

Two-User MIMO Broadcast Channel with Transmit Correlation Diversity: Achievable Rate Regions

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Abstract—In a multiple-input multiple-output (MIMO) broadcast channel (BC), the difference in spatial transmit correlation matrices of different users is called *transmit correlation diversity*. Recently, several works have extended this concept beyond its original scope, to include channels whose transmit correlation matrices have non-overlapping eigenspaces. In contrast to earlier analyses of overlapping eigenspaces that were mostly described in terms of degrees-of-freedom, this work presents achievable rate regions. These achievable regions are derived by rate-splitting, product superposition, or a combination thereof. Our rate expressions make explicit the contribution of the common parts and individual (non-overlapping) parts of the correlation eigenspaces toward the achievable rate region. As a by-product, a result of Hassibi and Hochwald on MIMO channel training is extended to channels with spatial correlation.

I. INTRODUCTION

In practical wireless channels, the fading coefficients between different antennas are often correlated. This correlation arises from the propagation environment causing the received signal gains to be larger in some spatial directions, and also from the spatially dependent patterns of the antennas [1]. The effect of spatial correlation on the capacity of multiple-input multiple-output (MIMO) links has been a subject of long-standing interest [2], [3]. It was shown in [4] that correlation is detrimental to the sum rate scaling with various transmission schemes, assuming that all users experience *identical* correlation. In practice, however, the users may have *different* correlation matrices because they are typically not co-located [5], [6].

The difference between the spatial correlation observed by different users in the system is called *transmit correlation diversity*. It was originally conceived for transmit spatial correlation matrices that have mutually exclusive, i.e., *nonoverlapping*, eigenspaces. Under this condition, a joint spatial division multiplexing (JSDM) transmission scheme was proposed in [7], [8] that reduces the overhead needed for channel estimation. Nonoverlapping correlation eigenspaces may occur in, e.g., severely rank-deficient MIMO links. However, in many other scenarios, transmit correlation matrices have *partially overlapping* eigenspaces, motivating to understand and exploit transmit correlation diversity in this more general setting.

This paper investigates a two-user MIMO broadcast channel (BC) with partially overlapping correlation eigenspaces. We consider the noncoherent setting in which the correlation matrices are deterministic and known, but the channel realizations

are not known *a priori* to either the transmitter or users. For this channel, we have previously proposed some achievable degrees of freedom (DoF), i.e., *rate pre-log*, regions in [9]–[11]. In this paper, we make further progress by characterizing some achievable *rate* regions.

Section III begins with an achievable rate region with orthogonal transmission, e.g., time-division multiple access (TDMA), which was shown to be DoF-optimal in the absence of spatial correlation [12]. As a by-product, we find the rate achieved with pilot-based schemes for the single-user channel, thus generalize the results in [13] to correlated fading. Section IV derives rate regions achieved with different superposition techniques. We flexibly employ rate splitting, product superposition, and a composition thereof to effectively create multiple data streams in both common and private parts of the correlation eigenspaces. Numerical results in Section V show that these superposition techniques significantly enlarge the achievable rate region upon orthogonal transmission. We note that most of our results do not require the fading to be Rayleigh. Due to the space limit, we only provide sketches of proofs of the theorems. For the full proofs, please refer to [14].

Notation: For random quantities, we use non-italic letters with sans serif fonts, e.g., a scalar x , a vector $\mathbf{v}$, and a matrix $\mathbf{M}$. Deterministic quantities are denoted with italic letters, e.g., a scalar x , a vector $\mathbf{v}$, and a matrix $\mathbf{M}$; $\mathbf{M}_{[i:j]}$ denotes the sub-matrix containing columns from i to j of $\mathbf{M}$. All rates are measured in bits/channel use.

II. SYSTEM MODEL

We consider a MIMO BC in which a transmitter equipped with M antennas transmits to two users, user k with N_k antennas, $k \in \{1, 2\}$. The channel between the transmitter and user k is flat and block fading with equal-length and synchronous coherence interval (across the users) of T channel uses. That is, the channel matrix $\mathbf{H}_k \in \mathbb{C}^{N_k \times M}$ remains constant during each length- T block and changes independently between blocks. Let $\mathbf{X}[b] \in \mathbb{C}^{M \times T}$ be the transmitted signal during a coherence block b . The received signal matrix at user k is

$$\mathbf{Y}_k[b] = \mathbf{H}_k[b]\mathbf{X}[b] + \mathbf{Z}_k[b], \quad k \in \{1, 2\}, \quad b = 1, 2, \dots,$$

where $\mathbf{Z}_k[b] \in \mathbb{C}^{N_k \times T}$ is the additive noise with independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ entries. The input is subject to the power constraint $\frac{1}{\nu} \sum_{b=1}^{\nu} \|\mathbf{X}[b]\|_F^2 \leq PT$, where ν is the number of blocks spanned by a codeword. Thus P is

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the signal-to-noise ratio (SNR) of the channel. Hereafter, we omit the block index b whenever confusion is not likely.

Channel Spatial Correlation: We assume that the channel is spatially correlated at the transmitter's side according to the Kronecker model, thus the channel matrices are expressed as

$$\mathbf{H}_k = \check{\mathbf{H}}_k \mathbf{R}_k^{\frac{1}{2}}, \quad k \in \{1, 2\},$$

where $\mathbf{R}_k = \frac{1}{N_k} \mathbb{E}[\mathbf{H}_k^H \mathbf{H}_k] \in \mathbb{C}^{M \times M}$, $\text{tr}(\mathbf{R}_k) = M$, is the transmit correlation matrix¹ of user k with rank r_k , and $\check{\mathbf{H}}_k \in \mathbb{C}^{N_k \times M}$ is drawn from a generic distribution satisfying $h(\check{\mathbf{H}}_k) > -\infty$, $\mathbb{E}[\check{\mathbf{H}}_k^H \check{\mathbf{H}}_k] = N_k \mathbf{I}_M$, $k \in \{1, 2\}$. We consider the eigendecomposition $\mathbf{R}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{U}_k^H$ where $\mathbf{\Sigma}_k$ is a $r_k \times r_k$ diagonal matrix containing r_k nonzero eigenvalues of $\mathbf{R}_k$, and $\mathbf{U}_k \in \mathbb{C}^{M \times r_k}$ contains the corresponding eigenvectors of $\mathbf{R}_k$. The rows of $\mathbf{H}_k$ belong to $\text{Span}(\mathbf{U}_k)$, so-called the *eigenspace* of user k . The matrix $\mathbf{H}_k$ can be expanded as

