



A relativistic quantum approach to neutrino and antineutrino emission via the direct Urca process in strongly magnetized neutron-star matter

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ABSTRACT

We study neutrino and antineutrino emission from the direct Urca process in neutron-star matter in the presence of strong magnetic fields. We calculate the neutrino emissivity of the direct Urca process, whereby a neutron converts to a proton, an electron and an antineutrino, or a proton-electron pair converts to a neutron-neutrino pair. We solve exact wave functions for protons and electrons in the states described with Landau levels. We find that the direct Urca process can satisfy the kinematic constraints even in density regions where this process could not normally occur in the absence of a magnetic field.

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Neutron stars (NSs) are cooled by neutrino and/or anti-electron neutrino emission. At very low temperatures the neutrino mean-free-path is very long. In this case neutrinos are easily emitted from the core regions of NSs. Since neutrino emission rates depend on circumstances inside NSs, the study of NS cooling through neutrino emission gives important information for constraining internal NS structure [1].

There are several kinds of the cooling processes [2]. Previous works have studied decay processes for the neutrino or the anti-neutrino emission such as the direct Urca (DU) process ($n \rightarrow p + e^- + \bar{\nu}_e$, $p + e^- \rightarrow n + \nu_e$), the modified Urca process (MU) ($n + N \rightarrow p + e^- + N' + \bar{\nu}_e$, $p + e^- + N \rightarrow n + N' + \nu_e$) [3], the neutrino-pair emission process ($N_1 + N_2 \rightarrow N'_1 + N'_2 + \nu + \bar{\nu}$) [4,5] and so on. Furthermore, there are many approaches which combine the above processes with other mechanisms. For example, one of them is to consider the superfluidity [6] produced by neutron and proton pairing correlations. This is known to lead to a strong reduction of the neutrino emissivity and affect the neutrino-antineutrino emission by the Cooper pair breaking and formation mechanism [7].

The DU process is one of the most feasible candidates to explain the rapid cooling of NSs [8,9]. One usually examines the rapid cooling associated with the DU process to compare temperatures and ages of isolated NSs. Recently another method has been realized by observing NSs in binary systems. Indeed, by analyzing the x-ray emission in MXB 1659-29, evidence of the direct Urca process has been found [10].

Because of energy-momentum conservation and Fermi statistics for NS matter composed of protons, neutrons and electrons, the DU process occurs in a density region, where the proton density ρ_p is larger than 1/8 of the neutron density ρ_n ($\rho_p \geq \rho_n/8$); this condition cannot be achieved in low density regions. However, if the symmetry energy is proportional to the density, and its slope is sufficiently

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large, the condition for the DU process can be satisfied even at rather low density. Many other approaches have been proposed to satisfy this condition by considering other phases such as pion [11] or kaon condensation [12,13] and matter including hyperon degrees of freedom [6,14]. However, the latter case turns out to affect the masses of NSs, which is closely related to the hyperon puzzle in NSs [15].

On the other hand, magnetars, which are associated with a very strong magnetic field [16,17], have properties different from normal NSs. The strength of their magnetic field is about $10^{14} - 10^{15}$ G in the surface region, and can reach 10^{17} G inside the star. Soft gamma repeaters (SGR) and anomalous X-ray pulsars (AXPs) correspond to magnetars [18]. Magnetars emit energetic photons. Furthermore, the surface temperature of magnetars is $T \approx 0.28 - 0.72$ keV, which is larger than those of normal NSs $T \approx 0.01 - 0.15$ keV at a similar age [19].

There are a number of theoretical works that show the influence of the magnetic field on the equation of state (EOS) of NS matter [20–27] and how the structure of NSs is altered by including magnetic fields [28,29]. We note however, that since our purpose here is to explore effects of the DU in NS matter and not the structure of NSs, we do not re-examine NS structure in this study. For the reader interested in magnetic effects on NS structure a list of works can be found in Ref. [28] (see also the “Lorene” website: <https://lorene.obspm.fr/data/magnetNS.html>).

A strong magnetic field can supply energy and momentum into the cooling process and changes the restriction caused by energy-momentum conservation by introducing transitions between Landau levels [30–32]. Indeed, the magnetic field influences the beta-decay process in NSs [24] and axion emission from magnetars [33]. Leinson [34] studied the effect of magnetic fields perturbatively and showed that a magnetic field increases the antineutrino emissivity. Thus, the associated strong magnetic fields may play a significant role in the cooling of magnetars.

In a previous paper [30] we have studied $\nu\bar{\nu}$ -pair emissions from the transition between two Landau levels in strongly magnetized NS matter. In that work, when the strength of the magnetic field is about $B \sim 10^{15}$ G, emission energies of this process are larger than those of the MU process with zero magnetic field, $B = 0$. All of this suggests that the magnetic field may increase the emission of particles because the strong magnetic field can supply momentum into the emission processes.

