{\lambda}) \tag{15b}$$

Substituting (15a) in (15b) yields:

$$\begin{aligned}&\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right)\overline{\boldsymbol{A}}^{-1}\boldsymbol{b} - \left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right)\overline{\boldsymbol{A}}^{-1}\left(\frac{\boldsymbol{A}_1}{\boldsymbol{A}_2}\right)^T\left(\frac{\boldsymbol{\pi}}{\overline{\boldsymbol{\mu}}}\right) \\ &= \left(\frac{\boldsymbol{B}_1}{\boldsymbol{B}_2}\right)\boldsymbol{D}(\boldsymbol{\lambda}) - \left(\frac{\boldsymbol{b}_1}{\boldsymbol{b}_2}\right)\end{aligned} \tag{16}$$

According to (6), the LMP can be expressed as:

$$\boldsymbol{\lambda} = \partial\overline{L}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu})/\partial\boldsymbol{D}(\boldsymbol{\lambda}) = -\boldsymbol{B}_1^T\boldsymbol{\pi} - \overline{\boldsymbol{B}_2^T}\overline{\boldsymbol{\mu}} \tag{17}$$

Therefore, the LMP response matrix $\boldsymbol{O}$ can be represented as:

$$\begin{aligned}\boldsymbol{O} &= \frac{\partial\boldsymbol{\lambda}}{\partial\boldsymbol{D}(\boldsymbol{\lambda})} = \frac{\partial^2\overline{L}(\boldsymbol{\pi}, \boldsymbol{\mu}, \boldsymbol{\nu})}{\partial\boldsymbol{D}^2(\boldsymbol{\lambda})} = -\Big(\boldsymbol{B}_1^T, \overline{\boldsymbol{B}_2^T}\Big)\left(\frac{\partial\boldsymbol{\pi}}{\partial\boldsymbol{D}(\boldsymbol{\lambda})}, \frac{\partial\overline{\boldsymbol{\mu}}}{\partial\boldsymbol{D}(\boldsymbol{\lambda})}\right)^T \\ &= \Big(\boldsymbol{B}_1^T, \overline{\boldsymbol{B}_2^T}\Big)\Big(\boldsymbol{B}_1^T, \boldsymbol{B}_2^T\Big)\Big(\big(\boldsymbol{A}_1^T, \boldsymbol{A}_2^T\big)^T\overline{\boldsymbol{A}}^{-1}\big(\boldsymbol{A}_1^T, \boldsymbol{A}_2^T\big)\Big)^{-1}\Big(\boldsymbol{B}_1^T, \overline{\boldsymbol{B}_2^T}\Big)^T\end{aligned} \tag{18}$$

From (1a) with $a_i > 0 \ i = 1, \ldots, N$, it is yielded that $\overline{\boldsymbol{A}}$ and $\boldsymbol{A}$ are positive definite and symmetric. Thus, $\boldsymbol{O}$ in (18) is symmetric. In addition, since the inverse of a positive definite matrix is positive definite [7], $\overline{\boldsymbol{A}}^{-1}$ is positive definite. Thus, considering the fact that a matrix product P^TMP is positive semi-definite if matrix M is positive semi-definite [7], $\boldsymbol{O}$ becomes positive semi-definite.

Finally, we give a condition for the existence of equilibrium point, i.e., the convergence of the proposed bilevel iterative approach. Since the iterations of the proposed approach are conducted in a local area around the initial point and the LMP response matrix $\boldsymbol{O}$ in (18) is constant in such a local area, the LMP can be expressed as a linear function of $\boldsymbol{D}(\boldsymbol{\lambda})$:

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}_0 + \boldsymbol{O}\boldsymbol{D}(\boldsymbol{\lambda}) \tag{19}$$

Thus, the LMP at k^{th} iteration is presented as:

$$\boldsymbol{\lambda}^k = \boldsymbol{\lambda}_0 + \boldsymbol{O}\boldsymbol{D}^k(\boldsymbol{\lambda}) \tag{20}$$

