IMPACT OF LESSON DESIGN ON MATHEMATICAL QUESTIONS

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How does the design of lessons impact the types of questions teachers and students ask during enacted high school mathematics lessons? In this study, we present data suggesting that lessons designed with the mathematical story framework in order to elicit a specific aesthetic response ("MCLEs") have a positive influence on the types of teacher and student questions asked during the lesson. Our findings suggest that when teachers plan and enact lessons with the mathematical story framework, teachers and students are more likely to ask questions that explore mathematical relationships and focus on meaning-making. In addition, teachers are less likely to ask short recall or procedural questions in MCLEs. These findings point to the role of lesson design in the quality of questions asked by teachers and students.

Keywords: classroom discourse, mathematics curriculum, aesthetic, mathematical story.

There is a broad consensus that the quality of the questions asked in mathematics classrooms influence student learning as it can constrain or broaden learning opportunities (Doyle, 1983; Chapin, O'Connor, & Anderson, 2009; Smith & Stein, 2011; Sullivan & Clarke, 1991). Despite this importance, there is evidence that mathematically rich questioning in high school mathematics lessons is rare (e.g., Boaler & Brodie, 2004; Hiebert & Wearne, 1993).

Responses to this challenge have been two-fold. First, one approach is to provide professional learning opportunities for teachers to learn more about productive mathematical discourse (e.g., Breyfogle & Herbel-Eisenmann, 2004; Chapin et al., 2009). A second approach has focused attention on incorporating teacher questioning as a part of designing lessons with high cognitive demand tasks (e.g., Smith & Stein, 2011) since such tasks provide potential opportunities for questions that involve meaningful mathematical connections and reasoning.

In this study, we describe how the mathematical questions posed by teachers and students in enacted lessons shifted through a third approach, namely, by having teachers design mathematically captivating learning experiences ("MCLEs") using the mathematical story framework (Dietiker, 2013, 2015). What is novel about this approach is that the professional learning and lesson design, which focused on how the ideas unfolded across a lesson as a form of narrative, included no attention toward the types of questions that would be asked during the lesson. This study is a part of a larger research project where lessons that students identified as highly interesting on a post-lesson survey were compared with those described by the same students as low interest in order to identify the characteristics of lessons that students find captivating. Since captivating mathematical stories should, in theory, provide opportunities for curiosity and questioning, the current study compares the types of questions asked during MCLEs and non-MCLEs. We report the ways in which this approach shifted the mathematical questions of both teachers and students, asking *How are the questions asked during MCLE enactments the same or different than those posed during non-MCLE enactments?* We end with a discussion on how this lesson design approach likely led to these improvements.

Theoretical Framework

This study interprets a sequence of mathematical events (e.g., tasks, discussions) that occur across a lesson as a *mathematical story* (Dietiker, 2015). Similar to literary stories, mathematical stories can differ in ways the content unfolds throughout the story and impacts the experiences of the audience (i.e., a student experiencing the lesson). Through the release and withholding of information, a mathematical story dynamically shifts a reader's recognition of what they know and enables them to recognize what they do not yet know. This shift in tension can compel a reader to wonder how the story will progress and end. For example, a mathematical story can provide a hint of a future revelation, thereby supporting the formulation and pursuit of questions (e.g., "Can leading coefficients of a polynomial help to identify its roots? How?"), similar to how a reader of a literary story might wonder how the story will advance. Although the MCLEs in this study were designed using the mathematical story framework, the non-MCLEs can also be interpreted for how the content unfolds in ways to inspire inquiry (or not).

Methods

The data for this study includes transcripts of lessons that were designed and taught during the 2018-2019 school year. Immediately after each lesson, students individually rated their interest in the lesson. The analysis of this survey data enabled the identification of lessons for each teacher that students described as most (and least) interesting. The six lessons with the highest interest measures were MCLEs, and the six lessons with the lowest interest measures were all non-MCLEs. These 12 lessons were taught by six experienced teachers with a minimum of four years of experience, and who taught in three high schools in New England with different curricula and demographic settings. The lessons were designed by teachers, with researchers, and represent a range of mathematical topics for both honors and non-honors courses spanning from Integrated Math 1 to calculus. To support the teachers' designing process of MCLE lessons, the teachers attended a two-week professional development during the summer of 2018 to learn about the mathematical story framework and begin the design process.