Leinson and Pérez [35] calculated the neutrino emissivity from the DU process in strong magnetic fields, and showed that the emissivity becomes larger as the magnetic field increases. In that calculation the magnetic field was so strong that all charged particles populated the lowest Landau level. This means that one has not yet estimated realistic magnetic effects in NSs with realistic magnetic field strengths.

In the present paper, therefore, we apply our quantum theoretical approach to the DU process in strong magnetic fields and calculate it through the transition between Landau levels for electrons and protons. As noted above, only this quantum approach can exactly describe the momentum transfer from the magnetic field.

The low energy Lagrangian of the weak interaction between baryons and leptons is written as

$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \sum_{l_1, l_2} \bar{\psi}_{l_1} \gamma_\mu (1 - \gamma_5) \psi_{l_2} \sum_{\alpha_1 \alpha_2} \bar{\psi}_{\alpha_1} \gamma_\mu (c_V - c_A \gamma_5) \psi_{\alpha_2}, \quad (1)$$

where $\psi_{l_{1,2}}$ is the field of the lepton $l_{1,2}$ which denotes the electron or muon or neutrino, and ψ_α is the field of the baryon α which denotes the proton or neutron, G_F is the Fermi coupling constant, while c_V and c_A are the vector and axial-vector coupling constants which include the contribution from the Cabbibo angle [36].

In the single particle model, the l -neutrino emissivity of the DU process is written as

$$\epsilon_{DU} = 2 \sum_{\alpha_n, \alpha_p, \alpha_l, \alpha_\nu} \mathcal{F}_n(E_n, E_p, E_l) e_\nu (2\pi) \delta(E_n - E_p - E_l - e_\nu) \times \left| G_F \int d^3\mathbf{r} \left[\bar{\psi}_{\alpha_l}^{(l)}(\mathbf{r}) \gamma_\mu (1 - \gamma_5) \psi_{\alpha_\nu}^{(\nu)}(\mathbf{r}) \right] \left[\bar{\psi}_{\alpha_p}^{(p)}(\mathbf{r}) \gamma_\mu (c_V - c_A \gamma_5) \psi_{\alpha_n}^{(n)}(\mathbf{r}) \right] \right|^2, \quad (2)$$

with

$$\mathcal{F}_n(E_n, E_p, E_l) = n_n(E_n) [1 - n_p(E_p)] [1 - n_e(E_l)], \quad (3)$$

where the subscript l indicates the electron or muon, while ν indicates the ν_e or ν_μ , ψ_a is the single particle wave-function, E_a is the energy of the particle $a (= n, p, e, \mu, \nu_e, \nu_\mu)$, and α_a denotes the quantum numbers such as momentum, spin, iso-spin and Landau-number. The quantity $n_a(E)$ is the Fermi-distribution function for the particle a ,

$$n_a(E) = \frac{1}{1 + e^{(E - \mu_a)/T}}, \quad (4)$$

where T is the temperature, and μ_a is the chemical potential of particle a .

We assume a uniform magnetic field along the z -direction, $\mathbf{B} = (0, 0, B)$, and take the electro-magnetic vector potential A^μ to be $A = (0, 0, xB, 0)$ at the position $\mathbf{r} \equiv (x, y, z)$. The relativistic wave function ψ is obtained from the following Dirac equation:

$$\left[\gamma_\mu \cdot (i\partial^\mu - \zeta e A^\mu - U_0 \delta_0^\mu) - M + U_s - \frac{e\kappa}{4M} \sigma_{\mu\nu} (\partial^\mu A^\nu - \partial^\nu A^\mu) \right] \psi_a(x) = 0, \quad (5)$$

where κ is the anomalous magnetic moment (AMM), e is the elementary charge and $\zeta = \pm 1$ or 0 is the sign of the particle charge. U_s and U_0 are the scalar field and time component of the vector field, respectively. These are determined from relativistic mean-field (RMF) theory [37], although the AMM and the mean-fields are taken to be zero for charged leptons.

