Transportation Research Part B 151 (2021) 22–41

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Mobility service design via joint optimization of transit networks and demand-responsive services

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ARTICLE INFO

Keywords:
Mobility-as-a-service
Public transit
Demand-responsive transportation service
Queuing network model

ABSTRACT

This paper proposes a modeling framework to design an integrated mobility service system that is composed of a local demand-responsive transportation (DRT) component and a fixed-route transit network component. The region is partitioned into disjoint local zones, and a trip is characterized into an intra- or inter-zonal one based on whether its origin and destination belong to the same zone. The transit network provides backbone line-haul service to inter-zonal trips, while the DRT system targets intra-zonal trips as well as the first-mile and last-mile legs of inter-zonal trips. The system components can be broadly defined, and this paper considers, as examples, conventional non-shared taxi or ride-sharing for the DRT service, and regular bus, bus rapid transit (BRT), or metro for the backbone network. For quantitative analysis of the DRT services, new aspatial queuing network models are developed to capture various DRT operations for serving trips with randomly distributed origins and destinations (i.e., intra-zonal trips) and those with origins or destinations concentrated at a set of fixed transit stations (i.e., first- or last-mile trip legs) at the same time. Each queuing model is integrated with the transit network design model, through proper service region partition, to derive closed-form formulas for the agency and passenger costs. Then, we formulate constrained non-linear programs that simultaneously optimize the zone partition, the fleet size and repositioning operation for the local DRT service, as well as the spacing and headway of a grid transit network. We apply the proposed models to a variety of transit technologies and application scenarios, so as to demonstrate applicability of the models, and to show promising performance of the proposed systems.

1. Introduction

The advancement of information and communication technologies and the boom of shared transportation modes, such as bike-sharing, ride-sharing and car-sharing, have facilitated the emergence of the Mobility as a Service (MaaS) concept in recent years. It advocates for seamless integration of transportation services across multiple modes, such as those traditionally categorized as transit service and demand-responsive transportation (DRT) service (e.g., taxi, ride-sharing, dial-a-ride), so as to enhance passenger experience, and in so doing further encourage a modal shift from privately-owned vehicles toward more sustainable public transportation services. The effectiveness of MaaS on reducing private vehicle mode share has been corroborated by several pilot programs in Europe (Sochor et al., 2015, 2016; Karlsson et al., 2017). However, concerns over MaaS are also rising. In some situations, the passenger-centric principle of MaaS and particularly its DRT component tend to pose as a competitor that diverts

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https://doi.org/10.1016/j.trb.2021.06.005
Received 17 October 2020; Received in revised form 23 April 2021; Accepted 15 June 2021
Available online 22 July 2021

travel demand away from public transit service (Hensher, 2017; Clewlow and Mishra, 2017; Sadowsky and Nelson, 2017; Pangbourne et al., 2018; Zhu et al., 2020). This is in the opposite direction of achieving sustainable mobility.

Better integration of conventional transit service and DRT service has been seen as a promising way to avoid such an undesirable situation, since it combines the efficiency of public transit and the convenience and flexibility of demand-responsive services. Thus, operation and design of such an integrated system have attracted considerable attention and have been continuously improved with new enabling technologies. Readers are referred to Lee and Savelsbergh (2017), Stiglic et al. (2018), Bian and Liu (2019), and Zhu et al. (2020) for recent efforts.

The design of transit networks has been extensively studied. Traditionally, the detailed network layout can be optimized with discrete network design models, such as those in Ceder and Wilson (1986), Baaj and Mahmassani (1995), and Zhao and Zeng (2008). At the system topology level, typical networks use either a grid structure (Holroyd, 1967) or a hub-and-spoke structure (Newell, 1979). Recently, a hybrid network topology has been proposed to take advantages of both grid and hub-and-spoke structures; see Daganzo (2010). This concept has been used for real-world design implementation in Barcelona (Estrada et al., 2011), and adjusted for a ring-radial road network (Badia et al., 2014), a flexible-route system for low-demand areas (Nourbakhsh and Ouyang, 2012), and a heterogeneous network structure to account for heterogeneous demand distributions (Ouyang et al., 2014; Chen et al., 2018; Petit and Ouyang, 2020). There are also efforts dedicated to exploring hierarchical systems using transit as feeders, e.g., deploying fixed-route buses to collect passengers and feed them to the backbone network (Sivakumaran et al., 2014), and overlapping local and express networks (Fan et al., 2018), where passengers may be served by the local service only, the express service only, or inter-modal services.

Studies on DRT services have also been emerging rapidly. Representative efforts include those on conventional taxi services (see Salanova et al., 2011 for a comprehensive review), ride-sharing (Agatz et al., 2012; Zha et al., 2016; Daganzo et al., 2020), and bike-sharing (Fishman, 2016; Dell'Amico et al., 2014; Lei and Ouyang, 2018; Jiang et al., 2020). Very recently, Daganzo and Ouyang (2019b) propose an overarching analytical framework to provide closed-form formulas to relate the agency fleet investment and the passengers' experience for multiple types of DRT services. It examines one type of mobility service at a time, and hence does not address the possibility of letting multiple transportation systems work together to serve a pool of travel demand. It also does not reveal the conditions under which coordination and joint service across multiple systems could be useful (e.g., in big cities).

Researchers have also investigated the feasibility of integrating DRT into a fixed-route transit network as a feeder service. Earlier efforts have explored the opportunity of using shuttle or dial-a-ride service as feeder systems in local areas. Quadrifoglio and Li (2009), and Li and Quadrifoglio (2010) build analytical models to determine the suitable conditions for using fixed-route vs. DRT systems for feeder service. Then, Li and Quadrifoglio (2011) optimize the number of feeder zones in the local residential area near a transit station. These three studies focus on the local feeder service but does not consider optimal design of the backbone network. Aldaihani et al. (2004) propose an integrated transit network design model, where local non-shared taxi service is confined to the catchment area of each transit station, such that trips crossing catchment area borders must use the transit service. Chen and Nie (2016, 2017) adapt this model to compare its performance with the that of a line-based coupled feeder service. A recent study of Wu et al. (2020) investigates the scenario of using dock-based shared bikes to provide the feeder service; the model jointly optimizes the transit network design and the bike-sharing system. In these recent efforts, DRT is used mainly as a feeder service for the transit system.

To the best of our knowledge, there are no studies in the current literature that systematically optimize the integration of local DRT services and the backbone transit network, so as to serve both long distance trips (via the multi-modal MaaS service) and local trips (via DRT service only). The distinction between "long-distance" vs. local trips should not be automatically dependent on the transit station catchment area, either. The decision on the size of "local DRT service zone" should be a critical part of the design process, because, as one could anticipate, larger "local" service areas tend to improve passenger experience by serving more passengers with point-to-point DRT service, and reduce transit agency cost per trip by pooling demand for the transit network, but it may at the same time impose a higher resource burden on the DRT system. A holistic model is needed to systematically cast insights on the interactions and inter-dependencies among intra-zonal vs. inter-zonal passengers, as well as transit vs. DRT system components.

In light of the aforementioned challenges, this paper focuses on optimizing an integrated mobility service system that simultaneously considers service region partition, transit network design, and characteristics of local DRT operations, so as to serve both intra- and inter-zonal passengers on a zone-based system. To stay focused, the transit network structure is set to be a grid, and options for the local DRT service include conventional taxi (that does not allow sharing) and ride-sharing (that pairs passengers based on spatiotemporal proximity of their origins). To quantitatively analyze the local DRT operations, we first introduce a new variant of the conventional taxi model in Daganzo (1978) and Daganzo and Ouyang (2019b). The taxi service in the new model serves not only trips with distributed origins and destinations (ODs), but also those with ODs concentrated at a set of fixed locations (i.e., transit stations). This model is further extended to describe the performance of the ride-sharing service. The characteristics of both proposed queuing models are derived in closed analytical form and integrated into the transit network design model. Then, we formulate a constrained non-linear program to minimize the system-wide sum of agency and passenger costs, and apply it to a series of hypothetical examples to cast insights into the properties of the optimal design and the performance of the proposed systems. Three counterparts, including a taxi-only system, a traditional transit system, and a fixed-route bus feeder system (Sivakumaran et al., 2014), are used as benchmarks. In addition, we integrate the DRT services with different transit technologies, and evaluate the system performance in different application scenarios. The results show that the proposed integrated systems, especially the combination of ride-sharing service and transit network, outperform three benchmarks for a wide range of scenarios. From the passenger point of view, the passenger travel pace via the proposed systems, when properly configured, could be comparable to private vehicles for

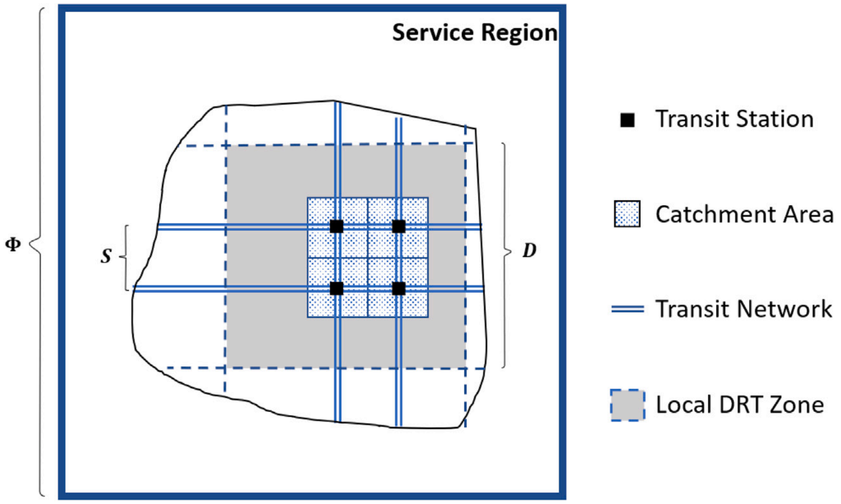

Fig. 1. Schematic illustration of mobility service layout.

certain cases, but the cost for providing such services is much lower. This is favorable because it helps to encourage passengers to shift from private modes to more sustainable shared mobility service systems. In addition, the optimal design in many of our studied scenarios involves multiple stations within each local DRT service zone, which emphasizes the importance of optimizing the size of local zones.

The rest of the paper is organized as follows. Section 2 introduces a basic system composed of a transit network and a conventional taxi system, and Section 3 presents an extension with ride-sharing service. Section 4 briefly presents, as benchmarks, the counterpart taxi-only, transit-only, and the fixed-route bus feeder systems. Section 5 presents the numerical experiments and insights. Finally, Section 6 summarizes this study and discusses future work.

2. Transit + non-shared taxi

We consider a $\Phi \times \Phi$ [km^2] square service region that generates steady travel requests at a rate of λ [pax/km^2-h], whose origins and destinations are independently and uniformly distributed according to a homogeneous Poisson process. The travel demand is exogenous and inelastic. Passengers in the service region have the same value-of-time β [$/h]. The streets in the service region form a very dense network, in which vehicles can travel in both N–S and E–W directions; thus, the distance is measured in $L-1$ metric. A table of key notation is summarized in Appendix A.

The study region is partitioned into a set of disjoint zones. The integrated mobility service system includes a DRT service running within each zone, with fully compliant drivers, and a conventional transit network (e.g., regular bus, bus rapid transit (BRT), or metro) spanning across the entire region, with spacing S [km] and headway H [h]. See Fig. 1 for illustration.

