Allocating Distributed Generators for Resilient Distribution System Under Uncertain Probability Distribution of Natural Disasters

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***Abstract*—In this paper, a new distributionally robust defender-attacker-defender (DAD) model is proposed for the planning of line hardening and allocating distributed generators (DG), to hedge against the risk of disruptions brought by natural disasters or extreme weather conditions. In our approach, we consider the case that the true probability distribution of extreme weather is ambiguous, and minimize the load shedding with the worst-case scenario of weather distribution. Unlike conventional robust models, our approach takes advantage of moment information of the weather distribution that is learned from historical data. We reformulate the proposed model as a tractable two-stage robust optimization and employ a column-and-constraint generation algorithm to solve it. Case study on the IEEE 32-node distribution test system illustrates the effectiveness of the proposed method.**

***Index Terms*—Distribution network, distributionally robust optimization, natural disaster, power system resilience.**

I. Introduction

Despite expanded and continued efforts on improving the survivability of power grids under natural disasters, U.S. energy infrastructure is still increasingly vulnerable to severe natural disasters such as earthquake [1], flood [2], hurricane [4], heat-wave [3] and other extreme weather conditions, which impact on economy and society significantly. According to a study by Congressional Research Service [15], U.S. incurs 20–55 billion dollars annually because of weather-related power outages. Only during years of 2003-2012, about 679 weather-related power outages occurred in the U.S. and each affected more than 50,000 customers [16]. Projections indicate that with changes in climate such as increasing temperatures and sea-level rise, the intensity and frequency of extreme weather events are more inevitable, which pose a serious threat to power infrastructure assets [14]. This calls for an accelerating need for improving power grids reliability and resiliency against disasters. Recent U.S. government initiatives and programs like Grid modernization Multi-Year Program Plan also reveal the importance of power system resilience under severe weather conditions [13].

Currently, the North American Electricity Reliability Corporation enforces $N-1$ security criteria in power systems [20], which requests that the power system needs to continue meeting the electricity loads under any single contingency. The logic behind this policy is that the probability of occurring single contingency is much more than a set of simultaneous multiple contingencies. However, since contingencies are usually dependent events, it appears that the likelihood of multiple contingencies is not negligible. Hence, various mathematical models have been proposed in the literature to provide optimal protection strategy under more general reliability criteria like $N-k$ security criteria.

From contingency analysis perspective, resilient power network planning models in the literature can be divided in two categories: deterministic worst-case (robust) models and uncertain interdiction models. In robust models, the interdiction planner launches a disruption that imposes the most significant loss to the defender. The trilevel defender-attacker-defender (DAD) games have been recently gained more popularity in this area [5], [6]. This sequential game involves three stages. In the first stage, the system operator, as a defender, prepares the grid by hardening the system components before power interruptions occur. In the second stage, the natural disaster, as an attacker, determines to attack a set of components to inflict maximum possible damage. Finally, the system operator responds to the disruptions by adjusting power flow accordingly to mitigate the adverse consequences. Reference [8] studies a two-stage robust optimization model for the planning of a resilient distribution network. It extends the traditional $N-k$ security criteria to capturing the spatial and temporal dynamics of the hurricane. Reference [7] adopts a DAD model for allocating defensive resources on power grids. It customizes Column-and-Constraint Generation algorithm to efficiently solve the problem. Even though robust models provide reliable decisions by considering the worst-case contingency scenario, this approach overestimates the chance of severe disruption scenario and therefore too conservative. In uncertain interdiction models, a predefined probability is assigned to each contingency scenario to handle stochastic dynamics of catastrophic events. Reference [10] develops two models for optimizing design and service restoration in power transmission networks under stochastic line disruptions. It resorts to a finite set of scenarios with equal probability of realization to capture uncertainty in the problem. Reference [11] models a mixed integer programming for the stochastic network interdiction problem, where transmission line disruption follows Bernoulli distribution. The sample average approximation method is used to reformulate the problem and

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a decomposition-based approach is proposed to solve the problem. While the stochastic nature of contingency occurrences is captured by a finite set of representative scenarios or a pre-assumed distribution in uncertain interdiction models, any inaccurate estimation of underlying probability distribution would trigger to a suboptimal and biased protection planning. Therefore, both of the robust and stochastic approaches have disadvantages.