$$\mathbf{H}_k = \check{\mathbf{H}}_k \mathbf{U}_k \mathbf{\Sigma}_k^{\frac{1}{2}} \mathbf{U}_k^H = \mathbf{G}_k \mathbf{\Sigma}_k^{\frac{1}{2}} \mathbf{U}_k^H, \quad k \in \{1, 2\},$$

where $\mathbf{G}_k := \check{\mathbf{H}}_k \mathbf{U}_k$ is equivalently drawn from a generic distribution satisfying $h(\mathbf{G}_k) > -\infty$, $\mathbb{E}[\mathbf{G}_k^H \mathbf{G}_k] = N_k \mathbf{I}_{r_k}$.

We assume that $\mathbf{R}_k$ is known to both the transmitter and user k , but the realizations of $\mathbf{H}_k$ are not known *a priori* at any node. User k might attempt to estimate $\mathbf{H}_k$ using known pilot symbols inserted in $\mathbf{X}$. Hereafter, we assume that $r_k \leq \min\{N_k, T\}$, $k \in \{1, 2\}$, and, without loss of generality, $r_1 \geq r_2$.

III. ACHIEVABLE RATES WITH ORTHOGONAL TRANSMISSION

A. The Single-User Case

We first consider the single-user case and, for simplicity, drop the user's index. The received signal is $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ where the assumptions for $\mathbf{X}$, $\mathbf{Z}$, and $\mathbf{H}$ are as before. In particular, $\mathbf{H}$ is block fading with coherence interval T and has rank- r correlation matrix $\mathbf{R} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^H$.

Theorem 1. *For the single-user spatially correlated channel, the rate R is achievable with a pilot-based scheme, where*

1) *if the transmitter does not know $\mathbf{R}$:*

$$R = \left(1 - \frac{M}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_N + \frac{P_\delta P_\tau \hat{\mathbf{H}} \hat{\mathbf{H}}^H}{P_\delta \text{tr}((\mathbf{\Sigma}^{-1} + P_\tau \mathbf{I}_r)^{-1}) + M} \right) \right], \quad (1)$$

where the rows of $\hat{\mathbf{H}} \in \mathbb{C}^{N \times M}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \mathbf{R}(\mathbf{I}_M + P_\tau \mathbf{R})^{-1} \mathbf{R})$, and (P_τ, P_δ) satisfies $P_\tau M + P_\delta(T - M) \leq PT$;

2) *if the transmitter knows $\mathbf{R}$ and uses orthogonal pilots:*

$$R = \left(1 - \frac{r}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_N + \frac{P_\delta P_\tau \hat{\mathbf{\Omega}} \hat{\mathbf{\Omega}}^H}{P_\delta \text{tr}((\bar{\mathbf{R}}^{-1} + P_\tau \mathbf{I}_r)^{-1}) + r} \right) \right], \quad (2)$$

where the rows of $\hat{\mathbf{\Omega}} \in \mathbb{C}^{N \times r}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \bar{\mathbf{R}}(\mathbf{I}_r + P_\tau \bar{\mathbf{R}})^{-1} \bar{\mathbf{R}})$, $\bar{\mathbf{R}} := \mathbf{V}^H \mathbf{R} \mathbf{V}$ for a truncated unitary matrix $\mathbf{V} \in \mathbb{C}^{M \times r}$ such that (s.t.) $\text{Span}(\mathbf{V}) = \text{Span}(\mathbf{U})$, and (P_τ, P_δ) satisfies $P_\tau r + P_\delta(T - r) \leq PT$.

¹Our results can be extended to the case where the users experience different large-scale fading, e.g., path loss and shadowing, by considering different values of $\text{tr}(\mathbf{R}_k)$.

Allowing non-orthogonal pilots improves the rate to

$$R = \left(1 - \frac{r}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_N + \frac{P_\delta \hat{\mathbf{\Omega}} \hat{\mathbf{\Omega}}^H}{r P_\delta (P_\tau + \frac{1}{r} \text{tr}(\bar{\mathbf{R}}^{-1}))^{-1} + r} \right) \right], \quad (3)$$

where the rows of $\hat{\mathbf{\Omega}} \in \mathbb{C}^{N \times r}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \bar{\mathbf{R}} - (P_\tau + \frac{1}{r} \text{tr}(\bar{\mathbf{R}}^{-1}))^{-1} \mathbf{I}_r)$.

Remark 1. *The optimal power allocation maximizing the rate in (3) is characterized by $P_\tau = \frac{(1-\alpha)PT}{r}$ and $P_\delta = \frac{\alpha PT}{T-r}$ with $\alpha = \frac{1}{2}$ if $T = 2r$ and $\alpha = b - \sqrt{b(b-a)}$ if $T > 2r$, where $a = 1 + \frac{\text{tr}(\bar{\mathbf{R}}^{-1})}{PT} - \frac{r^2}{PT \text{tr}(\bar{\mathbf{R}})}$ and $b = \frac{T-r}{T-2r} \left(1 + \frac{\text{tr}(\bar{\mathbf{R}}^{-1})}{PT}\right)$.*

Corollary 1. *If $\mathbf{R} = \mathbf{I}_M$, the achievable rate is*

$$R = \left(1 - \frac{M}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_N + \frac{P_\delta P_\tau}{M(1 + P_\delta + P_\tau)} \mathbf{H} \mathbf{H}^H \right) \right],$$

where $\mathbf{H} \in \mathbb{C}^{N \times M}$ is the uncorrelated channel matrix. This coincides with [13, Eq.(21)].