These two services, as well as the partition of the region, are simultaneously optimized in the design phase to minimize the system-wide cost, composed of passenger travel cost and agency's infrastructure investment and operation cost. The local DRT service zones are homogeneous, and each has an area of $D \times D$ [km^2]. Here, we assume Φ to be an integer multiple of D, and D an integer multiple of S, for modeling simplification and ease of implementation. In addition, we set $D \leq (\Phi/2)$, to rule out the degenerated situation $D = \Phi$, where the study region forms one local zone and there is no backbone network. Intra-zonal passengers will be served by the DRT service only, while inter-zonal passengers will be jointly served by DRT vehicles (as a feeder) and the backbone transit network, and each trip shall include three legs: (i) first-mile: travel by a DRT vehicle from the origin to a nearby backbone transit station; (ii) line-haul: travel on the transit network from the near-origin station to the near-destination station; and (iii) last-mile: travel by a DRT vehicle from a backbone transit station to the final destination.[1] Under uniform demand, the intra-zonal demand density, λ_1 [pax/km^2-h], and inter-zonal demand density, λ_2 [pax/km^2-h], are,

$$\lambda_1 = \frac{D^2 \lambda}{\Phi^2}, \quad \lambda_2 = \frac{(\Phi^2 - D^2)\lambda}{\Phi^2}. \tag{1}$$

[1] To stay focused, walking is not considered as a travel option in this study, mainly due to the limited impact of such trips (a small portion of travel demand, with a shorter trip distance) on the overall system cost under uniformly distributed demand. Appendix B derives formulas for the expected percentage of "local" trips (i.e., intra-zonal, first- and last-mile trips) that may consider walking, and briefly discusses the influence of such trips in the integrated mobility service systems.

Simple geometry shows that the expected length for intrazonal and interzonal trips are as follow:

$$L_1 = \frac{2D}{3}, \quad L_2 = \frac{\frac{2}{3}\Phi \cdot \lambda - \frac{2}{3}D \cdot \lambda_1}{\lambda_2}. \tag{2}$$

2.1. Backbone transit network

Similar to Daganzo (2010), we assume for simplicity that in the transit network: (i) passengers always board a transit vehicle at the station nearest to the origin and alight at the station nearest to the final destination; (ii) passengers always choose the most direct path between stations with the least number of transfers; (iii) if there are multiple routes resulting in the same cost, passengers will randomly break the tie; and (iv) passengers submit travel requests regardless of the transit schedule, and they do not have an appointment at the destination.

To ensure spatial coverage, in Fig. 1, we force the transit stations in the grid network to form square-shaped catchment areas.[2] With the station spacing S, it is easy to verify that the average access distance to the station is $S/2$. All routes are served by transit vehicles with a common headway of H, and the transit operations across lines are not synchronized. Let v_B [km/h] denote the cruising speed of transit vehicles, and t_s [h] the fixed dwell time at a station.

The passenger's generalized cost in the backbone transit network includes waiting time, transfer penalty, and in-vehicle travel time. The access to transit stations is provided by DRT service, and the associated cost will be evaluated in Section 2.2. The passenger expects a waiting time of $H/2$ each time when he/she needs to board a transit vehicle (e.g., at the near-origin station and at the transfer station). To simplify the analysis, we assume every inter-zonal passenger experiences exactly one transfer during the line-haul leg of the trip, which adds an extra inconvenience of Δ [h]. Given the expected passenger travel distance on transit vehicles L_2 as in Eq. (2),[3] the expected in-vehicle travel time includes cruising time (L_2/v_B), and waiting time ($t_s L_2/S$) at en-route stations.

The agency cost is composed of two parts: (i) the infrastructure investment, which is related to the two-way guideway length $L_B = 2\Phi^2/S$ [km] by coefficient c_g [\$/km-h], and to the number of stations in the network (Φ^2/S^2) by c_r [\$/station-h]; and (ii) the operation cost, including the mechanical part (vehicle depreciation, maintenance, etc.) that is related to the total vehicle-distance per hour, ($2L_B/H$) [veh-km/h] by c_v [\$/veh-km], and the payroll for crew, which is related to the total vehicle-hour of operation per hour, $(2L_B/H)(1/v_B + t_s/S)$ [veh-h/h], by c_m [\$/veh-h]. See more discussions in Daganzo and Ouyang (2019a). Thus, the average system-wide cost of the backbone transit network, Z_B [\$/pax], is as follows:

$$Z_B = \beta\left(H + \Delta + \frac{L_2}{v_B} + \frac{t_s L_2}{S}\right) + \frac{1}{\lambda_2 \Phi^2}\left[c_g L_B + c_r\frac{\Phi^2}{S^2} + c_v\frac{2L_B}{H} + c_m\frac{2L_B}{H}\left(\frac{1}{v_B} + \frac{t_s}{S}\right)\right]. \tag{3}$$

The design of transit network should satisfy capacity constraints, including (i) the number of on-board passengers should not exceed the transit vehicle capacity K [pax/veh], i.e., $(\lambda_2 \Phi S H/4) \le K$ (Sivakumaran et al., 2014; Daganzo and Ouyang, 2019a); and (ii) the headway H should have a minimum threshold $H_{\min}$ [h], e.g., to accommodate the needed dwell time at each station t_s and to avoid clogging the city streets (especially if the right-of-way is shared with other traffic).

2.2. Local taxi service

Taxis are provided within each local zone, as the local DRT service, to serve intra-zonal passengers as well as the first- and last-mile legs of inter-zonal passengers. An intra-zonal (or first-mile) passenger, with the trip starting at a random location, submits the travel request via calls or mobile phone applications. Then, the agency assigns the nearest randomly located idle taxi to this passenger, which picks up the passenger and delivers him/her to the final destination (or the nearest backbone station). A last-mile passenger alights from a transit vehicle at a backbone station, waits to be served by a DRT vehicle toward the final destination.[4] Let v_T [km/h] denote the cruising speed of taxis, and γ [\$/veh-h] denote the taxi operation cost per vehicle-hour.[5] Assume that the time for passengers to board, alight, or transfer to/from a taxi could be neglected.

A variant of the aspatial queuing network model in Daganzo and Ouyang (2019b) is proposed to quantify the relationship between the agency's cost for taxi service and the level of service experienced by passengers in an arbitrary zone; see Fig. 2. The model defines a set of possible workloads of service vehicles, denoted as $(i,j)_k$, where i is the number of passengers on board (to be dropped off), j is the number of passengers assigned to a vehicle (to be picked up), and k indicates the duty type of vehicles. The system state is described by the number of vehicles in each workload state, denoted $\{n_{ij,k}\}$.

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[2] Diamond-shaped catchment areas would minimize local feeder service distance, but its impact on the overall system-wide cost is minimum.

[3] Although the inter-zonal trip has three legs, the line-haul trip length shall be approximately the same as the distance between the origin and destination, since passengers go through the nearest transit stations.

[4] Here, we choose to let last-mile passengers wait at the station, and only use randomly located idle vehicles to pick up passengers at random locations. Alternatively, we could let some idle vehicles stay at each station as safety stocks, to (i) immediately serve the last-mile passengers once they arrive; and (ii) serve nearby intra-zonal trips and first-mile trips as suitable. This option can be suitable for DRT services with cheaper vehicle operation cost (e.g., for shared bikes).

[5] Since drivers of DRT service are fully compliant, the taxi operation cost per hour can be estimated as $\gamma = c_{T,m} + c_{T,v}v_T$, to capture the driver wage per hour $c_{T,m}$ [\$/veh-h], and the cost related to a vehicle's travel distance per hour. The latter is the product of the distance traveled per hour (i.e., the cruising speed) v_T, and the prorated cost $c_{T,v}$ [\$/veh-km] for vehicle acquisition, fuel, depreciation, etc., based on mileage. For stopped vehicles, we assume a comparable fee (e.g., for parking) applies, and hence we simply use $c_{T,v}v_T$ to capture the vehicle-related cost per hour.

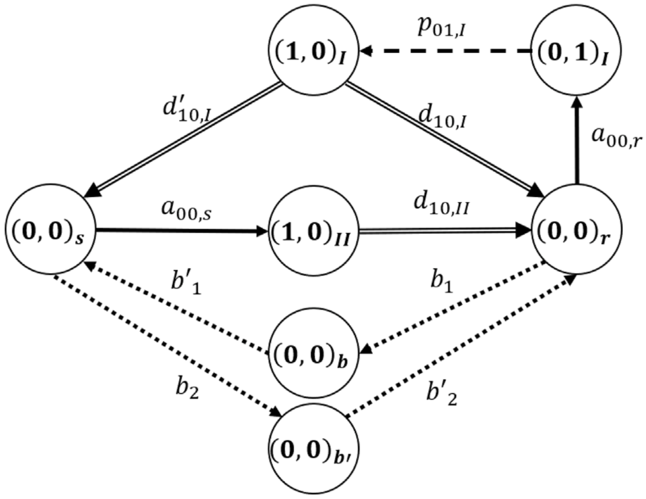


Fig. 2. State transition network for taxis inside a local zone.

An idle vehicle (with $i = j = 0$) could visit a transit station, denoted by workload $(0,0)_s$ with duty type $k = s$; or park at a random location, denoted by workload $(0,0)_r$ with duty type $k = r$. We set the expected number of vehicles idle at random locations to be the decision variable, i.e., $n_{00,r} \equiv n$, and express all other quantities as functions of it. To ensure service quality, we assume that there are sufficient idle vehicles at random locations, i.e., $n \gg 0$, and as a result, intra-zonal and first-mile passengers immediately get vehicle assignments. For dynamic balance of idle vehicles between stations and random locations, some randomly located idle vehicles, in state $(0,0)_r$, may need to be repositioned to their nearest backbone stations, with workload $(0,0)_b$ and duty type $k = b$; and some idle vehicles at stations, in state $(0,0)_s$, may be repositioned to random locations within the station's catchment area, with workload $(0,0)_{b'}$ and duty type $k = b'$.

The duty type of a non-idle vehicle (with $i + j > 0$) is characterized by the passenger trip it serves. For vehicles serving an intra-zonal trip or the first-mile leg of an inter-zonal trip, the duty type is denoted by $k = I$; for those serving the last-mile leg of an inter-zonal trip, the duty type is $k = II$.

A vehicle may change its state when it is assigned to, picks up, or drops off a passenger, or when it starts or ends its repositioning process. Each type of activity occurs at a certain rate. We define $a_{ij,k}$ [veh/h] to be the rate of assigning a new passenger to a vehicle in state $(i,j)_k$, as indicated by the solid arrows in Fig. 2; $p_{ij,k}$ [veh/h] the rate for a vehicle in state $(i,j)_k$ to pick up the assigned passenger, as indicated by the dashed arrows; and $d_{ij,k}$ [veh/h] the rate for a vehicle in state $(i,j)_k$ to drop off the on-board passenger, as indicated by the double arrows. We further define repositioning rates $\{b_1,\ b'_1,\ b_2,\ b'_2\}$ [veh/h], as indicated by the dotted arrows.

In Fig. 2, when an intrazonal or first-mile trip originates at a random location, it will be instantly assigned to the closet randomly located idle vehicle, with rate $a_{00,r}$ [veh/h]. After the assignment, the vehicle travels to pick up the assigned passenger with rate $p_{01,I}$ [veh/h], and drops off an intra-zonal passenger to his/her random destination at rate $d_{10,I}$ [veh/h], or a first-mile passenger to the nearest transit station at rate $d'_{10,I}$ [veh/h]. In the meantime, some $(0,0)_r$ vehicles will be repositioned to their nearest station at rate b_1 [veh/h]. A vehicle turns into an idle one at a backbone station, i.e., workload $(0,0)_s$, either after delivering a first-mile passenger with a transition rate of $d'_{10,I}$, or arriving at this station as a repositioned vehicle, with a transition rate of b'_1 [veh/h]. If there is at least one last-mile passenger waiting at the station, this idle vehicle will pick up a waiting passenger (which occurs at rate $a_{00,s}$ [veh/h]), and deliver the passenger to the final destination (which occurs at rate $d_{10,II}$ [veh/h]). Otherwise, if no passenger is waiting at the station, this vehicle will be repositioned to a random location within the station's catchment area, with a transition rate of b_2 [veh/h], and arrive at the designated location, with a transition rate of b'_2 [veh/h]. We do this type of dynamic repositioning to ensure steady vehicle arrivals at the station, and also because the vehicle cost is the same whether it is idling or in operation. Since vehicles do not wait at the stations, the expected number of vehicles in state $(0,0)_s$ is zero, i.e., $n_{00,s} = 0$.