Another track of works in the literature has been devoted to assess security of the power systems. The main objective of models in this area relies on the proper determination of both probability and undesirable consequence of contingency occurrences. However, most of these models do not provide any mechanism for identifying and defending those contingencies. Several statistical techniques have been studied to compute the probability of contingencies such as Markov chains, Poisson regression, and Bayesian models [12]. Reference [19] identities high risk $N-k$ contingencies based on probability analysis to protect system failures. It utilizes event tree technique to describe contingencies and rare event approximation method to evaluate associated probabilities. Reference [18] develops a Monte-Carlo nonsequential simulation framework to assess impacts of extreme weather events on failure rates of distribution lines. It formulates the probability of each contingency by a Poisson distribution function. Because of the complexity involved in forecasting weather-related outages, these kinds of models, however, have limited application in practice. For instance, simulation-based frameworks require a significant amount of computation resources and their convergency is not guaranteed in general.

In this paper, we propose a novel distributionally robust optimization approach to enhance the resilience of power grid planning. Our proposed model overcomes the disadvantages of both stochastic programming, in which the distribution of contingencies is predetermined and has no robustness respect to errors, and robust models which completely ignore the distribution of contingencies. Specifically, we consider the probability of contingency occurring, but instead of fixing the probability of contingency to any particular distribution, we construct an ambiguity set of contingency probability distributions. Then, we minimize the worst-case expected load shedding with respect to all probability distributions in the ambiguity set. More particularly, we concentrate on distribution power networks, which are totally sensitive to local weather conditions and can directly affect customer load points since they provide the connection between customers and the bulk system. Then, we will develop an efficient algorithm to detect the worst-case probability distribution of contingencies. Also, our algorithm can effectively provide valuable information to system operator to make an accurate judgment about security level of the system. The rest of this paper is organized as follows. In section II we describe mathematical formulation. Solution methodology is provided in section III. Computational experiments are conducted in section IV. Finally, we conclude the paper in section V.

II. Mathematical formulation

Using robust optimization models is quite prevalent in the literature to obtain the worst-case load shedding in the power grid. There are two essential criticisms against robust models. First, they treat all the components in the system with the same importance. However, when facing natural disasters, not all components are equally exposed to failure. For example, principal components may be of great attention by system operators and they likely arrange regular maintenance to ensure that these components are operating correctly. Thus, these assets are less susceptible to extreme weather conditions. But some other components like overhead lines are highly affected by environmental factors. Second criticism is that robust models are often over conservative, since they overlook the probability information of component failures. To overcome limitations of robust models, we propose a distributionally robust framework on resilient smart distribution system with the consideration of the probabilistic characteristics of natural disasters. Our model involves three levels. In the upper level, the defender intends to improve the resilience of the distribution power system by allocating defensive resources to power lines and locating distributed generators (DG) in proper buses. In the middle level, a natural disaster, as an attacker, randomly disrupts the system with its ambiguous distribution $\mathbb{P}$ aiming at damaging the system to the largest extent, i.e., maximum load shedding. Finally, in the lower level, the system operator seeks to minimize the load shedding by adjusting power flow throughout the grid. The mathematical formulation of the model is as follows.

$$\min_{\mathbf{g}\in\mathcal{G}} \max_{\mathbb{P}\in\mathfrak{D}_{(\mathbf{g})}} E_{\mathbb{P}}[Q(\mathbf{g},\mathbf{z})] \tag{1}$$