The single particle energy for a charged particle ($\zeta = \pm 1$) is then written as

$$E(n, p_z, s) = E^* + U_0 = \sqrt{p_z^2 + \left(\sqrt{2eBn + M^{*2}} + se\kappa B/2M \right)^2} + U_0, \quad (6)$$

with $M^* = M - U_s$, where n is the Landau number, p_z is the z -component of momentum, and $s = \pm 1$ is the spin. Furthermore, the wave-function overlap at points $\mathbf{r}_1$ and $\mathbf{r}_2$ for charged particles such as the proton and electron are written as

$$\psi_{n,s,p_z}^{(a)}(\mathbf{r}_1) \bar{\psi}_{n,s,p_z}^{(a)}(\mathbf{r}_2) = \frac{e^{i(p_y y + p_z z)}}{\sqrt{R_y R_z}} \hat{F}(x_1 - p_y/eB) \frac{\rho_M^{(a)}}{4E} \hat{F}(x_2 - p_y/eB), \quad (7)$$

with

$$\begin{aligned} \rho_M(n, s, P_z) &= \left[E^* \gamma^0 - \zeta \sqrt{2eBn} \gamma^2 - p_z \gamma^3 + M^* + \frac{e\kappa B}{2M} \Sigma_z \right] \\ &\quad \times \left[1 + \frac{s}{\sqrt{2eBn + M^{*2}}} \left(\frac{e\kappa B}{2M} + p_z \gamma_5 \gamma^0 - E^* \gamma_5 \gamma^3 \right) \right], \\ \hat{F} &= \text{diag}(f_n, f_{n-1}, f_n, f_{n-1}) = f_n \frac{1 + \Sigma_z}{2} + f_{n-1} \frac{1 - \Sigma_z}{2} \quad (\text{proton}), \end{aligned} \quad (8)$$

$$\hat{F} = \text{diag}(f_{n-1}, f_n, f_{n-1}, f_n) = f_{n-1} \frac{1 + \Sigma_z}{2} + f_n \frac{1 - \Sigma_z}{2} \quad (\text{electron}), \quad (9)$$

where $\Sigma_z = \text{diag}(1, -1, 1, -1)$, and R_a ($a = x, y, x$) is the size of the system along the a -direction.

For neutral particles ($\zeta = 0$) such as the neutron, the wave function is written as an eigenstate of momentum $\mathbf{p} \equiv (\mathbf{p}_T, p_z)$. So, the single particle energy is given by

$$E(\mathbf{p}, s) = E^* + U_0 = \sqrt{p_z^2 + \left(\sqrt{\mathbf{p}_T^2 + M^{*2}} + s e\kappa B/2M \right)^2} + U_0, \quad (10)$$

and the wave-function overlap becomes

$$\begin{aligned} \psi_{\mathbf{p},s}(\mathbf{r}_1) \bar{\psi}_{\mathbf{p},s}(\mathbf{r}_2) &= \frac{e^{i\mathbf{p} \cdot (\mathbf{r}_1 - \mathbf{r}_2)}}{\sqrt{R_x R_y R_z}} \left[E^* \gamma_0 - \mathbf{p} \cdot \boldsymbol{\gamma} + M^* + \frac{e\kappa B}{2M} \Sigma_z \right] \\ &\quad \times \left\{ 1 + \frac{s}{\sqrt{\mathbf{p}_T^2 + M^{*2}}} \left[\frac{e\kappa B}{2M} + \gamma_5 (p_z \gamma^0 - E^* \gamma^3) \right] \right\}. \end{aligned} \quad (11)$$

Substituting the above wave-functions into Eq. (2), we obtain

$$\begin{aligned} \epsilon_{DU} &= \frac{2(2\pi)^3 G_F^2 (eB)^2}{R_x^2 R_y^4 R_z^4} \sum_{n_p, n_e} \sum_{s_n, s_p, s_e} \left\{ \prod_{a=n, \nu} \left[R_x R_y R_z \int \frac{d^3 p_a}{(2\pi)^3} \right] \right\} \left\{ \prod_{b=p, e} \left[R_z \int \frac{dp_{bz}}{2\pi} R_y \int_{-eBR_x/2}^{eBR_x/2} \frac{dp_{by}}{2\pi} \right] \right\} \\ &\quad \times e_\nu \mathcal{F}_n(E_n, E_p, E_l) \delta(E_n - E_p - E_l - e_\nu) \delta(p_{ny} - p_{py} - p_{ly} - p_{vy}) \delta(p_{nz} - p_{pz} - p_{lz} - p_{vz}) \\ &\quad \times \frac{1}{16^3 E_n E_p E_l e_\nu} \sum_{i_1, j_1, i_2, j_2 (\pm 1)} \mathcal{M}(j_1, i_1) \mathcal{M}^*(j_2, i_2) L_{\mu\nu} N^{\mu\nu} \\ &= \frac{G_F^2 (eB)^3}{4(2\pi)^5} \sum_{n_p, n_l} \sum_{s_n, s_p, s_l} \int \frac{dp_{nT} p_{nT} dp_{pz} dp_{lz} de_\nu e_\nu^3 d\Omega_\nu}{E_n^* E_p^* E_l e_\nu} \frac{d\Omega_\nu}{4\pi} \mathcal{F}_n(E_n, E_p, E_l) \\ &\quad \times \delta(E_n - E_p - E_l - e_\nu) \sum_{i_1, j_1, i_2, j_2} \mathcal{M}(j_1, i_1) \mathcal{M}^*(j_2, i_2) L_{\mu\nu}(i_1, i_2) N^{\mu\nu}(j_1, j_2), \end{aligned} \quad (12)$$