Flow conservation at each workload node of the queuing network dictates that the transition rates satisfy the following system of equations,

$$a_{00,r} = p_{01,I} = \lambda D^2, \quad d_{10,I} = \lambda_1 D^2, \quad d'_{10,I} = \lambda_2 D^2, \quad a_{00,s} = d_{10,II} = \lambda_2 D^2, \quad b_1 = b'_1 = b_2 = b'_2. \tag{4}$$

Next we obtain the expected travel distance for vehicles at each workload state. Note that the expected travel distance is $S/2$ when a vehicle is delivering a first- or last-mile passenger, or is being repositioned between a random location and the nearest transit station.[6] The expected distance to deliver an intra-zonal passenger is L_1 as in Eq. (2). For vehicles en-route to pick up an assigned passenger, i.e., in state $(0,1)_I$, its expected travel distance is $0.63(D^2/n)^{1/2}$ (Daganzo and Ouyang, 2019a).

[6] Here, we ignore the rare case that there is no $(0,0)_r$ vehicle located in a station's catchment area. This assumption is acceptable when the number of stations within a local zone is small or the number of randomly located idle vehicles in a local zone is large. As such, for every station, the operator can always find an idle vehicle within its catchment area for repositioning.

Then, according to Little's formula (Little, 1961), the average number of vehicles in each state can be computed as follows,

$$\begin{aligned} n_{01,I} &= p_{01,I}\frac{0.63D}{v_T n^{1/2}} = \frac{0.63\lambda D^3}{v_T n^{1/2}}, \quad n_{10,I} = \frac{d'_{10,I}S}{2v_T} + \frac{d_{10,I}L_1}{v_T} = \frac{\lambda_2 D^2 S}{2v_T} + \frac{2\lambda_1 D^3}{3v_T}, \\ n_{10,II} &= \frac{d_{10,II}S}{2v_T} = \frac{\lambda_2 D^2 S}{2v_T}, \quad n_{00,b} = n_{00,b'} = \frac{b_1 S}{2v_T}. \end{aligned} \tag{5}$$

Finally, we connect the repositioning rates and last-mile passengers' waiting time. Here, for simplicity, we make a conservative assumption that all last-mile passengers of a station during a headway arrive at the same time. For a deterministic system, we could have $b_1 = b_2 = 0$, which already satisfies the flow conservation, and the average waiting time of last-mile passengers is $H/2$. However, in real-world operation, additional waiting time is expected, due to randomness in both the "batch size" (i.e., the number of last-mile passengers arriving at the same time) and the random "arrival" of idle vehicles at the station. For model simplicity, we further assume (somewhat conservatively) that the passengers' extra waiting time due to stochasticity (beyond the deterministic wait) can be approximated by an M/M/1 queue at each station. There are (D^2/S^2) stations within each local zone, so the "customer" arrival rate of this M/M/1 queue, i.e., the rate of requests for vehicles at each station, is $a_{00,s}(D^2/S^2)^{-1}$. Similarly, the service rate of this M/M/1 queue, i.e., the rate for idle vehicles to become available at each station, is $(d'_{10,I} + b'_1)(D^2/S^2)^{-1}$. Thus, the expected extra waiting time due to stochasticity, i.e., the mean sojourn time in the M/M/1 model, can be calculated from a well-known equation (Kleinrock, 1975),

$$\left[\left(d'_{10,I} + b'_1\right)\left(\frac{D^2}{S^2}\right)^{-1} - a_{00,s}\left(\frac{D^2}{S^2}\right)^{-1}\right]^{-1} = \frac{D^2}{b'_1 S^2} = \frac{D^2}{b_1 S^2}.$$

Note that the M/M/1 queuing system requires $b_1 = b_2 > 0$ to ensure stability. Otherwise, when $b_1 = b_2 = 0$, the expected randomness-related waiting time goes to infinity. As such, the last-mile passengers' total expected waiting time is $\frac{H}{2} + \frac{D^2}{b_1 S^2}$, when $b_1 \in \mathbb{R}^+$. On the other hand, it should be noted that large repositioning rates would increase the expected vehicle operating cost, as in Eq. (5).

Since the taxi drivers are fully compliant, the agency cost only depends on the fleet size $m = \sum_{i,j,k} n_{ijk} = n_{00,s} + n_{00,r} + n_{00,b} + n_{00,b'} + n_{01,I} + n_{10,I} + n_{10,II}$.[7] Thus, the agency cost of taxi service per trip is $(\gamma m)/(\lambda_1 D^2 + 2\lambda_2 D^2)$.

Per Little's formula, under steady-state operations, the average passenger travel time and waiting time after assignment in the local zone can be calculated as the ratio of the number of passengers in the system, $n_{01,I} + n_{10,I} + n_{10,II}$, to the total demand generation rate, $(\lambda_1 + 2\lambda_2)D^2$. In addition, the last-mile passengers' expected waiting time for vehicle assignment is $\left(\frac{H}{2} + \frac{D^2}{b_1 S^2}\right)$, as aforementioned. Thus, the average travel cost per trip is,

$$\frac{\beta}{(\lambda_1 + 2\lambda_2)D^2}\left[n_{10,I} + n_{01,I} + n_{10,II} + \lambda_2 D^2\left(\frac{H}{2} + \frac{D^2}{b_1 S^2}\right)\right].$$

Then, with Eq. (5), the system-wide cost per trip of local taxi service, including both the agency cost and the passenger cost, Z_L [\$/trip], is,

$$Z_L = \frac{\beta+\gamma}{\lambda_1 + 2\lambda_2}\left(\frac{0.63\lambda D}{v_T n^{1/2}} + \frac{2D\lambda_1 + 3S\lambda_2}{3v_T}\right) + \frac{\beta\lambda_2}{\lambda_1 + 2\lambda_2}\left(\frac{H}{2} + \frac{D^2}{b_1 S^2}\right) + \frac{\gamma}{\lambda_1 + 2\lambda_2}\left(\frac{n}{D^2} + \frac{b_1 S}{v_T D^2}\right). \tag{6}$$

2.3. Optimization model

We seek the optimal values of $(D,\ S,\ H,\ n,\ b_1)$ that minimize the system-wide total cost per passenger, Z [\$/pax], including the cost of backbone transit system given by Eq. (3), and that of local DRT systems given by Eq. (6). The optimization problem is as follows,

$$\min_{D,\ S,\ H,\ n,\ b_1} \quad Z = \frac{Z_L(\lambda_1 + 2\lambda_2)\Phi^2 + Z_B\lambda_2\Phi^2}{\lambda\Phi^2} \tag{7}$$

subject to (1), (2), (3) and (6),

$$D_{\min} \le D \le \Phi/2, \tag{8}$$

$$S_{\min} \le S \le D, \tag{9}$$

$$H \ge H_{\min}, \tag{10}$$

$$\lambda_2\Phi S H/4 \le K, \tag{11}$$

$$n,\ D,\ S,\ H,\ b_1 \in \mathbb{R}^+. \tag{12}$$

Constraints (8) and (9) impose minimum values of transit network spacing and DRT zone size for practical considerations; e.g., $S_{\min}$ could be related to city block size, and $D_{\min}$ could be related to city districts. Constraints (10) and (11) restrict the minimum

[7] For simplicity, the fleet size within each local zone is allowed to be any non-negative real number. The error from such a treatment is insignificant because the orders of magnitude of fleet size in practice are expected to be far larger than one.

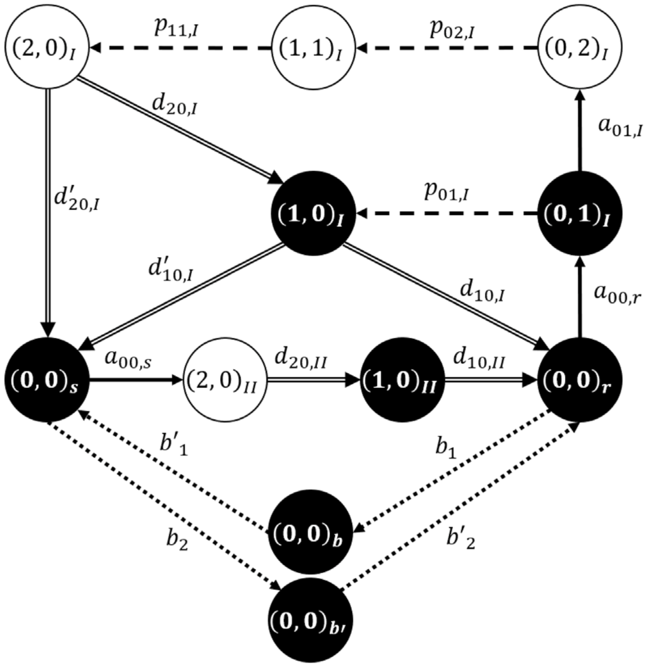


Fig. 3. State transition network for ride-sharing vehicles in a local zone.

headway and the transit vehicle capacity for the backbone network; see the discussion in Section 2.1. Constraint (12) defines the value domains of all decision variables. Here, since n is the expected number of vehicles at state $(0,0)_r$, its value is considered as a positive real number.

Due to the integer multiple relationship between Φ, D and S as per our assumption, the near-optimum value of decision variables D and S can be obtained through simple enumeration. Note that constraints (10)–(11) together form a new constraint for S, i.e.,

$$S \le 4K/(\lambda_2 \Phi H_{\min}). \tag{13}$$

In addition, it is easy to verify that the objective function is convex with respect to decision variables n, b_1 and H, respectively. Thus, the optimal values of the expected number of $(0,0)_r$ vehicles and the repositioning rate can be found via the first-order conditions,

$$n^* = \left[\frac{0.63(\beta+\gamma)\lambda D^3}{2\gamma v_T}\right]^{2/3}, \quad b_1^* = \left(\frac{\lambda_2 D^4 \beta v_T}{\gamma S^3}\right)^{1/2}. \tag{14}$$

Considering constraints (10)–(11), the optimal headway is:

$$H^* = \operatorname{mid}\left\{H_{\min},\ \left[\frac{4L_B c_v}{3\beta\lambda_2\Phi^2} + \frac{4L_B c_m}{3\beta\lambda_2\Phi^2}\left(\frac{1}{v_B}+\frac{t_s}{S}\right)\right]^{1/2},\ \frac{4K}{\lambda_2\Phi S}\right\}. \tag{15}$$

3. Transit + shared rides

In this section, we explore the opportunity of replacing local taxi service with the ride-sharing (RS) service as studied in Daganzo and Ouyang (2019b). To stay focused on the system structure, we consider that each RS vehicle carries at most two passengers at any time, and the characteristics of vehicles, including cruising speed and operation cost, remain the same as those of the taxis.

The queuing model for the ride-sharing system is more complex, as shown in Fig. 3. The workload states similar to the transit + taxi system are marked by shaded nodes, while the others represent new states for the RS service. The duty types are the same as those for the taxi service.

With RS service, an intra-zonal trip or the first-mile leg of an inter-zonal trip is instantly assigned to a suitable vehicle that is closest to the trip's origin. For better service we do not assign a new passenger to a vehicle already carrying an on-board passenger, and hence a suitable vehicle can be either idle (in the $(0,0)_r$ state), or currently en-route to pick up a passenger (in the $(0,1)_I$ state). We set the number of suitable vehicles to be the decision variable, i.e., $n_{00,r} + n_{01,I} \equiv n$. We further assume, by choice, that a vehicle always picks up the assigned passenger(s) before dropping off any on-board passenger(s); see discussion in Daganzo and Ouyang (2019b).