Proof of Theorem 1. We present here a sketch of the proof. For the full proof, see [14, Appendix A]. The proof follows by extending [13] to correlated generic fading. First, if the transmitter does not know $\mathbf{R}$, let it transmit $\mathbf{X} = \left[\sqrt{\frac{P_\tau}{M}} \mathbf{X}_\tau \quad \sqrt{\frac{P_\delta}{M}} \mathbf{X}_\delta \right]$, where $\mathbf{X}_\tau \in \mathbb{C}^{M \times M}$ is an orthogonal pilot matrix and $\mathbf{X}_\delta \in \mathbb{C}^{M \times (T-M)}$ is a data matrix with i.i.d. $\mathcal{CN}(0, 1)$ entries. The receiver performs minimum-mean-square-error (MMSE) channel estimation, thus the residual estimation error is uncorrelated with the input. By evoking the argument that the worst-case uncorrelated noise is Gaussian [13], we obtain (1). Second, by exploiting $\mathbf{R}$, the transmitter can project the signal onto the eigenspace of $\mathbf{R}$ using precoder $\mathbf{V}$. The transmitted signal is $\mathbf{X} = \mathbf{V} \left[\sqrt{\frac{P_\tau}{r}} \mathbf{X}_\tau \quad \sqrt{\frac{P_\delta}{r}} \mathbf{X}_\delta \right]$ with pilot matrix $\mathbf{X}_\tau \in \mathbb{C}^{r \times r}$ and Gaussian data matrix $\mathbf{X}_\delta \in \mathbb{C}^{r \times (T-r)}$. Again, using MMSE channel estimation and the worst-case uncorrelated noise argument, we obtain the achievable rate (2) with orthogonal pilots and (3) with optimized pilots. $\square$

B. The Two-User Case with Orthogonal Transmission

We consider a baseline scheme based on orthogonal transmission, e.g., TDMA, that activates only one user per time-frequency resource unit. From Theorem 1, the following corollary demonstrates an achievable rate with this strategy.

Corollary 2. *For the two-user BC, by activating only user $k \in \{1, 2\}$, user k can achieve a rate R_k given by R in Theorem 1 with $\mathbf{R} = \mathbf{R}_k$, $r = r_k$, $N = N_k$, while the other user achieves zero rate. The convex hull of $(0, 0)$, $(R_1, 0)$, and $(0, R_2)$ is an achievable rate region with orthogonal transmission.*

IV. ACHIEVABLE RATES WITH RATE SPLITTING AND SUPERPOSITION

For any nonnegative integers s_1, s_2, s_0 satisfying $s_0 \leq r_0$, $s_1 \leq r_1 - r_0$ and $s_2 \leq r_2 - r_0$, one can find eigendirections $\mathbf{V}_0 \in \mathbb{C}^{M \times s_0}$, $\mathbf{V}_1 \in \mathbb{C}^{M \times s_1}$, $\mathbf{V}_2 \in \mathbb{C}^{M \times s_2}$ that are aligned

with a part of the common and noncommon sections of the two channel eigenspaces s.t.²

$$\begin{aligned}\text{Span}(\mathbf{V}_0) &\subset (\text{Span}(\mathbf{U}_1) \cap \text{Span}(\mathbf{U}_2)), \\ \text{Span}(\mathbf{V}_1) &\subset (\text{Span}(\mathbf{U}_1) \cap \text{Span}(\mathbf{U}_2)^\perp), \\ \text{Span}(\mathbf{V}_2) &\subset (\text{Span}(\mathbf{U}_2) \cap \text{Span}(\mathbf{U}_1)^\perp).\end{aligned}$$

In our achievable schemes, we will use $\mathbf{V}_0$, $\mathbf{V}_1$ and $\mathbf{V}_2$ as precoders to create multiple data streams in both the common and private sections of the correlation eigenspaces. By tuning the parameters s_1 , s_2 , and s_0 , we explore the trade-off between the number of data dimensions (indicating the pilot overhead) and the amount of channel uses for data transmission within each section. For $k \in \{1, 2\}$, let us denote

$$\begin{aligned}\bullet \Phi_{k0} &:= \mathbf{U}_k^H \mathbf{V}_0, \Phi_{kk} := \mathbf{U}_k^H \mathbf{V}_k, \Phi_k := [\Phi_{k0} \ \Phi_{kk}] = \mathbf{U}_k^H [\mathbf{V}_0 \ \mathbf{V}_k], \\ \bullet \bar{\mathbf{R}}_{k0} &:= \Phi_k^H \Sigma_k \Phi_{k0}, \bar{\mathbf{R}}_{kk} := \Phi_k^H \Sigma_k \Phi_{kk}, \bar{\mathbf{R}} := [\bar{\mathbf{R}}_{k0} \ \bar{\mathbf{R}}_{kk}] = \Phi_k^H \Sigma_k \Phi_k, \\ \bullet \check{\mathbf{R}}_{k0} &:= \Phi_k^H \Sigma_k \Phi_{k0}, \check{\mathbf{R}}_{kk} := \Phi_k^H \Sigma_k \Phi_{kk}.\end{aligned}$$