with

$$\mathcal{M}(j_p, j_l) = \int dx f_{n_e + (j_l - 1)/2} \left(x + \frac{p_{nT}}{\sqrt{2eB}} \right) f_{n_p + (j_p - 1)/2} \left(x - \frac{p_{nT}}{\sqrt{2eB}} \right), \quad (13)$$

$$L_{\mu\nu}(j_1, j_2) = \frac{1}{16} \text{Tr} \left\{ \rho_M^{(e)}(n_f, s_f, k_e) (1 + j_1 \Sigma_z) \gamma_\mu (1 - \gamma_5) \not{k}_\nu \gamma_\nu (1 - \gamma_5) (1 + j_2 \Sigma_z) \right\}, \quad (14)$$

$$N_{\mu\nu}(j_1, j_2) = \frac{1}{16} \text{Tr} \left\{ (1 + j_2 \Sigma_z) \rho_M^{(p)}(n_i, s_i, p_{iz}) (1 + j_1 \Sigma_z) \gamma_\mu (c_V - c_A \gamma_5) \rho_M^{(n)} \gamma_\nu (c_V - c_A \gamma_5) \right\}, \quad (15)$$

where $R_x = R_y = R_z$ is assumed, and f_n is the n -th harmonic oscillator wave function. In addition, the neutron momentum is taken to be $p_n = (E_n, 0, p_{nT}, p_{nz})$, where without loss of generality the neutron transverse momentum is assumed to be directed along the y -axis.

In cool NSs the temperature is less than 1 keV, and the emitted neutrino energy is of the order of the temperature $e_\nu \sim T$, so that we can take the lowest order term of the temperature by using the following approximation

$$e_\nu^3 \mathcal{F}_n(E_n, E_p, E_l) \approx \mathcal{I}_{DU} \delta(E_n - \mu_n) \delta(E_p - \mu_p) \delta(e_e - \mu_e), \quad (16)$$

with

$$\mathcal{I}_{DU} = \int dE_1 dE_2 dE_3 (E_1 - E_2 - E_3)^3 \mathcal{F}_n(E_1, E_2, E_3) \approx \frac{457}{5040} \pi^6 T^6. \quad (17)$$

Therefore, the neutrino emissivity in the low temperature limit can be written as

$$\epsilon_{DU} = \frac{457\pi}{10080} G_F^2 T^6 \sum_{n_l, n_p} \frac{p_{nT}}{p_{pz} p_{lz} \sqrt{p_{nT}^2 + M_n^{*2}}} \times \frac{1}{2^6} \sum_s \sum_{i,j} \int \frac{d\Omega_v}{4\pi} \mathcal{M}(j_1, i_1) \mathcal{M}^*(j_2, i_2) \frac{L_{\mu\nu} N^{\mu\nu}}{e_\nu}. \quad (18)$$

In addition, we assume chemical equilibrium for the system. Hence, the chemical potentials satisfy the following relations:

$$\mu_n = \mu_n^* + U_0(n) = \mu_p + \mu_l = \mu_p^* + \mu_l + U_0(p). \quad (19)$$

At the low temperature limit, the chemical potentials are taken to be the Fermi energies at zero temperature, and the effective chemical potentials for baryons μ^* in the presence of a magnetic field are written as

$$\mu_p^*(n_p, k_{pz}, s) = \sqrt{\left(\sqrt{2eBn_p + M_p^{*2}} + se\kappa_p/2M_p\right)^2 + p_{pz}^2}, \quad (20)$$

$$\mu_n^*(k_{nT}, k_{nz}, s) = \sqrt{\left(\sqrt{k_{nT}^2 + M_n^{*2}} + se\kappa_n/2M_n\right)^2 + k_{nz}^2}, \quad (21)$$

where $s = \pm 1$, n_p is the Landau number of the proton, and k_{pz} is the z-component of the proton Fermi momentum which depends on n_p . In addition, k_{nT} and k_{nz} are the transverse component and z-component of the neutron Fermi momentum, whose values are not the same. That is the momentum-distribution breaks the spherical symmetry through the AMM.

