Non-Hermitian Dynamics in a Hermitian System

Noah Flemens and Jeffrey Moses
School of Applied and Engineering Physics, Cornell University, Ithaca, New York 14853, USA.
nrf33@cornell.edu

ABSTRACT: The emergence of an exceptional point and spontaneous PT-symmetry breaking when optical parametric amplification and idler second harmonic generation are simultaneously phase matched leads to behavior characteristic of non-Hermitian systems in a fully Hermitian system.

The emergence of exceptional points that divide regions of broken and unbroken PT-symmetry in the presence of gain and loss have been the subject of intense investigation over the past two decades [1]. Recently, several studies have induced these non-Hermitian features in nonlinear optical systems without the use of material gain, loss, or optical scattering [2-5]. Instead, strong laser fields are used to amplify/de-amplify two modes that are coupled by the nonlinear interaction. Provided the exchange of energy between the strong fields and the two-mode subsystem is negligible, the interaction within the subsystem is well approximated by non-Hermitian linear coupled mode theory. This leads to regions of purely real or imaginary eigenvalues in the two-mode subsystem's eigenspectra, resulting in oscillatory or non-oscillatory behavior, respectively. However, when the energy exchange with the strong fields is no longer negligible, the system becomes nonlinear and will undergo a cyclic exchange of energy between all fields resulting in a loss of the non-Hermitian characteristics of the system. Crucially, this precludes applications in efficient frequency conversion where it is desirable to damp out or eliminate these conversion cycles entirely to enable uniform conversion between the spatiotemporal profiles of two or more fields.

Here, we report a phenomenon with significant implications for the development of efficient and sustainable photonic devices: a Hermitian nonlinear optical system where a *complete* exchange of energy between four modes exhibits dynamics characteristic of non-Hermitian systems – regions of damped oscillatory and non-oscillatory exchange separated by an exceptional point. We find that a non-Hermitian subsystem consisting of half of the four coupled modes is responsible for the unconventional dynamics, impressing its behavior upon the larger system.

The Hermitian system is composed of simultaneous optical parametric amplification (OPA) and idler second harmonic generation (SHG) (Fig. 1a), which possesses dynamics that proceed initially as OPA, with energy being transferred from a strong pump field to a weakly seeded signal and unseeded idler field, but as the amplitude of the signal and idler grow, idler photons are unidirectionally displaced to the idler second harmonic (SH) field by SHG. The removal of idler photons prevents recombination with signal photons to form pump photons thus eliminating the OPA conversion cycles. In previous work, we found this to be a potential route to overcoming conventional OPA efficiency limitations with behavior akin to dissipative parametric amplifiers [6,7], but remarkably possessing damped behavior without any dissipation [8,9].