A. Rate Splitting

With rate splitting, we let the transmitter transmit

$$\mathbf{X} = \mathbf{V}_0 \mathbf{X}_0 + \mathbf{V}_1 \mathbf{X}_1 + \mathbf{V}_2 \mathbf{X}_2,$$

where $\mathbf{X}_0$, $\mathbf{X}_1$, and $\mathbf{X}_2$ are independent and satisfy the power constraint $\mathbb{E}[\text{tr}(\mathbf{X}^H \mathbf{X})] \leq PT$. Thanks to the precoders, $\mathbf{X}_k$ is seen by user k only, while $\mathbf{X}_0$ is seen by both users. Specifically,

$$\mathbf{Y}_k = \mathbf{G}_k \Sigma_k^{\frac{1}{2}} \Phi_{k0} \mathbf{X}_0 + \mathbf{G}_k \Sigma_k^{\frac{1}{2}} \Phi_{kk} \mathbf{X}_k + \mathbf{Z}_k, \quad k \in \{1, 2\},$$

where the equivalent channels $\mathbf{G}_k \Sigma_k^{\frac{1}{2}} \Phi_{k0} \in \mathbb{C}^{N_k \times s_0}$ and $\mathbf{G}_k \Sigma_k^{\frac{1}{2}} \Phi_{kk} \in \mathbb{C}^{N_k \times s_k}$ are correlated and unknown. The received signal at user k is similar to the output of a two-user multiple-access channel (MAC) with (s_0, s_k) equivalent transmit antennas and N_k receive antennas. The two MACs share a common signal $\mathbf{X}_0$. From the capacity region of the MACs [16], we know that the rate pairs (R_0, R_1^p) and (R_0, R_2^p) are simultaneously achievable for the MAC 1 and MAC 2, respectively, if the rates $R_0 \geq 0$, $R_1^p \geq 0$, $R_2^p \geq 0$ satisfy

$$\begin{cases} R_0 &\leq \frac{1}{T} I(\mathbf{Y}_1; \mathbf{X}_0 | \mathbf{X}_1), \\ R_1^p &\leq \frac{1}{T} I(\mathbf{Y}_1; \mathbf{X}_1 | \mathbf{X}_0), \\ R_0 + R_1^p &\leq \frac{1}{T} I(\mathbf{Y}_1; \mathbf{X}_0, \mathbf{X}_1), \\ R_0 &\leq \frac{1}{T} I(\mathbf{Y}_2; \mathbf{X}_0 | \mathbf{X}_2), \\ R_2^p &\leq \frac{1}{T} I(\mathbf{Y}_2; \mathbf{X}_2 | \mathbf{X}_0), \\ R_0 + R_2^p &\leq \frac{1}{T} I(\mathbf{Y}_2; \mathbf{X}_0, \mathbf{X}_2). \end{cases}$$

Then for the BC, user 1 achieves rate R_1^p with private signal $\mathbf{X}_1$, user 2 achieves rate R_2^p with $\mathbf{X}_2$, and both users can achieve rate R_0 with common signal $\mathbf{X}_0$. Let R_{0k} be user k 's share in R_0 , then the BC can achieve the rate pair $(R_1, R_2) = (R_{01} + R_1^p, R_{02} + R_2^p)$. Replacing $R_0 = R_{01} + R_{02}$, $R_1^p = R_1 - R_{01}$, and $R_2^p = R_2 - R_{02}$ in (4) and applying Fourier-Motzkin elimination, we obtain the following achievable rate region.

² $\mathbf{V}_0$ can be calculated from $\mathbf{U}_1$ and $\mathbf{U}_2$ using, e.g., Zassenhaus algorithm [15]. $\mathbf{V}_1$ and $\mathbf{V}_2$ can be found similarly from $\mathbf{U}_1$ and $\text{null}(\mathbf{U}_2)$, and $\text{null}(\mathbf{U}_1)$ and $\mathbf{U}_2$, respectively, where $\text{null}(\mathbf{U}_k)$ is s.t. $[\mathbf{U}_k \ \text{null}(\mathbf{U}_k)]$ is unitary.

Lemma 1. *With rate splitting, the two-user BC can achieve any rate pair (R_1, R_2) satisfying*

$$\begin{cases} R_1 \leq \frac{1}{T} \min\{I(\mathbf{Y}_1; \mathbf{X}_1, \mathbf{X}_0), I(\mathbf{Y}_1; \mathbf{X}_1 | \mathbf{X}_0) + I(\mathbf{Y}_2; \mathbf{X}_0 | \mathbf{X}_2)\}, \\ R_2 \leq \frac{1}{T} \min\{I(\mathbf{Y}_2; \mathbf{X}_2, \mathbf{X}_0), I(\mathbf{Y}_2; \mathbf{X}_2 | \mathbf{X}_0) + I(\mathbf{Y}_1; \mathbf{X}_0 | \mathbf{X}_1)\}, \\ R_1 + R_2 \leq \frac{1}{T} \min\{I(\mathbf{Y}_1; \mathbf{X}_1 | \mathbf{X}_0) + I(\mathbf{Y}_2; \mathbf{X}_2, \mathbf{X}_0), \\ I(\mathbf{Y}_1; \mathbf{X}_1, \mathbf{X}_0) + I(\mathbf{Y}_2; \mathbf{X}_2 | \mathbf{X}_0)\}, \end{cases}$$

for input distributions $p_{\mathbf{X}_0}$, $p_{\mathbf{X}_1}$, and $p_{\mathbf{X}_2}$ satisfying the power constraint $\mathbb{E}[\|\mathbf{X}_0\|_F^2 + \|\mathbf{X}_1\|_F^2 + \|\mathbf{X}_2\|_F^2] \leq PT$.

We consider the input distribution characterized by

$$\begin{aligned}\mathbf{X}_0 &= \left[\sqrt{P_{0\tau}} \mathbf{I}_{s_0} \ \mathbf{0}_{s_0 \times s_1} \ \sqrt{\frac{P_{0\delta}}{s_0}} \mathbf{S}_0 \right], \\ \mathbf{X}_1 &= \left[\mathbf{0}_{s_1 \times s_0} \ \sqrt{P_{1\tau}} \mathbf{I}_{s_1} \ \sqrt{\frac{P_{1\delta}}{s_1}} \mathbf{S}_1 \right], \\ \mathbf{X}_2 &= \left[\mathbf{0}_{s_2 \times s_0} \ \sqrt{P_{2\tau}} \mathbf{I}_{s_2} \ \sqrt{\frac{P_{2\delta}}{s_2}} \mathbf{S}_2 \right],\end{aligned}$$

where the data matrices $\mathbf{S}_0 \in \mathbb{C}^{s_0 \times (T-s_1-s_0)}$, $\mathbf{S}_1 \in \mathbb{C}^{s_1 \times (T-s_1-s_0)}$, and $\mathbf{S}_2 \in \mathbb{C}^{s_2 \times (T-s_2-s_0)}$ have i.i.d. $\mathcal{CN}(0, 1)$ entries. This signaling corresponds to a pilot-based scheme where pilots are sent simultaneously in the mutually exclusive parts of the correlation eigenspaces. With this input distribution, we obtain the following achievable rate region.