For comparison we also show the neutrino emissivity from the DU process when $B = 0$ as

$$\epsilon_{DU} = \frac{457\pi}{10080} G_F^2 T^6 \left\{ \frac{(c_V - c_A)^2}{2} \mu_p^* (M_n^{*2} - M_p^{*2} + m_l^2 - 2\mu_p^* \Delta U_0 - \Delta U_0^2) \right. \\ \left. + \frac{(c_V + c_A)^2}{2} \mu_n^* (M_n^{*2} - M_p^{*2} - m_l^2 - 2\mu_n^* \Delta U_0 + \Delta U_0^2) - (c_V^2 - c_A^2) \mu_l M_p^* M_n^* \right\}, \quad (22)$$

where $\Delta U_0 = U_0(p) - U_0(n)$; this expression is equivalent to that of Ref. [38]. This emissivity is non-zero $\epsilon_{DU} > 0$ when $p_F(n) \leq p_F(p) + p_F(l)$ with $p_F(a)$, the Fermi momentum of the particle a . This momentum matching condition determines the density region for the DU process when $B = 0$. In contrast, for $B \neq 0$, the magnetic field carries momentum and breaks the momentum matching condition, and the DU process can occur in any density region.

In this work we discuss the relation between the DU emissivity and the density dependence of the symmetry energy as well as the effects of the magnetic field. For the calculation of nuclear matter, we use the following Lagrangian density in the relativistic mean-field approach:

$$\mathcal{L}_{RMF} = \bar{\psi} (i\gamma \cdot \partial - M + g_s \sigma) \psi + \tilde{U}[\sigma] + \frac{g_V^2}{2m_V^2} (\bar{\psi} \gamma \psi)^2 - \frac{C_s^{IV}}{2M^2} (\bar{\psi} \tau \psi)^2 - \frac{C_v^{IV}}{2M^2} (\bar{\psi} \gamma \tau \psi)^2, \quad (23)$$

where $\tilde{U}[\sigma]$ is the self-energy term of the sigma meson which includes the non-linear effect for the scalar mean-field, and the other terms are written as the zero-range interaction between two nucleons. In addition, C_s^{IV} and C_v^{IV} are the coupling constants for the iso-vector interaction in the Lorentz scalar and vector channels, respectively [39]. Then, the scalar-field U_s and the time component of the vector-field are given by

$$U_s = g_s \sigma + \frac{C_s^{IV}}{2M^2} (\rho_s(p) - \rho_s(n)) \tau_z, \quad (24)$$

$$U_0 = \frac{g_V^2}{m_V^2} (\rho_p + \rho_n) + \frac{C_v^{IV}}{2M^2} (\rho_p - \rho_n) \tau_z. \quad (25)$$

In the above formalism we omit the magnetic field energy density $B^2/(8\pi)$ in the EOS. This term contributes to the pressure and affects the composition of NS matter significantly when $B \gtrsim 10^{18}$ G [27]. In the present calculation we adopt a smaller strength of the magnetic field, and this term does not contribute to the neutrino emissivity.

We use the parameter-set PM1 for symmetric matter as given in Ref. [40]. In addition, we give the three parameter-sets, SF1, SF2 and SF3, for the other parameters of the iso-vector parts, C_s^{IV} and C_v^{IV} , whose detailed values are shown in Table 1. The symmetry energy is fixed to be $e_{sym} = 32$ MeV at normal nuclear density though the parameter-sets give different density dependences of the symmetry energy. The SF1 includes only the Lorentz vector channel ($C_s^{IV} = 0$), the SF2 includes only the iso-vector Lorentz scalar channel ($C_v^{IV} = 0$), and the SF3 includes the negative value of the Lorentz vector channel ($C_v^{IV} < 0 < C_s^{IV}$). In addition, we list the values of the slope parameter L for these parameter-sets.

In Fig. 1 we show the density-dependence of the symmetry energy. We see that the three parameter-sets give quite different density dependences of the symmetry energy though all of them satisfy 32 MeV for the symmetry energy at normal nuclear density. This means that the parameter-sets, SF1, SF2 and SF3, represent different EOSs.

Table 1

The parameter-sets for the iso-vector parts of the mean-fields. All parameter-sets give a symmetry energy of 32 MeV at normal nuclear density. The fourth column shows the slope parameter of the symmetry energy.

Parameter-Set	C_s^{IV}	C_v^{IV}	L (MeV)
SF1	0	20.42	92.7
SF2	23.61	0	84.1
SF3	33.02	-8.154	81.0

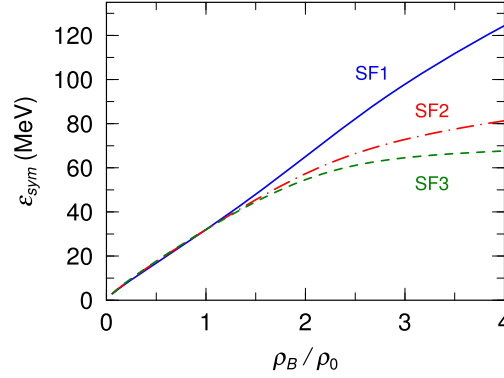


Fig. 1. The density dependence of the symmetry energy. The solid, dot-dashed and dashed lines represent the results with the parameter sets SF1, SF2 and SF3, respectively.