Given n vehicles available for a new assignment, the likelihood of assigning a $(0,0)_r$ vehicle or a $(0,1)_I$ vehicle is proportional to the relative vehicle density, and hence,

$$a_{00,r} = \lambda D^2 \frac{n_{00,r}}{n}, \quad a_{01,I} = \lambda D^2 \frac{n_{01,I}}{n}. \tag{16}$$

The pickup transition happens at rate $p_{ij,I}$, when a vehicle with type $k = I$ changes its state from $(i,j)_I$ to $(i+1, j-1)_I$; with n available vehicles, the expected distance for any pickup is approximated by $0.63(D^2/n)^{1/2}$, as in the transit + taxi model. The drop-off transition rate for type $k = I$ vehicles depends on the composition of on-board passengers. Here, we assume that interzonal trips are always given priority. For example, for a vehicle at state $(2,0)_I$, three cases could happen: (i) if both on-board passengers are intrazonal, which occurs with probability $(\lambda_1/\lambda)^2$, the vehicle drops off the passenger whose destination is nearer to vehicle's current location, and the expected travel distance is $0.63(D^2/2)^{1/2}$; (ii) if there is only one first-mile passenger, which occurs with probability $2(\lambda_1/\lambda)(\lambda_2/\lambda)$, this passenger is dropped off first at the station nearest to vehicle's current location, and the expected travel distance is $S/2$; (iii) if both on-board passengers are inter-zonal, which occurs with probability $(\lambda_2/\lambda)^2$, they are dropped off at the nearest station together, with the expected travel distance $S/2$. In cases (i) and (ii), since there is one intra-zonal passenger on board after the first delivery, their transitions (to the same workload state) are combined and denoted by $d_{20,I}$. The delivery process for case (iii) is denoted by $d'_{20,I}$. The analysis for vehicles at state $(1,0)_I$ is similar, except that the expected distance to drop off the on-board intra-zonal passenger is L_1 from Eq. (2). Thus,

$$d_{20,I} = \left[1 - \left(\frac{\lambda_2}{\lambda}\right)^2\right] p_{11,I}, \quad d'_{20,I} = \left(\frac{\lambda_2}{\lambda}\right)^2 p_{11,I}, \quad d_{10,I} = \left(\frac{\lambda_1}{\lambda}\right) p_{01,I} + d_{20,I}, \quad d'_{10,I} = \left(\frac{\lambda_2}{\lambda}\right) p_{01,I}. \tag{17}$$

The expected travel distance of vehicles in state $(2,0)_I$, denoted as $E[l_d^{20,I}]$ [km], and that of $(1,0)_I$ vehicles, $E[l_d^{10,I}]$ [km], are as follows,

$$\begin{aligned} E[l_d^{20,I}] &= S/2 \cdot \left[(\lambda_2/\lambda)^2 + 2(\lambda_1/\lambda)(\lambda_2/\lambda)\right] + 0.63(D^2/2)^{1/2} \cdot (\lambda_1/\lambda)^2, \\ E[l_d^{10,I}] &= \frac{S/2 \cdot (\lambda_2/\lambda) p_{01,I} + L_1 \cdot \left[(\lambda_1/\lambda) p_{01,I} + d_{20,I}\right]}{p_{01,I} + d_{20,I}}. \end{aligned} \tag{18}$$

Again, the last-mile legs of all interzonal trips start from a transit station. Passengers arrive in batches per transit headway, and wait at the station to be served by $(0,0)_s$ vehicles. We pair up these passengers and serve them with shared rides; i.e., by solving a simple vehicle routing problem (VRP) with vehicle capacity being two. Since the destination of each inter-zonal trip is known before its last-mile leg starts, the pairing process does not impose additional waiting time on the passengers.

Since every vehicle that serves last-mile trips carries two passengers away from the station, its assignment rate is $a_{00,s} = \lambda_2 D^2/2$. The expected length between two adjacent last-mile passenger destinations can be estimated with the asymptotic formula (Beardwood et al., 1959; Robuste et al., 1990):

$$\frac{0.95\sqrt{S^2 \cdot \lambda_2 S^2 H}}{\lambda_2 S^2 H} = \frac{0.95}{\sqrt{\lambda_2 H}},$$

where 0.95 is a constant for L-1 metric; S^2 is the station catchment area size; and $\left(\lambda_2 S^2 H\right)$ is the number of last-mile passengers per station per headway. Thus, the expected distances to deliver the first and second on-board passengers are respectively $E[l_d^{20,II}] = (S/2)$, and $E[l_d^{10,II}] = (0.95/\sqrt{\lambda_2 H})$.

According to the Little's formula, we express the pickup, delivery, and repositioning flow rates as functions of the fleet size in each workload state,

$$\begin{aligned} &p_{01,I} = \frac{n_{01,I} n^{1/2} v_T}{0.63D}, \quad p_{02,I} = \frac{n_{02,I} n^{1/2} v_T}{0.63D}, \quad p_{11,I} = \frac{n_{11,I} n^{1/2} v_T}{0.63D}, \quad d_{20,I} + d'_{20,I} = \frac{n_{20,I} v_T}{E[l_d^{20,I}]}, \\ &d_{10,I} + d'_{10,I} = \frac{n_{10,I} v_T}{E[l_d^{10,I}]}, \quad d_{20,II} = \frac{n_{20,II} v_T}{E[l_d^{20,II}]}, \quad d_{10,II} = \frac{n_{10,II} v_T}{E[l_d^{10,II}]}, \quad b_1 = \frac{n_{00,b} v_T}{S/2}, \quad b_2 = \frac{n_{00,b'} v_T}{S/2}. \end{aligned} \tag{19}$$

Flow conservation at each workload state gives the following equations,

$$\begin{aligned} &a_{00,r} = a_{01,I} + p_{01,I}, \quad a_{01,I} = p_{02,I} = p_{11,I} = d_{20,I} + d'_{20,I}, \quad p_{01,I} + d_{20,I} = d_{10,I} + d'_{10,I}, \\ &a_{00,s} = d_{20,II} = d_{10,II} = \lambda_2 D^2/2, \quad b_1 = b'_1, \quad b_2 = b'_2, \quad d'_{10,I} + d'_{20,I} + b'_1 = a_{00,s} + b_2. \end{aligned} \tag{20}$$

Here, similar to the taxi case, we again model the stochastic part of last-mile passenger waiting time at each station using an M/M/1 queue, with arrival rate $a_{00,s}\left(\frac{D^2}{S^2}\right)^{-1}$ and service rate $\left(d'_{10,I} + d'_{20,I} + b_1\right)\left(\frac{D^2}{S^2}\right)^{-1}$. Thus, the expected sojourn time is $\left(\frac{D^2}{S^2}\right)\left(d'_{10,I} + d'_{20,I} + b_1 - a_{00,s}\right)^{-1}$. We set $d'_{10,I} + d'_{20,I} + b_1 - a_{00,s} > 0$, and $b_1 \geq 0$, to ensure that the service rate of the M/M/1 queue is strictly greater than its arrival rate, and in turn to guarantee stability.

Solution to the system of equations in Eq. (20) is expressed in terms of $n \equiv n_{00,r} + n_{10,I}$. For simplicity, we define,

$$B \equiv \frac{v_T n^{3/2}}{0.63\lambda D^3} + 2, \tag{21}$$

and then, the numbers of vehicles in all workload states are,

$$\begin{aligned} &n_{00,r}=\frac{(B-1)n}{B}, \quad n_{01,I}=\frac{n}{B}, \quad n_{02,I}=n_{11,I}=\frac{0.63\lambda D^3}{v_T B n^{1/2}}, \quad n_{20,II}=\frac{\lambda_2 D^2 S}{4v_T}, \quad n_{10,II}=\frac{0.95 D^2 \lambda_2^{1/2}}{2v_T H^{1/2}}, \\ &n_{20,I}=\frac{D^2}{\lambda v_T B}\left[\frac{S\left(\lambda_2^2+2\lambda_1\lambda_2\right)}{2}+\frac{0.63\lambda_1^2 D}{\sqrt{2}}\right], \quad n_{10,I}=\frac{(B-2)D^2}{v_T B}\left(\frac{\lambda_2 S}{2}+\frac{2\lambda_1 D}{3}\right)+\frac{2\lambda_1 D^3(\lambda_1+2\lambda_2)}{3\lambda v_T B}, \\ &n_{00,b}=\frac{b_1 S}{2v_T}, \quad n_{00,b'}=\left(\frac{\lambda_2}{\lambda B}+\frac{B-2}{B}+\frac{b_1}{\lambda_2 D^2}-\frac{1}{2}\right)\frac{\lambda_2 D^2 S}{2v_T}. \end{aligned} \tag{22}$$

Thus, the system-wide cost per trip of local RS service, Z_L' [\$/trip], is,

$$Z_L'=\frac{1}{\lambda_1+2\lambda_2}\left[\sum_i\sum_j\sum_k \frac{\gamma+(i+j)\beta}{D^2}n_{ijk}+\beta\left(\frac{\lambda_2 S^2}{\lambda B}+\frac{(B-2)S^2}{B}+\frac{b_1 S^2}{\lambda_2 D^2}-\frac{S^2}{2}\right)^{-1}+\frac{\lambda_2 H\beta}{2}\right]. \tag{23}$$

This system can be optimized by replacing Z_L in (7) with Z_L' and adding two new constraints $d_{10,I}'+d_{20,I}'+b_1-a_{00,s}>0$, and $b_1\ge 0$ to guarantee stability. Let Z' [\$/pax] denote the new system-wide objective function, which is convex with respect to b_1, and hence the optimal b_1^* can be easily obtained by checking the first-order condition and the newly added constraints, as follows,

$$b_1^*=\max\left\{0,\ \frac{\lambda_2 D^2}{2}-\frac{(B-2)\lambda_2 D^2}{B}-\frac{\lambda_2^2 D^2}{\lambda B}+\left(\frac{\lambda_2 D^4\beta v_T}{\gamma S^3}\right)^{1/2}\right\}. \tag{24}$$

However, function Z' is no longer a convex function of n, and does not have a closed-form solution for headway H any more, due to the newly added term $H^{-1/2}$ in $n_{10,II}$; see Eq. (22). Thus, their near-optimum values are solved numerically using the Brent method (Brent, 1971), as implemented in the Python Scipy package (Virtanen et al., 2020), with multiple starts.

Other variables, i.e., D and S, can be optimized with the same method as in the transit + taxi model.

4. Benchmarks

In this section, we provide closed-form formulas for the performance of conventional taxi service, and that of the conventional transit service. We also borrow the analytical model for a fixed-route bus feeder system from Sivakumaran et al. (2014). They serve as benchmarks for the proposed integrated mobility systems.

4.1. Taxi only

With traditional taxi service, there is no transit station for taxis to park, and passengers go directly from their origins to destinations in the study region. In such scenario, the taxi fleet size, m, and passenger door-to-door travel time, T [h], are functions of the number of idle vehicles, n (Daganzo and Ouyang, 2019b):

$$m=n+\frac{0.63\lambda\Phi^3}{n^{1/2}v_T}+\frac{2\lambda\Phi^3}{3v_T}, \quad T=\frac{1}{\lambda\Phi^2}\left(\frac{0.63\lambda\Phi^3}{n^{1/2}v_T}+\frac{2\lambda\Phi^3}{3v_T}\right). \tag{25}$$

The minimum system cost of taxi-only service, Z_T' [\$/pax], is found by solving the following:

$$\min_{n\in\mathbb{Z}^+} \quad Z_T'=\frac{\gamma}{\lambda\Phi^2}\left(n+\frac{0.63\lambda\Phi^3}{n^{1/2}v_T}+\frac{2\lambda\Phi^3}{3v_T}\right)+\frac{\beta}{\lambda\Phi^2}\left(\frac{0.63\lambda\Phi^3}{n^{1/2}v_T}+\frac{2\lambda\Phi^3}{3v_T}\right). \tag{26}$$

Similar to the transit + taxi model, this is also a convex problem. The optimal value $n^*=\left[\frac{0.63(\beta+\gamma)\lambda\Phi^3}{2\gamma v_T}\right]^{2/3}$ from the first order condition.

4.2. Transit only

Without feeder service, we consider a simple grid structure with square-shape catchment areas. Passengers need to walk from/to nearest stations at both ends of the trip, incurring an average total walking distance of S. Let v_w [km/h] denote the walking speed. The counterpart of Eq. (3) for the transit network, Z_B' [\$/pax], is,

$$\min_{S,H\in\mathbb{R}^+} \quad Z_B'=\beta\left[H+\Delta+\frac{S}{v_w}+\frac{2\Phi}{3}\left(\frac{1}{v_B}+\frac{t_s}{S}\right)\right]+\frac{1}{\lambda}\left[\frac{2c_g}{S}+\frac{c_r}{S^2}+\frac{4c_v}{SH}+\frac{4c_m}{SH}\left(\frac{1}{v_B}+\frac{t_s}{S}\right)\right] \tag{27}$$

$$\text{subject to} \quad H\ge H_{\min},\ S\ge S_{\min}, \tag{28}$$

$$\lambda_2\Phi SH/4\le K. \tag{29}$$

Conditional on the value of S, the optimum headway is given by an EOQ tradeoff,

$$H^*=\mathrm{mid}\left\{H_{\min},\ \left[\frac{4c_v}{\lambda\beta S}+\frac{4c_m}{\lambda\beta S}\left(\frac{1}{v_B}+\frac{t_s}{S}\right)\right]^{1/2},\ \frac{4K}{\lambda_2\Phi S}\right\}, \tag{30}$$

and the spacing S can be optimized numerically.