Theorem 2. *With rate splitting, the two-user BC with $r_1 \geq r_2$ can achieve any rate pair (R_1, R_2) satisfying*

$$\begin{cases} R_1 &\leq \min\{R'_1, R_1^p + R_{02}\}, \\ R_2 &\leq \min\{R'_2, R_2^p + R_{01}\}, \\ R_1 + R_2 &\leq \min\{R_1^p + R'_2, R'_1 + R_2^p\}. \end{cases} \quad (6)$$

Here,

$$\begin{aligned}R'_1 &= \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_1} + \frac{\bar{\mathbf{Q}}_1 \bar{\mathbf{R}}_1 \mathbf{P}_{1\delta} \bar{\mathbf{R}}_1^H \bar{\mathbf{Q}}_1^H}{\text{tr}((\bar{\mathbf{R}}_1^{-1} + \mathbf{P}_{1\tau})^{-1} \mathbf{P}_{1\delta}) + 1} \right) \right], \\ R_1^p &= \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_1} + \frac{P_{1\delta} \bar{\mathbf{Q}}_1 \bar{\mathbf{R}}_{11} \bar{\mathbf{R}}_{11}^H \bar{\mathbf{Q}}_1^H}{s_1 [\text{tr}((\bar{\mathbf{R}}_1^{-1} + \mathbf{P}_{1\tau})^{-1} \mathbf{P}_{1\delta}) + 1]} \right) \right], \\ R_{01} &= \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_1} + \frac{P_{0\delta} \bar{\mathbf{Q}}_1 \bar{\mathbf{R}}_{10} \bar{\mathbf{R}}_{10}^H \bar{\mathbf{Q}}_1^H}{s_0 [\text{tr}((\bar{\mathbf{R}}_1^{-1} + \mathbf{P}_{1\tau})^{-1} \mathbf{P}_{1\delta}) + 1]} \right) \right],\end{aligned}$$

where $\mathbf{P}_{1\tau} := \begin{bmatrix} P_{0\tau} \mathbf{I}_{s_0} & \mathbf{0} \\ \mathbf{0} & P_{1\tau} \mathbf{I}_{s_1} \end{bmatrix}$, $\mathbf{P}_{1\delta} := \begin{bmatrix} \frac{P_{0\delta}}{s_0} \mathbf{I}_{s_0} & \mathbf{0} \\ \mathbf{0} & \frac{P_{1\delta}}{s_1} \mathbf{I}_{s_1} \end{bmatrix}$, and the rows of $\bar{\mathbf{Q}}_1 \in \mathbb{C}^{N_1 \times (s_0+s_1)}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \mathbf{P}_{1\tau}^{-\frac{1}{2}} (\mathbf{P}_{1\tau}^{\frac{1}{2}} \mathbf{P}_{1\delta} \mathbf{P}_{1\tau}^{\frac{1}{2}} + \mathbf{I}_{s_1+s_0})^{-1} \mathbf{P}_{1\tau}^{\frac{1}{2}})$; furthermore,

$$R'_2 = \frac{s_1 - s_2}{T} \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right) \right]$$

$$\begin{aligned}
 & + \frac{P_{2\delta} \bar{\Omega}_2 \bar{\mathbf{R}}_{22} \bar{\mathbf{R}}_{22}^H \bar{\Omega}_2^H}{P_{2\delta} \text{tr}(\bar{\mathbf{R}}_{22}^H (\bar{\mathbf{R}}_2 + \bar{\mathbf{R}}_2 \mathbf{P}_{2\tau} \bar{\mathbf{R}}_2)^{-1} \bar{\mathbf{R}}_{22}) + s_2} \Bigg) \\
 & + \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right. \right. \\
 & \quad \left. \left. + \frac{\bar{\Omega}_2 \bar{\mathbf{R}}_2 \mathbf{P}_{2\tau} \bar{\mathbf{R}}_2 \bar{\Omega}_2^H}{\text{tr}((\bar{\mathbf{R}}_2^{-1} + \mathbf{P}_{2\tau})^{-1} \mathbf{P}_{2\delta}) + 1} \right) \right], \\
 R_2^p &= \frac{s_1 - s_2}{T} \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right. \right. \\
 & \quad \left. \left. + \frac{P_{2\delta} \bar{\Omega}_2 \bar{\mathbf{R}}_2 \bar{\mathbf{R}}_{22} \bar{\mathbf{R}}_{22}^H \bar{\mathbf{R}}_2 \bar{\Omega}_2^H}{P_{2\delta} \text{tr}(\bar{\mathbf{R}}_{22}^H (\bar{\mathbf{R}}_2 + \bar{\mathbf{R}}_2 \mathbf{P}_{2\tau} \bar{\mathbf{R}}_2)^{-1} \bar{\mathbf{R}}_{22}) + s_2} \right) \right] \\
 & + \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right. \right. \\
 & \quad \left. \left. + \frac{P_{2\delta} \bar{\Omega}_2 \bar{\mathbf{R}}_2 \bar{\mathbf{R}}_{22} \bar{\mathbf{R}}_{22}^H \bar{\mathbf{R}}_2 \bar{\Omega}_2^H}{s_2 [\text{tr}((\bar{\mathbf{R}}_2^{-1} + \mathbf{P}_{2\tau})^{-1} \mathbf{P}_{2\delta}) + 1]} \right) \right], \\
 R_{02} &= \left(1 - \frac{s_1 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right. \right. \\
 & \quad \left. \left. + \frac{P_{0\delta} \bar{\Omega}_2 \bar{\mathbf{R}}_2 \bar{\mathbf{R}}_{20} \bar{\mathbf{R}}_{20}^H \bar{\mathbf{R}}_2 \bar{\Omega}_2^H}{s_0 [\text{tr}((\bar{\mathbf{R}}_2^{-1} + \mathbf{P}_{2\tau})^{-1} \mathbf{P}_{2\delta}) + 1]} \right) \right],
 \end{aligned}$$