$s.t.$

$$Q(\mathbf{g},\mathbf{z}) = \min_{\mathbf{u}\in\mathcal{H}(\mathbf{g},\mathbf{z})} \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} s_{nt} \tag{2}$$

$$\mathcal{G} = \Bigg\{ \sum_{(m,n)\in\mathcal{E}} y_{mn} \le B_1, \sum_{n\in\mathcal{N}} w_n \le B_2, \quad y_{mn}, w_n \in \{0,1\}, \forall (m,n)\in\mathcal{E}, \forall n\in\mathcal{N} \Bigg\} \tag{3}$$

$$\mathfrak{D}_{(\mathbf{g})} = \Bigg\{ \int d\mathbb{P} = 1, \quad 0 \le \int (1-z_{mn}) d\mathbb{P} \le \mu_{mn}^{max}, \quad \sum_{(m,n)\in\mathcal{E}} (1-z_{mn}) \le B_3, \quad z_{mn}\in\{0,1\}, \quad \forall (m,n)\in\mathcal{E} \Bigg\} \tag{4}$$

$$\mathcal{H}(\mathbf{g},\mathbf{z}) = \Bigg\{$$

$$0 \le p_{mn,t} \le K_{mn}(z_{mn}+y_{mn}), \quad \forall (m,n)\in\mathcal{E}, \quad \forall t\in\mathcal{T}, \tag{5}$$

$$0 \le q_{mn,t} \le R_{mn}(z_{mn}+y_{mn}), \quad \forall (m,n)\in\mathcal{E}, \quad \forall t\in\mathcal{T}, \tag{6}$$

$$0 \le x_{nt}^{p} \le w_n C_n^p, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \tag{7}$$

$$0 \le x_{nt}^{q} \le D_{nt}^{q}, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \tag{8}$$

$$0 \le s_{nt} \le D^p_{nt}, \quad \forall n \in \mathcal{N}, \quad \forall t \in \mathcal{T}, \tag{9}$$

$$\nu^{min} \le \nu_{nt} \le \nu^{max}, \quad \forall n \in \mathcal{N}, \quad \forall t \in \mathcal{T}, \tag{10}$$

$$\sum_{k|(n,k)\in\mathcal{E}} p_{nk,t} = p_{mn,t} - D^p_{nt} + x^p_{nt} + s_{nt},$$
$$\forall n \in N, \forall (m,n) \in \mathcal{E}, \quad \forall t \in \mathcal{T}, \tag{11}$$

$$\sum_{k|(n,k)\in\mathcal{E}} q_{nk,t} = q_{mn,t} - D^q_{nt} + x^q_{nt},$$
$$\forall n \in N, \forall (m,n) \in \mathcal{E}, \quad \forall t \in \mathcal{T}, \tag{12}$$

$$\left.\begin{aligned} &\nu_{nt} = \nu_{mt} - (\tau_{mn} p_{mn,t} + \eta_{mn} q_{mn,t})/V_0, \\ &\forall m, n \in \mathcal{N} | (m,n) \in \mathcal{E}, \quad \forall t \in \mathcal{T}, \end{aligned}\right\} \tag{13}$$

where, $\mathcal{T}$, $\mathcal{N}$, and $\mathcal{E}$ represent sets of time periods, nodes, and power lines, respectively. B_1 and B_2 denote available budget for hardening power lines and DG units and B_3 is the budget for line attackings. τ_{mn} and η_{mn} indicate resistance and reactance of the power line (m,n), K_{mn} and R_{mn} are upper limits of active and reactive power flow in line (m,n), D^p_{nt} and D^q_{nt} are active and reactive power demand at node n in time t, C^p_n is the capacity of DG unit at node n, ν^{max} and ν^{min} are upper and lower bounds of voltage respectively, and μ^{max}_{mn} represents upper bound of failure rate in line (m,n). Decision variables are binary variable for line hardening (y_{mn} equals 1 if line (m,n) is hardened), binary variable for DG placement (w_n equals to 1 if DG is located at node n), interdiction binary variable (z_{mn} equals to 0 if line (m,n) is attacked), active power flow across line (m,n) in period t ($p_{mn,t}$), reactive power flow across line (m,n) in period t ($q_{mn,t}$), active power generation of DG unit at node n in period t (x^p_{nt}), reactive power generation at node n in period t (x^q_{nt}), voltage magnitude at node n in period t (ν_{nt}), and load shedding at node n in period t (s_{nt}). To simplify the notations, we use vector $\mathbf{g}$ to represent the first-stage decision variables including y_{mn} and w_n and vector $\mathbf{u}$ to represent the second-stage decision variables including $p_{mn,t}$, $q_{mn,t}$, x^p_{nt}, x^q_{nt}, ν_{nt}, and s_{nt}.