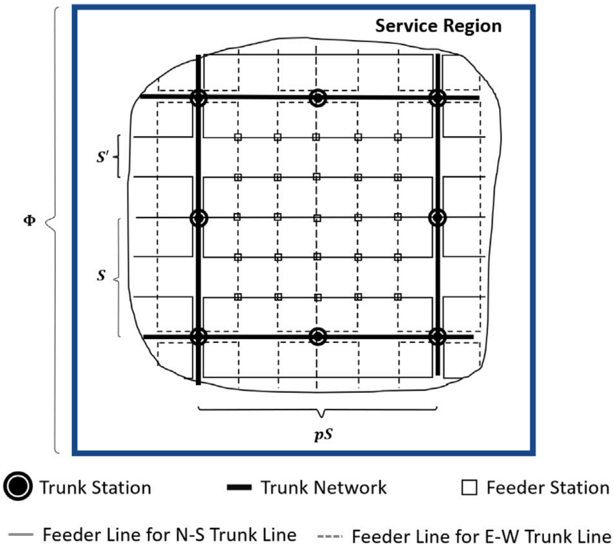


Fig. 4. Layout of the fixed-route feeder bus service.
Source: Adapted from Sivakumaran et al. (2014).

4.3. Fixed-route bus feeder

Sivakumaran et al. (2014) analyzed the performance of a hierarchical system. The backbone transit (e.g., bus, BRT, and metro) forms a grid network that covers the entire city. A set of "branch-and-trunk" feeder lines branch out of each backbone station, travel perpendicular to the backbone line(s) to collect passengers, and merge into a neighboring backbone station. The backbone network's line spacing is an integer multiple p of the backbone station spacing S; see Fig. 4 for an example with $p = 2$. Other parameters or variables of the backbone transit network use the same notation as those in Section 2.1; those of the feeder system are marked with an apostrophe.

This reference assumes that any trips longer than an empirical critical distance, l_c [km], will use the backbone network (i.e., feeder–backbone–feeder), and all others will not (i.e., feeder-only). Inconvenience associated with transfers between feeder lines is ignored, while that associated with transfers between a feeder line and a backbone line is still considered, denoted as Δ' [h]. In a square city, when $l_c < \Phi$, Sivakumaran et al. (2014) show that the demand rates and the expected travel distances of feeder–feeder and feeder–backbone–feeder passengers, as counterparts of (1) and (2), are as follows:

$$\begin{aligned} \lambda'_1 &= \frac{l_c^2\left(12\Phi^2 - 8\Phi l_c + l_c^2\right)}{6\Phi^4}\lambda, \quad \lambda'_2 = \lambda - \lambda'_1, \\ L'_1 &= \frac{l_c(40\Phi^2 - 30\Phi l_c + 4l_c^2)}{5(12\Phi^2 - 8\Phi l_c + l_c^2)}, \quad L'_2 = \frac{4\Phi^5 - 8\Phi^2 l_c^3 + 6\Phi l_c^4 - 0.8 l_c^5}{6\Phi^4 - 12\Phi^2 l_c^2 + 8\Phi l_c^3 - l_c^4}. \end{aligned} \tag{31}$$

The system-wide cost of the feeder system per passenger, Z_F [\$/pax], including those for passenger waiting, walking, transfer penalty (between feeder and backbone only), riding, as well as those for the agency (ignoring the guideway infrastructure cost), is shown to be the following,

$$\begin{aligned} Z_F =& \beta\left[\frac{S'}{v_w} + H' + \frac{\lambda'_2 S}{\lambda}\left(\frac{p+1}{2v'_B} + \frac{pt'_s}{2S'} + 2\Delta'\right) + \frac{\lambda'_1 L'_1}{\lambda}\left(\frac{1}{v'_B} + \frac{1}{2pv'_B} + \frac{t'_s}{S'}\right)\right] + \\ & \frac{1}{\lambda\Phi^2}\left[c'_r\frac{2\Phi^2}{S'^2} + c'_v\frac{L'_B}{H'} + c'_m\left(\frac{L'_B}{H'v'_B} + \frac{4\Phi^2 t'_s}{S'^2 H'}\right)\right], \end{aligned} \tag{32}$$

where $L'_B = \frac{\Phi^2(4+2/p)}{S'}$ is the total guideway length of the feeder network.

The system-wide cost of the backbone transit system per passenger, Z''_B [\$/pax], including the agency infrastructure and operation costs, the feeder–backbone–feeder passengers' waiting time at backbone stations, transfer penalty between backbone lines, and the in-vehicle riding time, is shown to be the following,

$$Z''_B = \beta\left[\frac{\lambda'_2}{\lambda}\left(H + \Delta + \frac{L'_2}{v_B} + \frac{L'_2 t_s}{S}\right)\right] + \frac{1}{\lambda\Phi^2}\left[c_g\frac{2\Phi^2}{pS} + c_r\frac{2\Phi^2}{pS^2} + c_v\frac{4\Phi^2}{pSH} + c_m\frac{4\Phi^2}{pSH}\left(\frac{1}{v_B} + \frac{t_s}{S}\right)\right]. \tag{33}$$

Then, the design model for this type of system can be written as follows.

$$\min_{S,H,S',H'\in\mathbb{R}^+,p\in\mathbb{N}^+} \quad Z_H = Z_F + Z''_B \tag{34}$$

Table 1
System parameters.

Modes	t_s (s)	v_B or v_T (km/h)	Δ (min)	K (pax/veh)	c_g ($/km-h)	c_r ($/station-h)	c_v ($/veh-km)	c_m ($/veh-h)	γ ($/veh-h)
Bus	45	25	0.9	120	9	0.01	2	40	–
BRT	45	40	0.9	150	90	0.01	2	40	–
Metro	45	60	3	1000	900	0.03	6	120	–
DRT	0	25	0	–	–	–	–	–	52

$$\text{subject to} \quad pS \leq \Phi/2, \tag{35}$$

$$p\lambda_2' \Phi SH/4 \leq K, \; (\lambda_1' \Phi S' H' + p\lambda_2' SS' H')/4 \leq K', \tag{36}$$

$$H \geq H_{\min}, \; S' H'/S \geq H'_{\min}. \tag{37}$$

Constraint (35) ensures that there are at least two backbone lines in each travel direction; constraints (36) enforce vehicle capacities; and constraints (37) stipulate the minimum headways. This model can be solved as a geometric program, as suggested in Sivakumaran et al. (2014).

5. Numerical analysis

In this section, we numerically quantify the performance of the proposed mobility service systems for two purposes: (i) casting insights into the impacts of various parameters on the optimal designs; (ii) comparing with benchmark cases to reveal the benefit from integrated mobility services.

The backbone network service could be provided by bus, BRT, or metro technologies, and local DRT service is provided by taxis or ride-sharing vehicles. All parameter values, as exhibited in Table 1, are borrowed from, or made consistent with, Daganzo (2010) and Daganzo and Ouyang (2019a).[8] We use a relatively large fixed dwell time per stop for all three transit modes. The default DRT vehicle speed is assumed to be the same as that of the bus, 25 km/h, unless specified otherwise.[9] For DRT operations, the cost for crew is the same as that for the bus, $c_{T,m} = 40$ \$/veh-h, and the variable cost is approximately $c_{T,v} = 0.48$ \$/km (Zoepf et al., 2018). Thus, for DRT vehicles, $\gamma = c_{T,m} + c_{T,v} v_T = 52$ \$/veh-h. Finally, we set $D_{\min} = 2$ km, $S_{\min} = 0.25$ km and $H_{\min} = 2$ min, and a relatively low value of walking speed $v_W = 2$ km/h to capture possible inconvenience related to walking (Daganzo, 2010).

We focus on the optimal design, system characteristics (fleet size in the study region $M \equiv m\Phi^2/D^2$, and density of available DRT vehicles n/D^2), and performance indicators (system-wide cost per passenger Z, agency cost per passenger, and passengers' average door-to-door pace[10]) of the proposed mobility systems, and compare them to those of the bench-marking transit-only and taxi-only counterparts, when we vary a set of system parameters (including demand density, region size, passenger value-of-time, vehicle cruising speed, and the transit technology type). At the end of this section, we compare the performance of the proposed models with that of a fixed-route feeder system from Sivakumaran et al. (2014).

5.1. Baseline solutions

In this subsection, we consider a base-line city with $\Phi = 10$ km, $\beta = 20$ \$/h, and the demand density λ varies from 10 to 1000 pax/km^2-h. Buses are used to provide the backbone transit service. Table 2 summarizes the optimal designs and system characteristics of both proposed mobility systems. It shows that, as demand density increases, for both proposed mobility systems, the optimal transit network spacing and headway decrease and the size of the local DRT zone shrinks. This is expected, as a city with higher demand density tends to (i) enhance the passenger experience by providing better spatio-temporal accessibility, while at the same time, (ii) control the amount of intra-zonal passengers that are served solely by the more "costly" DRT service. In addition, the local zone tends to contain multiple station catchment areas (i.e., $D > S$), and for all scenarios in Table 2, one local zone contains four station catchment areas. This suggests the importance of optimizing the local zone size as a decision variable rather than refraining the DRT service to be inside each station catchment area. In addition, the density of available vehicles n/D^2 increases with the demand, which should yield a shorter passenger waiting time for DRT pickup, and in turn a higher level of service. The optimal repositioning rate b_1^* of the bus + taxi model is very large, as needed to reduce last-mile passengers' waiting time. For the bus + RS model, the vehicles arriving at the stations to deliver first-mile passengers, with transition rates $d'_{10,I}$ and $d'_{20,I}$, are usually enough to serve the last-mile trips, i.e., $b_1^* = 0$ (except when demand is very low $\lambda = 10$ pax/km^2-h).

[8] In Daganzo (2010), Daganzo and Ouyang (2019a), the demand rate is measured in passengers per hour. To be compatible with our proposed models, we convert the original value to be the demand density, as in passengers per unit area per hour.

[9] The cruising speed of DRT vehicles is usually slightly higher than that of buses in practice, even in mixed traffic, possibly due to larger and heavier vehicles for bus operation, as well as the safety concerns for passengers standing in the bus. In this paper we use equal cruising speed by default, as a conservative treatment, to ensure that the performance of the proposed mobility service systems does not get overestimated. Section 5.4 will discuss the impacts of various traffic conditions, as well as different cruising speeds of buses and DRT vehicles.

[10] The passengers' equivalent door-to-door pace is calculated as the ratio of the average door-to-door travel time (i.e., passengers' average generalized cost divided by the value-of-time) to the average travel distance $2\Phi/3$. It is a direct measurement of the passenger experience; the smaller the pace, the better the passenger experience.

Table 2
Optimal design and system characteristics under varying demand density (Φ = 10 km, β = 20 \$/h).