where $\mathbf{P}_{2\tau} := \begin{bmatrix} P_{0\tau} \mathbf{I}_{s_0} & \mathbf{0} \\ \mathbf{0} & P_{2\tau} \mathbf{I}_{s_2} \end{bmatrix}$, $\mathbf{P}_{2\delta} := \begin{bmatrix} P_{0\delta} \mathbf{I}_{s_0} & \mathbf{0} \\ \mathbf{0} & \frac{P_{2\delta}}{s_2} \mathbf{I}_{s_2} \end{bmatrix}$, and the rows of $\bar{\Omega}_2 \in \mathbb{C}^{N_2 \times (s_0 + s_2)}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \mathbf{P}_{2\tau}^{-1} (\mathbf{P}_{2\tau}^{-1} \bar{\mathbf{R}}_2 \mathbf{P}_{2\tau}^{-1} + \mathbf{I}_{s_2 + s_0})^{-1} \mathbf{P}_{2\tau}^{-1})$. The integers s_0, s_1, s_2 satisfy $s_0 \leq r_0$, $s_1 \leq r_1 - r_0$, and $s_2 \leq r_2 - r_0$. The power components $P_{i\tau}, P_{i\delta}$, $i \in \{0, 1, 2\}$, satisfy

$$P_{0\tau} s_0 + P_{0\delta} (T - s_1 - s_0) + \sum_{i=1}^2 [P_{i\tau} s_i + P_{i\delta} (T - s_i - s_0)] \leq PT. \quad (7)$$

The convex hull of (6) over all feasible values of s_0, s_1, s_2 and all possible power allocations satisfying (7) is an achievable rate region for the two-user BC.

Proof. From Lemma 1, the achievable rate region is fully characterized by the mutual information terms $I(\mathbf{Y}_k; \mathbf{X}_k, \mathbf{X}_0)$, $I(\mathbf{Y}_k; \mathbf{X}_k | \mathbf{X}_0)$, and $I(\mathbf{Y}_k; \mathbf{X}_0 | \mathbf{X}_k)$, $k \in \{1, 2\}$. With the help of the pilots, user k first MMSE-estimates the equivalent channel $\mathbf{G}_k \Sigma_k^{\frac{1}{2}} \Phi_k$ by $\bar{\Omega}_k \bar{\mathbf{R}}_k$ and then decodes the data in $\mathbf{S}_0$ and $\mathbf{S}_k$. To analyze the conditional mutual information $I(\mathbf{Y}_k; \mathbf{X}_k | \mathbf{X}_0)$, and $I(\mathbf{Y}_k; \mathbf{X}_0 | \mathbf{X}_k)$, consider that the receiver removes partly the interference caused by $\mathbf{X}_0$ and $\mathbf{X}_k$, respectively, using the knowledge of these terms and the channel estimate, before data decoding. Finally, by using repeatedly the worst-case uncorrelated noise argument in a similar manner as in the single-user case, we can show that $I(\mathbf{Y}_k; \mathbf{X}_k, \mathbf{X}_0) \geq R'_k$, $I(\mathbf{Y}_k; \mathbf{X}_k | \mathbf{X}_0) \geq R_k^p$, and $I(\mathbf{Y}_k; \mathbf{X}_0 | \mathbf{X}_k) \geq R_{0k}$, $k \in \{1, 2\}$. Substituting these bounds into Lemma 1, we obtain (6). The full proof can be found in [14, Appendix B]. $\square$

B. Product Superposition

With product superposition, we transmit

$$\mathbf{X} = [\mathbf{V}_0 \ \mathbf{V}_2] \mathbf{X}_1 \mathbf{X}_2,$$

with

$$\begin{aligned}
 \mathbf{X}_1 &= \begin{bmatrix} \sqrt{\rho_{1\tau}} \mathbf{I}_{s_0} & \sqrt{\frac{\rho_{1\delta}}{s_0}} \mathbf{S}_1 \\ \mathbf{0} & \sqrt{\rho_{1a}} \mathbf{I}_{s_2} \end{bmatrix}, \\
 \mathbf{X}_2 &= \begin{bmatrix} \sqrt{P_{2\tau}} \mathbf{I}_{s_2 + s_0} & \sqrt{\frac{P_{2\delta}}{s_2 + s_0}} \mathbf{S}_2 \end{bmatrix},
 \end{aligned} \quad (8)$$

where the data matrices $\mathbf{S}_1 \in \mathbb{C}^{s_0 \times s_2}$ and $\mathbf{S}_2 \in \mathbb{C}^{(s_2 + s_0) \times (T - s_2 - s_0)}$ have i.i.d. $\mathcal{CN}(0, 1)$ entries. In this way, the signal $\mathbf{X}_1$ for user 1 is embedded in the pilot of user 2, thus user 1 communicates in the first $s_2 + s_0$ channel uses only. On the other hand, $\mathbf{X}_1$ constitutes the equivalent channel of user 2. This input distribution leads to the following achievable rate.