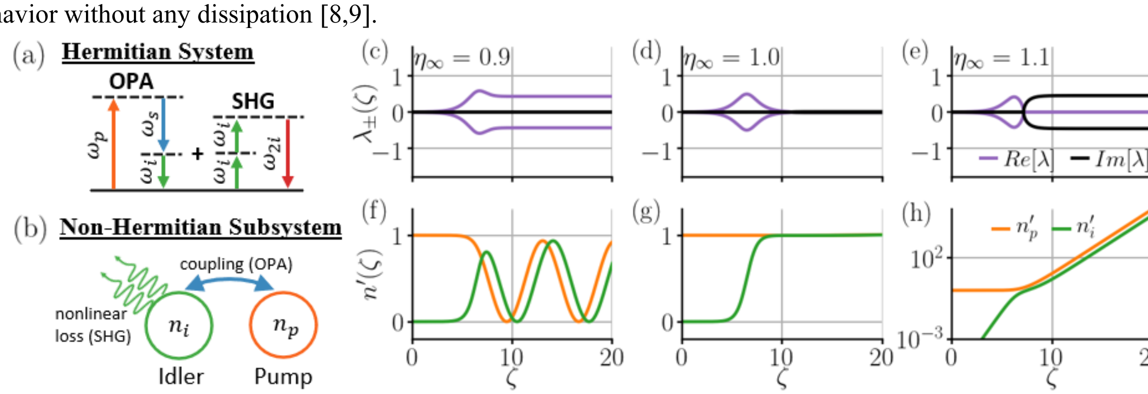

Fig. 1. (a) Virtual energy level diagrams for the Hermitian process of OPA and simultaneous idler SHG. (b) The non-Hermitian pump-idler subsystem where energy is lost to the idler SH field. Eigenvalues of H when (c) $\eta_\infty < 1$ leading to purely real values for all propagation, (d) $\eta_\infty = 1$ where an exceptional point is reached at infinity, and (e) $\eta_\infty > 1$ leading to eventual crossing of the exceptional point resulting in a spontaneous transition to purely imaginary values. Correspondingly, the pump and idler fractional photon dynamics in the gauge-transformed frame are (f) oscillatory, (g) coalescent, and (h) exponential.

Here we explain the phenomenon. The dynamics of the interaction can be understood first by examining the pump-idler subsystem (Fig. 1b). This subsystem (in a gauge-transformed frame where amplitudes grow in proportion to idler photons displaced) is described by the propagation dependent non-Hermitian Hamiltonian $H(\zeta) = \sqrt{n_s(\zeta)}\sigma_x - i\gamma_0\sqrt{n_{2i}(\zeta)}\sigma_z$ where ζ is a non-dimensional propagation coordinate, σ_x and σ_z are the Pauli spin matrices, and n_s and n_{2i} are the fraction of photons in the signal and idler SH fields respectively. γ_0 is a constant that depends on the material and field parameters, which can be tuned through choice of signal wavelength or by structuring the material via quasi-phase matching to vary the effective nonlinear coefficients for phase-matched OPA and SHG. The eigenvalues of this Hamiltonian are $\lambda = \pm\sqrt{n_s(\zeta) - \gamma_0^2 n_{2i}(\zeta)}$. We define the parameter $\eta(\zeta) \equiv \gamma_0\sqrt{n_{2i}(\zeta)/n_s(\zeta)}$ so that the eigenspectra of the pump-idler subsystem has an exceptional point when $\eta(\zeta) = 1$. Qualitatively, $\eta(\zeta)$ is the relative strength of the idler SHG and OPA processes. Whether or not the exceptional point is crossed depends on the value of this parameter in steady-state, $\eta_\infty \equiv \eta(\zeta \to \infty)$. If $\eta_\infty < 1$, the exceptional point will not be crossed, the system is PT-symmetric leading to real eigenvalues (Fig. 1c), and the pump and idler fields will oscillate forever (Fig. 1f). When $\eta_\infty = 1$, the exceptional point is approached at infinity (Fig. 1d), where the pump and idler amplitudes coalesce (Fig. 1g). When $\eta_\infty > 1$, the exceptional point is crossed after finite propagation, leading to spontaneous PT-symmetry breaking, imaginary eigenvalues (Fig. 1g), and exponential grown of the pump and idler fields (Fig. 1h).

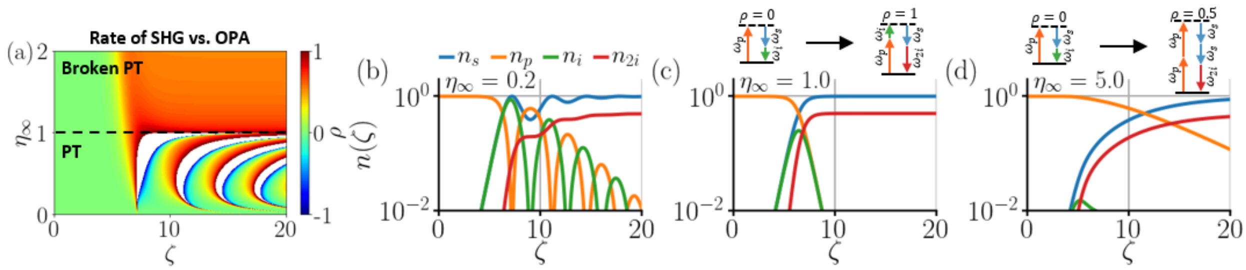

Fig. 2: (a) Rate of SHG vs. OPA, ρ, as a function of η_∞ and non-dimensional propagation coordinate ζ showing the regimes of oscillatory and non-oscillatory behavior separated by $\eta_\infty = 1$ (exceptional point at $\zeta \to \infty$). Full dynamics of the Hermitian system (c) in the oscillatory regime, (d) when the exceptional point is reached at $\zeta \to \infty$ in the pump-idler subsystem, (e) and for the non-oscillatory regime. When $\eta_\infty \geq 1$, ρ reaches a static steady state value, resulting in a transition from OPA photon exchange dynamics to new hybrid exchange dynamics as depicted by the virtual level diagrams. Notably, for $\zeta \to \infty$ in all cases, full nonlinear conversion of the pump has occurred (steady state).