In the above formulation, set (3) indicates budget constraints for possible hardening lines and DG units. Set (4) characterizes the uncertainty set of the probability distribution $\mathbb{P}$ of contingency occurrence. More particularly, it determines an upper bound for the failure rate of each line and also restricts the number of attacks to be at most B_3. Constraints (5) and (6) enforce an upper bound for active and reactive power flow over a line, respectively. Whenever a line is attacked and not protected, a contingency occurs, i.e., the probability for non-contingency for that line is zero. Constraints (7) and (8) restrain active and reactive power generation at each node, respectively. Constraints (9) restrict the amount of load shedding to be less than the real demand. Constraints (10) confine the voltage level at each node to be within $[\nu^{min}, \nu^{max}]$. Constraints (11)-(13) represent linearized DistFlow equations that have been employed in the literature [21]. In the following, we obtain a tractable reformulation for the problem (1)-(13).

Proposition 1. *For any fixed first stage decision* $\mathbf{g}$*, the worst-case distribution problem* $\max_{\mathbb{P}\in\mathfrak{D}_{(\mathbf{g})}} E_{\mathbb{P}}[Q(\mathbf{g},\mathbf{z})]$ *is equivalent to:*

$$\max_{\mathbb{P}\in\mathfrak{D}_{(\mathbf{g})}} E_{\mathbb{P}}[Q(\mathbf{g},\mathbf{z})] = \min_{\boldsymbol{\beta}\ge 0} \max_{\mathbf{z}} Q(\mathbf{g},\mathbf{z}) + \sum_{(m,n)\in\mathcal{E}} (\mu^{max}_{mn} + z_{mn} - 1)\beta_{mn} \tag{14}$$

Proof.

$$\max_{\mathbb{P}\in\mathfrak{D}_{(\mathbf{g})}} E_{\mathbb{P}}[Q(\mathbf{g},\mathbf{z})] = \max_{\mathbb{P}} \int Q(\mathbf{g},\mathbf{z}) d_{\mathbb{P}} \tag{15}$$

$$s.t. \int d\mathbb{P} = 1 \tag{16}$$

$$\int (1 - z_{mn}) d\mathbb{P} \le \mu^{max}_{mn} \tag{17}$$

Problem (15)-(17) is always feasible. Moreover, because all the variables in $\mathcal{H}(\mathbf{g},\mathbf{z})$ are bounded, $Q(\mathbf{g},\mathbf{z})$ is bounded and consequently problem (15)-(17) is always bounded. Therefore, we can apply strong duality. The dual problem can be written as follows:

$$\min_{\boldsymbol{\beta}\ge 0,\gamma} \gamma + \sum_{(m,n)\in\mathcal{E}} \mu^{max}_{mn}\beta_{mn} \tag{18}$$

$$s.t.$$

$$\gamma + \sum_{(m,n)\in\mathcal{E}} (1 - z_{mn})\beta_{mn} \ge Q(\mathbf{g},\mathbf{z}), \ \forall \mathbf{z}, \tag{19}$$

where γ and $\boldsymbol{\beta}$ are dual variables corresponding to constraints (16) and (17), respectively. In the above formulation, we can observe that the optimal solution γ should satisfy:

$$\gamma = \max_{z} \{Q(\mathbf{g},\mathbf{z}) - \sum_{(m,n)\in\mathcal{E}} (1 - z_{mn})\beta_{mn}\}. \tag{20}$$