Theorem 3. With product superposition, the two-user BC can achieve any rate pair (R_1, R_2) of the form

$$\begin{aligned}
 R_1 &= \frac{s_2}{T} \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_1} \right. \right. \\
 & \quad \left. \left. + \frac{\rho_{1\delta} P_{2\tau} \hat{\Omega}_{10} \hat{\Omega}_{10}^H}{\rho_{1\delta} P_{2\tau} \text{tr}((\bar{\mathbf{R}}_{k0}^{-1} + \rho_{1\tau} P_{2\tau} \mathbf{I}_{s_0})^{-1}) + s_0} \right) \right],
 \end{aligned}$$

where the rows of $\hat{\Omega}_{10} \in \mathbb{C}^{N_1 \times s_0}$ are i.i.d. according to $\mathcal{CN}(\mathbf{0}, \rho_{1\tau} P_{2\tau} \bar{\mathbf{R}}_{10} (\rho_{1\tau} P_{2\tau} \bar{\mathbf{R}}_{k0} + \mathbf{I}_{s_0})^{-1} \bar{\mathbf{R}}_{k0})$; and

$$\begin{aligned}
 R_2 &= \left(1 - \frac{s_2 + s_0}{T}\right) \mathbb{E} \left[\log \det \left(\mathbf{I}_{N_2} \right. \right. \\
 & \quad \left. \left. + \frac{P_{2\delta} \hat{\mathbf{G}}_{2e} \hat{\mathbf{G}}_{2e}^H}{P_{2\delta} \text{tr}((\bar{\mathbf{R}}_{2e}^{-1} + P_{2\tau} \mathbf{I}_{s_2 + s_0})^{-1}) + s_2 + s_0} \right) \right]
 \end{aligned}$$

where $\hat{\mathbf{G}}_{2e} \in \mathbb{C}^{N_2 \times (s_2 + s_0)}$ has distribution imposed by

$$\begin{aligned}
 \hat{\mathbf{G}}_{2e} &= \sqrt{P_{2\tau}} (\sqrt{P_{2\tau}} \mathbf{G}_2 \Sigma_2^{\frac{1}{2}} \Phi_2 \mathbf{X}_1 + \mathbf{Z}_{[1:s_2 + s_0]}) \\
 &\quad \times (P_{2\tau} \bar{\mathbf{R}}_{2e} + \mathbf{I}_{s_2 + s_0})^{-1} \bar{\mathbf{R}}_{2e}. \quad (9)
 \end{aligned}$$

The integers s_0 and s_2 satisfy $s_0 \leq r_0$ and $s_2 \leq r_2 - r_0$. The power constraint is

$$(s_0 \rho_{1\tau} + s_2 (\rho_{1\delta} + \rho_{1a})) \left(P_{2\tau} + \frac{T - s_2 - s_0}{s_2 + s_0} P_{2\delta} \right) \leq PT. \quad (10)$$

In (9), $\mathbf{X}_1$ is given by (8) and $\bar{\mathbf{R}}_{2e}$ is defined as

$$\bar{\mathbf{R}}_{2e} := \begin{bmatrix} \rho_{1\tau} \bar{\mathbf{R}}_{20} & \sqrt{\rho_{1\tau} \rho_{1a}} \Phi_{20}^H \Sigma_2 \Phi_{22} \\ \sqrt{\rho_{1\tau} \rho_{1a}} \Phi_{22}^H \Sigma_2 \Phi_{20} & \frac{\rho_{1\delta}}{s_0} \text{tr}(\bar{\mathbf{R}}_{20}) \mathbf{I}_{s_2} + \rho_{1a} \bar{\mathbf{R}}_{22} \end{bmatrix}.$$

By swapping the users' roles, a similar rate pair is achievable. The convex hull of the origin and these rate pairs over all feasible values of s_0, s_1, s_2 and all possible power allocations (10) is an achievable rate region for the two-user BC.

Proof. In the first $s_2 + s_0$ channel users, user 1 MMSE-estimates the equivalent channel $\Omega_{10} := \mathbf{G}_1 \Sigma_1^{\frac{1}{2}} \Phi_{10}$ by $\hat{\Omega}_{10}$ and then decode the data in $\mathbf{S}_1$. User 2 first MMSE-estimates the equivalent channel $\mathbf{G}_{2e} := \mathbf{G}_2 \Sigma_2^{\frac{1}{2}} \Phi_2 \mathbf{X}_1$ by $\hat{\mathbf{G}}_{2e}$ and then decode the data in $\mathbf{S}_2$. The achievable rates $I(\mathbf{Y}_k; \mathbf{X}_k)$ are lower bounded by R_k , $k \in \{1, 2\}$, using the worst-case uncorrelated noise as before. For the full proof, see [14, Appendix C]. $\square$

C. Hybrid Superposition

We consider a composite scheme that involves both rate splitting and product superposition. The transmitted signal is

$$\mathbf{X} = [\mathbf{V}_0 \ \mathbf{V}_1] \mathbf{X}'_2 \mathbf{X}_1 + \mathbf{V}_2 \mathbf{X}_2$$

with

$$\begin{aligned} \mathbf{X}_1 &= \left[\sqrt{P_{1\tau}} \mathbf{I}_{s_1+s_0} \sqrt{\frac{P_{1\delta}}{s_1+s_0}} \mathbf{S}_1 \right], \\ \mathbf{X}_2 &= \begin{bmatrix} \mathbf{0}_{s_2 \times s_0} & \sqrt{P_{2\tau}} \mathbf{I}_{s_2} & \sqrt{\frac{P_{2\delta}}{s_2}} \mathbf{S}_2 \end{bmatrix}, \\ \mathbf{X}'_2 &= \begin{bmatrix} \sqrt{\rho_{2\tau}} \mathbf{I}_{s_0} & \begin{bmatrix} \mathbf{0}_{s_0 \times s_2} & \sqrt{\frac{\rho_{2\delta}}{s_0}} \mathbf{S}'_2 \end{bmatrix} \\ \mathbf{0} & \sqrt{\rho_{2a}} \mathbf{I}_{s_1} \end{bmatrix}, \end{aligned}$$

where the data matrices $\mathbf{S}_1 \in \mathbb{C}^{(s_1+s_0) \times (T-s_1-s_0)}$, $\mathbf{S}_2 \in \mathbb{C}^{s_2 \times (T-s_2-s_0)}$, and $\mathbf{S}'_2 \in \mathbb{C}^{s_0 \times (s_1-s_2)}$ have i.i.d. $\mathcal{CN}(0,1)$ entries. In this way, user 1 communicates in its full eigenspace. A part ($\mathbf{X}'_2$) of user 2's signal is embedded in the pilot of user1, and the remaining part $\mathbf{X}_2$ is sent in the private eigenspace of user 2. With hybrid superposition, we obtain an achievable rate region reported in [14, Theorem 7]. The expressions are omitted here due to the space limit.