λ (pax/km^2-h)	Service Type	Decision Variable					System Characteristics	
		D (km)	S (km)	H (min)	n (veh)	b_1 (veh/h)	M (veh)	n/D^2 (veh/km^2)
10	Bus + Taxi	5.00	2.50	9.86	7.81	53.71	206.14	0.31
	Bus + RS	5.00	2.50	10.48	11.42	32.61	178.53	0.46
100	Bus + Taxi	3.33	1.67	3.55	16.10	150.97	1,136.19	1.45
	Bus + RS	3.33	1.67	3.85	23.25	0.00	893.88	2.09
500	Bus + Taxi	2.00	1.00	2.02	16.95	271.75	3,334.25	4.24
	Bus + RS	2.50	1.25	2.00	31.29	0.00	2,855.04	5.01
1000	Bus + Taxi	2.00	1.00	2.00	26.91	384.31	6,081.94	6.73
	Bus + RS	2.00	1.00	2.00	30.70	0.00	4,431.56	7.67

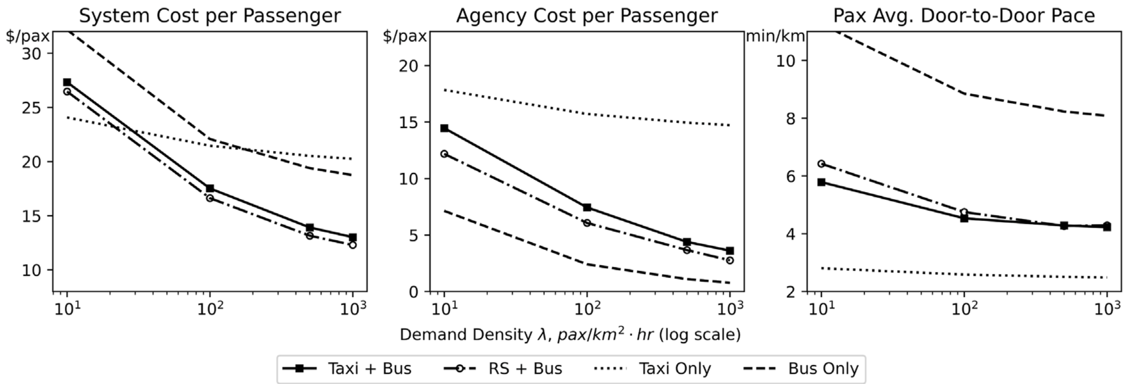


Fig. 5. System performance vs. demand density λ.

Note that the optimal spacings in all scenarios are relatively larger than what one may expect in currently existing transit systems. This is reasonable, since the local DRT service expedites the first- and last-mile legs of the inter-zonal trips and mitigates the negative impact of large spacing on passenger accessibility.

If we compare the optimal designs of the two proposed mobility service systems, we see that the optimal spacing of the bus + RS system is always greater than, or at least equal to, that of the corresponding bus + taxi system. This may be due to more efficient first- and last-mile service of RS operation. In addition, given the same D and S, the optimal bus headway of the bus + RS system is larger than that of the bus + taxi system. The difference can be attributed to the last-mile delivery tours of local RS service, which becomes more efficient due to demand pooling under a larger value of headway. In addition, the optimal density of available vehicles in the bus + RS model is always higher than that of the bus + taxi model, by utilizing both the idle vehicles and the vehicles with one assignment to serve passengers.

Fig. 5 plots the system-wide cost, agency cost, and users' average pace of the four systems: the two proposed mobility systems, the transit-only system, and the taxi-only system. It can be seen that the total cost per passenger monotonically decreases with demand for all systems. The varying slopes of these curves demonstrate very strong economies of scale in the proposed mobility service – the largest portion of the saving comes from the decreasing agency cost per passenger, as more passengers share the agency investment and smaller DRT service zone reduces the burden on local DRT operation. The passengers' average pace generally decreases, but slightly increases in certain cases, e.g., when demand increases from λ = 500 to 1000 pax/km^2-h for the bus + RS service. This fluctuation shows that, when demand density is extremely high, it may be beneficial to sacrifice passenger experience slightly, e.g., by reducing the ratio of intrazonal passengers, to achieve a significant reduction of DRT operation cost.

When the demand is very low ($\lambda \leq 30$ pax/km^2-h), taxi-only system is advantageous since such low demand density cannot justify the relatively large investment needed for running a transit system, let alone the use of DRT vehicles for feeder services. With a larger travel demand, i.e., $\lambda \geq 30$ pax/km^2-h, the proposed mobility systems, especially the bus + RS system, become significantly superior. Although the RS passengers may experience some detours, these two systems provide a similar level-of-service to the passengers when demand approaches λ = 1000 pax/km^2-h. This is probably because (i) the RS system can easily find good passenger pairs under such high demand; (ii) the higher density of available vehicles in the bus + RS model can further reduce the expected pick up distance; and (iii) with the small DRT zone size, the magnitude of RS detours is small. Moreover, the local RS service also experiences a lower average travel distance needed to deliver the (second) last-mile passengers as the demand density increases.

In the following subsections, we fix the demand density at λ = 200 pax/km^2-h (following the Barcelona case in Daganzo, 2010), and conduct a series of sensitivity analysis by varying system parameters one at a time.

Table 3
Optimal design and system characteristics under varying region size ($\beta = 20$ \$/h, $\lambda = 200$ pax/km^2-h).

Φ (km)	Service Type	Decision Variable					System Characteristics	
		D (km)	S (km)	H (min)	n (veh)	b_1 (veh/h)	M (veh)	n/D^2 (veh/km^2)
10	Bus + Taxi	2.50	1.25	2.86	14.38	189.89	1,735.09	2.30
	Bus + RS	2.50	1.25	3.09	20.22	0.00	1,338.87	3.23
20	Bus + Taxi	2.86	1.43	2.60	18.78	207.51	7,432.91	2.30
	Bus + RS	3.33	1.67	2.65	29.12	0.00	5,986.62	2.62
30	Bus + Taxi	3.33	1.67	2.38	25.56	225.05	18,325.74	2.30
	Bus + RS	2.00	2.00	2.41	9.13	0.00	14,533.18	2.28
40	Bus + Taxi	3.33	1.67	2.18	25.56	225.67	32,546.59	2.30
	Bus + RS	3.33	1.67	2.17	28.33	0.00	23,764.34	2.55

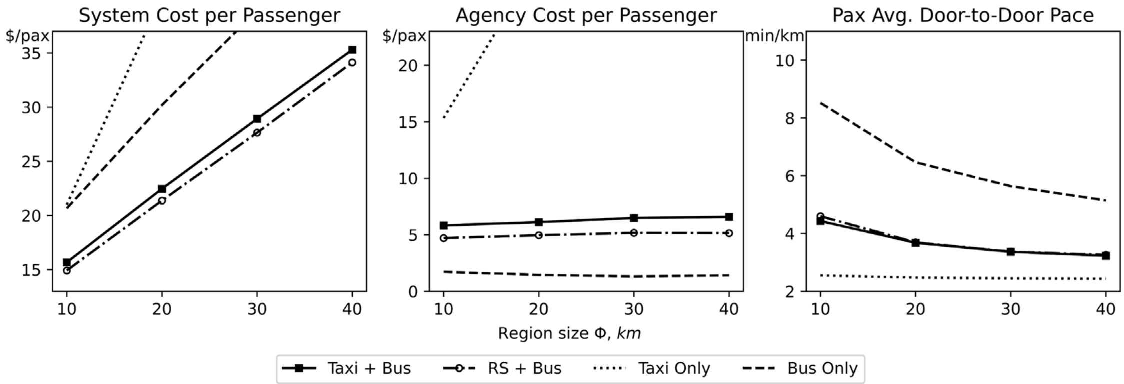


Fig. 6. System performance vs. region size Φ.

5.2. Impacts of region size

When the region side length Φ varies from 10 to 40 km, the designs and characteristics of the optimal integrated mobility systems are exhibited in Table 3, and the performance comparisons against benchmark systems are illustrated in Fig. 6. When the city size scales up, the direct impact is the longer expected travel distance of all passengers (especially for interzonal passengers). As such, it is advantageous to have a sparser bus network with a larger spacing to reduce the inter-zonal passengers' in-vehicle travel time and the transit agency cost. Accordingly, the optimal local zone size D^* should generally increase as well, as observed when changing region size from 10 km to 30 km. However, for a really large study region, i.e., $\Phi = 40$ km, the spacing is restricted by the vehicle capacity, i.e., Eq. (13). In this case, if S continues to increase, the transit vehicle capacity constraint may be violated, which will force the local zone size D to expand (to reduce the inter-zonal travel demand λ_2), and as a result, lead to a higher local DRT operation cost. Thus, the spacing and zone size remain the same as those of smaller regions. The optimal number of idle vehicles per unit area is insensitive to the region sizes in the bus + taxi model, which is expected because of the constant demand density and passenger value-of-time. However, this not observed in the bus + RS model, due to the complex interactions between the number of idle vehicles n and the repositioning rates b_1 and b_2.

As the region size increases, the system-wide cost increases almost linearly due to the longer expected travel distance, as shown in Fig. 6. Yet, the proposed systems have the least slopes among all. The agency cost of the taxi-only system is very sensitive to the region size, while those of other three systems remain stable due to the backbone network's economies of scale. The passenger experience, as indicated by the average door-to-door travel pace in Fig. 6, is slightly improved as the region expands.

5.3. Impacts of value-of-time

The passenger value-of-time β determines the relative weights of passenger experience vs. agency cost. Table 4 summarizes the results for a range of β values. As β increase, the optimal design is inclined toward providing a better passenger experience, by: (i) increasing D, which allows more passengers to use DRT service to travel directly from their origins to their destinations; (ii) increasing S, to reduce inter-zonal passengers' travel cost for the line-haul legs (i.e., the major part of their trips), and the benefits dwarf the negative impacts on the local access distance; (iii) decreasing H, which reduces passenger waiting time at transit stations; and (iv) increasing the density of available DRT vehicles to reduce the distance needed to pick up a passenger. As a result, for both proposed systems, the passenger experience is enhanced under a higher β, demonstrated by the decreased passenger's average

Table 4
Optimal design and system characteristics under varying passenger value-of-time (Φ = 10 km, λ = 200 pax/km^2-h).

β ($/h)	Service Type	Decision Variable					System Characteristics	
		D (km)	S (km)	H (min)	n (veh)	b_1 (veh/h)	M (veh)	n/D^2 (veh/km^2)
1	Bus + Taxi	2.00	0.50	21.45	7.50	108.70	1,036.59	1.88
	Bus + RS	2.00	0.67	19.39	8.68	0.00	883.14	2.17
10	Bus + Taxi	2.00	1.00	4.53	8.33	121.53	1,489.69	2.08
	Bus + RS	2.50	1.25	4.38	16.34	0.00	1,267.29	2.61
20	Bus + Taxi	2.50	1.25	2.86	14.38	189.89	1,735.09	2.30
	Bus + RS	2.50	1.25	3.09	20.22	0.00	1,338.87	3.23
30	Bus + Taxi	5.00	1.25	2.61	62.72	832.05	2152.16	2.51
	Bus + RS	3.33	1.67	2.26	37.48	0.00	1,651.19	3.37
40	Bus + Taxi	5.00	1.67	2.00	67.72	624.04	2,410.19	2.71
	Bus + RS	3.33	1.67	2.00	41.99	0.00	1,704.48	3.78

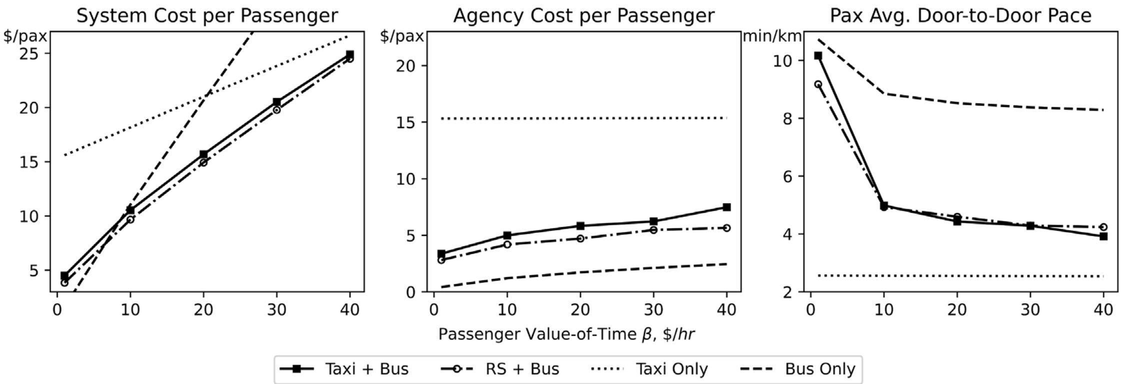


Fig. 7. System performance vs. passenger value-of-time β.

Table 5
Optimal design and system characteristics under various traffic conditions (Φ = 10 km, λ = 200 pax/km^2-h, β = 20 $/h).