Substituting γ from (20) to objective function (18) will complete the proof. □

By Proposition 1 and combining two minimization operations, problem (1)-(13) is equivalent to the following program:

$$\min_{\boldsymbol{\beta}\ge 0, \mathbf{g}\in\mathcal{G}} \max_{\mathbf{z}} Q(\mathbf{g},\mathbf{z}) + \sum_{(m,n)\in\mathcal{E}} (\mu^{max}_{mn} + z_{mn} - 1)\beta_{mn} \tag{21}$$

III. Solution Methodology

In order to solve Problem (21), we employ a Column-and-Constraint Generation (CCG) framework that is introduced in [22]. It is illustrated in reference [22] that CCG optimality cuts are more powerful than commonly used Benders decomposition cuts and thus converge more quickly. To use CCG framework, we need to reformulate the original problem to master problem and subproblem. The master problem is defined as below:

$$\min_{\boldsymbol{\beta}\ge 0, \mathbf{g}\in\mathcal{G}} \lambda + \sum_{(m,n)\in\mathcal{E}} (\mu^{max}_{mn} - 1)\beta_{mn} \tag{22}$$

$$s.t.$$

$$\lambda \ge \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{N}} s^j_{nt} + \sum_{(m,n)\in\mathcal{E}} \beta_{mn} z_{mn}, \forall j = 1, ..., r \tag{23}$$

$$\mathbf{u}^j \in \mathcal{H}(\mathbf{g}, \mathbf{z}^j), \forall j = 1, ..., r. \tag{24}$$

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Since the master problem is a relaxation of the original problem, it generates a lower bound for problem (1)-(13). The next step is to obtain the worst-case scenario and add the corresponding cuts to the master problem. With a given $\mathbf{g}$ and $\boldsymbol{\beta}$ from the master problem, to detect the worst-case scenario, we can solve the following subproblem:

$$\max_{\mathbf{z}} \min_{\mathbf{u}\in\mathcal{H}(\mathbf{g},\mathbf{z})} \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} s_{nt} + \sum_{(m,n)\in\mathcal{E}} \beta_{mn} z_{mn}. \quad (25)$$

Since for the given $\mathbf{g}$ and $\boldsymbol{\beta}$ the subproblem produces a feasible solution for the trilevel model, it yields to an upper bound for the original problem. Furthermore, the lower level program in the subproblem is always feasible and bounded. Hence, there is no duality gap and we can dualize the inner minimization problem to transform the subproblem into the following maximization problem:

$$\max_{\boldsymbol{\pi},\mathbf{z}} \sum_{(m,n)\in\mathcal{E}} \beta_{mn} z_{mn} + \sum_{t\in\mathcal{T}}\sum_{(m,n)\in\mathcal{E}} K_{mn}\pi^1_{mn,t}(z_{mn} + y_{mn}) + \sum_{t\in\mathcal{T}}\sum_{(m,n)\in\mathcal{E}} R_{mn}\pi^2_{mn,t}(z_{mn}+y_{mn}) - \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} D^p_{nt}\pi^3_{nt} - \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} D^q_{nt}\pi^4_{nt} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} w_n C^p_n \pi^5_{nt} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} D^q_{nt}\pi^6 + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} D^p_{nt}\pi^7_{nt} + \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} v^{max}\pi^8_{nt} - \sum_{t\in\mathcal{T}}\sum_{n\in\mathcal{N}} v^{min}\pi^9_{nt} \quad (26)$$