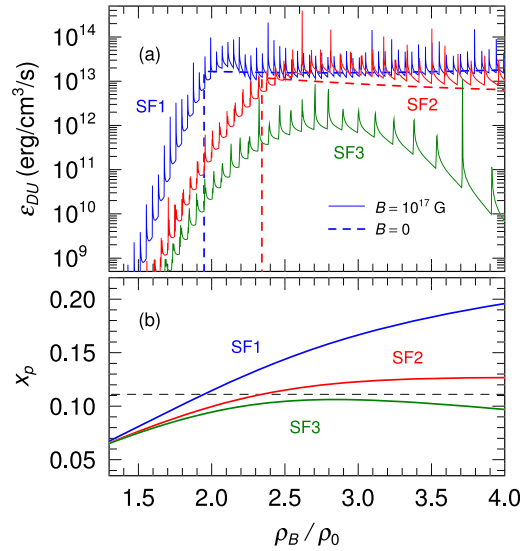


Fig. 2. (a) The density dependence of the neutrino emissivity in the DU process at $T = 0.50$ keV and (b) that of the proton fraction of NS matter at $B = 0$. The vertical lines represent the results with the SF1, SF2 and SF3, respectively. In the upper panel (a) the solid and dashed lines indicate the results when $B = 10^{17}$ G and $B = 0$, respectively. Note that the dashed line for the SF3 is not shown because $\epsilon_{DU} = 0$.

In actual calculations we substitute the mean-fields and the chemical potentials at $B = 0$ with these parameter-sets into Eq. (18) and calculate the neutrino emissivity. As mentioned above, surface temperatures of magnetars are $T \approx 0.28 - 0.72$ keV [19]. Thus, we choose $T = 0.50$ keV for the present calculations.

First, we restrict the matter to consist of only protons, neutrons and electrons. In Fig. 2 we show the density dependence of the neutrino emissivity in the DU process (a) and the density dependence of the proton fraction in NS matter, x_p (b). In Fig. 2(a), the dashed line indicates the results for $B = 0$, which appear in the density region of $x_p \geq 1/9$, where $x_p = 1/9$ is denoted by the horizontal dashed line in the lower panel (b). For future reference, we define ρ_{DU} as the critical density at which the proton fraction $x_p = 1/9$.

The calculations result in large fluctuations, which also appears in the $\nu\bar{\nu}$ -pair [30] and axion emission [31]. These fluctuations reflect the density of states in the xy -plane given by the Landau levels [32,41].

When $B = 0$, the results of the neutrino emissivity for SF1 and SF2 suddenly appear at the density ρ_{DU} . They increase very rapidly and become almost flat as the density increases. When $B = 10^{17}$ G, the density dependence of the neutrino emissivity has comb-like-shapes which are due to the fact that the neutrinos are emitted through transitions between Landau levels. The emissivity appears at a density of $\rho < \rho_{DU}$ and becomes larger gradually with increasing density. In the density region $\rho \geq \rho_{DU}$, they undergo large oscillations though their local minima almost agree with the $B = 0$ results.

For SF3, on the other hand, the proton fraction does not exceed $1/9$, and the DU does not occur when $B = 0$. When $B = 10^{17}$ G, however, neutrinos are emitted, and thus, we can confirm that the magnetic-field increases the neutrino emission. In addition, the peak

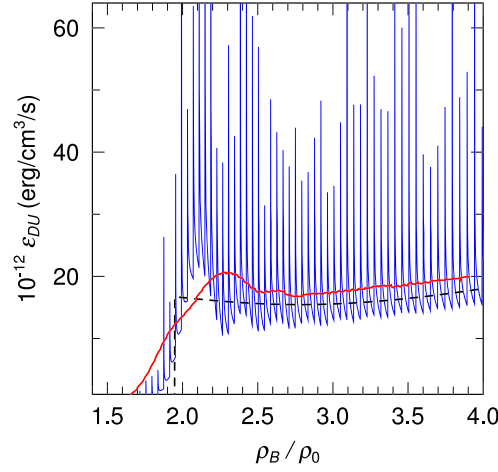


Fig. 3. The density dependence of neutrino emissivity in the DU process at $T = 500$ eV and $B = 10^{17}$ G (a) for PM1-SF1. The thin solid blue and dashed lines indicate the same quantities as those in Fig. 2a while the scale is linear. The thick solid red line represents averaging over the density dependence of the emissivities (thin solid blue line) using a width of 0.01 fm^{-3} .