Traffic Condition v_B, v_T (km/h)	Service Type	Decision Variable					System Characteristics	
		D (km)	S (km)	H (min)	n (veh)	b_1 (veh/h)	M (veh)	n/D^2 (veh/km^2)
Severely Congested	Bus + Taxi	2.50	0.83	4.50	27.27	237.70	3,127.28	4.36
(10, 10)	Bus + RS	2.00	1.00	4.46	23.32	0.00	2,569.49	5.83
Mildly Congested	Bus + Taxi	3.33	1.11	3.29	29.93	359.21	2,067.39	2.69
(20, 20)	Bus + RS	2.50	1.25	3.27	22.85	0.00	1,615.32	3.66
Base Case (25, 25)	Bus + Taxi	2.50	1.25	2.86	14.38	189.89	1,735.09	2.30
	Bus + RS	2.50	1.25	3.09	20.22	0.00	1,338.87	3.23
Faster DRT	Bus + Taxi	5.00	1.67	2.74	41.09	523.11	1,538.92	1.64
(25, 40)	Bus + RS	3.33	1.67	2.69	24.86	0.00	1,056.92	2.24

door-to-door pace, but this requires a bit more agency operation cost correspondingly. In addition, as shown in Fig. 7, the transit-only system is superior among all when $\beta \le 8$ $/h, and the taxi-only system is potentially advantageous when $\beta > 40$ $/h. When $8 < \beta \le 40$ $/h, the bus + RS system outperforms other systems.

5.4. Impacts of DRT vehicle cruising speed

In previous subsections, the cruising speeds of DRT vehicles and buses are assumed to be both 25 km/h. The city could get congested, and it is also possible that DRT vehicles have a higher speed than buses. Hence, this subsection analyzes the impacts of different traffic conditions through the following four scenarios: scenario 1 (S1) "severely congested" with $v_T = v_B = 10$ km/h, scenario 2 (S2) "mildly congested" with $v_T = v_B = 20$ km/h, scenario 3 (S3) "base case" with $v_T = v_B = 25$ km/h, and scenario 4 (S4) "faster-DRT" with $v_B = 25$ km/h and $v_T = 40$ km/h. The DRT operation cost coefficient, γ, is updated in each scenario accordingly.

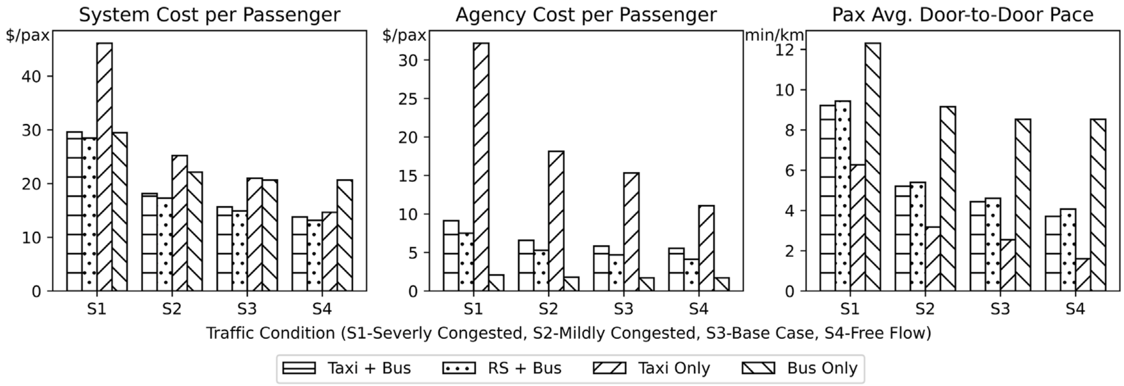
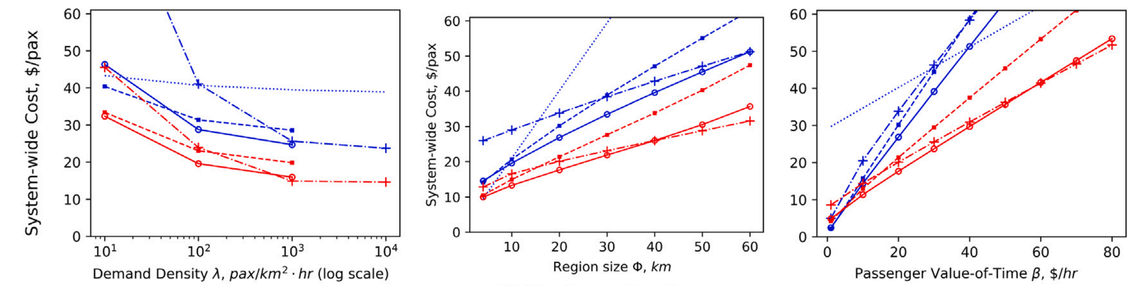


Fig. 8. System performance vs. traffic conditions (v_B and v_T).



(a) Varying demand density (b) Varying region size (c) Varying value-of-time

Fig. 9. System performance vs. transit technologies ($\beta = 20$ \$/h, $\Phi = 20$ km, $\lambda = 200$ pax/km^2-h, $v_T = v_B = 25$ km/h).

The optimal designs of proposed systems are summarized in Table 5. As the cruising speed increases, the marginal improvement of DRT service is more significant, and hence both the optimal spacing S^* and optimal zone size D^* generally increase for both proposed mobility service systems. The fluctuation of the optimal local zone size is observed in base case for the bus + taxi system, and a possible reason is that the DRT speed is not high enough to provide affordable taxi service in a local zone with size $4S^* = 5$ km. The density of idle/available vehicles decreases since the higher cruising speed expedites the pickup process and provides the opportunity to reduce the number of idle vehicles. The system performance is plotted in Fig. 8. As expected, both the agency cost and the passenger experience are improved under better traffic conditions. The bus + RS model is superior in all scenarios. However, for a very congested city, the bus-only system is competitive, since the advantages of using "expensive" DRT vehicles as feeder service is insignificant when its speed is low; and under the "faster DRT" scenarios, the taxi-only service becomes very advantageous, as expected.

5.5. *Impacts of transit technology*

In this subsection, we explore the applicability of the proposed systems when the transit network uses different technologies such as BRT and metro. Only the transit + RS system is presented here because it dominates the transit + taxi system for all scenarios in Sections 5.1–5.4. We also compare its performance with that of transit-only and taxi-only systems. In total, we consider seven transportation system options, including: taxi-only, bus-only, BRT-only, Metro-only, Bus + RS, BRT + RS, and Metro + RS. To make the expensive transit technologies (BRT and Metro) competitive, we consider a base case with extended region size, i.e., $\Phi = 20$ km, $\lambda = 200$ pax/km^2-h, and $\beta = 20$ \$/h.

The results are shown in Fig. 9. The BRT + RS system outperforms all systems when demand density $\lambda < 1000$ pax/km^2-h. However, if the demand continues to increase, bus and BRT are no longer suitable to operate, considering the constraints on transit vehicle capacity, minimum headway, and minimum spacing. In this case, Metro + RS becomes the optimal transportation system, as shown in Fig. 9(b). For a small city with $\Phi \le 5$ km, taxi-only is preferred since the transit system is not cost-effective in such a small region. When $5 \le \Phi \le 40$ km, which is the case for most real-world cities, the proposed BRT + RS system is superior. For a mega metropolitan area, such as the Greater Tokyo area, Metro + RS is the most efficient transportation system, as expected. With regard to wealth level, the bus-only system and the BRT-only system are competitive for the extremely poor condition of $\beta \le 5$ \$/h, BRT + RS system is the most suitable system for common cities with $5 \le \beta \le 60$ \$/h, and Metro + RS is advantageous for extremely wealthy cities with $\beta \ge 60$ \$/h.

Table 6
Transit system parameters for high-wage cities in Sivakumaran et al. (2014).

Modes	t_s (s)	v_B or v'_B (km/h)	Δ (s)	Δ' (s)	K (pax/veh)	$H_{\min}$ (min)	c_g ($/km-h)	c_r ($/station-h)	c_v ($/veh-km)	c_m ($/veh-h)
Feeder	20	20	n/a	0	80	n/a	0	0.56	0.47	50.40
Bus	30	25	10	20	80	3	10	0.7	0.59	63
BRT	30	40	20	60	120	2	270	7	0.66	84
Metro	45	60	60	120	1000	4	990	490	2.20	201

Table 7
Comparison of system performance: Transit + RS vs. Fixed-Route Feeder Bus. ($\beta = 20$ $/h).

System-wide cost per passenger ($/pax)		λ [pax/km^2-h]								
		50			150			250		
		Transit+RS	Fixed-Route Feeder	Diff.	Transit+RS	Fixed-Route Feeder	Diff.	Transit+RS	Fixed-Route Feeder	Diff.
Φ [km]	10	Bus + RS 18.57	Bus + Walk 20.67	10.16%	BRT + RS 15.01	Bus + Walk 17.33	13.38%	BRT + RS 13.31	Bus + Walk 16.00	16.82%
	20	BRT + RS 24.54	Bus + Walk 27.67	11.31%	BRT + RS 18.95	BRT + Feeder 23.33	18.78%	BRT + RS 17.24	BRT + Feeder 21.33	19.19%
	30	BRT + RS 28.29	BRT + Feeder 33.33	15.13%	BRT + RS 22.81	BRT + Feeder 27.00	15.51%	BRT + RS 21.16	BRT + Feeder 25.33	16.46%

5.6. Comparison with Fixed-Route Bus Feeder (Sivakumaran et al., 2014)

In this subsection, we compare the performance of the proposed transit + RS mobility service with that of the fixed-route feeder system in Section 4.3. For ease of comparison, we apply our model to the same case study scenarios with the same parameter values as those in Sivakumaran et al. (2014), so that the results can be directly compared. The key parameter values for feeder bus, bus, BRT and metro backbone networks in "high-wage cities" (i.e., $\beta = 20$ $/h), as in Sivakumaran et al. (2014), are listed in Table 6. In addition, following this reference, the critical distance for the fixed-route bus feeder system is $l_c = 8$ km.

The system-wide costs per passenger of the proposed transit + RS service for a range of demand densities and city sizes are presented in Table 7. For each scenario, we report the optimal backbone transit technology (bus, BRT, or metro) to be integrated with RS service. The counterpart costs of the fixed-route feeder systems are copied from the case study section of Sivakumaran et al. (2014),[11] based on the best one among the following options: (i) BRT backbone plus bus feeders, (ii) metro backbone plus bus feeders, as well as (iii) bus for the backbone and walking for local access.

The percentage differences in Table 7 show clear superiority of the proposed transit + RS service as compared to the benchmark fixed-route feeder bus service. In all investigated scenarios, the transit + RS service outperforms the fixed-route feeder bus service by over 10%, and the improvement is most significant when the city is moderately large (e.g., $\Phi = 20$ km) and the demand is high (e.g., $\lambda = 250$ pax/km^2-h). As the study region size continues to grow (e.g., $\Phi = 30$ km), the relative difference shrinks, possibly due to the better economies of scale of fixed-route feeder buses in very large cities.

6. Conclusion

This paper proposes a modeling framework for integrated mobility service systems, composed of a fixed-route backbone transit network and a zone-based local DRT service system, to serve both local and long-distance travel demand. The system design includes the determination of local zones, the DRT service fleet size and repositioning operations, as well as the transit network spacing and headway. Based on the service region partition, we first categorize the travel demand into intra-zonal trips and inter-zonal trips, and obtain their demand density and the expected travel distance. Taxi and ride-sharing are considered as examples of DRT services, and new aspatial queuing network models are proposed to quantify the system performance for each DRT system to serve trips with uniformly distributed O/D and those with O/D concentrated at transit stations. We formulate the system design problem as a constrained non-linear program, where the objective function includes the transit agency cost, DRT agency cost, and passenger travel cost. Numerical experiments are carried out to cast insights into the optimal system design under a series of scenarios. As observed from our example scenarios, it is optimal to provide DRT service in zones that may contain multiple station catchment areas. In addition, our system performance comparison shows that the proposed mobility service systems generally outperform the conventional transportation systems, as well as the fixed-route feeder bus service, with significant improvements, except for

[11] The results in Sivakumaran et al. (2014) were originally reported as travel times, with the unit of minutes. To be consistent, these values are divided by 60 (min/h) and multiplied by β ($/h) to obtain monetary costs. In addition, the constraint on feeder bus' minimum headway is relaxed in Sivakumaran et al. (2014).

very poor, very small, or extremely congested cities. More specifically, the transit + RS system always dominates the transit + taxi system, by offering comparable or acceptable service at lowers costs. When we consider different transit technologies for the backbone service, the BRT is generally advantageous for most big cities in the world, and the metro is preferable for very large, very densely populated, and very wealthy cities. In summary, the proposed mobility service systems provide a very promising service quality that is comparable to privately-owned vehicles, while the costs are much lower.