$s.t.$

$$\pi^1_{mn,t} + \pi^3_{mt} - \pi^3_{nt} + \frac{\tau_{mn}}{V_0}\pi^{10}_{nt} \le 0, \quad \forall m,n\in\mathcal{N}|(m,n)\in\mathcal{E}, \quad \forall t\in\mathcal{T}, \quad (27)$$

$$\pi^2_{mn,t} + \pi^4_{mt} - \pi^4_{nt} + \frac{\eta_{mn}}{V_0}\pi^{10}_{nt} \le 0, \quad \forall m,n\in\mathcal{N}|(m,n)\in\mathcal{E}, \quad \forall t\in\mathcal{T}, \quad (28)$$

$$-\pi^3_{nt} + \pi^5_{nt} \le 0, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \quad (29)$$

$$-\pi^4_{nt} + \pi^6_{nt} \le 0, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \quad (30)$$

$$\pi^8_{nt} - \pi^9_{nt} - \pi^{10}_{nt} \le 0, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \quad (31)$$

$$-\pi^3_{nt} + \pi^7_{nt} \le 1, \quad \forall n\in\mathcal{N}, \quad \forall t\in\mathcal{T}, \quad (32)$$

$\pi^3_{nt}, \pi^4_{nt}, \pi^{10}_{nt}$ are free and other variables are nonpositive.

The corresponding CCG Algorithm is described as follows:

Step 0: Initialization. Set $LB = -\infty$, $UB = \infty$, the set of attacks $F = \emptyset$, an optimality gap ϵ, and iteration index $r = 1$.

Step 1: Solve the master problem (22)-(24), obtain optimal value $objMP$ and optimal hardening decision $\hat{\mathbf{g}}^r$ and $\hat{\boldsymbol{\beta}}^r$, and update $LB = objMP$.

Step 2: Solve the subproblem (26)-(32), obtain optimal value $objSP$ and optimal attack scenario $\hat{\mathbf{z}}^r$, and update $UB = \min\{UB, objSP + \sum_{(m,n)\in\mathcal{E}}(\mu^{max}_{mn} - 1)\beta_{mn}\}$. Then, add $\hat{\mathbf{z}}^r$ to F, create dispatch variables $\mathbf{u}^r$ and corresponding constraints $\mathbf{u}^r \in \mathcal{H}(\mathbf{g}, \hat{\mathbf{z}}^r)$ and add them to the master problem.

Step 3: If $Gap = (UB - LB)/LB \le \epsilon$, terminate; otherwise, $r = r + 1$ and go to step 1.

In order to obtain the worst-case distribution, we first run the CCG algorithm to get the optimal attack set F. Then, we solve the problem (18)-(19). The optimal dual solutions will provide the probability of each attack scenario.

IV. Case study

We evaluate the effectiveness of our proposed model by performing experiments on the IEEE 33-node distribution system. Fig. 1 illustrates the impact of hardening budget

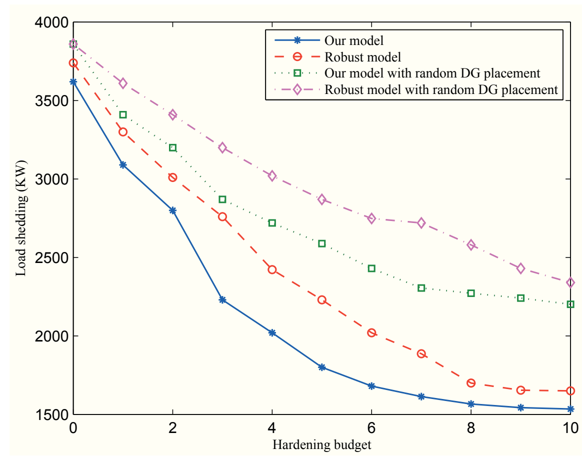

Fig. 1. Load shedding for different hardening budget

on load shedding. As it can be seen the magnitude of load shedding decreases when the hardening budget increases. For instance, by expanding hardening budget from 1 to 10, load shedding decreases more than 57% and 55% in our model and robust model, respectively. Nevertheless, as the budget rises, the influence of line hardening becomes smaller since we only observe nearly 2% and 3% reduction in our model and robust model when the budget is enlarged from 8 to 10. This indicates that it is not necessary to do line hardening as much as possible as it may just cause more costs without improving the reliability of the system significantly. We also compare the performance of our model against robust model in Fig. 1. For the same hardening budget, the load shedding in our model is significantly less than the robust model in all cases. This obviously shows that our model outperforms the robust model. Furthermore, in order to investigate the impact of locating DG units in the grid, we randomly assign DG units to nodes and then solve optimization models. Results in Fig. 1 clearly verify the key role of the optimal DG placement in load shedding reduction. Comparing four different curves in Fig. 1 reveals that considering both line hardening and optimal DG assignment is essential to enhance distribution system resilience.