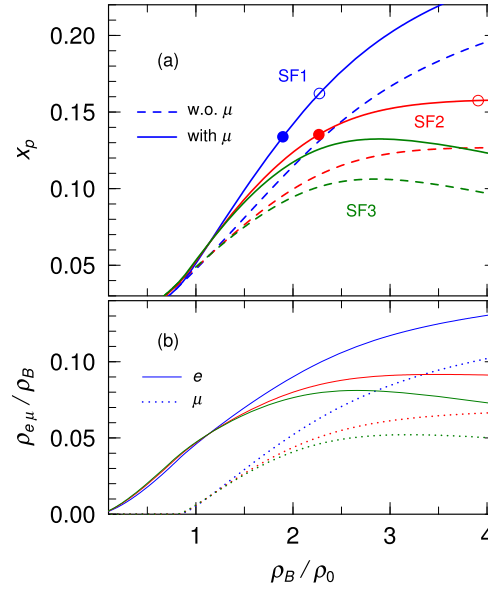


Fig. 4. Proton fractions (a) and lepton fractions (b) in NS matter with muons. In the upper panel the solid and dashed lines represent the proton fractions in NS matter with and without muons, respectively. In the lower panel the thin solid and dotted lines represent the electron and muon fractions. Lines represent the results from the SF1, SF2 and SF3 EOS as labeled.

position of the neutrino emissivity agrees with that of the proton fraction, namely the effect of the magnetic field is related to the proton fraction and can appear when the proton fraction is close to $1/9$.

In Fig. 3 we plot the results for SF1 on a linear scale. Here one can see that the spike of the neutrino emissivity is very narrow, and that its height is much larger than that with zero-magnetic field. To estimate the strength of the neutrino emission, we smooth the emissivity according to the following equation:

$$\bar{\epsilon}_{DU}(\rho_B) = \frac{1}{\Delta\rho} \int_{\rho_B - \Delta\rho/2}^{\rho_B + \Delta\rho/2} d\rho \epsilon_{DU}(\rho). \quad (26)$$

We show this result for SF1 when $\Delta\rho = 0.01 \text{ fm}^{-3}$ with a solid line. The smoothed value for the emissivity is still larger than that when $B = 0$, but the difference is not very drastic.

Here, we add the following comment. In the study of pion [42,43], axion [31] and $\nu\bar{\nu}$ -pair production [30] we showed a very large difference in the results between cases including the AMM of the protons and those not including it. In this calculation there is no significant difference between the cases with and without the AMM. Without a magnetic field the above particle production is forbidden by energy-momentum conservation. Thus, only the argument of the overlap function of Eq. (13) in the tail region contributes to the transition strength, and a small difference in input values makes a very large difference in the calculation results. In the DU process the transitions are not completely forbidden by the kinematics, and the value of the argument $p_{nT}/\sqrt{2eB}$ is not located in the tail region.

As a next step we study the DU process in NS matter including muons. In Fig. 4 we show the proton fraction in such matter in the upper panel and the electron and muon densities per baryon in the lower panel. The solid and open circles show the critical densities of

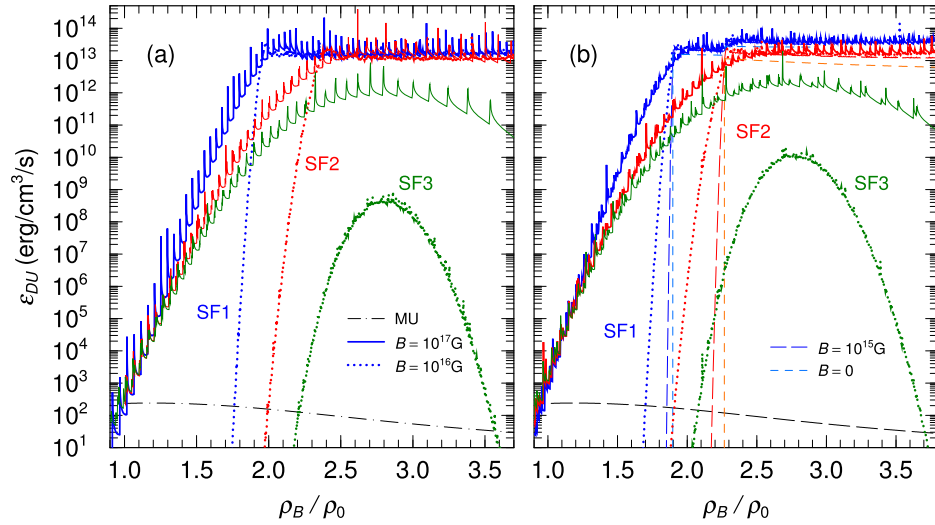


Fig. 5. Density dependence of the neutrino emissivity in the DU process at $T = 500$ eV for the PM1-SF1 EOS and for matter containing only protons, neutrons and electrons (a) and matter also containing muons (b). The solid, dotted and long-dashed lines represent the results for magnetic field strengths, $B = 10^{17}$, 10^{16} and 10^{15} G, respectively, and the dashed lines indicate the results without a magnetic field. The long-dashed line shows the neutrino emissivity from the MU process.