The work in this paper can be extended in several directions. First, it will be great to seek opportunities to verify the proposed models via simulations, and possibly implement them in a real-world city. To make the system more desirable for interzonal passengers whose trips just cross a zone boundary, the idea of "overlapping zones" (Liu et al., 2020) can be adopted together with the option of allowing transfer between DRT vehicles at the zone border. Another interesting option is to switch the zone-based model to a distance-based model, i.e., passengers with trip length shorter than a critical value (a new decision variable), will be served by DRT service only (analogous to the "intra-zonal trips"), while others will be jointly served by the DRT and transit services (analogous to the "inter-zonal trips"). The critical length can be implemented in real-world operations through proper pricing strategies, such as "MaaS packages." This idea will eliminate undesirable services for those aforementioned inter-zonal passengers near zone boundaries, and can be easily incorporated into the proposed transit + taxi model. However, the distance-based transit + RS model still requires further investigation, due to the complicated matching requirements and passenger detour issues. In this paper, a grid network is assumed for the backbone transit system. Additional backbone network structures could also be explored, such as the hybrid structure proposed in Daganzo (2010), or grid with local routes in Ouyang et al. (2014), or bimodal transit network in Fan et al. (2018). In addition, the aspatial queuing models developed in this study can be further extended to more types of DRT services, such as those with non-chauffeured vehicles as feeders (e.g., bike-sharing); those with a larger DRT vehicle occupancy but longer passenger riding time (e.g., dial-a-ride); and those explicitly control the maximum waiting and detour experienced by passengers (e.g., ride-matching in Daganzo et al., 2020). The latter one pairs passengers considering the proximity of both their origins and destinations, which is a potential solution to the passengers' detour under current ride-sharing operation. Furthermore, this paper only considers homogeneous trip demand distribution; the models can be easily generalized to allow heterogeneous demand distribution, whereas the local zone size, network spacing, and/or DRT operation can vary across zones according to the demand variation. It would be interesting to consider the social and environmental sustainability of the proposed MaaS system, such as including energy consumption and the costs of emissions from different types of system components as a part of the objective function. In addition, this study focuses on system optimum design without considering individual passengers' response (e.g., mode choice), which is an important aspect of the mobility service, especially when passengers exhibit heterogeneous socioeconomic characteristics. Related issues such as equity in accessibility, social fairness of passenger choices, and possible pricing strategies are nevertheless critical. Future work on integrated mobility services should address such issues from the passengers' perspectives. Finally, this paper only considers the economic benefit of the proposed mobility systems when transit agencies and DRT service providers are cooperative and share the same objective; yet, this may not be the case in the real world – it will be important to explore possible incentive mechanisms and policy instruments for these non-cooperative situations.

CRediT authorship contribution statement

Yining Liu: Conceptualization, Methodology, Algorithm design, Software, Data curation, Writing - original draft. **Yanfeng Ouyang:** Conceptualization, Methodology, Algorithm design, Supervision, Funding acquisition, Writing - reviewing & editing.

Acknowledgments

The authors sincerely thank the editors and three anonymous reviewers for sharing many valuable insights and providing very constructive suggestions that helped us significantly improve the models and numerical experiments in this paper. This research was supported in part by the U.S. National Science Foundation via Grant CMMI-1662825, and a grant from the USDOT Region V University Transportation Center, United States.

Appendix A. Table of key notation

See Table 8.

Appendix B. Impacts of walking trips in integrated mobility system

This appendix presents a quick analysis on the impacts of considering walking in the proposed mobility service systems. Intuitively, walking will be favorable only when the distance of a trip (or a trip leg) is small, and thus, walking is taken as an alternative for local DRT services only. To stay focused, we take transit + taxi as an example. Suppose that a passenger has a walking speed of v_w [km/h] and a trip (or a leg of a trip) of length l [km]. The cost for the passenger to walk is $\frac{\beta l}{v_w}$. Also suppose that the expected time for a passenger to wait for taxi assignment is T [h] (if any), and the expected pick-up distance is l_p [km]. The cost for this passenger to use the taxi service is $\frac{(\gamma+\beta)(l_p+l)}{v_T}+\beta T$, where $\frac{\gamma(l_p+l)}{v_T}$ is the cost related to the taxi vehicle-hour of operations to serve this passenger.

The values of T, l_p and l vary across trip types. Intra-zonal and first-mile passengers are assigned to an idle vehicle instantly, i.e., $T = 0$, and after assignment, the expected pickup distance is $l_p = 0.63D/\sqrt{n}$. Thus, the maximum trip length that makes

Table 8
List of key notation defined in this paper.

Notation	Description
D (D_{min}) [km]	(Minimum) Side length of a local DRT zone.
l_c [km]	Empirical critical distance that distinguishes the feeder-only passengers and the feeder–backbone–feeder passengers in the fixed route bus feeder model.
l_d [km]	Expected distance for a DRT vehicle to drop off its on-board passenger(s).
L_1, L_2 [km]	Expected trip length for intra- and inter-zonal passengers, respectively.
L_B [km]	Two-way guideway length of the backbone transit network.
L'_B [km]	One-way guideway length of feeder network in the fixed route bus feeder model.
Φ [km]	Side length of a square study region.
S (S_{min}) [km]	(Minimum) Station spacing of backbone transit network.
S' [km]	Station spacing of feeder network in the fixed route bus feeder model.
λ [pax/km^2-h]	Demand density.
λ_1, λ_2 [pax/km^2-h]	Intra- and inter-zonal demand density, respectively.
Δ [h]	Passenger inconvenience associated with transfers between backbone transit lines (beyond waiting time).
Δ' [h]	Passenger inconvenience associated with transfers between a feeder line and a backbone line (beyond waiting time) in the fixed route bus feeder model.
H (H_{min}) [h]	(Minimum) Service headway of backbone transit network.
H' (H'_{min}) [h]	(Minimum) Service headway of feeder bus in the fixed route bus feeder model.
t_s [h]	Backbone transit vehicle fixed lost time at each trunk station.
t'_s [h]	Feeder bus fixed lost time at each feeder station in the fixed route bus feeder model.
K [pax/veh]	Backbone transit vehicle capacity.
K' [pax/veh]	Feeder bus capacity in the fixed route bus feeder model.
m [veh]	Fleet size for DRT service in a local zone.
M [veh]	Fleet size for DRT service in the study region.
n [veh]	Number of randomly located idle DRT vehicles.
$n_{ij,k}$ [veh]	Expected number of DRT vehicles currently having i passengers on-board and j passengers to be picked up, with assignment type k.
$a_{ij,k}, p_{ij,k}, d_{ij,k}, b_1, b'_1, b_2, b'_2$ [veh/h]	Transition rates of DRT vehicles out of state $(i,j)_k$ due to assignment, pickup, delivery or repositioning.
v_B [km/h]	Backbone transit vehicle cruising speed.
v'_B [km/h]	Feeder bus cruising speed in the fixed route bus feeder model.
v_T [km/h]	DRT vehicle cruising speed.
v_w [km/h]	Passenger walking speed.
β [$/h]	Passenger value-of-time.
γ [$/veh-h]	DRT operation cost per vehicle-hour.
c_g [$/km-h]	Backbone transit infrastructure cost related to two-way guideway (amortized).
c_r (c'_r) [$/station-h]	Backbone transit (feeder bus) infrastructure cost related to stations (amortized).
c_m (c'_m) [$/veh-h]	Backbone transit (feeder bus) operation cost related to total vehicle-hour.
c_v (c'_v) [$/veh-km]	Backbone transit (feeder bus) operation cost related to total vehicle-distance.
$c_{T,m}$ [$/veh-h]	DRT operation cost related to total vehicle-hour of operation.
$c_{T,v}$ [$/veh-km]	DRT operation cost related to total vehicle-distance.
Z_B [$/pax]	System-wide cost of the backbone transit network per passenger.
Z'_B [$/pax]	System-wide cost of the transit network per passenger in the transit-only model.
Z''_B [$/pax]	Backbone network system-wide cost per passenger in the fixed route bus feeder model.
Z_F [$/pax]	Feeder bus system-wide cost per passenger in the fixed route bus feeder model.
Z'_T [$/pax]	System-wide cost of taxi-only service per passenger.
Z_L [$/trip]	System-wide cost of local taxi service per trip.
Z'_L [$/trip]	System-wide cost of local RS service per trip.
Z [$/pax]	System-wide cost of transit + taxi service per passenger.
Z' [$/pax]	System-wide cost of transit + RS service per passenger.
B	An intermediate term defined for model convenience, $B \equiv (v_T n^{3/2})/(0.63\lambda D^3) + 2$.

walking favorable, l_1^*, can be determined by $\frac{(\gamma+\beta)(l_p+l_1^*)}{v_T} - \frac{\beta l_1^*}{v_w} = 0$, which yields $l_1^* = \frac{0.63 v_w(\gamma+\beta)Dn^{-1/2}}{v_T\beta - v_w(\gamma+\beta)}$. Last-mile passengers, instead, have $l_p = 0$ and $T = \frac{H}{2} + \frac{D^2}{b_1 S^2}$. Then, the maximum trip length, l_2^*, can be determined by $\frac{(\gamma+\beta)l_2^*}{v_T} + \beta\left(\frac{H}{2} + \frac{D^2}{b_1 S^2}\right) - \frac{\beta l_2^*}{v_w} = 0$, which yields $l_2^* = \frac{v_w v_T \beta}{v_T\beta - v_w(\gamma+\beta)}\left(\frac{H}{2} + \frac{D^2}{b_1 S^2}\right)$.

As such, an intra-zonal passenger will walk only if the destination is within a diamond-shaped area around the origin, with both diagonal lengths equal to $2l_1^*$. A first-mile passenger will walk only if the trip origin is within the same diamond-shaped area around a transit station. A last-mile passenger will walk only if the destination is with a diamond-shaped area around a station with both diagonal lengths being $2l_2^*$. Thus, the proportion of intra-zonal, first-mile, and last-mile passengers who choose to walk are $2(l_1^*)^2/D^2$, $2(l_1^*)^2/S^2$, and $2(l_2^*)^2/S^2$, respectively. For the entire system, the proportion of potential walking trips in a local zone is $\frac{1}{\lambda_1+2\lambda_2}\left[\lambda_1\frac{2(l_1^*)^2}{D^2} + \lambda_2\frac{2(l_1^*)^2}{S^2} + \lambda_2\frac{2(l_2^*)^2}{S^2}\right]$.

Clearly, each of these potential walking trips has an average length of $\frac{2l_1^*}{3}$ or $\frac{2l_2^*}{3}$ inside the corresponding diamond area. Hence, the potential passenger-kilometers-traveled (PKT) by walking, as a fraction of the total PKT in a local zone, is only $\frac{1}{\lambda_1\ L_1+2\lambda_2(S/2)}\left[\frac{4\lambda_1(l_1^*)^3}{3D^2} + \frac{4\lambda_2(l_1^*)^3}{3S^2} + \frac{4\lambda_2(l_2^*)^3}{3S^2}\right]$.

Table 9
Fractions of potential walking trips and PKT for transit + taxi service ($\Phi = 10$ km, $\beta = 20$ \$/h).

λ [pax/km^2-h]	10	100	500	1000
% Walking	5.75	2.41	2.29	1.52
% PKT	1.11	0.34	0.33	0.18

Using the parameter values from Section 5, we can compute the fractions of potential walking trips and PKT under different demand rates, as shown in Table 9. It shows that only about 1%–5% of local trips and no more than 1% of the total local PKT, would possibly be replaced by walking. Hence, if the total cost is the only concern, it seems reasonable to ignore walking in our analysis.

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