Another important parameter that can affect the results is the upper bound of failure rate for each line. To examine the impact of this parameter we run three different random cases. In each case, we first pick a line arbitrary and set its

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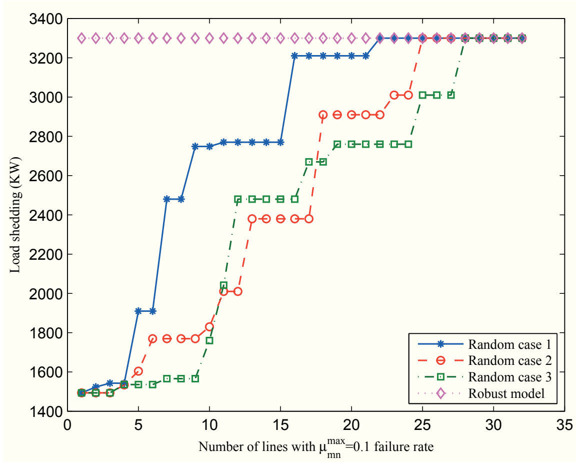

Fig. 2. Impact of increasing number of components with potential failure probability on load shedding

TABLE I
WORST-CASE DISTRIBUTION CONFIGURATION

attacked lines	probability
15-16-17	0.09
2-18-31	0.1
3-20-23	0.07
4-18-24	0.05
7-26-30	0.6
6-23-25	0.09

upper bound of failure rate equals to $\mu_{mn}^{max} = 0.1$ and other upper bounds to be zero. We iteratively add one more random line (m, n) with $\mu_{mn}^{max} = 0.1$ at each step until all lines are selected. Fig. 2 shows the amount of load shedding at each step of this procedure. As the number of selected lines increases, the results converge to the robust model. This finding is aligned with our intuition that the robust model treat with all lines in the same manner.

Finally, we drive the worst-case distribution of attack scenario in Table I. It can be noticed that there are some scenarios with low probability of occurrence. Our approach can successfully recognize these scenarios with their associated probabilities and thus it can serve as a useful tool for the system operator to improve the security and reliability of the distribution power system under natural disasters.

V. CONCLUSION

This paper proposes a novel model to enhance the distribution power system resilience under natural disasters. Results reveal that optimal planning of hardening lines and DG unit placements can effectively reduce the load shedding in the grid. Comparing with commonly used robust models, our approach is promising as it yields to less load shedding. As the number of components with potential high failure probability increases, our results converge to the robust model. Furthermore, our model provides a useful tool for system operators to identify the probability of each disaster scenario in the worst-case distribution and plan to protect the grid accordingly in a cost-effective manner while ensuring security and reliability of the system. Therefore, our proposed approach is a practical approach.

REFERENCES

[1] Bingham, J., 2011. Japan earthquake: country on the brink of massive blackouts. [Online]. Available: http://www.telegraph.co.uk/news/worldnews/asia/japan/8388279/Japan-earthquake-country-on-brink-of-massive-blackouts.html.

[2] Fox News, 2013. Fox News. [Online]. Available: http://www.foxnews.com/world/2013/02/26/heavy-rains-flood-homes-cut-power-and-water-supplies-to-villages-in-macedonia/.

[3] Merco Press, 2012. Heat causes massive blackout and the collapse of Buenos Aires city. [Online]. Available: http://en.mercopress.com/2012/11/08/heat-causes-massive-blackout-and-the-collapse-of-buenos-aires-city.