ρ_{DU} for the electron and muon neutrino emission, respectively. In matter containing muons the proton fraction becomes larger, and the critical density ρ_{DU} becomes a little lower than in matter without muons. Here, we note that the proton fraction in the SF3 EOS is also larger and exceeds $1/9$, but the DU does not appear.

In order to examine the dependence on the magnetic field strength, in Fig. 5 we show the results at $B = 10^{17}$, $B = 10^{16}$ and $B = 10^{15}$ G for NS matter without muons (a) and with muons (b). The results for SF3 when $B = 10^{15}$ G are too small to appear in this figure. For comparison, we also show the neutrino luminosities from the MU process [44] with the long-dashed lines. When $B \lesssim 10^{15}$ G, the emissivities do not appear for $\rho_B < \rho_{DU}$, and the magnetic field does not play an important role for the DU process.

The emissivities from the NS matter with muons are larger than those without muons. Particularly, the emissivity for SF3 at $B = 10^{16}$ G is much larger in the matter with muons than that without muons though the contribution from the muon process is negligibly small.

As the strength of the magnetic field decreases, the amplitudes of the oscillations in the density dependence become smaller for both magnetic field strengths in NS matter. In the density region, $\rho_B < \rho_{DU}$, the neutrino emissivities are largely suppressed, and the DU process dominates in a narrower density region, as shown in the results by SF3, where the emissivities in the DU process are still much larger than that of the MU process.

Here, we observe the following facts: As the magnetic field strength decreases, the emission rate becomes larger for the $\nu\bar{\nu}$ -pairs [30] despite a smaller momentum transfer from the magnetic field. The $\nu\bar{\nu}$ -pairs are produced via the transition of the proton and electron between different Landau levels; namely the initial and final particles are the same. As the magnetic field strength decreases, the energy interval between the initial and final states also becomes smaller. Hence, the emission strength becomes larger. In the DU process, however, the initial particle is a neutron which does not stay in the Landau level, and the energy interval is continuous, so that this effect does not appear.

In summary, we have used a relativistic quantum approach to study neutrino and anti-neutrino emission in the DU process from NS matter with strong magnetic fields, $B = 10^{15} - 10^{17}$ G. We use three parameter-sets for the symmetric nuclear force to illustrate the relation between the proton fraction and the neutrino emissivity. If the proton fraction satisfies the DU condition $x_p \geq 1/9$, the neutrino emissivities are not much different from the case of $B = 0$, and the magnetic field does not significantly amplify the emission, though it causes very large fluctuations in the density dependence of the neutrino emissivity. In the usual forbidden region $x_p < 1/9$, however, the magnetic field contributes to the emission and changes the kinematical condition. The effect is larger when the magnetic field strength is larger and the proton fraction is closer to $1/9$.

If the magnetic field is rather weak $B \lesssim 10^{15}$ G, the DU process does not appear when $x_p < 1/9$, so that it does not occur in the surface region of NSs. However, neutrino emission from the Landau quantization may occur in the inner core of magnetars which are thought to have such strong magnetic fields. Those neutrinos are able to escape from the matter because they have very low energy, and their mean-free-paths are very long.

Magneto-hydrodynamic proto-NS simulations [45–47] have demonstrated that the magnetic field inside a NS can obtain a toroidal configuration. It has also been demonstrated [46] that the field strength of toroidal magnetic fields can be ~ 100 times stronger than that of a poloidal magnetic field. Indeed, by analyzing the hard X-ray data and precession of pulsars, Makishima et al. [48] suggested evidence for the existence of a toroidal magnetic field whose strength is about one hundred times that of poloidal magnetic fields for cold NSs.

In the present calculation the neutrino emissivities vanish when $x_p \ll 1/9$ because we use the low temperature approximation whereby the Fermi distribution functions are approximated by a step function. As mentioned above, we exactly calculated the $\nu\bar{\nu}$ -pair production in strong magnetic fields [30] and showed that the neutrino emissivity is larger than that of the MU process in the case of zero magnetic field, even though the production rates described here are zero in the low temperature approximation. This implies that there may be much larger neutrino emissivities in the DU process when $x_p \ll 1/9$; it may be possible to explicitly show that with an exact calculation. We defer exploring this point to a future work.

Our final remark is about the EOS as there are many different versions from many nuclear models. A salient feature is the density-dependence of the symmetry energy, which is related to the proton fraction in NS matter. One needs to explore using different EOSs [49,50] which we also defer to a future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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