We now consider the full Hermitian nonlinear system and find that the features characteristic of non-Hermitian systems are still present. The full system dynamics are dictated by the rate of change in photon flux densities due to idler SHG versus OPA, ρ. Fig. 2a shows that ρ is divided into two qualitatively different regions based on whether PT-symmetry is broken in the pump-idler subsystem. When the pump-idler subsystem is PT-symmetric ($\eta_\infty < 1$), ρ is oscillatory due to conversion cycles of the OPA, leading to a damped oscillatory exchange of energy between the four modes (Fig. 2b). However, when PT-symmetry is broken in the pump-idler subsystem ($\eta_\infty \geq 1$), conversion cycles are eliminated and ρ approaches a static steady state resulting in a new fixed photon exchange interaction as depicted by the virtual level diagrams in Fig. 2c,d. For $\eta_\infty = 1$, the new hybrid state is a perfect balance of OPA and idler SHG, while for $\eta \gg 1$, the idler SHG process dominates. Remarkably, this results in a direct conversion of pump photons to signal and idler SH photons. Notably, in all cases the pump is asymptotically fully depleted by nonlinear conversion in steady state ($\zeta \to \infty$), and in the $\eta_\infty > 1$ case, the purely imaginary subsystem eigenvalues beyond the exceptional point cause the full four-wave Hermitian system to entirely lose its oscillatory behavior.

These qualitatively new dynamics could lead to an order of magnitude gain in efficiency for some OPA devices by suppressing spatiotemporally inhomogeneous conversion [8]. More generally, inducing non-Hermitian features in fully nonlinear Hermitian systems represents a new route toward efficient and sustainable photonic devices.

REFERENCES

[1] R. El-Ganainy, M. Khajavikhan, D.N. Christodoulides, and S. K. Ozdemir, “The dawn of non-Hermitian optics,” Commun Phys **2**, 37 (2019).
[2] M.-A. Miri, and A. Alù, “Nonlinearity-induced PT-symmetry without material gain,” New J. Phys. **18**, 065001 (2016).
[3] Y. Jiang, Y. Mei, Y. Zuo, Y. Zhai J. Li, J. Wen, and S. Du, “Anti-parity-time symmetric optical four-wave mixing in cold atoms,” Phys. Rev. Lett. **123**, 193604 (2019).
[4] F. Zhang, Y. Feng, X. Chen, L. Ge, and W. Wan, “Synthetic anti-PT symmetry in a single microcavity,” Phys.Rev. Lett. **124**, 053901 (2020).
[5] A. Bergman, R. Duggan, K. Sharma, M. Tur, A. Zadok, and A. Alù, “Observation of anti-parity-time-symmetry, phase transitions and exceptional points in an optical fibre,” Nat. Commun. **12**, 486 (2021).
[6] R. El-Ganainy, J. I. Dadap, and R. M. Osgood, "Optical parametric amplification via non-Hermitian phase matching,"Opt. Lett. **40**, 5086 (2015).
[7] J. Ma, J. Wang, P. Yuan, G. Xie, K. Xiong, Y. Tu, X. Tu, E. Shi, Y. Zheng, and L. Qian, "Quasi-parametric amplification of chirped pulses based on a Sm3+-doped yttrium calcium oxyborate crystal," Optica **2**, 1006–1009 (2015).
[8] Flemens, N., Swenson, N., and Moses, J., “Second harmonic amplification,” in OSA Advanced Photonics Congress (AP) 2020, NpM4D.6, Optical Society of America (2020).
[9] Flemens, N. and Moses, J., “Non-hermitian dynamics mimicked by a fully parametric hybrid nonlinear optical system,” in OSA Advanced Photonics Congress (AP) 2020, NpM3E.2, Optical Society of America (2020).