[4] Reuters UK, 2012. Superstoprm Sandy cuts power to 8.1 million homes. [Online]. Available: http://uk.reuters.com/article/2012/10/30/us-storm-sandy-powercuts-idUSBRE89T10G20121030.

[5] G. Brown, M. Carlyle, J. Salmeron, and K. Wood, *Defending critical infrastructure*, Interfaces, vol. 36, no. 6, pp. 530–544, 2006.

[6] V. M. Bier, E. R. Gratz, N. J. Haphuriwat, W. Magua, and K. R. Wierzbicki, *Methodology for identifying near-optimal interdiction strategies for a power transmission system*, Reliab. Eng. Syst. Saf., vol. 92, no. 9, pp. 1155–1161, 2007.

[7] W. Yuan, L. Zhao, and B. Zeng, *Optimal power grid protection through a defender–attacker–defender*, Reliab. Eng. Syst. Saf., vol. 121, pp. 83–89, 2014.

[8] W. Yuan, J. Wang, F. Qiu, C. Chen, C. Kang, and B. Zeng , *Robust optimization–based resilient distribution network planning against natural disasters*, IEEE Trans. Smart Grid, vol. 7, no. 6, pp. 2817–2826, 2016.

[9] E. Yamangil, R. Bent, and S. Backhaus, *Designing resilient electrical distribution grids*, Tech. Rep., 2014. [Online]. Available: https://arxiv.org/abs/1409.4477.

[10] S. Shen, *Optimizing designs and operations of a single network or multiple interdependent infrastructures under stochastic arc disruption*, Computers & Operations Research, vol. 40, no. 11, pp. 2677–2688, 2013.

[11] U. Janjarassuk and J. T. Linderoth, *Reformulation and sampling to solve a stochastic network interdiction problem*, Networks, vol. 52, no. 3, pp. 120–132, 2008.

[12] Z. Yujia, A. Pahwa, and Y. Shie-Shien, *Modeling weather-related failures of overhead distribution lines*, IEEE Trans. Power Syst. , vol. 21, no. 4, pp. 1683–1690, 2006.

[13] U.S. Department of Energy, Grid Modernization Multi-Year Program Plan, November 2015.

[14] U.S. Department of Energy, Climate Change and the U.S. Energy Sector: Regional Vulnerabilities and Resilience Solutions, October 2015.

[15] R. J. Campbell, *Weather-Related power outages and electric system resiliency*. Washington, DC, USA: Congressional Research Service, 2012.

[16] Executive office of the President, *Economic benefits of increasing electric grid resilience to weather outages–August 2013*. [Online]. Available: http://ieeexplore.ieee.org/xpl/articleDetails.jsp?arnumber=6978705.

[17] Q. Chen, C. Jiang, W. Qiu, and J. D. McCalley, *Probability models for estimating the probabilities of cascading outages in high-voltage transmission network*, IEEE Trans. Power Syst. , vol. 21, no. 3, pp. 1423–1431, 2006.

[18] R. Rocchetta, Y. F. Li, and E. Zio, *Risk assessment and risk-cost optimization of distributed power generation systems considering extreme weather conditions*, Reliab. Eng. Syst. Saf., vol. 136, pp. 47–61, 2015.

[19] Q. Chen and J. D. McCalley, *Identifying high risk N–k contingencies for online security assessment*, IEEE Trans. Power Syst. , vol. 20, no. 2, pp. 823–834, 2005.

[20] NERC, N., Standard TPL-001-01, system performance under normal conditions, 2011. [Online]. Available: http://www.nerc.com/docs/standards/sar/Project_2006-02_TPL-00-1.pdf.

[21] M. E. Baran and F. F. Wu, *Network reconfiguration in distribution systems for loss reduction and load balancing*, IEEE Trans. Power Del., vol. 4, no. 2, pp. 1401–1407, 1989.

[22] B. Zeng and L. Zhao, *Solving two-stage robust optimization problems using a column-and-constraint generation method*, Oper. Res. Lett., vol. 41, no. 5, pp. 457–46, 2013.

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