Data Analysis

Each of these lessons was transcribed and later coded for the mathematical plot by a team of researchers. The team identified the mathematical questions that were raised, explicitly or implicitly, by teachers and students throughout the lesson. Any questions that were non-mathematical, such as "Can you show it under the document camera?", were not included. We also did not formulate questions that clearly involved content from earlier grades, as we interpreted these as "checking answers" (e.g., "Is 18 times 3 is 54?"). Also, repeated questions were not counted as additional questions in this study. After identifying all the questions, we identified the acts during which each question was open and unanswered.

Then, each of the three researchers independently coded each question raised in the mathematical plots for its mathematical qualities. To distinguish the types of questions, we adapted Boaler and Brodie's (2004) categories of questions. We began with including six categories, namely *gathering information: procedural and factual* (GIPF), *inserting terminology* (IT), *exploring mathematical meanings and/or relationships* (EMMR), *probing for an explanation of thinking* (PET), *linking and applying* (LA), *extending thinking* (ET). We excluded three categories because either these categories are non-mathematical and thus not part of the mathematical story (in the case of *establishing context)* or they could be merged with other categories (in the case of *generating discussion* and *orienting and focusing*).

This coding framework was used to distinguish the qualities of the questions raised explicitly or implicitly by teachers and students. An important distinction of our coding scheme is that we coded questions based on how they were taken up and addressed within the story arcs as opposed to deciding the intent of the question independent of how the question was answered. Based on our initial analysis of questions in non-MCLEs and MCLEs lessons, we added two new categories: struggling with recently learned procedure and facts (SIPF) and problem-solving without known procedures (PSWP). SIPFs also are recall type questions; however, unlike GIPFs, students do not provide quick responses but instead struggle to recall the recently learned facts and perform the procedures. PSWPs a range of problems; both novel (unfamiliar to students) and challenging (perhaps familiar, but students choose to reason their way through the problem instead of applying a familiar procedure).

After questions were coded, the researchers met to resolve differences and to find consensus. To learn whether MCLEs have different proportions of each type of question when compared to non-MCLEs, we conducted a paired samples t-test for each teacher (pairing their MCLE and non-MCLE). Significance was determined when $p < 0.05$.

Findings

Overall, there were 417 teacher questions and 176 student questions in the 12 lessons, out of which 182 teacher and 99 student questions were from six non-MCLEs and 235 teacher and 77 student questions were from six MCLEs. On average, MCLEs have 30% more teacher questions in comparison to the non-MCLEs. In contrast, students asked approximately 27% more questions in non-MCLEs in comparison to MCLEs. However, the numbers of teacher and student questions were not significantly different between these two types of lessons. Note that we found only one *extended thinking* question across all the lessons. It was asked by a student in an MCLE lesson. Because of the lack of this type of question this category was excluded from further analysis. Following are our findings on the shifts of teacher and student questioning when comparing MCLEs to non-MCLEs.

Shifts in Types of Teacher Questions

The data (Table 3) show a stark difference for the types of questions that emerged in lessons that were MCLEs, as compared to those that were non-MCLEs. Overall, in MCLEs lessons, approximately one-fourth of the teachers' questions were for encouraging students to explore the mathematical meaning and reasoning (EMMR) in comparison to their non-MCLEs with only 1.6% of such questions. In fact, only two teachers asked either one or two EMMR questions in their non-MCLEs. These preliminary findings suggest that teachers tend to ask significantly more EMMR questions in their MCLEs (M=23.8, SD=10.4), which encourage students to explore underlying mathematical meaning and relationships, as compared to their non-MCLEs (M=1.3, SD=2.1) lessons (t (5) =5.6, p=0.003). On the other hand, in non-MCLEs, teachers tend to ask twice as many recall questions (i.e., GIPF and SIPF types) (M= 27.6, SD=12.5), which is significantly different as compared to their MCLEs (M= 56.6, SD=18.9) lessons (t (5) = -3.5, p=0.018).

Our data also suggests that teachers probe students to explain their thinking more often in their MCLEs (M= 18.6, SD=16.2) as compared to their non-MCLEs (M=6.4, SD=5.2), though this difference is not statistically significant (t (5) =1.97, p=0.11). The "problem solving with logic and unspecified procedures" (PSWP) type of questions were slightly higher in non-MCLEs (M=35.6, SD=19.2) in comparison to MCLEs (M=29.6, SD=15.7). However, there was no significant difference between MCLEs and non-MCLEs for this question type, (t (5) = - 6.8, p=0.53), as both types of lessons have questions that require problem-solving where students felt challenged and used multiple strategies (e.g., logic, guess and check).