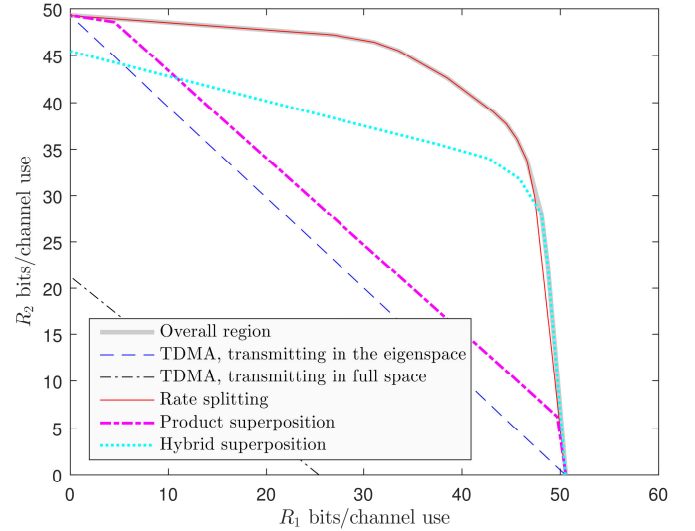
Remark 2. Hybrid superposition utilizes both rate splitting and product superposition but is not a generalization, in the sense that the results of pure rate splitting and product superposition cannot be recovered from the hybrid scheme.

V. NUMERICAL RESULTS

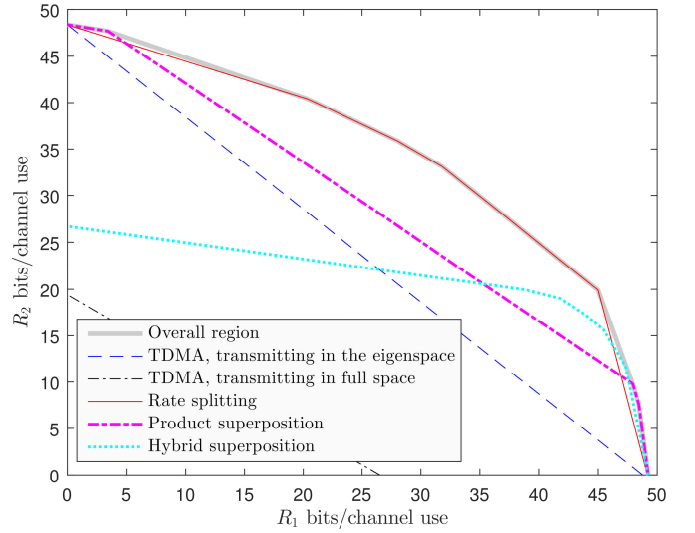
We assume Rayleigh fading, i.e., $\mathbf{G}_k$ has independent $\mathcal{CN}(0,1)$ entries, and generate the correlation matrices $\mathbf{R}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{U}_k^H$, $k \in \{1, 2\}$, as follows:

- The eigenvalues in $\mathbf{\Sigma}_k$ are drawn from the joint distribution of the nonzero eigenvalues of a Wishart matrix $\mathbf{B}\mathbf{B}^H$ where $\mathbf{B}$ is a $M \times r_k$ matrix with independent $\mathcal{CN}(0,1)$ entries, and normalized so that $\text{tr}(\mathbf{\Sigma}_k) = M$. This is suggested by the maximum-entropy channel modeling approach [17].
- The eigenvectors $\mathbf{U}_k$ are generated as $\mathbf{U}_k = \hat{\mathbf{U}}_k \mathbf{\Xi}_k$, where $\hat{\mathbf{U}}_1$ and $\hat{\mathbf{U}}_2$ are drawn respectively by selecting randomly r_1 and r_2 columns of a random unitary matrix $\mathbf{U}$ which is uniformly distributed in the space of $M \times M$ unitary matrix, and the rotation matrix $\mathbf{\Xi}_k$ is drawn uniformly from the space of $r_k \times r_k$ unitary matrix.

In Fig. 1, we plot the rate regions for the BC achieved with the considered schemes in a setting of $T = 24$, $M = 16$, $N_1 = N_2 = 12$, and $(r_1, r_2, r_0) \in \{(9, 7, 2), (10, 6, 4)\}$, at SNR $P = 30$ dB. We observe that the rate region of TDMA while transmitting in the channel eigenspace $\text{Span}(\mathbf{U}_k)$ is much larger than that while transmitting in full space $\mathbb{C}^M$. This is because the former scheme spends less time (r_k channel uses) for channel estimation than the latter scheme (M channel uses), while both schemes essentially communicate through the same effective channel. The rate region can be largely improved with rate splitting and superposition. Rate splitting achieves a large region with respect to other schemes, especially when the ranks of the two eigenspaces are relatively similar (as $r_1 = 9, r_2 = 7$



(a) $r_1 = 9, r_2 = 7, r_0 = 2$



(b) $r_1 = 10, r_2 = 6, r_0 = 4$

Fig. 1. The rate regions of various schemes for the spatially correlated BC with $T = 24$, $N_1 = N_2 = 12$, $M = 16$, and at SNR $P = 30$ dB.

in Fig. 1(a)). The improvement by product superposition is more pronounced when the rank difference between two eigenspaces is more significant (as $r_1 = 10, r_2 = 6$ in Fig. 1(b)) since the gain achieved by product superposition come from the nonoverlapping part of the eigenspaces.

VI. CONCLUSION

We study the two-user noncoherent MIMO BC with spatial correlation eigenspaces that partially overlap. We derive some achievable rate regions with pilot-based signaling together with rate splitting, product superposition, and a composition thereof. These schemes exploit effectively both the common and mutually exclusive parts of the correlation eigenspaces, thus provide methods to make use of transmit correlation diversity using only statistical channel knowledge. A next step is to find outer bounds on the rate region.

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