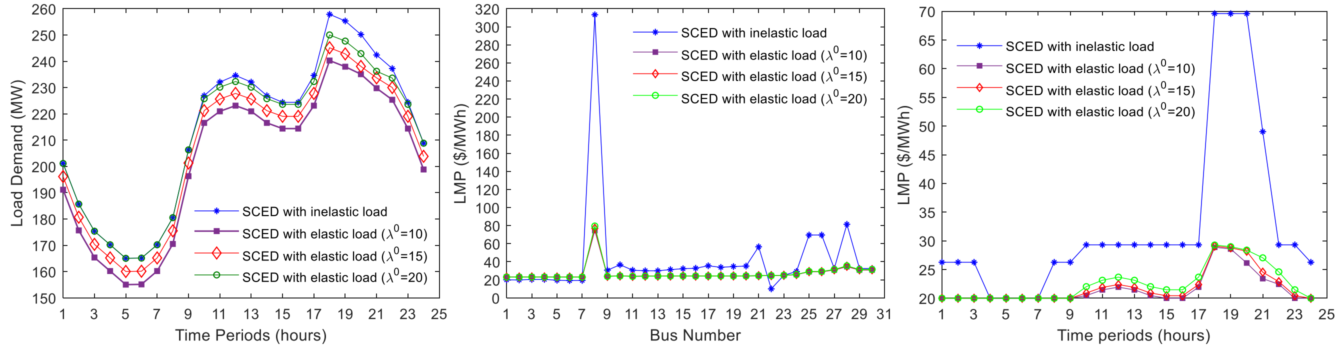
Then, the demand is changed by the RDR function (9) as:

$$\boldsymbol{D}^{k+1}\Big(\boldsymbol{\lambda}^k\Big) = \boldsymbol{a} - \boldsymbol{K}\boldsymbol{\lambda}^k \tag{21}$$

Based on (20) and (21), we have

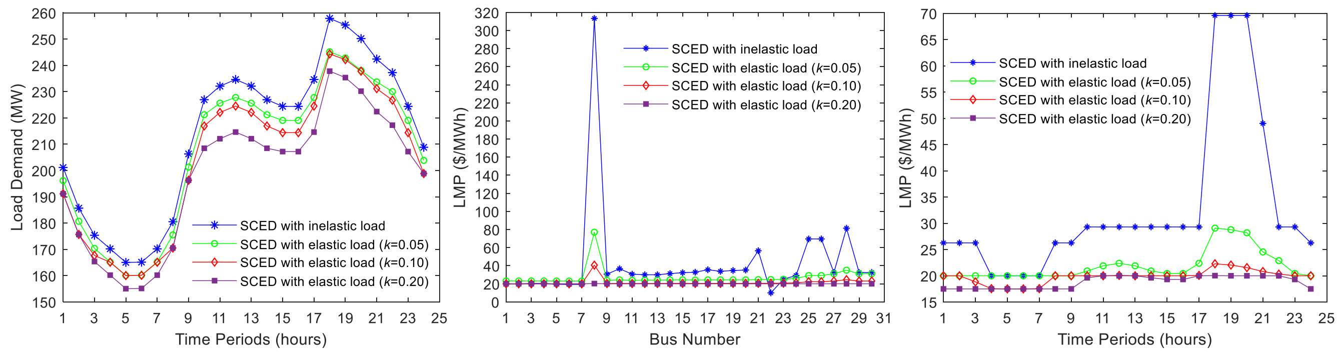
$$\begin{aligned}\boldsymbol{\lambda}^{k+1} &= \boldsymbol{\lambda}_0 + \boldsymbol{O}\boldsymbol{D}^{k+1}(\boldsymbol{\lambda}) = \boldsymbol{\lambda}_0 + \boldsymbol{O}\Big(\boldsymbol{a} - \boldsymbol{K}\boldsymbol{\lambda}^k\Big) \\ &= \boldsymbol{\lambda}_0 + \mathrm{Oa} - \boldsymbol{O}\boldsymbol{K}\boldsymbol{\lambda}^k\end{aligned} \tag{22}$$