Table 3. Proportions of Student and Teacher Questions in MCLEs and non-MCLEs

	Teacher Questions		Student Question	
	MCLE mean (SD)	non-MCLE mean (SD)	MCLE mean (SD)	non-MCLE mean (SD)
No. of questions/ lesson	39.2 (5)	30.3(8.5)	12.8 (7.3)	16.5 (6.8))
EMMR	23.8 (10.4) *	1.3 (2.1)	25.8 (24.1) *	0.06 (1.6)
GIPF	27.2 (12.7) *	50.7 (16)	19.4 (13.5)	38.9 (17.8)
SIPF	0.4 (0.97) *	5.9 (5.8)	6.2 (13.4)	11.6 (18)
PET	18.7 (16.2)	6.4 (5.2)	18.5 (14.1)	23.5 (15.8)
PSWP	29.6 (15.7)	35.6 (19.2)	23.4 (14.4)	22.4 (14.8)
IT	0.38 (0.9)	0.0 (0.0)	6.7 (10.3)	3.02 (5.7)

Note: *Reflects a statistically significant difference (alpha < .05)

Shifts in Types of Student Questions

Similar to teachers, students also asked a higher proportion of exploring the mathematical meaning and reasoning (EMMR) questions in MCLEs (M=25.8, SD =24.1) in comparison to non-MCLE lessons (M=0.06, SD=1.6), and this difference was significant (t (5) =2.6, p=0.048). In contrast, the proportion of student recall questions (GIPF and SIPF, combined) in non-MCLEs (M=50.6, SD=31) was nearly twice that of MCLEs (M=25.6, SD=19.5). However, this difference was not significant (t (5) = - 1.8, p=0.14).

Discussion

We are encouraged to find that when teachers design lessons as MCLEs, it also results in a richer and wider variety of teacher and student questions during enacted lessons. This unexpected benefit of designing lessons with the mathematical story framework raises new questions for mathematics teacher education; namely, rather than training teachers what types of questions to ask during instruction, might it be better to prepare teachers to design lessons that encourage student curiosity and inquiry? We suspect that the teachers' intentional focus on how and when to enable certain mathematical ideas to emerge throughout a lesson in order to spur student curiosity and inquiry in MCLEs likely supported both teachers and students asking a rich and wide variety of questions.

Across all our lessons, we note the lack of *linking and applying* and *extended thinking type* questions in both types of lessons. This finding is supported by other studies (e.g., Kosko, Rougee & Herbst, 2014) where secondary school teachers did not include these types of questions in their planned lessons. One possibility is that the data of this study did not include any consecutive lessons, so therefore these question types did not appear in these lessons. The reason for non-significant differences in question types, such as teachers' *probing* questions, was likely due to the limited number of lessons in our study. Future research is needed to better understand the impact of designing lessons using a mathematical story framework on teacher and student questioning during enacted lessons.

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References

Breyfogle, M. L., & Herbel-Eisenmann, B. A. (2004). Focusing on students' mathematical thinking. *The Mathematics Teacher*, *97*(4), 244-247.

Boaler, J., & Brodie, K. (2004, October). The importance, nature and impact of teacher questions. In *Proceedings of the twenty-sixth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 774-782).

Chapin, S. H., O'Connor, C., O'Connor, M. C., & Anderson, N. C. (2009). *Classroom discussions: Using math talk to help students learn, Grades K-6*. Math Solutions.

Dietiker, L. (2013). Mathematics texts as narrative: Rethinking curriculum. *For the Learning of Mathematics*, 33(3), 14–19.

Dietiker, L. (2015). Mathematical story: A metaphor for mathematics curriculum. Educational Studies in Mathematics, 90(3), 285–302. https://doi.org/10.1007/s10649-015-9627-x

Doyle, W. (1983). Academic work. *Review of educational research*, *53*(2), 159-199.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American educational research journal*, *30*(2), 393-425.

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom? *Mathematics Education Research Journal*, *26*(3), 459-476.

Smith, M. S., & Stein, M. K. (2011). Five practices for orchestrating productive mathematics discussions. Reston, VA: NCTM.

Sullivan, P., & Clarke, D. J. (1991). Communication in the classroom: The importance of good questioning. Geelong: Deakin University.