Since $\boldsymbol{\lambda}_0 + \boldsymbol{O}\boldsymbol{a}$ is a constant vector, to guarantee the convergence of the proposed iterative approach, the spectral radius of the coefficient matrix $\boldsymbol{O}\boldsymbol{K}$ in (22) should be smaller than 1, i.e., $\rho(\boldsymbol{O}\boldsymbol{K}) < 1$ [8]. According to [9], let $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ be positive semi-definite matrices, then we have $\rho(\boldsymbol{C}\boldsymbol{B}) \leq \rho(\boldsymbol{C}\boldsymbol{A})$, if $\boldsymbol{B} \preceq \boldsymbol{A}$. Thus, under the fact that both $\boldsymbol{O}$ and $\boldsymbol{K}$ are symmetric and positive semi-definite matrices, the spectral radius of $\boldsymbol{O}\boldsymbol{K}$ is bounded by $0 < \rho(\boldsymbol{O}\boldsymbol{K}) \leq \rho(\boldsymbol{O}\overline{\boldsymbol{K}})$, where $\overline{\boldsymbol{K}} = \mathrm{diag}(\max_{\forall j}(k_j))$ and $\overline{\boldsymbol{K}} \succeq \boldsymbol{K}$. Furthermore, we have $\rho(\boldsymbol{O}\overline{\boldsymbol{K}}) = \rho(\max_{\forall j}(k_j)\boldsymbol{O}\boldsymbol{I}) = \max_{\forall j}(k_j)\rho(\boldsymbol{O}\boldsymbol{I}) = \max_{\forall j}(k_j)\rho(\boldsymbol{O})$, where $(\boldsymbol{O}) > 0$, to ensure the convergence condition $\rho(\boldsymbol{O}\boldsymbol{K}) < 1$, it is sufficient to have $\rho(\boldsymbol{O}\overline{\boldsymbol{K}}) < 1$ which gives $\max_{\forall j}(k_j) < 1/\rho\max_{\forall j}(k_j) < 1/\rho(\boldsymbol{O})$. This is a sufficient condition for the convergence of the proposed iterative approach. In particular, if the inherent demand elasticity of all loads is the same as k, we arrive at $\rho(\boldsymbol{O}\boldsymbol{K}) = k\rho(\boldsymbol{O})$ and the convergence condition of the proposed approach becomes $k < 1/\rho(\boldsymbol{O})$. ■



(a) Electricity demand (b) LMPs at time 18:00 (c) LMP at bus 25 over 24 hours

Fig. 1. Comparison of LMPs and load demand with and without RDR under different λ^0 values ($k = 0.05$).



(a) Electricity demand (b) LMPs at time 18:00 (c) LMP at bus 25 over 24 hours

Fig. 2. Comparison of LMPs and load demand with and without RDR under different k values ($\lambda^0 = 15$).

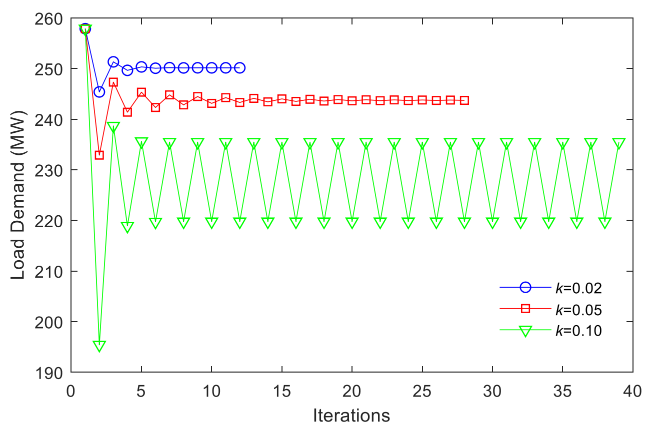


Fig. 3. Iterative process under different demand elasticity k.

III. Numerical Example

The proposed equilibrium point tracking model, implemented using CPLEX 12.6 solver within MATLAB software, is tested on the IEEE 30-bus system [10]. In order to test the proposed model under different load demand levels, we choose the load demands over 24 hours for study and the real-time RDR is performed at each hour. Besides, we choose different values for the inherent demand elasticity k as 0.05, 0.1 and 0.2, and different values for the fixed load side price λ^0 as 10, 15, and 20 \$/MWh.

The comparisons of LMPs and load demands with and without the proposed RDR are shown in Fig.1 and Fig. 2. Fig. 1(a) and Fig. 2(a) show that for any λ^0 and k values, the load demand can be effectively reduced at peak time when the LMP is high to avoid damages of price spikes. This verifies the effectiveness of the proposed RDR model. Moreover, it is observed that by decreasing λ^0 and increasing k, the proposed RDR further decreases loads.

Fig. 1(b) and Fig. 2(b) depict the LMPs during the peak load time periods (i.e., at 18:00), when the total electricity demand is about 1.4 times of the initial electricity demand and transmission line 6-8 is severely congested. Thus, the LMP at bus 8 is extremely high, rising to \$313/MWh. Using elastic electricity demand (i.e., the proposed RDR), the LMP at bus 8 is significantly reduced. Moreover, it is seen that decreasing λ^0 has slight impact on the LMPs in Fig. 1(b) and the LMP at bus 8 is always \$80/MWh for any λ^0 value, whereas increasing k has a remarkable impact on the LMPs in Fig. 2(b) and the LMP at bus 8 is reduced from \$80/MWh to \$20/MWh by increasing k from 0.05 to 0.2. In addition, by using the proposed RDR, LMPs become much smoother. For instance, with $\lambda^0 = 15$ and $k = 0.20$ in Fig. 2(b), all the LMPs become equal, which indicates that the congestion on line 6-8 is mitigated.

Fig. 1(c) and Fig. 2(c) illustrate the LMP at bus 25 over 24 hours. It can be observed that the price volatility of bus 25 with elastic electricity demand is much less than that without elastic electricity demand. Especially, at peak load time period, the LMP is reduced from \$70/MWh to less than \$30/MWh. Changing λ^0 does not significantly affect the LMP over 24 hours as the three curves with elastic demand in Fig. 1(c) have slight differences. However, increasing k more effectively reduces the LMP and the three curves with elastic demand in Fig. 2(c) have greater difference. In particular, when $k = 0.2$, the LMP over the whole day is not more than 20$/MWh.

Finally, the convergence performance of the proposed bilevel iterative approach under different demand elasticities is shown in Fig. 3, where $k = 0.02$ needs 13 iterations to converge, $k = 0.05$ needs 28 iterations to converge and $k = 0.1$ leads to non-convergence. Thus, smaller k results in better convergence performance, which is consistent with the convergence condition of the proposed iterative approach.

IV. Conclusion

In this letter, a bi-level iterative approach is proposed to characterize the interaction between ISO/RTO and load aggregators. Furthermore, an optimization model is set up to quantify the equilibrium point of this price-based RDR and the condition for the existence of the equilibrium point is given. Simulation results illustrate that the participation of the proposed RDR can flatten loads and LMPs, and mitigate price spikes at peak time. Moreover, increasing the inherent demand elasticity has higher impacts on LMPs than decreasing the load side price. Future work will extend the proposed model to consider the impact of uncertainties by using the stochastic game theory approach. In addition, extending the convergence condition of the proposed iterative approach and its mathematical proof to the case where elastic load limits are included can be considered as the future research work.

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